Can Lorentz-breaking fermionic condensates form in large N strongly-coupled LGT?

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The possibility of explicit or spontaneous violations of Lorentz symmetry has attracted a good deal of attention in recent years for a variety of phenomenological and theoretical reasons.

Here we will explore the possibility of spontaneous violation.

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Note: In such a gravity theory no cosmological constant can occur. (P. Kraus and T.T.)

Effective field theory low energy descriptions of the effects of such spontaneous LSB (assuming that it takes place) have been given in various recent works.

The central question, however, is:

Is it possible to have such dynamical SLSB in an UV complete theory?

To investigate this question we turn to strong-coupling LGT as a model of non-perturbative dynamics leading to fermionic condensates.

Standard U(N) (SU(N)) LGT with massless naive fermions

$$A = \sum_p \beta \mathrm{tr} U_p + \sum_b \bar{\psi}(x)\mathbf{M}_\mu(x)\psi(x+\hat{\mu}) \ ,$$

$$\bar{\psi}(x)\mathbf{M}_\mu\psi(x+\hat{\mu}) = \frac{1}{2}\left[\bar{\psi}(x)\gamma_\mu U_\mu(x)\psi(x+\hat{\mu}) - \bar{\psi}(x+\hat{\mu})\gamma_\mu U^\dagger_\mu(x)\psi(x)\right]$$

Strong coupling limit $\beta \to 0$ -- drop plaquette term

Small β plaquette term corrections easily taken into account within the convergent strong coupling expansion in β

In this strong coupling large N limit can we have

$$\left\langle \bar\psi(x)\,\Gamma^A\,\psi(x)\right\rangle \overset{?}{\neq} 0$$

Here

$$\Gamma^A = \big(1, \quad \gamma^5, \quad \gamma^\mu, \quad \gamma^5\gamma^\mu, \quad \ldots\big)$$

Add source term for the operator to the action:

$$\bar\psi(x)\,\Gamma^A\,\psi(x)\;K_A$$

$$K_A = K\,n_A, \qquad\qquad n^A n_A = 1$$

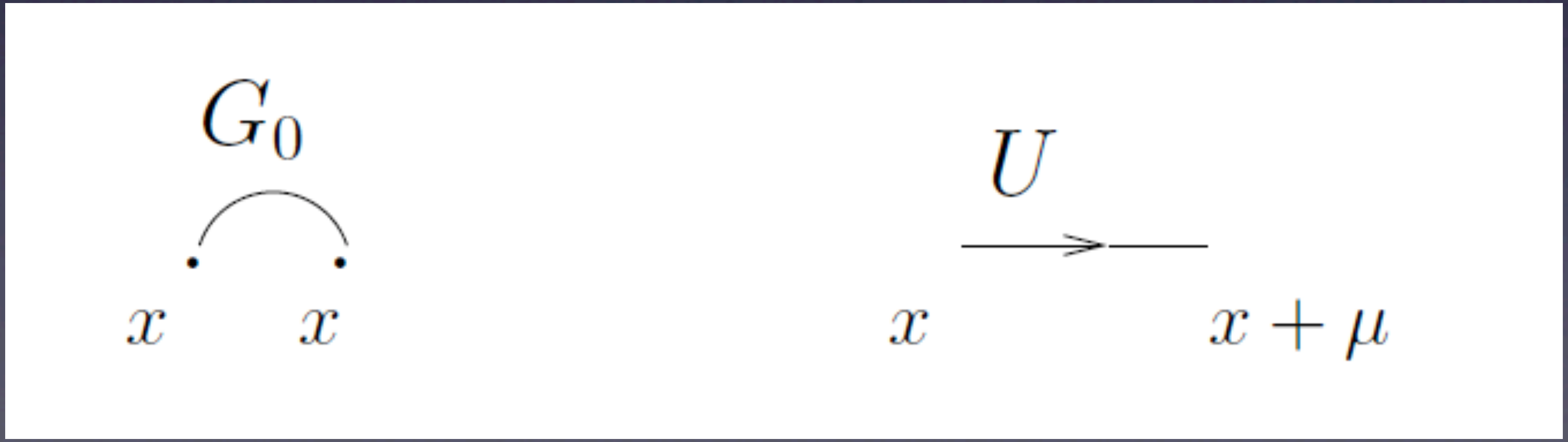
$$A = \sum_{b=(x.\mu)} \bar\psi(x)\mathbf{M}_\mu(x)\psi(x+\hat\mu) + \sum_x \bar\psi(x)\Gamma^A\psi\, K_A$$

Now perform standard Hopping Expansion with:

(i) Source defining the bare local `propagator':

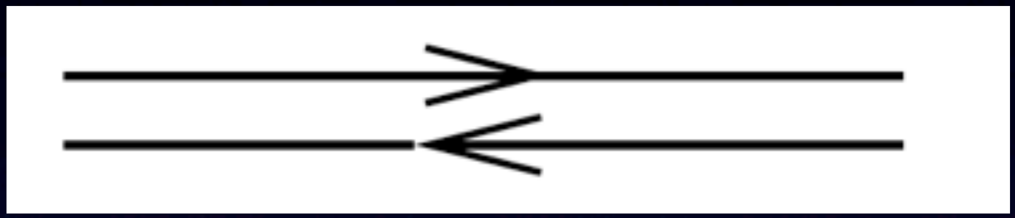
$$G_0 = [\Gamma^A K_A]^{-1} = \frac{1}{K}\,[\Gamma^A n_A]^{-1}$$

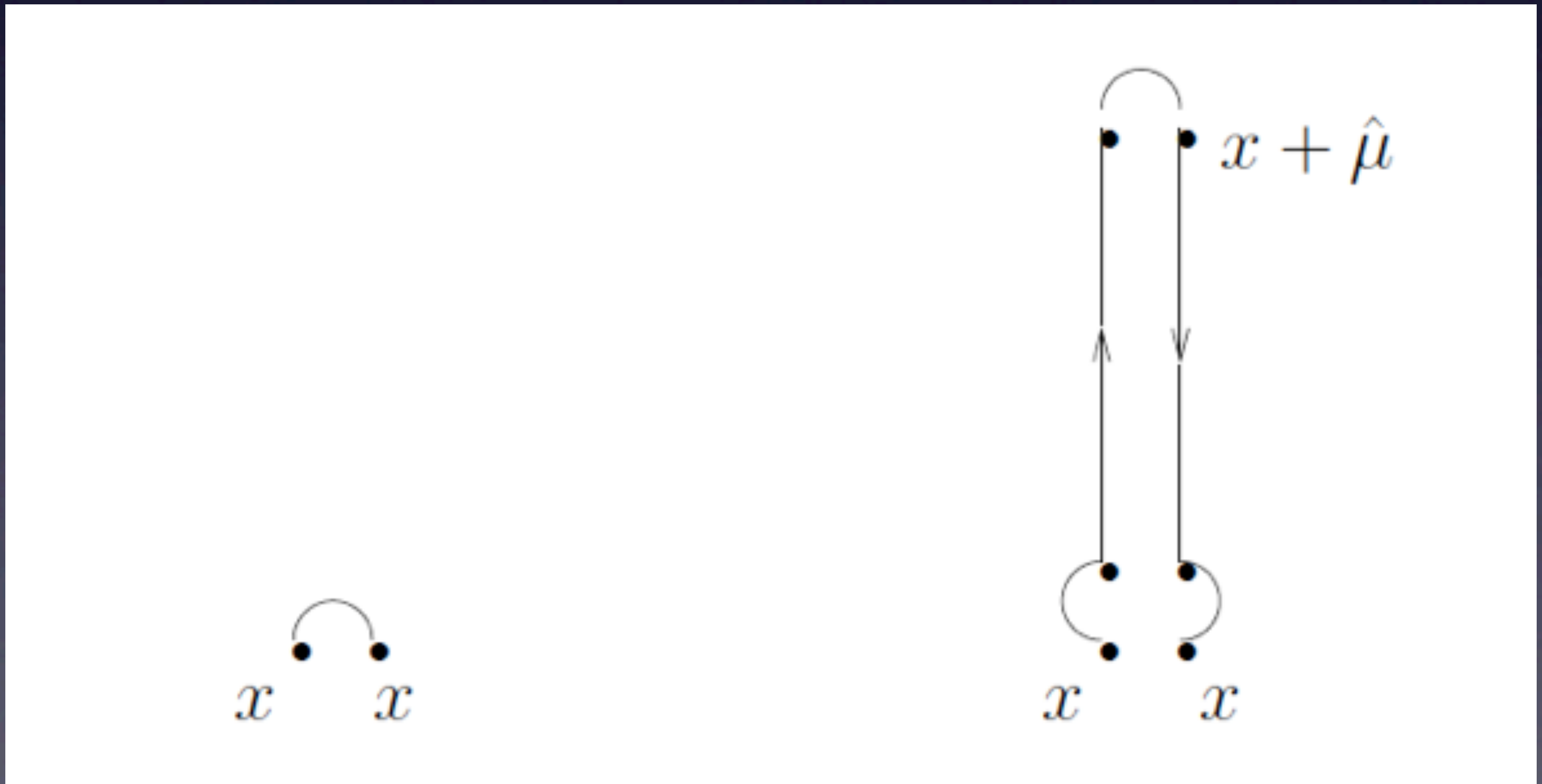
(ii) The bond hopping giving the (nearest neighbor) interaction



Dominant large N graphs are the `tree graphs':

Only graphs with U and $U^\dagger$ on same bond can contribute-- this creates an effective 4-fermion (nearest) neighbor interaction



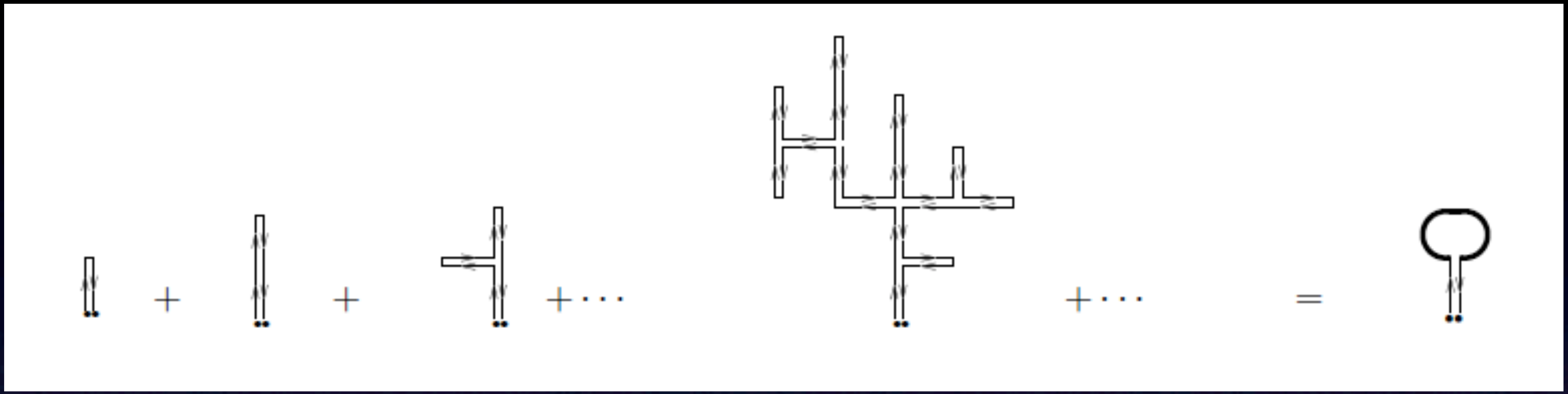


0-th order

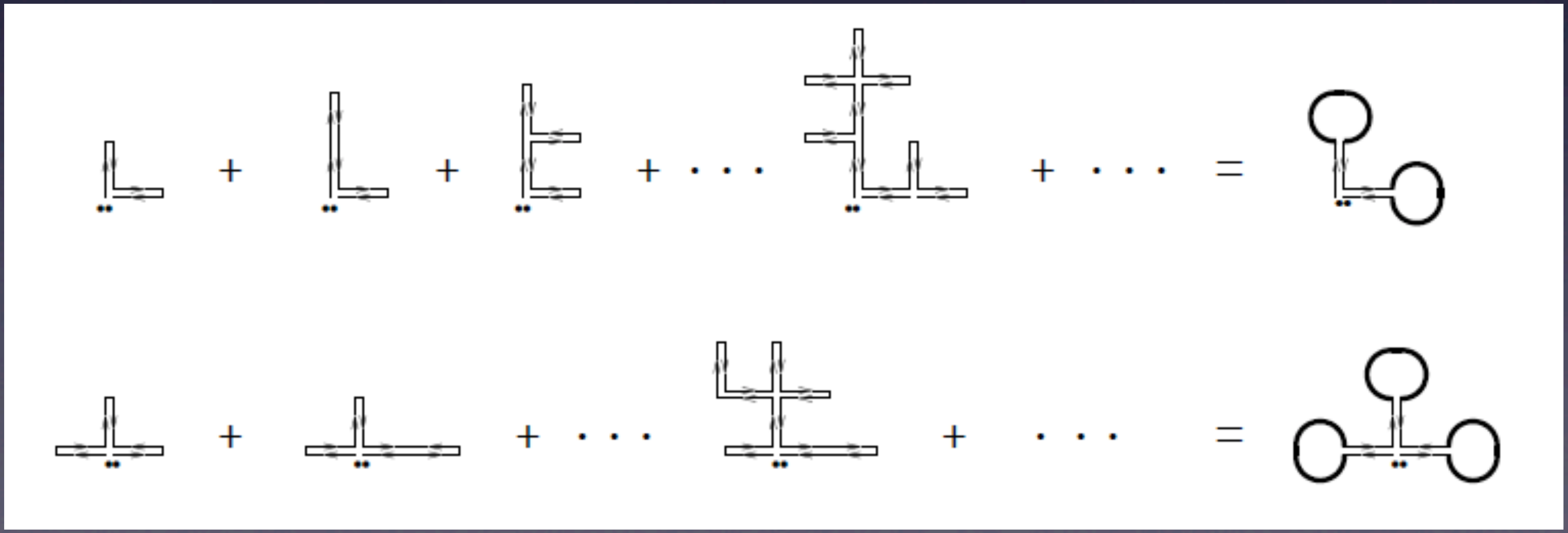
1st order

Tree graph structure

`1-trunk' trees



`2-trunk' and `3-trunk' trees



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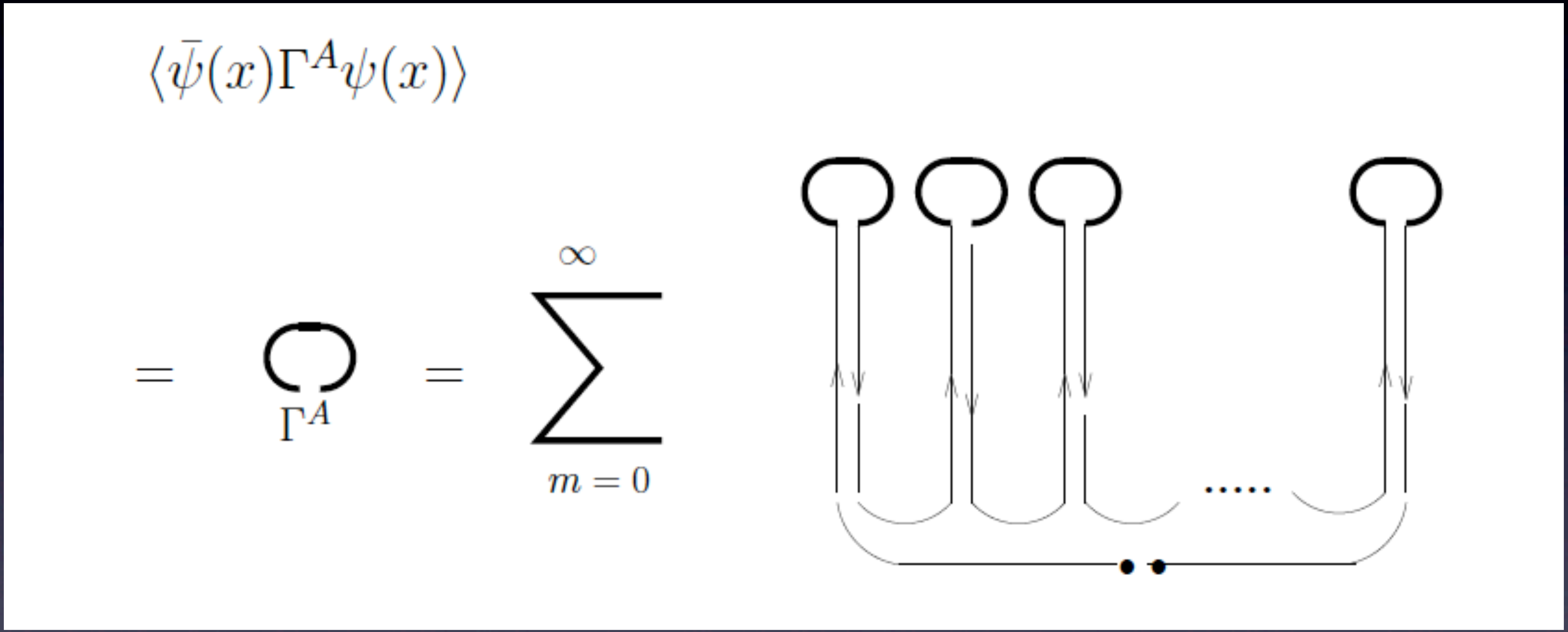
In this manner one arrives at the **self-consistent equation for the sum of the trees** giving the large N contribution to the expectation in the presence of the source:



One may now seek solutions in the limit of the source $K^A \to 0$

Solutions:

- $\Gamma^A = 1$: non-trivial solution exists (BBEG, ...):

$$\langle \bar{\psi}\psi \rangle \neq 0$$

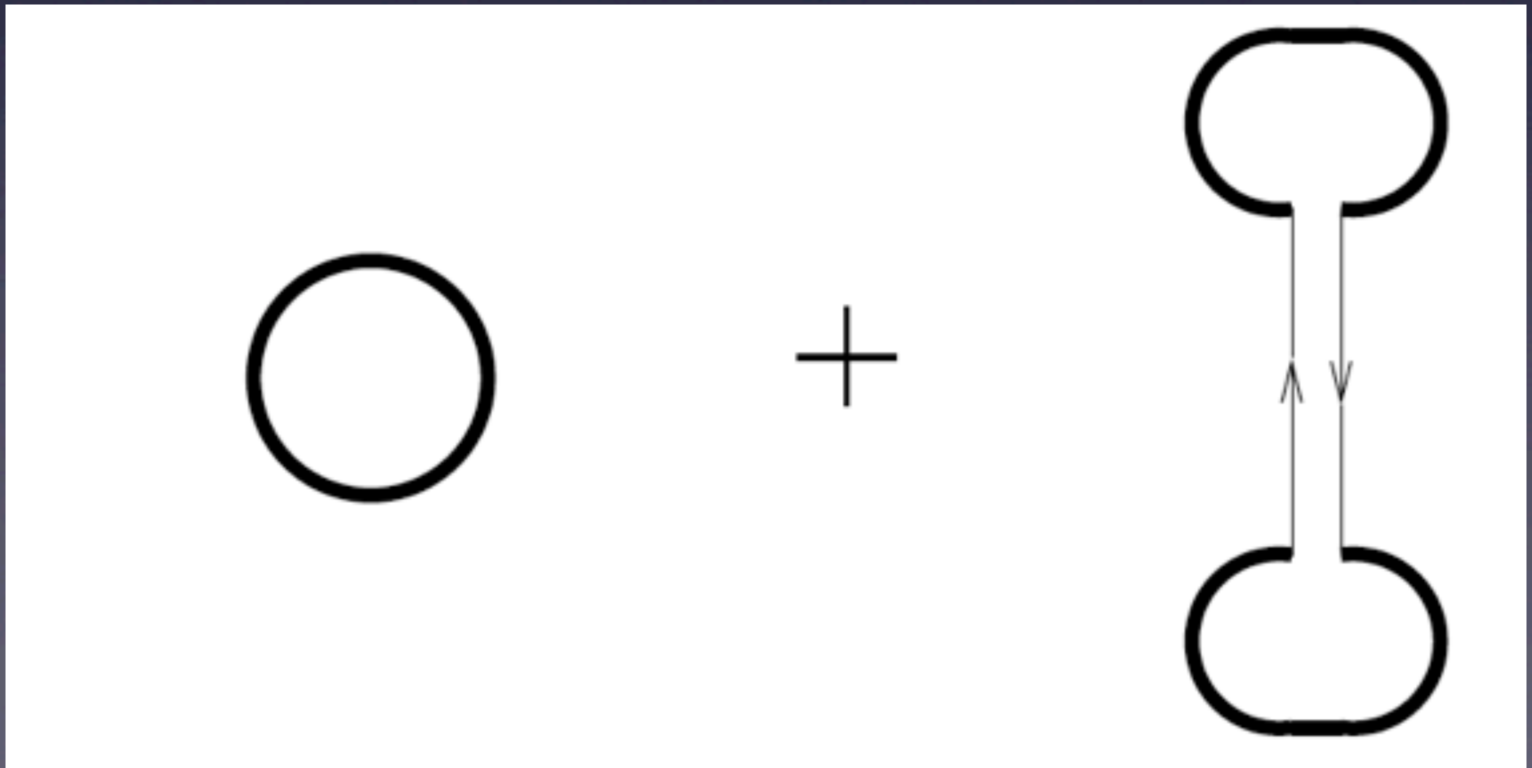
- $\Gamma^A = \gamma^5$: no non-trivial solution
- $\Gamma^A = \gamma^\mu$: no non-trivial solution
- $\Gamma^A = \gamma^5\gamma^\mu$: non-trivial solution exists:

$$\langle \bar{\psi}\gamma^5\gamma^\mu\psi \rangle \neq 0$$

Effective Action for Composite Operators

Instead of explicitly sum graphs, construct the corresponding effective action (CJT) for the operator $\bar{\psi}(x)\Gamma^A\,\psi(x)$ directly.

Result for the effective action = Legendre transform w.r.t. to source term $\bar{\psi}(x)\Gamma^A\,\psi(x)K_A$:



Variation leads to same equation for expectation as by graph summation

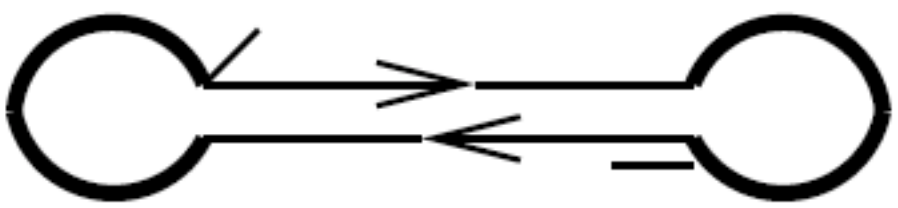
Once SB occurs by some condensate other (higher) condensates also form.

Of particular interest is the tensor operator:

$$\bar{\psi}(x)\gamma^{\nu}\partial_{\mu}\psi(x) \qquad \Longrightarrow \qquad \bar{\psi}(x)\gamma^{\nu}U_{\mu}\psi(x+\hat{\mu})$$

One finds that it has non-vanishing expectation once $\bar{\psi}(x)\,\gamma^{5}\gamma^{\mu}\,\psi(x)$ develops non-vanishing expectation:

$$\langle\bar{\psi}\gamma^{\nu}U_{\mu}\psi\rangle =$$



Locking of space-time and internal symmetries

Formation of condensates presents another interesting possibility.

Simple example: space-time Euclidean Lorentz SO(4) and internal `flavor' SO(4).

Condensate $$\langle\bar{\psi}(x)\,\gamma^m\gamma_\mu\,\psi(x)\rangle = C\;\delta^m_\mu\rangle$$

$$SO(4)_F \times SO(4)_L \to SO(4)_d$$

In this case the low energy effective theory would be invariant under an SO(4)`Lorentz' symmetry.

Such condensates indeed can be shown to arise in much the same manner as above.

But in Minkowski space: $SO(4) \rightarrow SO(3,1)$

i.e. one has non-compact spacetime groups. This implies that such (complete) locking would necessitate also non-compact internal groups.

This in turn would require fields transforming under unitary reps, i.e. infinite dimensional representations of the internal group, i.e. an infinite-component field theory.

More general example: $SL(4,R)_F \times SO(3,1)_L \rightarrow SO(3,1)_d$

Conclusions

- In the large N and strong coupling limit of U(N), SU(N) LGT certain Lorentz symmetry breaking fermionic condensates are found to occur, at least in the same approximation in which the chiral symmetry breaking condensates are found to occur.

- Another possibility is the formation of condensates locking internal and external groups.

- To decide, in any particular model, which of these condensates is actually dominant (global minimum) requires additional computations beyond those outlined above for individual condensates.