Search for the IR fixed point in the Twisted Polyakov Loop scheme (II)

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arXiv:0910.4196 and Work in progress

Lattice2010 @ Tanka Village 2010/6/15

Collaborators

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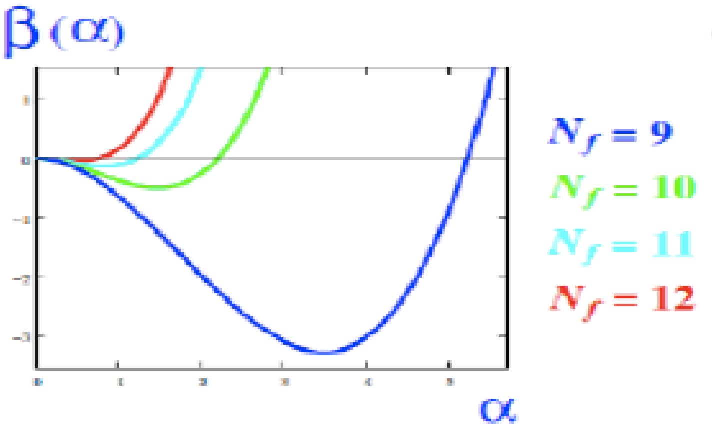
Numerical simulation was carried out on the vector supercomputer
NEC SX-8 in YITP, Kyoto University
and RCNP, Osaka University
SR and BlueGene in KEK

a : Nagoya University
b : Tohoku University
c : National Chio-Tung University, and
National Center for Theoretical Science
d : KEK
e : Osaka University
f : RIKEN-BNL Research Center
g: Tsukuba University

Introduction

large flavor SU(3) gauge theories:

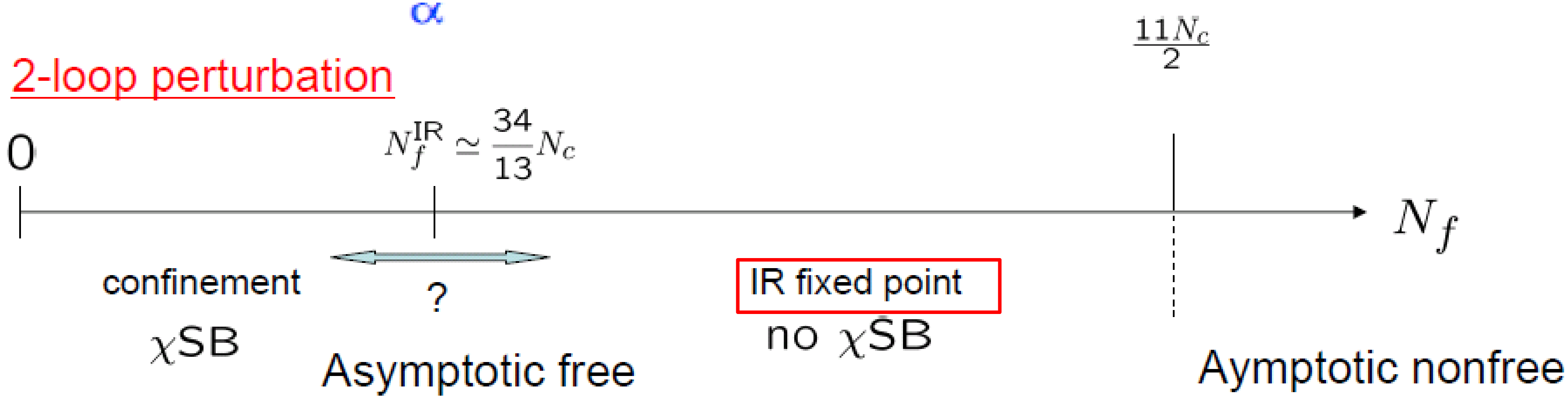


Two-loop perturbative beta function:

$$\beta(\alpha) = -\alpha^2(c_1 + c_2\alpha)$$

The signs of C1 and C2 depend on Nf.

2-loop perturbation



Gardi and Karliner Nucl.Phys.B529:383-423,1998.
Ncw~12 from resummation method of 3 terms of the beta fn

Miransky and Yamawaki Phys.Rev.D55:5051-5066,1997
Ncw=11.9 based on the same idea of Gardi et al.

Ryttov and Sannino Phys.Rev.D78:065001,2008
Ncw=8.25 from calclulation of the anomalous dimension

Previous and Recent studies in lattice QCD with Nf=12

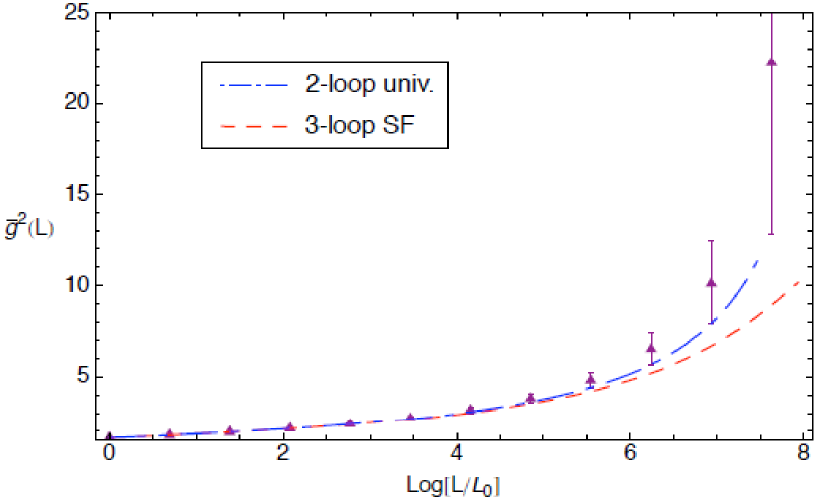
PHASE

- Iwasaki et al: Phys.Rev.D76:034504
- Deuzeman, Lombordo and Pallante arXiv:0904.4662 [hep-ph]
  there is a conformal phase...
- Fodor, Holland, Kuti, Nogragi, and Schroeder arXiv:0809.4890, 0911:2463 [hep-lat]
  Nf=12 is below the conformal window. (stout-smeared staggered fermion)

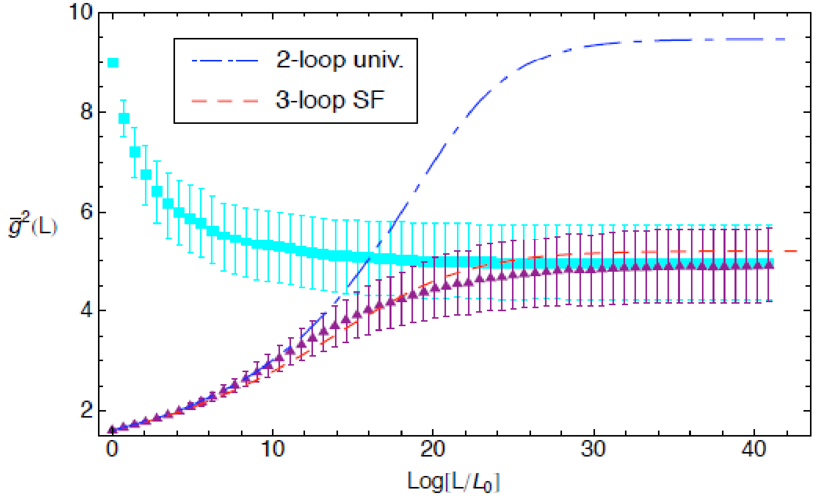
Coupling

- Appelquist, Fleming and Neil: Phys.Rev.Lett.100:171607(2008), PRD79:076010

The running coupling constant in Schrodinger functional scheme.



Nf=8 There is no edivence of fixed point.



Nf=12 : a flat region in low energy scale.

- Fodor, Holland, Kuti, Nogragi, and Schroeder
  study the renormalized coupling in Wilson loop scheme

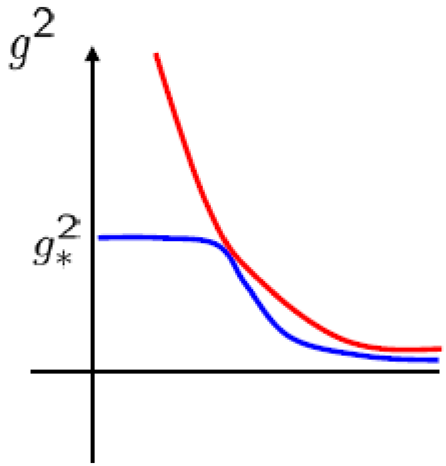
The difficulty to study low energy behavior comes from a large discretization error .

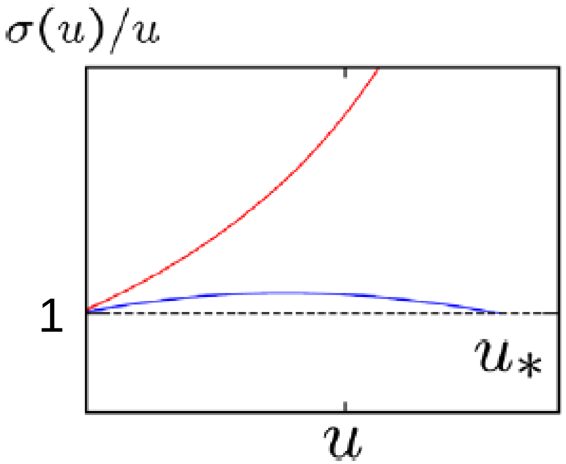
Summary of our work

- Measure the renormalized coupling in the Twisted Polyakov loop scheme (TPL).
  The existence of the fixed point does not depend on the renormalization scheme.

- Estimate the $O(a^2)$ discretization error.

- Study the growth rate of the renormalized coupling.
  We focus on the step scaling at each coupling rather than the beta function, since the latter suffers from accumulation of errors at low energy region.





For each step scaling procedure

input renormalized coupling
$u = g^2(1/L)$

output renormalized coupling
(scaling function)
$\sigma(u; s) = g^2(1/sL)$

s:scaling parameter

When the coupling stops running, the ratio (sigma/u) becomes “1”.

Twisted Polyakov loop scheme

Twisted Polyakov loop (TPL) scheme

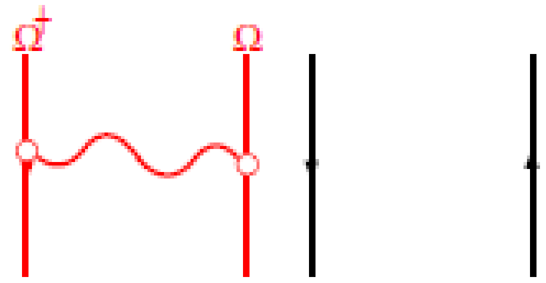
de Divitiis et al.:NPB422:382

TPL coupling: $$g^2_{TP} = \frac{1}{k}\frac{\langle \sum_{y,z} P_1(y,z,L/2a)P_1(0,0,0)^*\rangle}{\langle \sum_{x,y} P_3(x,y,L/2a)P_3(0,0,0)^*\rangle}$$

no O(a/L) error

At tree level, g^2_{TP} is proportional to bare coupling.

$$g^2_{TP}\Big|_{\text{tree}} = g_0^2$$
$$k = \frac{1}{24\pi^2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{n^2+(1/3)^2} = 0.0318$$



Remarks on TPL coupling

➢If the theory is conformal, the correlation length becomes infinity.
The divergence part of the OPE for Ploop correlator depends on the conformal dimension of each operator. It cancels out between numerator and denominator. The coefficient of OPE depends on boundary contribution and gives a nontrivial constant. The ratio of Polyakov loop gives a nontrivial constant ($\neq 1$).

➢However, even if there is no conformal field theory, the IR limit of TPL coupling goes to constant.

$$g^2_{TP} \xrightarrow[1/L\to 0]{} \frac{1}{k} \sim 32$$

This is a lattice artifact of the TPL coupling.

Simulation: Nf=12

Nf=12 QCD

- Wilson action + staggered fermion
- Hybrid Monte Carlo algorithm (exact algorithm)
- 50,000 ~ 100,000 Trj. for each parameter

Lattice size for step scaling

$$s = 1 : L = 4, 5, 6, 8$$
$$s = 2 : L = 8, 10, 12, 16$$

Parameter $4.5 \leq \beta \leq 20$

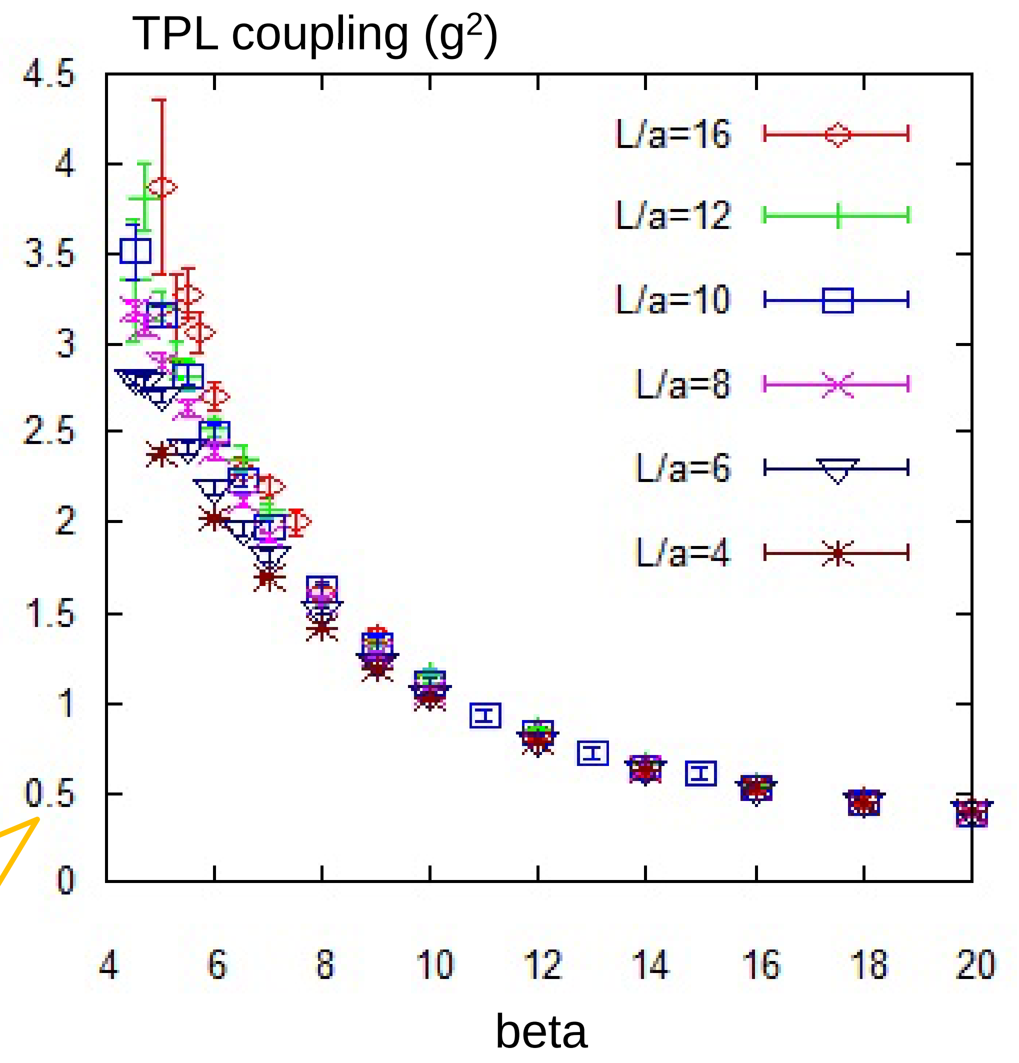
Interpolation of beta at fixed L/a

$$f(\beta) = \frac{C_1}{\beta} + \frac{C_2}{\beta^2} + \frac{C_3}{\beta^3} + \frac{C_4}{\beta^4}$$

Interpolation to L/a=5

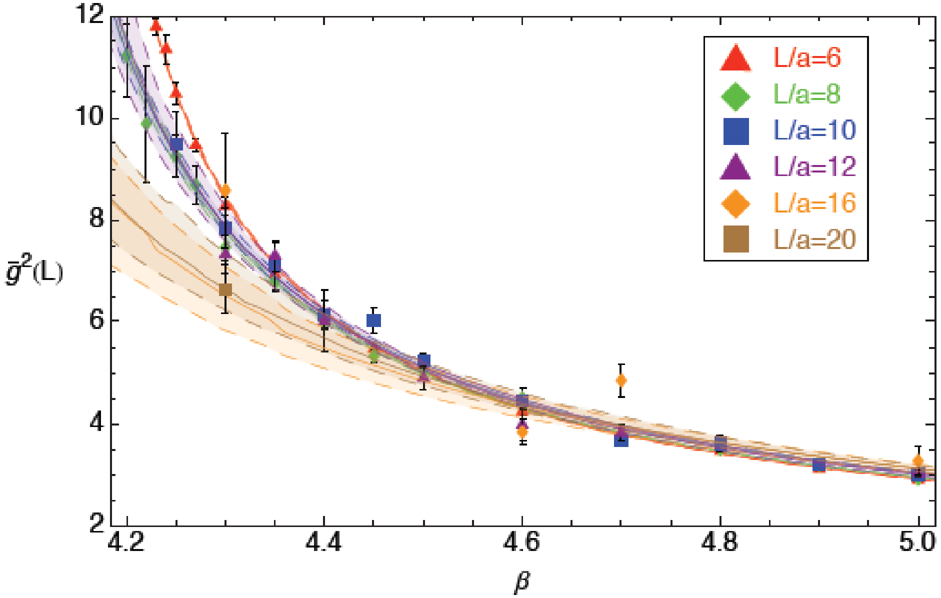
At high energy, the data of the large lattice size are larger than that of the small lattice .

This behavior is same as one of the SF scheme.



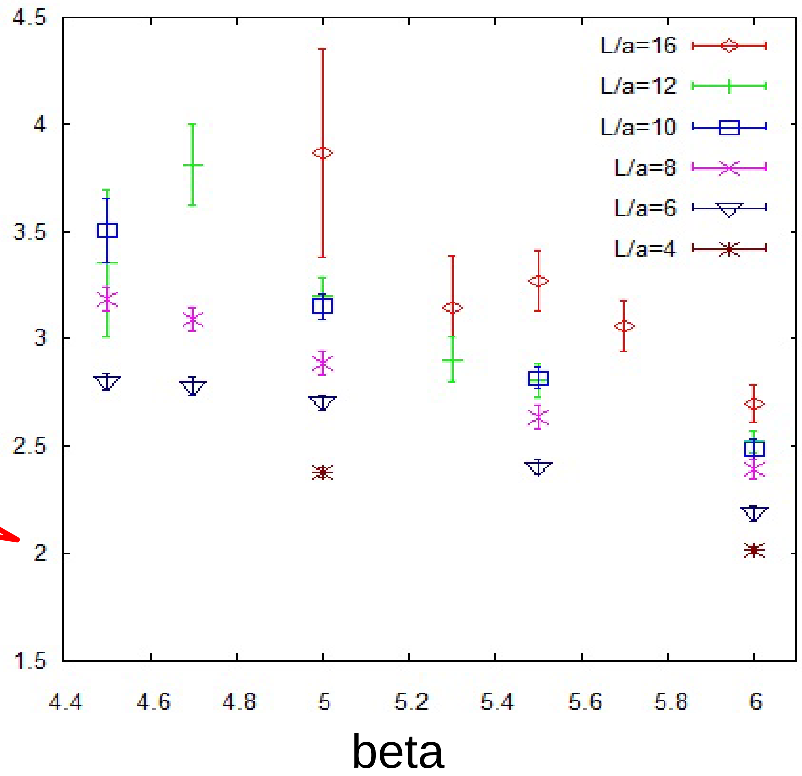
In low beta region (4.5 < beta < 6.0)

SF scheme (Appelquist et.al.)



In the low beta region (beta~4.6), the SF scheme shows the inversion of the order of lattice size dependence in the fixed beta.

TPL scheme

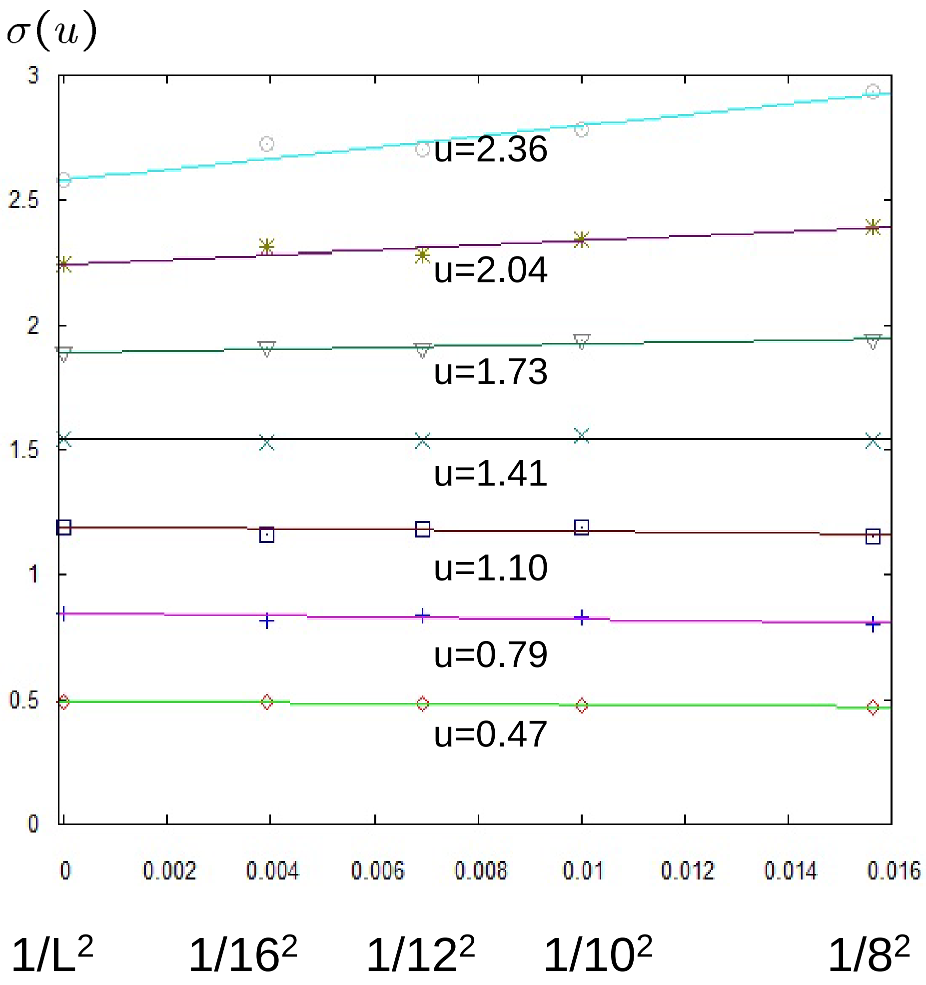


The TPL coupling does not show the inversion even in beta=4.5.

cf. Also, in Wilson loop scheme:
Fodor et al. @ SCGT09 conference

Continuum extrapolation for each step scaling

4-points linear extrapolation



Lattice size for step scaling

$$s = 1 : L = 4, 5, 6, 8$$
$$s = 2 : L = 8, 10, 12, 16$$

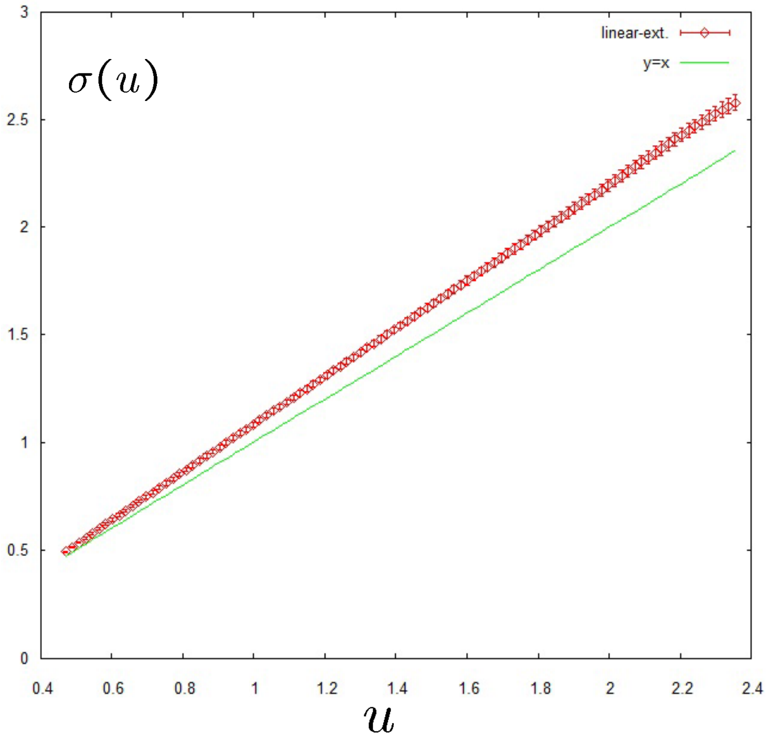
From u= 0.471 to 2.36, we measure the scaling function.
The region of these data values corresponds to 5< beta < 18.

We take the continuum limit using linear extraplation of a^2 .

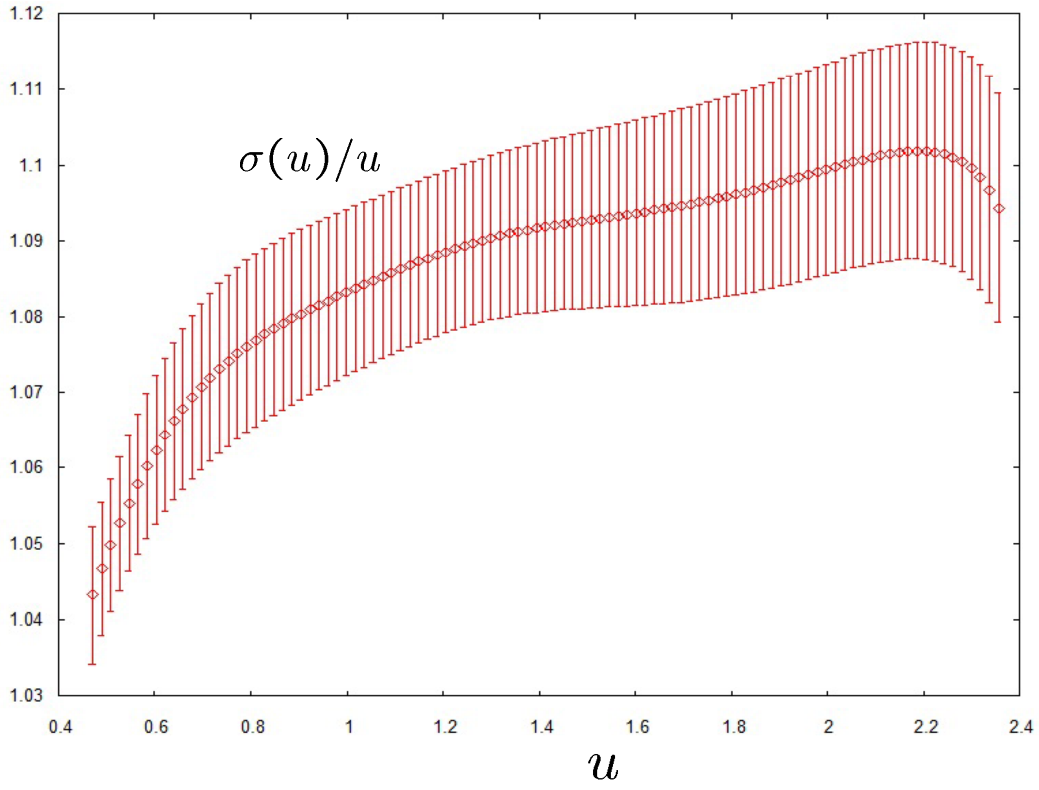
At the strong coupling region, there is a large scaling violation.

Statistical errors are very small (<3%)

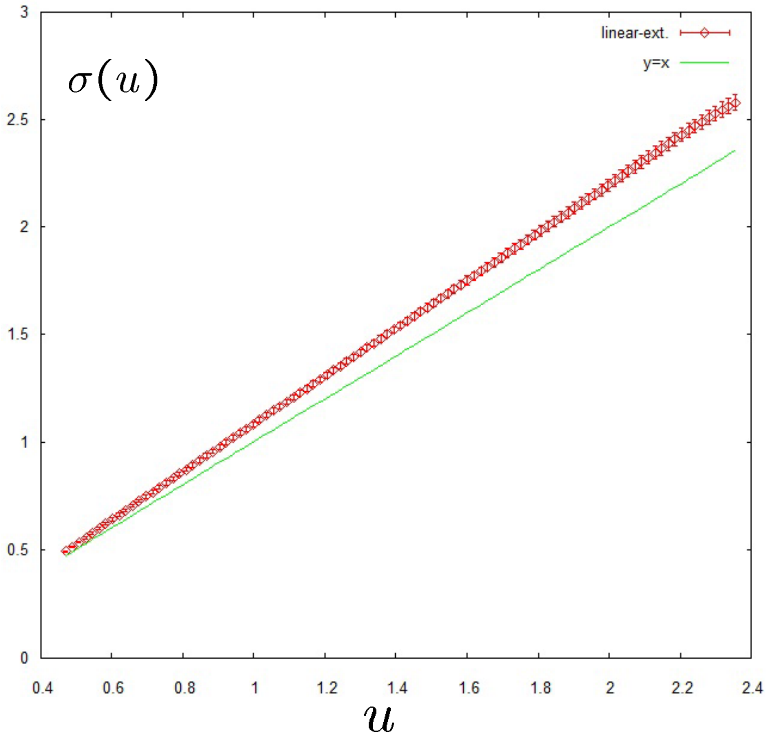
The scaling function for each step scaling



If there is a fixed point, the data will be consistent with y=x line.



The scaling function for each step scaling



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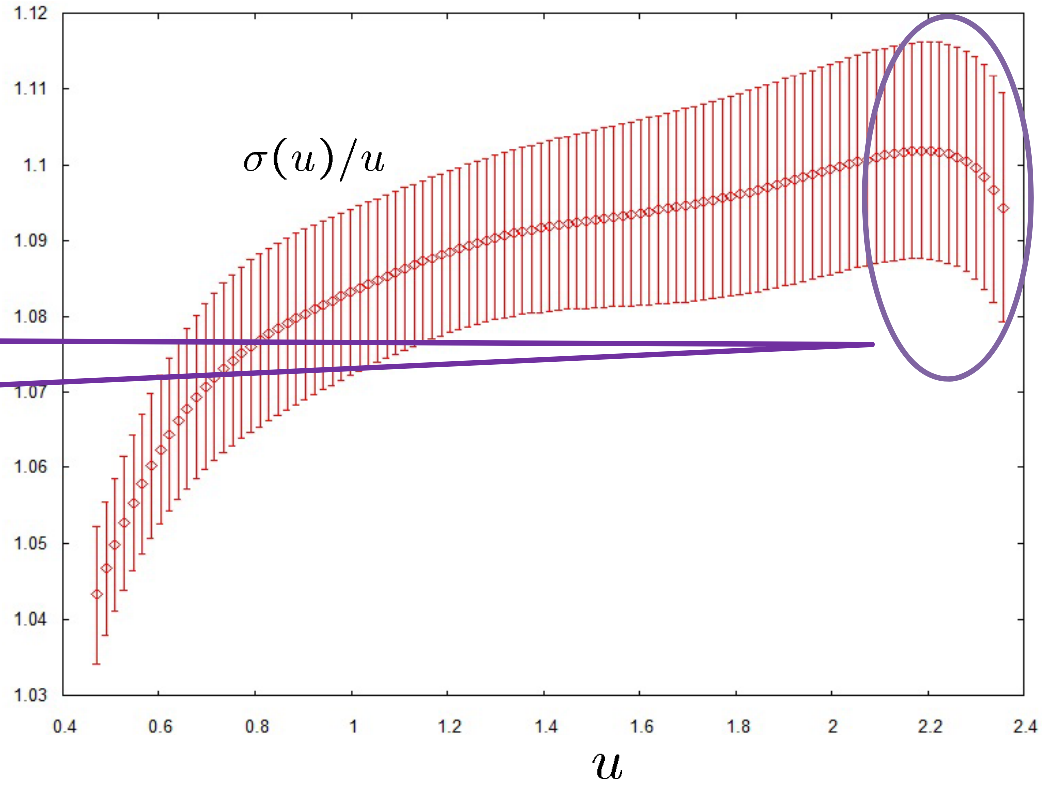


Is the growth rate starting to be small at low energy?

NO!

See Next Slide.

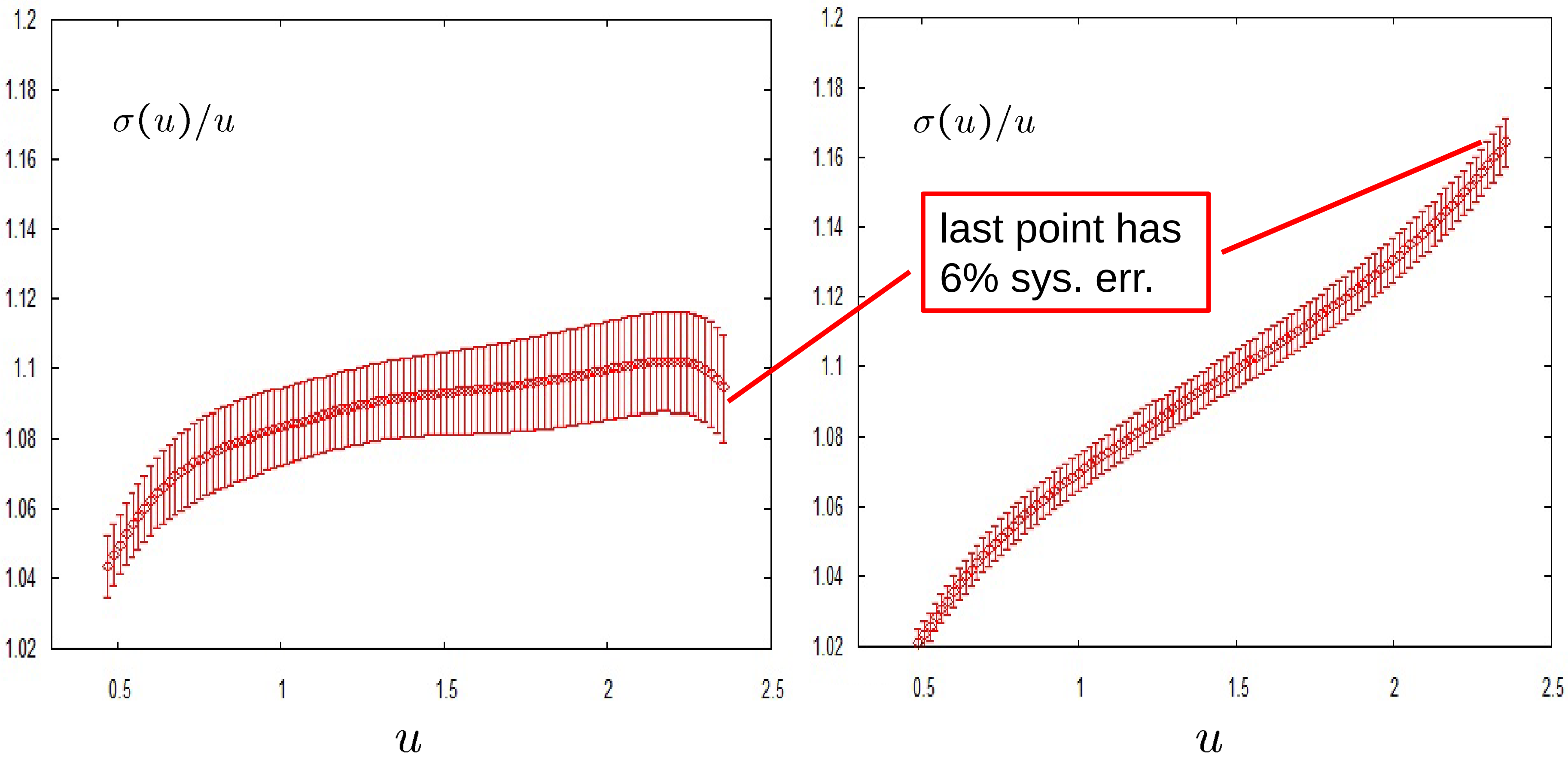
The differences between two extrapolations show systematic errors.
There is a large systematic error.

Lattice size for step scaling

$s = 1 : L = 4, 5, 6, 8$
$s = 2 : L = 8, 10, 12, 16$

4-points linear extrapolation

3-points constant extrapolation



There is no signal of the fixed point.
We have to do some improvements (larger lattice size simulation).

Summary

- In low beta, the TPL coupling's behavior is different from SF scheme.
- In our analyses, we estimated the discretization errors in $O(a^2)$.
- We study the growth rate of the renormalized coupling.
- The TPL coupling in the case of Nf=12 does not show a signal of IR fixed point.
- Now, we are working in progress to study the lower energy region and have to do some improvement to reduce the systematic error.

Future directions

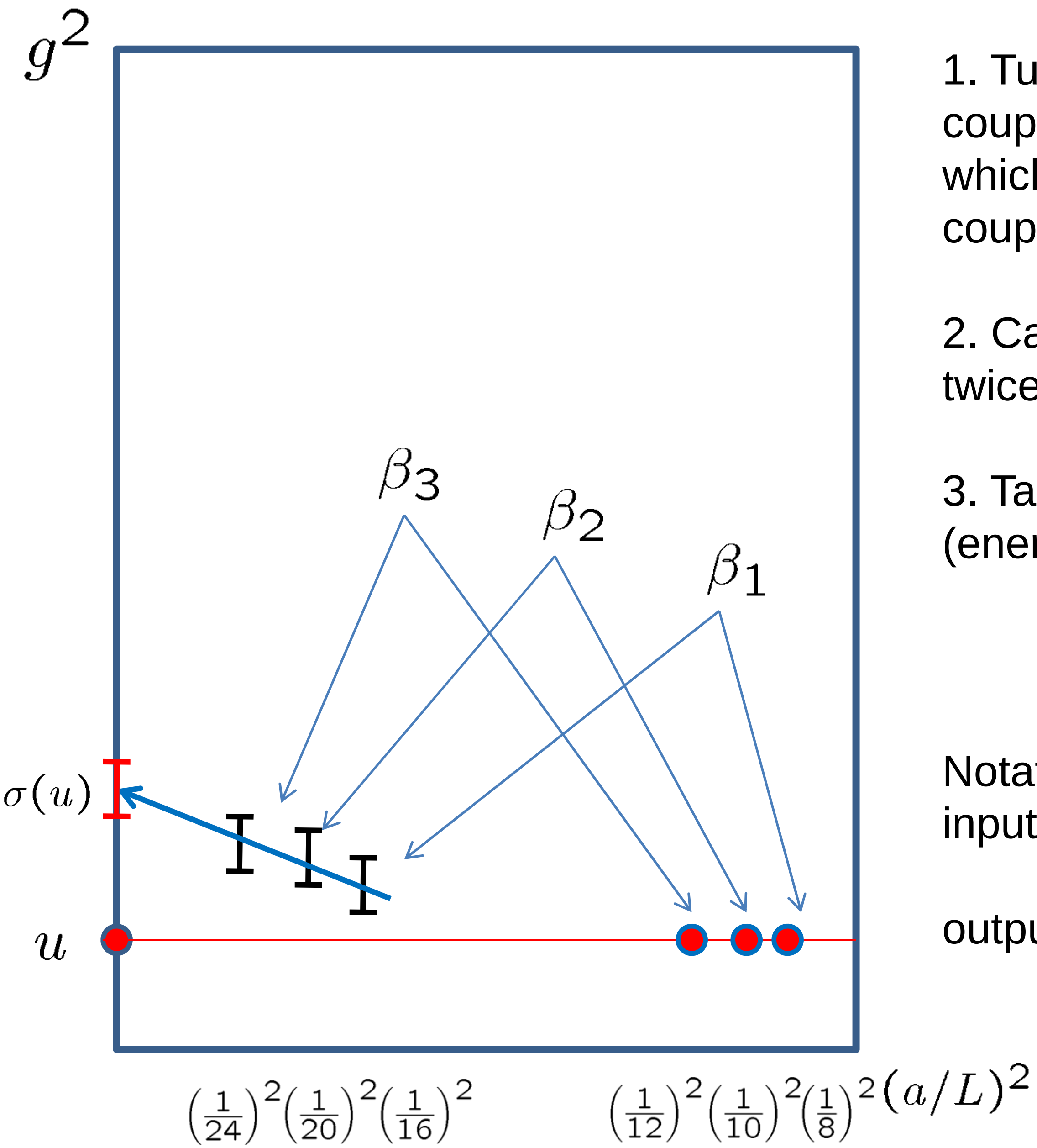
- Numerical measurement of the anomalous dimension at the IR point
  the composite operator of fermions is interesting (Del Debbio et al.)
- Study Nf dependence (for example Nf=8 or 16)
  to study arbitrary Nf, we need Overlap or Domain-wall fermion
- The other gauge group and representation of fermions
  many people are studying…(Sannino, Yamada, Ohki…)

SU(2) fund.Nf=8 case
(Twisted Wilson loop sheme)
H.Ohki's talk
(17 June pm5:40-)

Back up slides

$$\langle O^i(x) O^j(0) \rangle \sim \sum_k \frac{C^{ijk}}{|x|^{\Delta_i + \Delta_j - \Delta_k}} O^k$$

Step scaling procedure



1. Tune the value of beta (bare coupling) for a small lattice size, which gives the renormalized coupling "u".

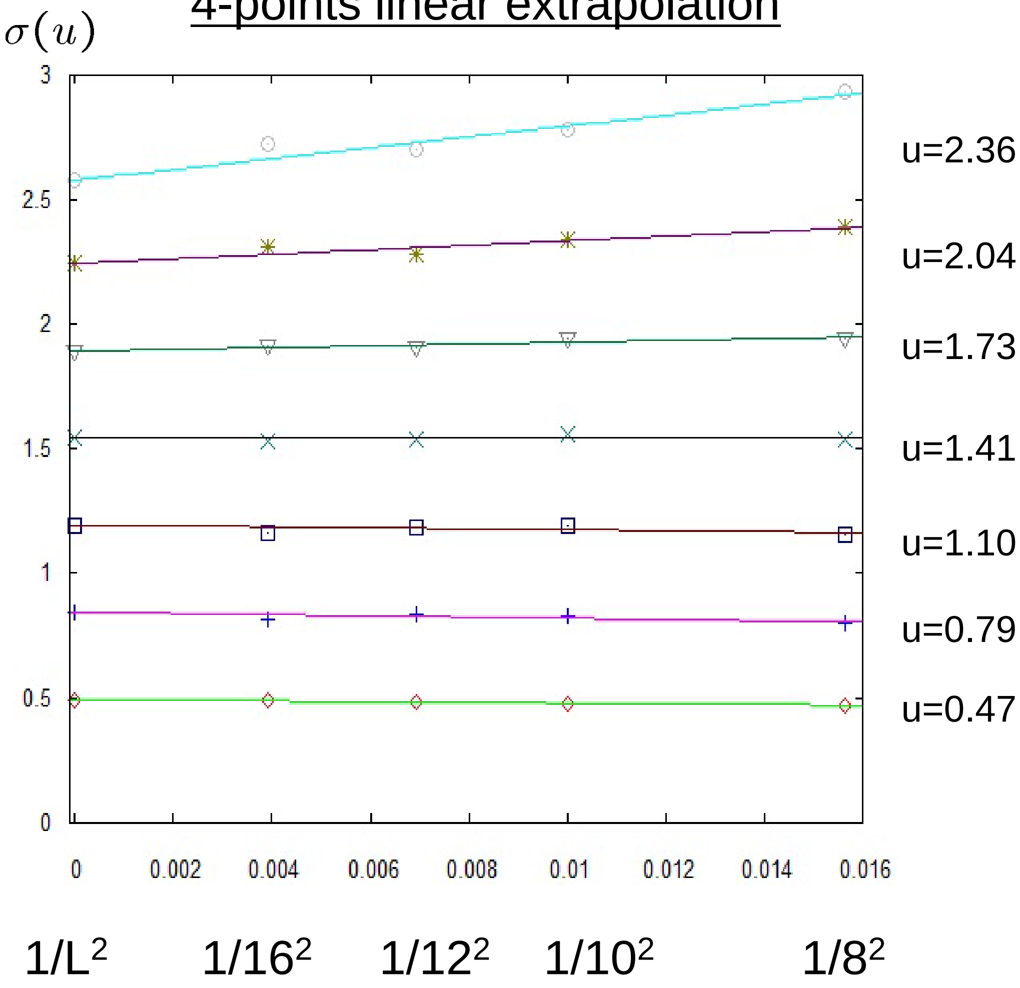
2. Carry out the simulation for the twice size of lattice.

3. Take the continuum limit (energy scale $\mu = 1/2L_0$)

Notation:
input renormalized coupling :
$$u = g^2(1/L)$$
output renormalized coupling :
$$\sigma(u) = g^2(1/sL)$$

Comparison the continuum extrapolations

4-points linear extrapolation



3-points constant extrapolation