Calculation of ρ meson decay width from the PACS-CS configurations

at Lat2010 2010/06/15

Naruhito Ishizuka for PACS-CS collaboration
University of Tsukuba

We present results of ρ meson decay width evaluated from SC. phase shift for $I=1$ two-pion system with PACS-CS configurations generated at

$N_f = 2+1$, Imp. Wilson

$a = 0.091 \text{ fm}$, $L = 2.9 \text{ fm}$

$m_\pi = 410 \text{ MeV}$

We use NEW Op. to reduce the computational cost.

1. Introduction

Previous works of ρ meson decay

1) From transition amp.

$$\langle 0|(\pi\pi)(t)\rho(0)|0\rangle \sim \left[\, 1 \;+\; \underline{\langle \pi\pi|\rho\rangle} \cdot t \;+\; O(t^2)\,\right] \times \mathrm{e}^{-m_\rho t}$$

$\sim$ calc. of WME.

$$\rightarrow \quad \Gamma = |\langle \pi\pi|\rho\rangle|^2 \cdot L^3 k E/(24\pi) = g^2_{\rho\pi\pi}/(6\pi) \cdot k^3/E^2$$

1) S. Gottlieb, P.B. Mackenzie, H.B. Thacker, D. Weingarten PL134B(84)346.

Quench , $m_\pi/m_\rho = 0.84 - 0.91$

2) R.D. Loft, T.A. DeGrand PRD9(1989)2692.

Quench , $m_\pi/m_\rho = 0.9$

3) C. McNeile, C. Michae + UKQCD PLB556(2003)177.

$N_f = 2$, $m_\pi/m_\rho = 0.578^{+13}_{-19}$ $\qquad g_{\rho\pi\pi} = 6.77^{+91}_{-56}$

4) K. Jansen, C. McNeile, C. Michael, C. Urbach (ETMC) PRD80(2009)054510.

$N_f = 2$, $m_\pi/m_\rho = 0.336 - 0.534$ $\qquad g_{\rho\pi\pi} = 5.2(1.3)$

Problem : $\langle \pi\pi|\rho\rangle$ at $\sqrt{s}$ Where is $\sqrt{s}$? **: ambiguous**

2) From SC. phase

Energy on lat. $\longrightarrow$ $\tan\delta$ $\longrightarrow$ $g_{\rho\pi\pi}$, Γ

by finite size formula

exp. :

$g_{\rho\pi\pi} = 5.98(2)$

$\Gamma = \frac{g^2_{\rho\pi\pi}}{6\pi} \frac{k^3_\rho}{m^2_\rho} = 149.1(8)$ MeV

1) CP-PACS PRD76(07)094506.

$N_f = 2$, $a = 0.21$ fm , $L = 2.5$ fm , $m_\pi = 330$ MeV

Imp.Wilson , Moving frame with $\mathbf{p} = \mathbf{e}_3 \cdot (2\pi/L)$

$g_{\rho\pi\pi} = 6.25(67)$

2) M. Göckeler, R. Horsley, Y. Nakamura et.al. (QCDSF) arXiv:0810.5337 (Lat08)

$N_f = 2$, $a = 0.072 - 0.084$ fm , $m_\pi = 240 - 810$ MeV

Imp.Wilson , CM. frame

$g_{\rho\pi\pi} = 5.3^{+2.1}_{-1.5}$

3) X. Feng, K. Jansen, D.B. Renner (ETMC) arXiv:0910.4871 (Lat09)

$N_f = 2$, $a = 0.086$ fm , $L = 2.1$ fm , $m_\pi = 391$ MeV

tmQCD, CM and Moving frame with $\mathbf{p} = \mathbf{e}_3 \cdot (2\pi/L)$

$g_{\rho\pi\pi} = 6.16(48)$

4) X. Feng, L. Jansen, D.B. Renner (ETMC) *Just previous talk*

5) J. Frison *Talk on 18(Fri.) 17:40 at Room1*

This work : from SC. phase shift

Gauge conf. :

$N_f = 2+1$, Imp. Wilson

$\beta = 1.9\ ,\ 32^3 \times 64\ ,\ K_{ud} = 0.13754\ ,\ K_s = 0.13640$

$a = 0.091\ \mathrm{fm}\ ,\ L = 2.9\ \mathrm{fm}$

$m_\pi = 410\ \mathrm{MeV}$

generated by PACS-CS col.
PRD79(2009)034503.

Calc. points :

tot. mom. $\mathbf{p}$	Γ	state $[\sqrt{s}/m_\rho]$ (without interaction) ground	1st Ex.	refered by
$(0,0,0)$ CM. F.	$\mathbf{T}_1^-$	$\rho_{1,2,3}(0,0,0)\,[\,1\,]$	$\pi(0,0,1)\,\pi(0,0,-1)\,[\,1.3\,]$	$\mathbf{T}_1(\rho(\mathbf{0}))$
$(0,0,1)$ Moving F.	$\mathbf{E}^-$	$\rho_{1,2}(0,0,1)\,[\,1\,]$	$\pi(0,1,1)\,\pi(0,-1,0)\,[\,1.4\,]$	$\mathbf{E}(\rho_T(\mathbf{p}))$
$(0,0,1)$ Moving F.	$\mathbf{A}_2^-$	$\rho_3(0,0,1)\,[\,1\,]$	$\pi(0,0,0)\,\pi(0,0,1)\,[\,1.02\,]$	$\mathbf{A}_2(\rho_3(\mathbf{p}),\pi(\mathbf{0})\pi(\mathbf{p}))$ ($\mathbf{p} = (0,0,1)$)

(by using variational method)

All calculations are carried out with PACS-CS in U. Tsukuba.

2. Method

Finite size formula :

M. Lüscher, NPB354(1991)531.
K.Rummainnen and S.Gottlib, NPB450(1995)397.

for $\mathbf{T}_1(\,\rho(\mathbf{0})\,)$

$$\frac{k}{\tan\delta(k)} = Z_{00}(q;\mathbf{0})$$

$$\left(\begin{array}{l} \sqrt{E^2 - p^2} = \sqrt{s} = 2\sqrt{m_\pi^2 + k^2} \;\;, \\ q = kL/(2\pi) \end{array} \right)$$

for $\mathbf{E}(\,\rho_T(\mathbf{p})\,)$

$$\frac{k}{\tan\delta(k)} = Z_{00}(q;\mathbf{p}) - \frac{1}{\sqrt{5}}\frac{1}{q^2} Z_{20}(q;\mathbf{p})$$

for $\mathbf{A}_2(\,\rho_3(\mathbf{p})\,,\,\pi(\mathbf{0})\pi(\mathbf{p})\,)$

$$\frac{k}{\tan\delta(k)} = Z_{00}(q;\mathbf{p}) + \frac{2}{\sqrt{5}}\frac{1}{q^2} Z_{20}(q;\mathbf{p})$$

$$Z_{lm}(q;\mathbf{p}) = 2/(\gamma L\sqrt{\pi}) \cdot \sum_{\mathbf{r}\in D(\mathbf{p})} \mathcal{Y}_{lm}(\mathbf{r}) \cdot (r^2 - q^2)^{-1}$$

: spherical zeta function

$$D(\mathbf{p}) = \{\, \mathbf{r} \,|\, \mathbf{r} = \hat{\gamma}^{-1}(\mathbf{n} + \mathbf{p}/2)\;,\; \mathbf{n} \in \mathbb{Z}^3 \,\}$$

$\gamma = E/\sqrt{s}$: Lorentz bost factor

Op. for $\mathbf{A}_2(\,\rho_3(\mathbf{p})\,,\,\pi(\mathbf{0})\pi(\mathbf{p})\,)$

Energies are extracted by variational method.

For $\mathbf{T}_1(\,\rho(\mathbf{0})\,)$ and $\mathbf{E}(\,\rho_T(\mathbf{p})\,)$
energies are extracted from time correlation function of ρ meson
as usual calculations of hadron masses.

Source op. :

$$\overline{O}_1(t_s) = \frac{1}{\sqrt{2}}\Big(\,\pi^-(\mathbf{0},t_s)\,\pi^+(\mathbf{p},t_s)\;-\;\pi^+(\mathbf{0},t_s)\,\pi^-(\mathbf{p},t_s)\,\Big)$$

$\pi(\mathbf{p},t_s)$: constructed with U(1) noise

$$\overline{O}_2(t_s) = \sum_{\mathbf{z}\in\Gamma}\frac{1}{\sqrt{2}}\Big(\,\overline{U}(\mathbf{z},t_s)\,\gamma_j\,U(\mathbf{z},t_s)\;-\;\overline{D}(\mathbf{z},t_s)\,\gamma_j\,D(\mathbf{z},t_s)\,\Big)\cdot\mathrm{e}^{i\mathbf{p}\cdot\mathbf{z}}$$

(: smearing OP. for ρ meson with $\mathbf{p}$) (on Coulomb gauge)

$$U(\mathbf{z},t_s) = \sum_{\mathbf{x}} u(\mathbf{z}+\mathbf{x},t_s)\cdot F(\mathbf{x})$$ (: smearing quark)

$$F(\mathbf{x}) = A\cdot\mathrm{e}^{-B|\mathbf{x}|}$$: smering function

$$\Gamma = \{\,\mathbf{z}\,|\,L/2\times(n_1,n_2,n_3)\,,\,n_j=(0\text{ or }1)\,\}$$

Sink op :

$$O_1(t) = \frac{1}{\sqrt{2}}\Big(\, \pi^-(\mathbf{0},t_1)\,\pi^+(\mathbf{p},t) \;-\; \pi^+(\mathbf{0},t_1)\,\pi^-(\mathbf{p},t)\,\Big) \cdot \mathrm{e}^{m_\pi\cdot(t_1-t)}$$

with fixed t_1

$$\langle 0|O_1^\dagger(t) \sim \langle 0|\pi(\mathbf{0})|\pi(\mathbf{0})\rangle\langle\pi(\mathbf{0})|\pi(\mathbf{p})|E\rangle\langle E| \;\times \mathrm{e}^{E\cdot t}$$

for $t_1 >> t$

$|E\rangle$: energy eigenstate $(\sim |\pi\pi; E\rangle\,)$

(: same *t* dependence as usual Heisenberg op.)

$$O_2(t) = \sum_{\mathbf{x}} \frac{1}{\sqrt{2}}\Big(\, \bar{u}(\mathbf{x},t)\,\gamma_3\,u(\mathbf{x},t) \;-\; \bar{d}(\mathbf{x},t)\,\gamma_3\,d(\mathbf{x},t)\,\Big) \cdot \mathrm{e}^{\mathrm{i}\mathbf{p}\cdot\mathbf{x}}$$

Variational method

2 x 2 correlation matrix :

$$G_{ij}(t) = \langle 0|\,O_i^\dagger(t)\,\overline{O}_j(t_s)\,|0\rangle$$

with

$$\left(\begin{array}{rl} \text{sink} & : \; O_1(t)\;,\; O_2(t) \\ \text{source} & : \; \overline{O}_1(t)\;,\; \overline{O}_2(t) \end{array}\right.$$

Assuming 2 state dominant,

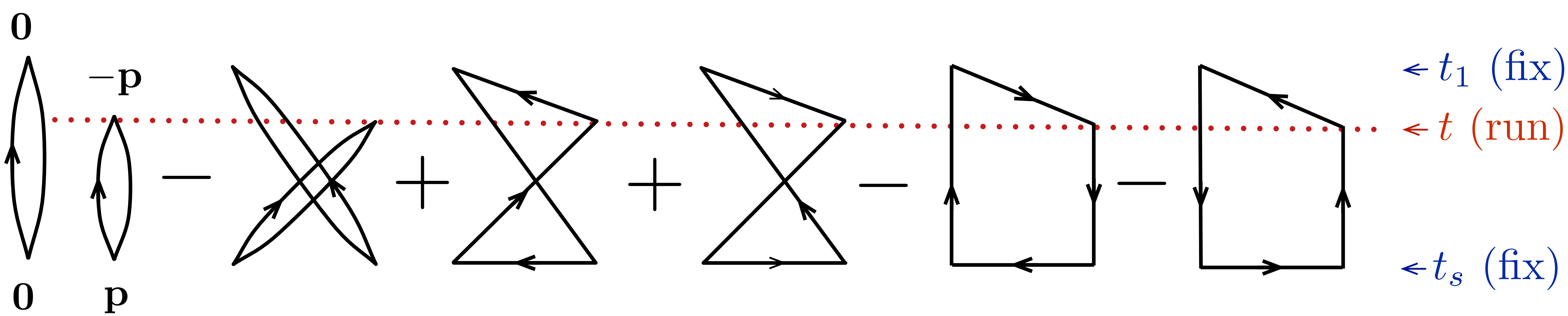
$$\mathrm{Ev}[\,G(t_R)^{-1}\,G(t)\,] \;=\; \Big\{\; \lambda_1(t) = \mathrm{e}^{-E_1\cdot t}\;,\;\; \lambda_2(t) = \mathrm{e}^{-E_2\cdot t}\;\Big\}$$

for $t_1 \gg t \gg t_s$

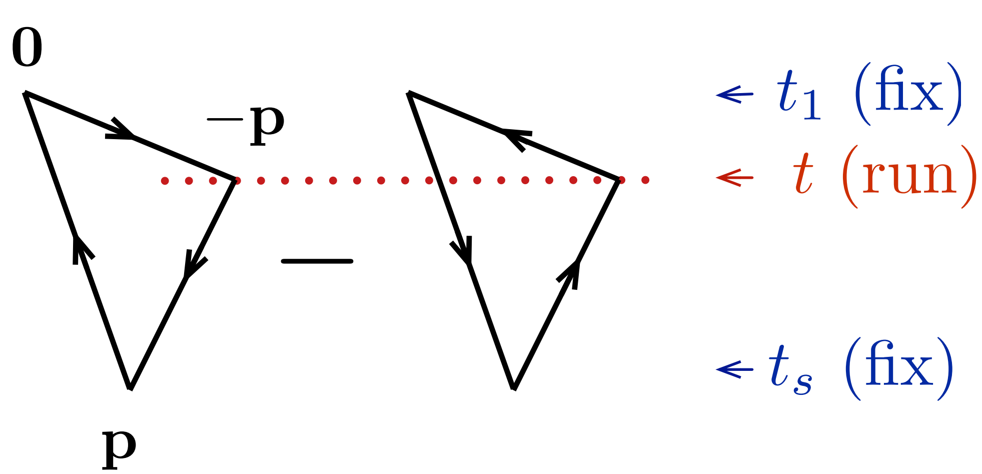
Calc. of $G(t)$ of $\mathbf{A}_2(\,\rho_3(\mathbf{p})\,,\,\pi(\mathbf{0})\pi(\mathbf{p})\,)$

Diagrams :

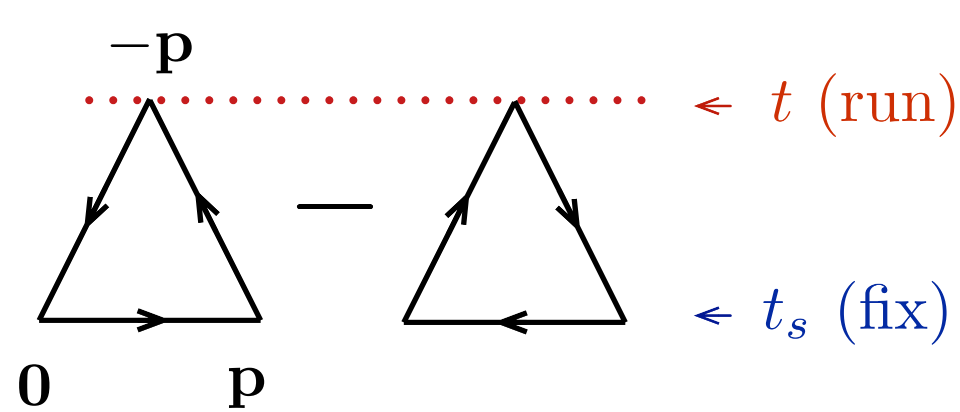
$G_{\pi\pi\to\pi\pi}(t) =$



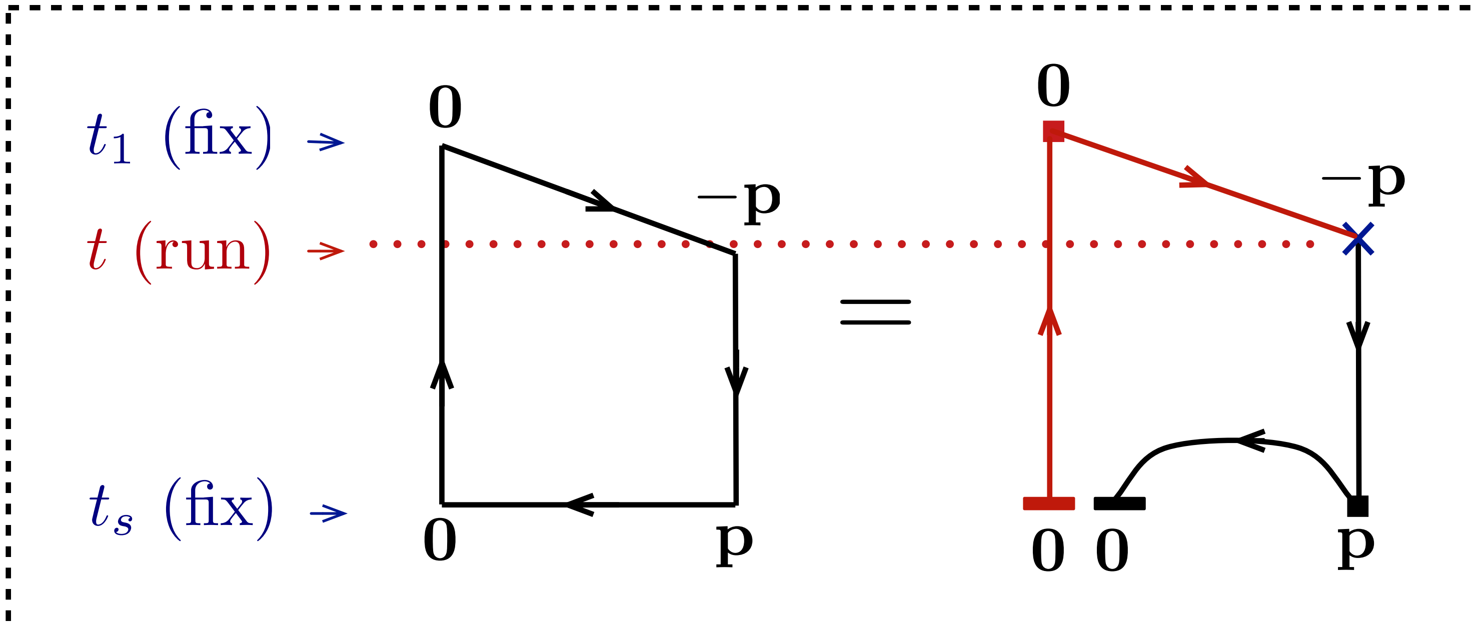
$G_{\rho\to\pi\pi}(t) =$



$G_{\pi\pi\to\rho}(t) =$



Example :



(1)

$$= \sum_{\mathbf{y}} D^{-1}(\mathbf{x}, t; \mathbf{y}, t_s) \cdot \xi(\mathbf{y}) e^{i\mathbf{q}\cdot\mathbf{y}}$$

$$\equiv Q(\mathbf{x}, t; \mathbf{q}, t_s)$$

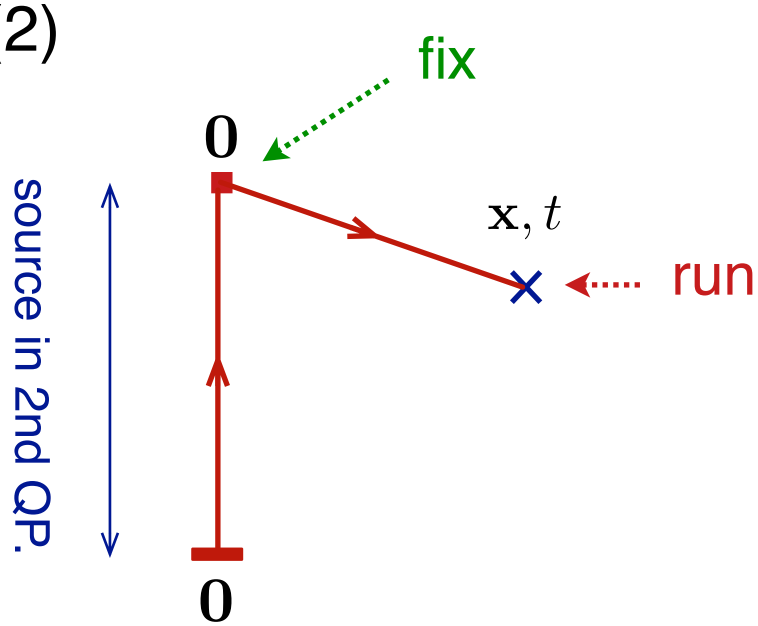
$\xi(\mathbf{x})$: U(1) noise

$$\sum_{j=1}^{N_R} \xi_j^\dagger(\mathbf{x})\xi_j(\mathbf{y}) = \delta^3(\mathbf{x}-\mathbf{y})$$

for $N_R \to \infty$

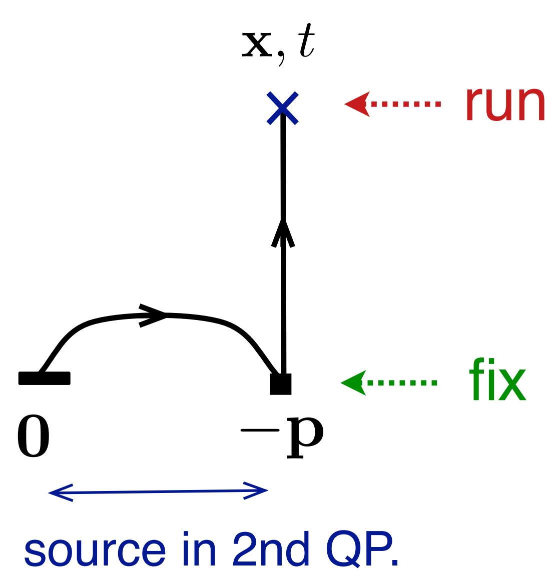
($N_R = 10$ (in present work))

(2)



$$= \sum_{\mathbf{y}} D^{-1}(\mathbf{x}, t; \mathbf{y}, t_s) \cdot \gamma_5 \, Q(\mathbf{y}, t_1; \mathbf{0}, t_s)$$

$$\equiv R(\mathbf{x}, t; \mathbf{0}, t_s)$$



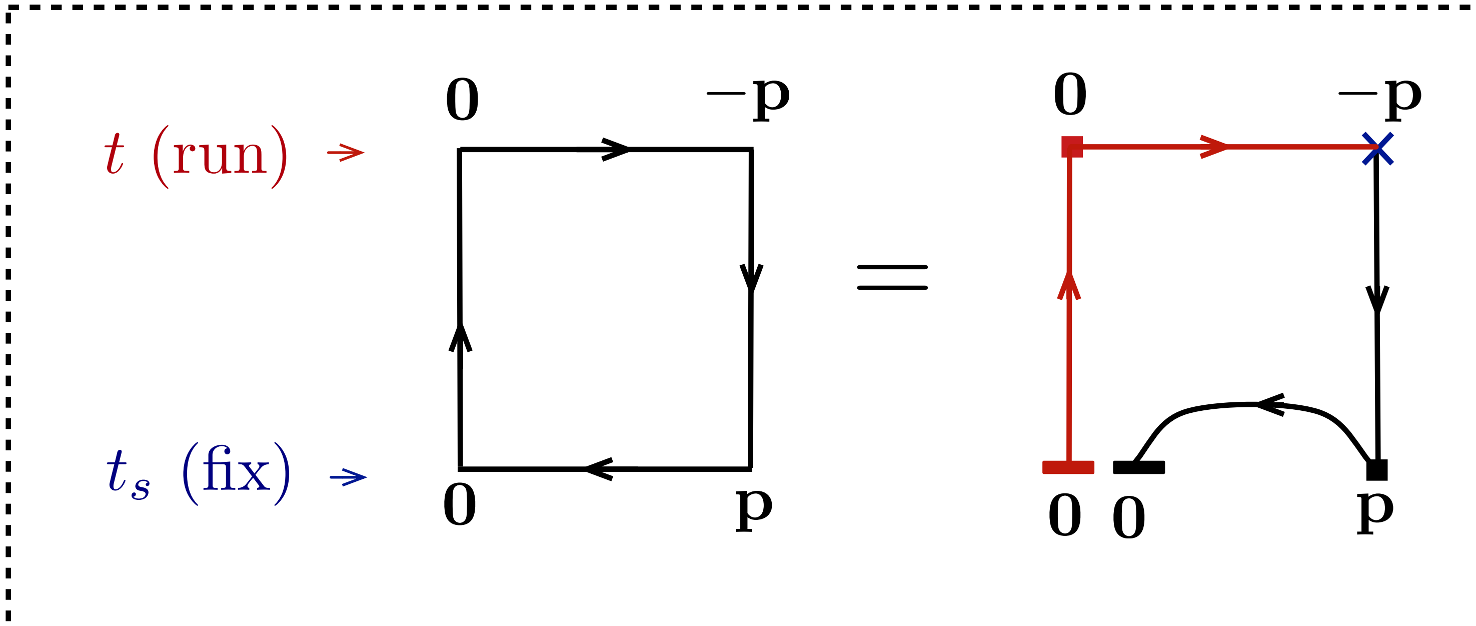
$$= \sum_{\mathbf{y}} D^{-1}(\mathbf{x}, t; \mathbf{y}, t_s) \cdot \gamma_5 \, e^{-i\mathbf{p}\cdot\mathbf{y}} Q(\mathbf{y}, t_s; \mathbf{0}, t_s)$$

$$\equiv B(\mathbf{x}, t; \mathbf{0}, t_s)$$

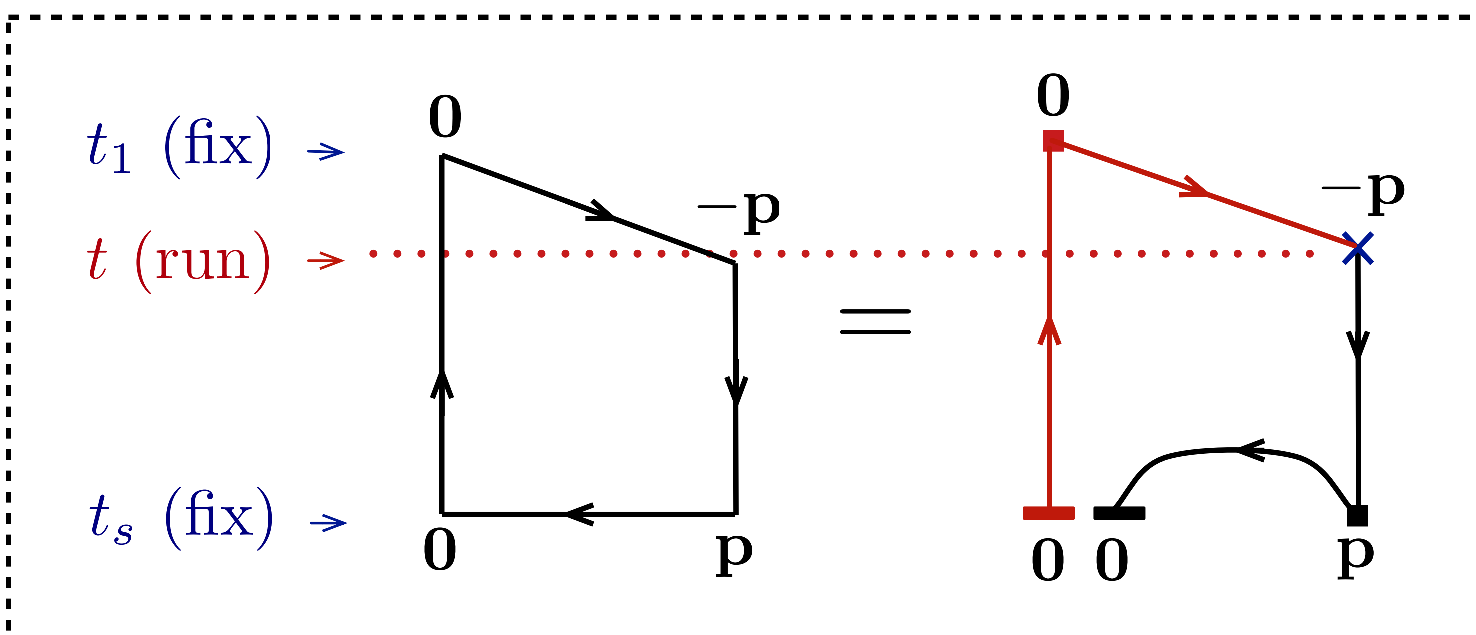
(3)

$$\sum_{\mathbf{y}} \mathrm{Tr}\left[R(\mathbf{x}, t; \mathbf{0}, t_s) \, B^\dagger(\mathbf{x}, t; \mathbf{0}, t_s) \right] \cdot e^{-i\mathbf{p}\cdot\mathbf{x}}$$

CP-PACS(2007) $\pi\pi$ op. : $\pi(\mathbf{0}, t)\pi(\mathbf{p}, t)$



This work $\pi\pi$ op. : $\pi(\mathbf{0}, t_1)\pi(\mathbf{p}, t)$



[# of Red type of QP.]

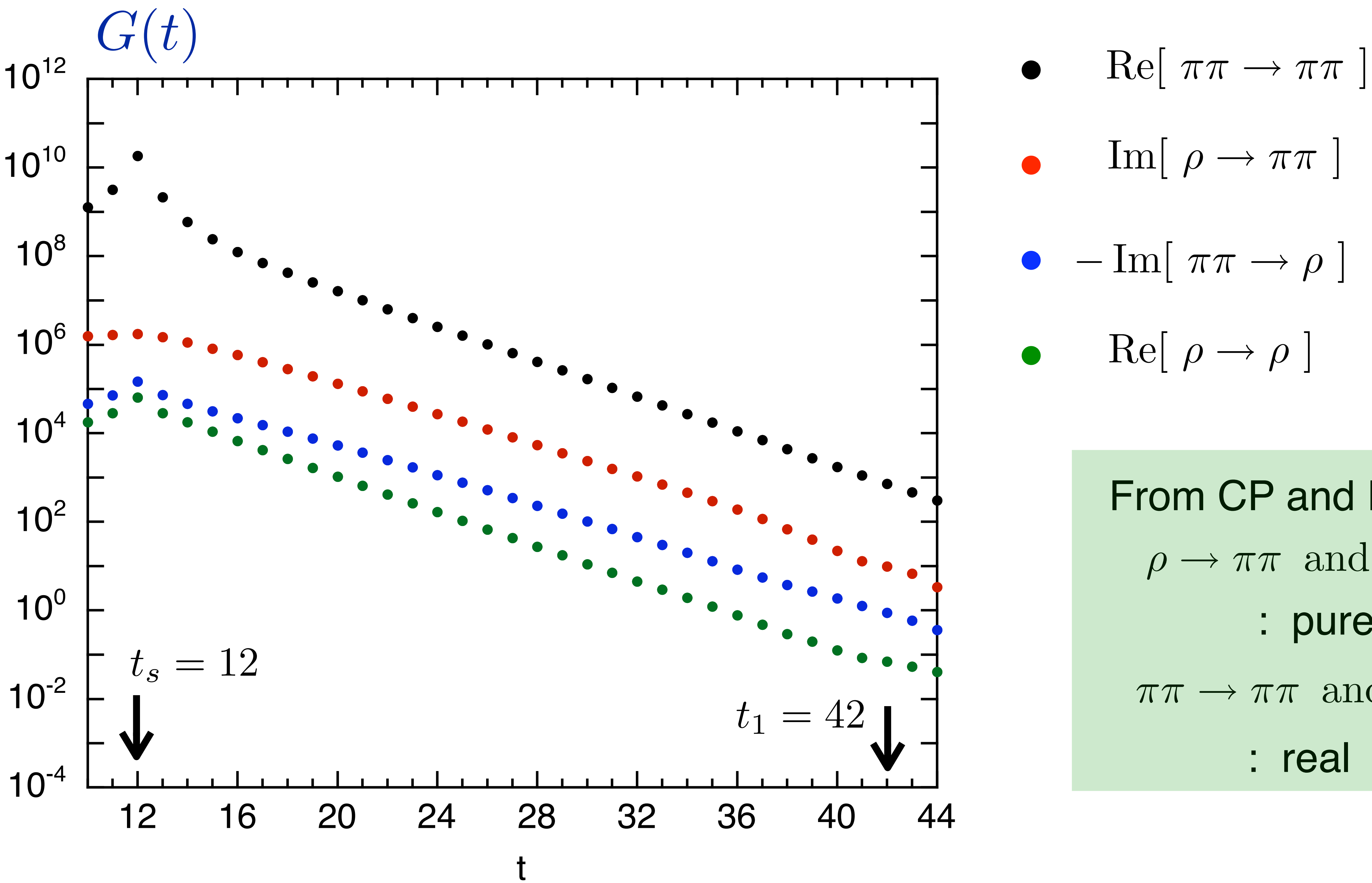
$\propto$ [# of time slice : T]

$\Rightarrow 1$

Computational cost
is reduced by T.

3. Results

1) Results of $\mathbf{A}_2\,(\,\rho_3(\mathbf{p})\,,\ \pi(\mathbf{0})\pi(\mathbf{p})\,)$



From CP and P sym.

$\rho \to \pi\pi$ and $\pi\pi \to \rho$

: pure imaginary

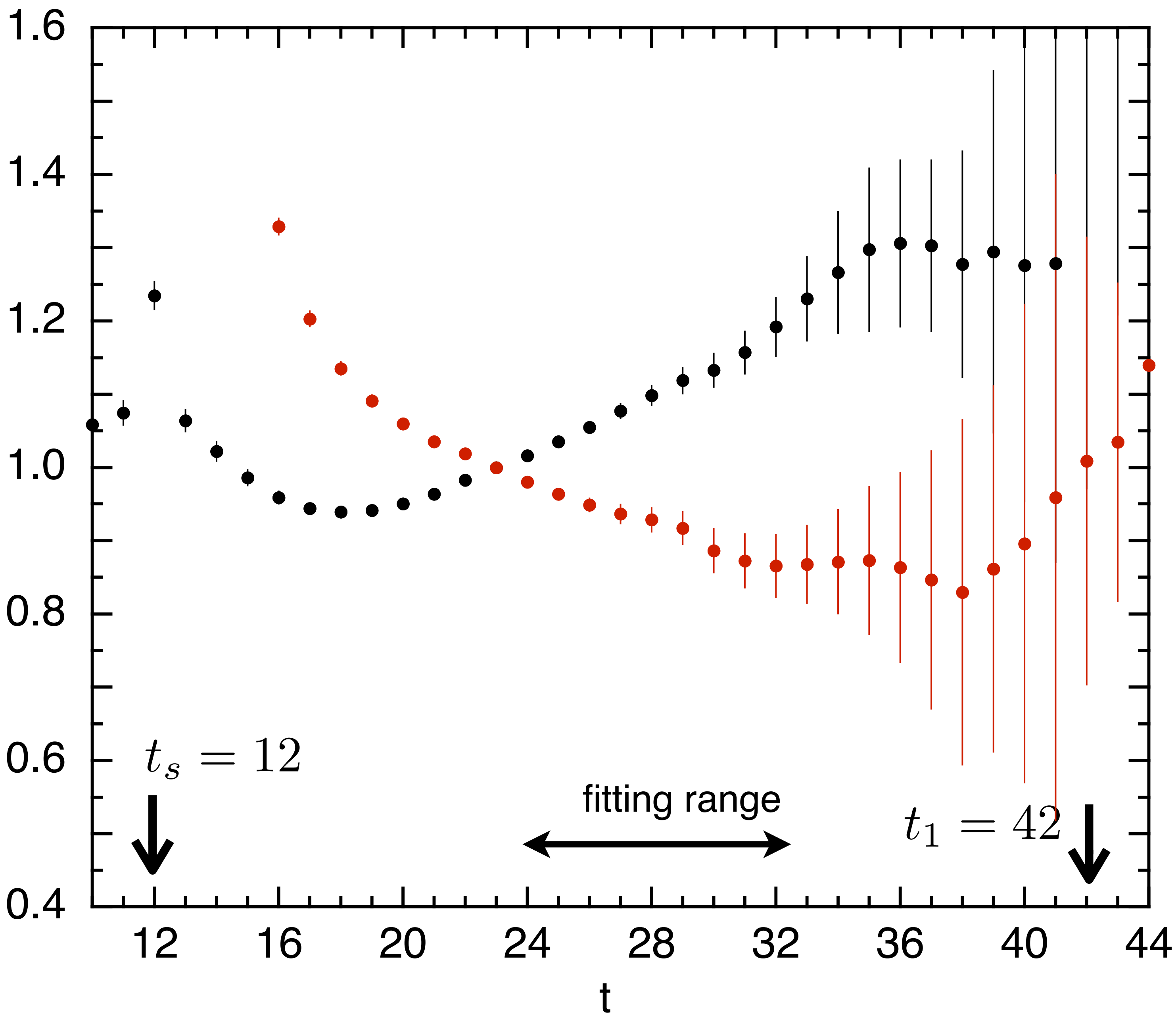
$\pi\pi \to \pi\pi$ and $\rho \to \rho$

: real

Eigenvalue $\lambda(t) = \mathrm{Ev}[\,G(t_R)^{-1}\,G(t)\,] \quad (\,t_R = 23\,)$

$\lambda(t)\,/\,\langle\,\pi(\mathbf{0},t)\,\pi(\mathbf{0},t_s)\,\rangle / \,\langle\,\pi(\mathbf{p},t)\,\pi(\mathbf{p},t_s)\,\rangle$

$\sim\ \mathrm{e}^{-\Delta E\cdot t}$



● ground state

$$\Delta E = -0.0191(27)$$
$$E = 0.4413(36)$$
$$\sqrt{s} = 0.3952(40)$$

$$\tan\delta(k) = +0.0942(47)$$

attractive

● 1st Ex. state

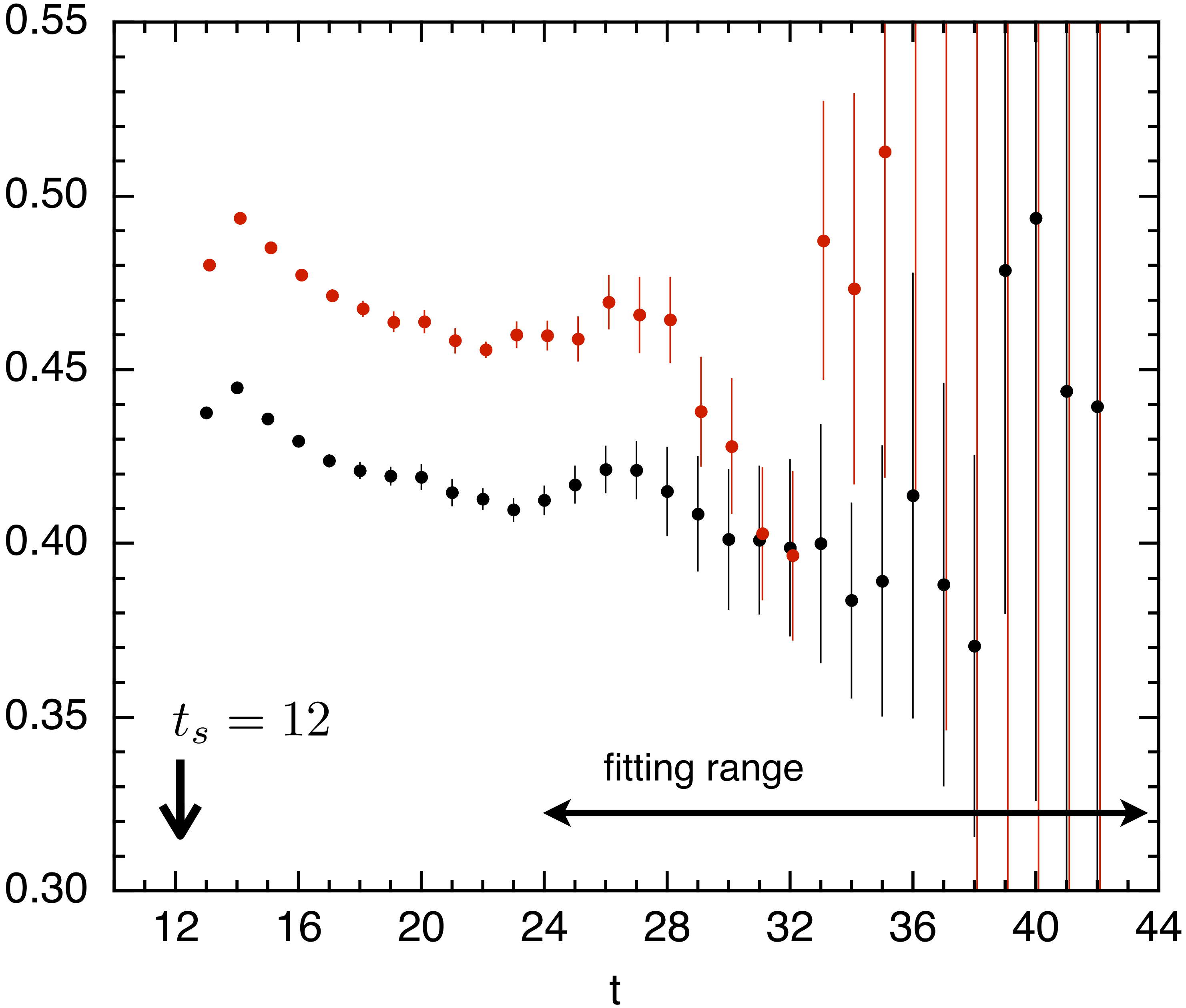
$$\Delta E = 0.0150(42)$$
$$E = 0.4755(44)$$
$$\sqrt{s} = 0.4330(48)$$

$$\tan\delta(k) = -0.165(50)$$

repulsive

2) Results of $\mathbf{T}_1\,(\,\rho(\mathbf{0})\,)$ and $\mathbf{E}\,(\,\rho_T(\mathbf{p})\,)$

Effective mass



● $\mathbf{T}_1\,(\,\rho(\mathbf{0})\,)$

$$E = 0.4122(84)$$
$$\sqrt{s} = 0.4122(84)$$

$$\tan\delta(k) = -0.74(24)$$

repulsive

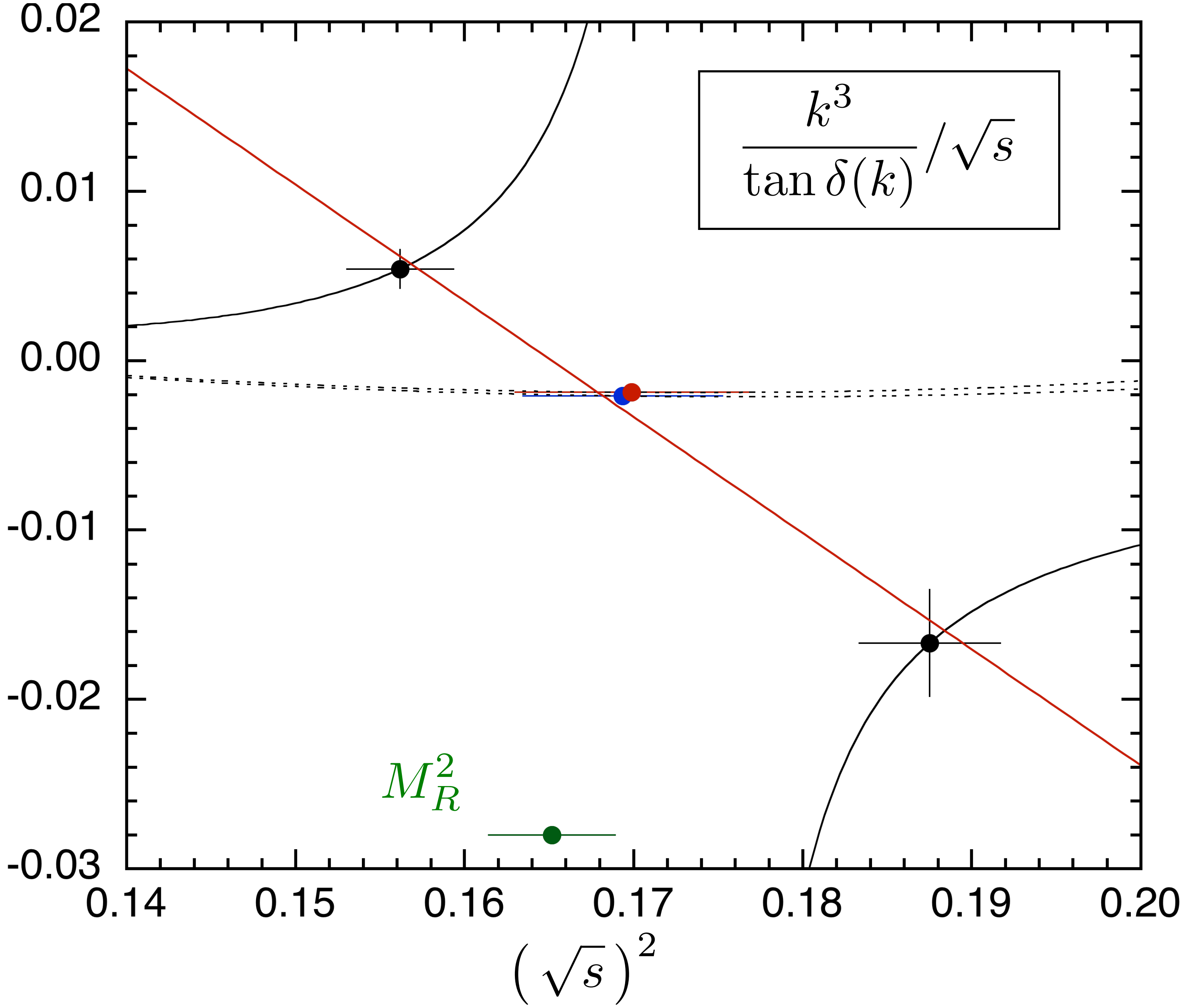
● $\mathbf{E}\,(\,\rho_T(\mathbf{p})\,)$

$$E = 0.4560(65)$$
$$\sqrt{s} = 0.4115(72)$$

$$\tan\delta(k) = -0.65(18)$$

repulsive

3) Results of decay width



- $\mathbf{A}_2\,(\,\rho_3(\mathbf{p})\,,\,\pi(\mathbf{0})\pi(\mathbf{p})\,)$ (black)
- $\mathbf{T}_1\,(\,\rho(\mathbf{0})\,)$ (red)
- $\mathbf{E}\,(\,\rho_T(\mathbf{p})\,)$ (blue)
- solid / dotted : finite size formula
- red line : fitting line

$$\frac{k^3}{\tan\delta(k)}/\sqrt{s} = \frac{6\pi}{g_{\rho\pi\pi}^2}\cdot\left(M_R^2-(\sqrt{s})^2\right) \quad \text{for } \sqrt{s}\sim M_R$$

$$g_{\rho\pi\pi} = 5.24(51)$$

$$M_R = 0.4064(46)$$

$$\Gamma = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{k_\rho^3}{m_\rho^2} = 113(22)\ \text{MeV}$$

exp. :

$$g_{\rho\pi\pi} = 5.98(2)$$

$$\Gamma = 149.1(8)\ \text{MeV}$$

4. Summary

We calculate SC. phase shift for *I=1* two-pion system
and evaluate $\rho\pi\pi$ coupling constant
with PACS-CS configurations generated at

$N_f = 2+1$, Imp. Wilson

$a = 0.091\ \mathrm{fm}$, $L = 2.9\ \mathrm{fm}$

$m_\pi = 410\ \mathrm{MeV}$

Our results :

$$g_{\rho\pi\pi} = 5.24(51)$$

$$M_R = 0.4064(46)$$

$$\Gamma = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{k_\rho^3}{m_\rho^2} = 113(22)\ \mathrm{MeV}$$

exp. :

$$g_{\rho\pi\pi} = 5.98(2)$$

$$\Gamma = 149.1(8)\ \mathrm{MeV}$$

: slightly smaller than exp.

possible reason :

(1) mass dependence

(2) O(a^2) error

: Future improvement

Trial calcs :

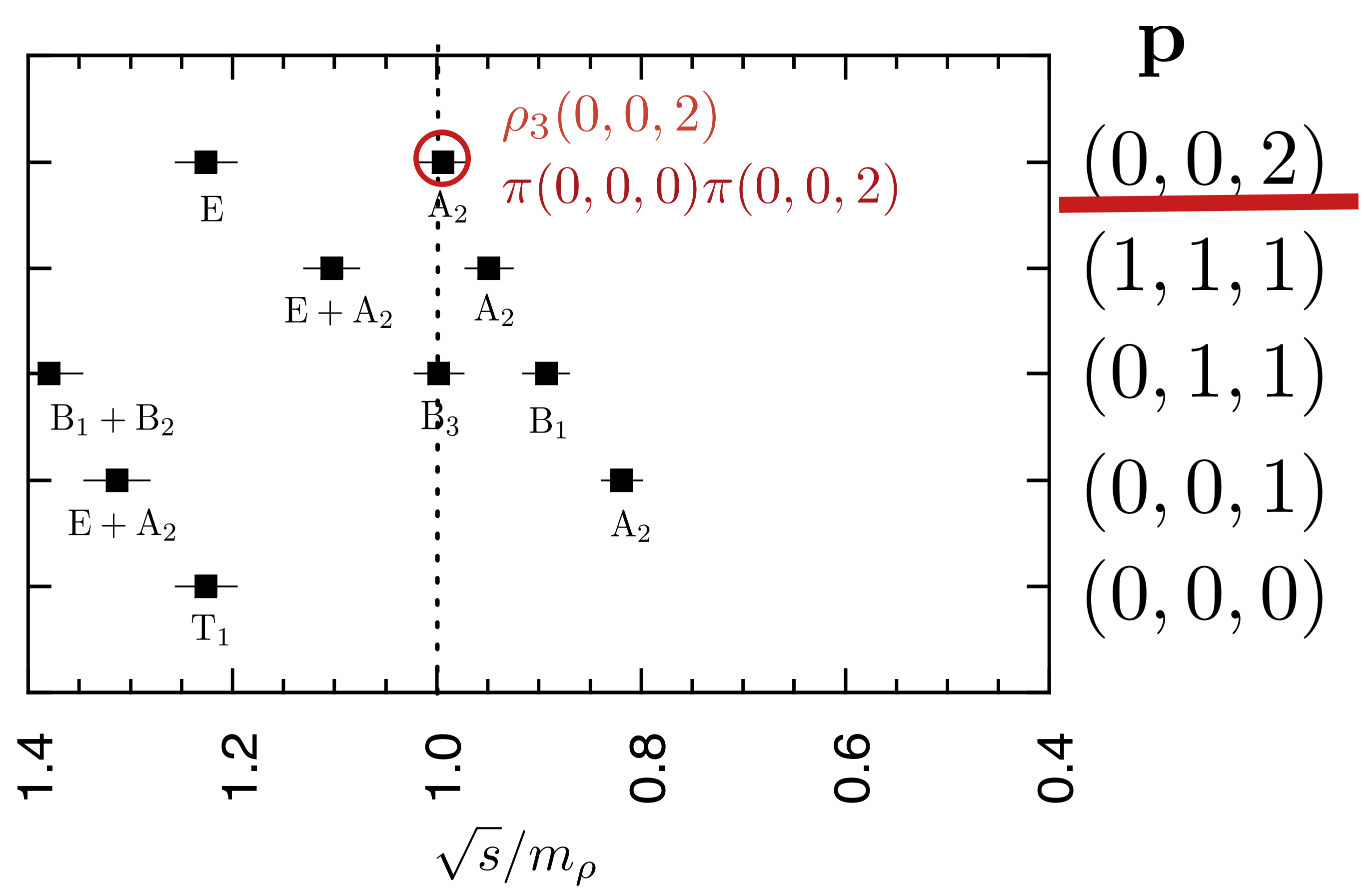
$N_f = 2+1$, Imp. Wilson

$\beta = 1.9$, $32^3 \times 64$

$a = 0.091$ fm , $L = 2.9$ fm

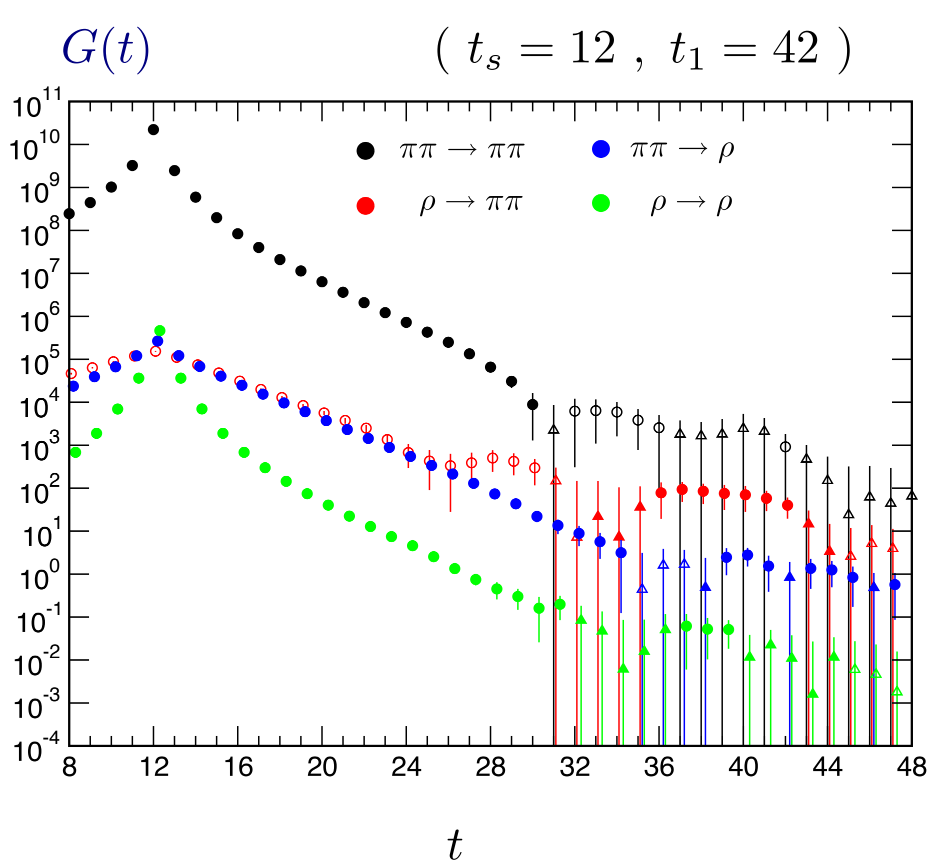
$m_\pi = 300$ MeV

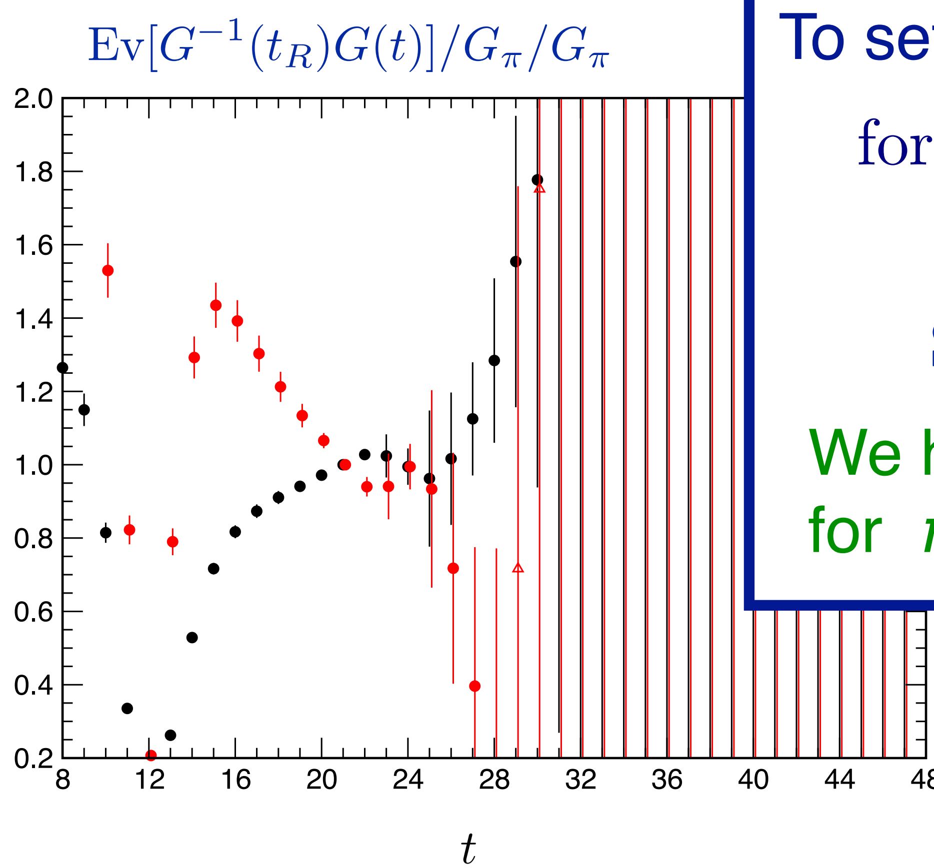
generated by PACS-CS col.
PRD79(2009)034503.



Results of #c. = 92

(70 day by 1.4Tf machine)





To set $\sqrt{s} \sim m_\rho$

for $m_\pi \to 0$

$\mathbf{p} \to$ larger

Stat. Error. $\to$ large

We have to improve method for π with high momentum.