Two-photon decay of π^0 from two flavor lattice QCD

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with JLQCD collaboration

Motivation

- Why we study $\pi^0 \to \gamma^{(*)}\gamma^{(*)}$ process ?
  - Lattice analysis of on- and off-shell photon decay amplitude

    Anomaly effect plays an important role

    → resent measurement [Jefferson Lab (2009)]

  - Non-perturbative study of $\gamma^*\gamma \to \pi^0$ transition form factor

    Comparison with pQCD analysis in deep Euclidean region

    → CELLO, CLEO and BaBar ($e^+e^- \to \gamma^* \to$ hadron(+γ))

  - Applicable to the calculation of NLO hadronic contribution to muon anomalous g-2

    → precision test of the SM, constraint on BSM

In this talk

- Analysis of lattice data in CHPT
  - Fit the lattice results of $\pi^0 \to \gamma\gamma$ transition form factor in low energy
    $\to$ an analysis of chiral behavior
  - Comparison with CHPT and VMD
    $\to$ correspondence to LECs and vector meson mass
  - Pion loop correction to the $\pi^0 \to \gamma\gamma$ amplitude

CHPT with anomalous term

- Wess-Zumino effective action

$$\mathcal{L}_{WZW} = -\frac{N_c}{96\pi^2 f_0}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}\pi^0$$

Wess, Zumino (1971), Witten (1983)

induced from axial-anomaly.

At N_c = 3 and in the massless limit:

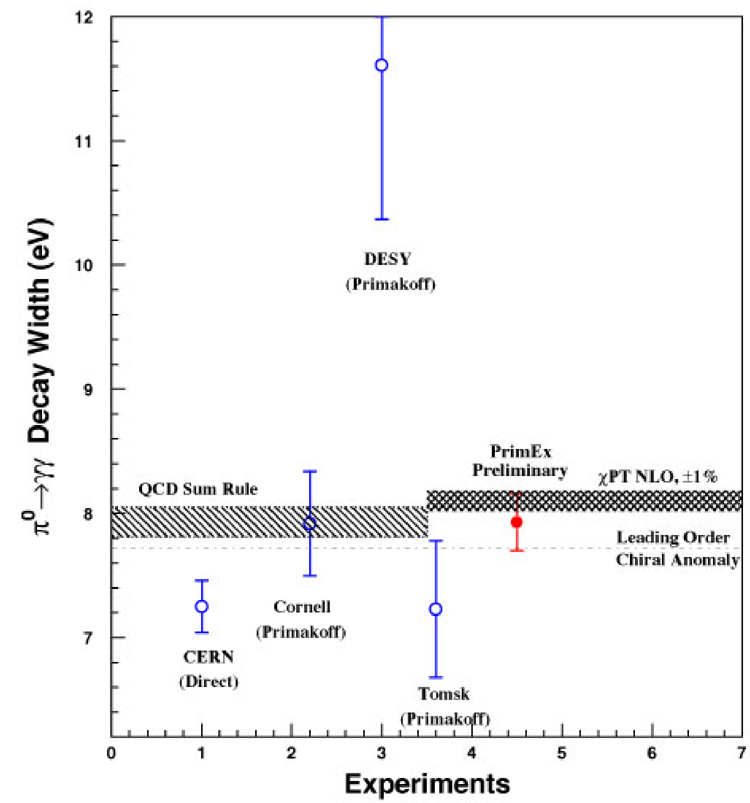
$$\lim_{m_q\to 0} f_{\pi^0\gamma\gamma}(0,0) = \frac{1}{4\pi^2 f_0}$$

- Decay width:
  - LO : $\Gamma^{\mathrm{LO}}_{\pi^0\to\gamma\gamma} = 7.73(4)\,\mathrm{eV}$
  - NLO: $\Gamma^{\mathrm{LO}}_{\pi^0\to\gamma\gamma} = 7.54(4)\,\mathrm{eV}$
  - Include SU(2) breaking

$$\Gamma^{\pi^0-\eta}_{\pi^0\to\gamma\gamma} = 7.93(12)\,\mathrm{eV}\ [\text{Ioffe, Oganesian (2007)}]$$

  - CHTP + $1/N_c$ expansion

$$\Gamma^{1/N_c}_{\pi^0\to\gamma\gamma} = 8.10(8)\,\mathrm{eV}\ [\text{Goity, Bernstein, Hostein, (2002)}]$$



K. de Jager (2008)

Lattice set-up

- Dynamical overlap fermion (Nf=2)
  - Exact chiral symmetry on the lattice
  - Dynamical configuration generated by JLQCD/TWQCD
  - $16^3 \times 32$, $1/a$=1.67 GeV, Iwasaki gauge action
  - m_q = 0.015, 0.025, 0.035, 0.050
  - Z_V =1.38 : non-perturbative value (RI-MOM)

$\pi^0 \to \gamma\gamma$ transition form factor on lattice

- Definition

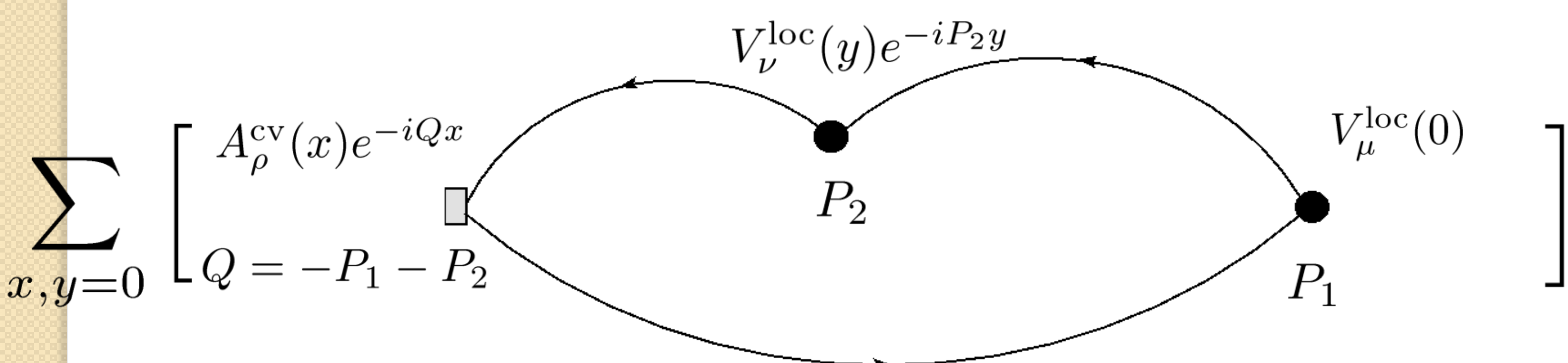
$$\int d^4x e^{ip_2 x} e^{ip_1 y} \langle \pi^0 | V_\nu^{\rm EM}(x) V_\mu^{\rm EM}(y) | 0 \rangle = \varepsilon_{\mu\nu\alpha\beta} p_1^\alpha p_2^\beta f_{\pi^0\gamma^*\gamma^*}(p_1, p_2)$$

<u>3-point function, V-V-A (Nf=2)</u>

$$\left\langle \nabla A^3 V_\nu^{\rm EM} V_\mu^{\rm EM} \right\rangle_c (P_1, P_2) = \sum_{x,y} \left\langle \nabla A^3(x) V_\nu^{\rm EM}(y) V_\mu^{\rm EM}(0) \right\rangle_c e^{-iQx} e^{-iP_2 y}$$

- Fourier transformed to 4D Euclidean momentum: $P^2 = -p^2$

  (fixed $P_2 = (0, 0, 0, 2\pi n/N_t)$)



$\pi^0 \to \gamma\gamma$ transition form factor on lattice

- Extraction of transition form factor
  - Using conserved axial current and local vector current
    → no contact term
  - In the low energy [~ $Q^2 \ll (10m_\pi)^2 \sim (m_{\pi'})^2$], where pion pole is dominant

$$\langle \nabla A V_\nu V_\mu \rangle (P_1, P_2) = \frac{f_\pi m_\pi^2}{Q^2 + m_\pi^2} \varepsilon_{\mu\nu\alpha\beta} P_1^\alpha P_2^\beta f_{\pi^0 \gamma^* \gamma^*}(P_1, P_2) + \cdots$$

$$\Rightarrow \langle \nabla A V_\nu V_\mu \rangle (P_1, P_2) / (\varepsilon_{\mu\nu\alpha\beta} P_{1\,\alpha} P_{2\,\beta}) = - \frac{f_\pi m_\pi^2}{Q^2 + m_\pi^2} f^{\mathrm{lat}}_{\pi^0 \gamma^* \gamma^*}(P_1, P_2)$$

- Momentum assignment

To avoid the lattice artifact due to Lorentz violation,

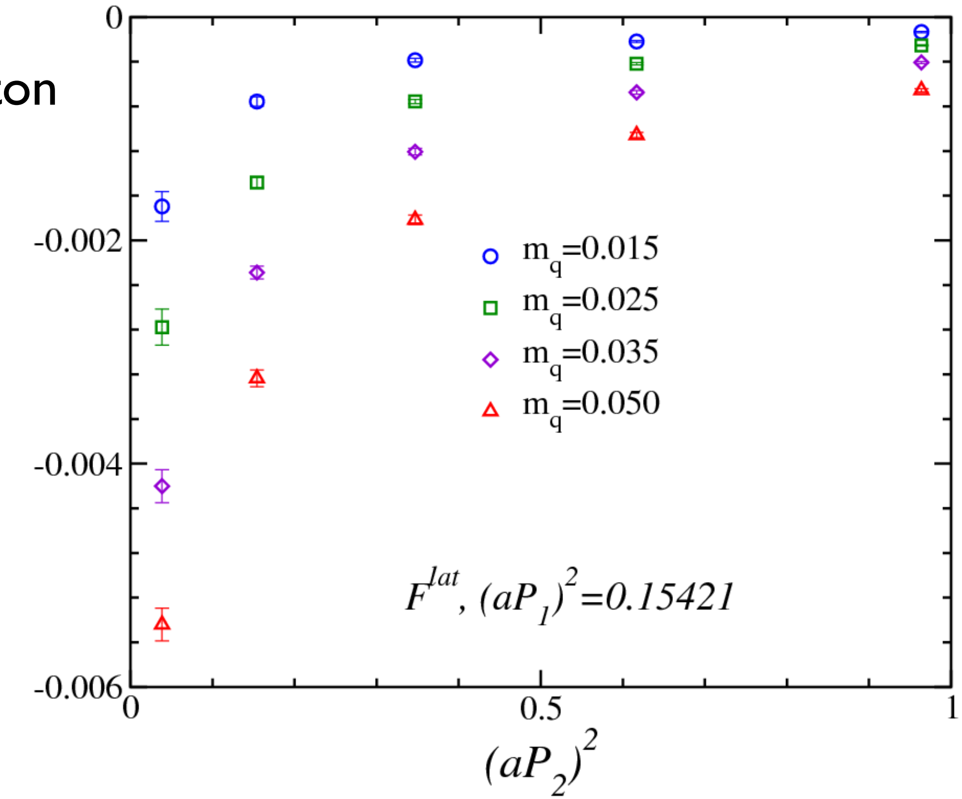
e.g. $\varepsilon_{\mu\nu\alpha\beta} P_{1\,\alpha} P_{2\,\beta} (P_{1\,\mu}^2 + P_{2\,\nu}^2) a^2$, we take a reduced momentum assignment:

$$P_1 = \left(0, 0, \frac{2\pi n_3}{L_3}, 0\right),\ P_2 = \left(0, 0, 0, \frac{2\pi n_4}{L_4}\right) \text{ at } \mu = 2,\ \nu = 1$$

$$P_1 = \left(0, \frac{2\pi n_2}{L_2}, 0, 0\right),\ P_2 = \left(0, 0, 0, \frac{2\pi n_4}{L_4}\right) \text{ at } \mu = 3,\ \nu = 1$$

Fit function to lattice results

- Based on VMD
  - Vector meson coupling to photon
  - $f_{\pi\gamma\gamma}$ represents vector meson mass and its coupling



- Based on CHPT
  - from WZW effective action
  - LECs and pion mass and decay constant
- CHPT combined with VMD
  - Low energy : chiral loops, High energy : tail of vector meson pole
  - LECs corresponds to vector mass and coupling

VMD fit ansatz

- VMD

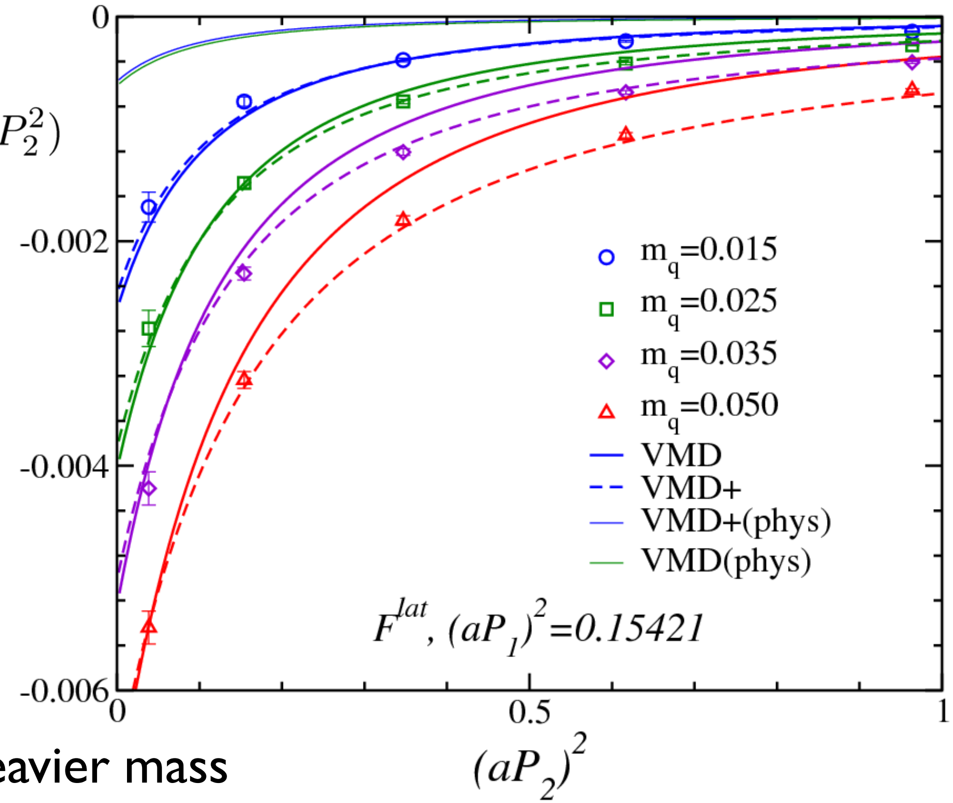
$$F^{VMD}(P_1, P_2) = -\frac{m_\pi^2 X_a}{Q^2 + m_\pi^2} G_v(P_1^2) G_v(P_2^2)$$

$$G_v(P^2) = \frac{m_v^2}{P^2 + m_v^2}$$

m_v is obtained from <VV>

$$X_a = f_{\pi^0\gamma\gamma}^{\rm lat}(0,0) = 0.02599(57)$$

cf. WZ model; $1/(4\pi^2)=0.025330\ldots$



- VMD + excited vector

Assumed excited state is much heavier mass

$$F^{VMD+}(P_1^2, P_2^2) = -\frac{m_\pi^2}{Q^2 + m_\pi^2} X_a \left[c_3 G_v(P_1^2) G_v(P_2^2) + \frac{c_4 - c_3}{2}\{G_v(P_1^2) + G_v(P_2^2)\} + 1 - c_4 \right]$$

$$X_a = 0.0243(29),\ c_4^0 = 1.19(21),\ c_3^m = -1.3(6.8),\ c_4^m = -8.4(8.6)$$

VMD well describe lattice data at the lowest momentum.

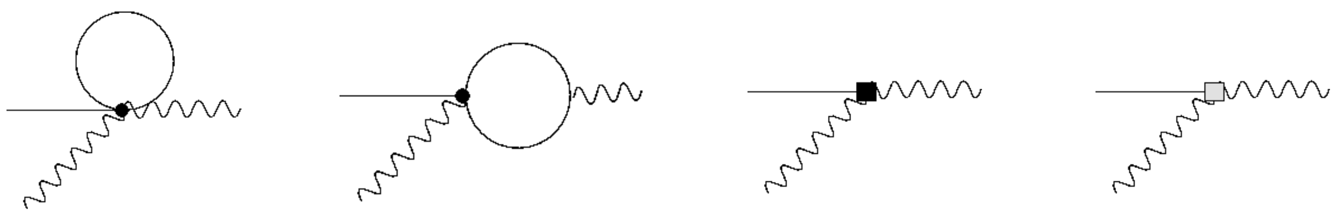
NLO in CHPT

- Chiral expansion at O(p^6)

Borasoy, Nissler (2004)

$$\mathcal{L}_{WZW}\epsilon^{\mu\nu\alpha\beta}d^4x = -\frac{\sqrt{2}}{8\pi^2 f_0^3}\Big(\mathrm{tr}\,\partial_\mu\pi[\pi,[\pi,v_\alpha\partial_\nu v_\beta]] - \mathrm{tr}\,\partial_\mu\pi[\pi,\partial\nu v_\alpha][\pi,v_\beta] - 2i\mathrm{tr}\,\partial_\mu\pi\partial_\nu\pi\partial_\alpha v_\beta + \cdots\Big)\epsilon^{\mu\nu\alpha\beta}d^4x$$

<u>Contribution to transition from factor</u>



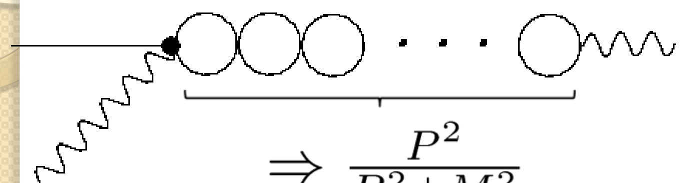
$$\Lambda_{\pi^0\gamma\gamma} = 4\pi^2 f_\pi f_{\pi^0\gamma\gamma}$$
$$= 1 + \frac{1}{f_\pi^2}\Big[-2\Delta_\pi + I(p_1^2,m_\pi^2) + I(p_2^2,m_\pi^2)\Big] - \frac{64\pi^2}{3}(p_1^2+p_2^2)w_1 - \frac{246\pi^2}{3}m_\pi^2 w_3$$

$$\Delta_\pi = \frac{m_\pi^2}{16\pi^2}\ln\frac{m_\pi^2}{\mu^2},\ I(p^2;m^2) = \frac{1}{3}\Delta_\pi + \frac{2}{3}\Big[\frac{-p^2}{64\pi^2}\sigma^2\Big\{-1+\ln\frac{m_\pi^2}{\mu^2}+2\sigma\tan^{-1}\frac{1}{\sigma}\Big\} - \frac{-p^2+6m^2}{96\pi^2}\Big]$$

Low energy constant: $w_{1,3}$ and we define $\sigma = \sqrt{1-4m^2/p^2}$

New fit ansatz - based on CHPT

- Re-summation of bubble diagram



$$= \frac{I}{f_\pi^2} + 2\frac{I}{f_\pi^2} + \left(2\frac{I}{f_\pi^2}\right)^2 + \cdots + [\text{counter term}]$$

$$\Rightarrow \frac{P^2}{P^2+M^2}$$

$$\to \Big[\frac{I(P^2;m^2)}{f_\pi^2} + \frac{64\pi^2}{3}P^2 w_1\Big]\Big[1 - 2\Big(\frac{I(P^2;m^2)}{f_\pi^2} + \frac{64\pi^2}{3}P^2 w_1\Big)\Big]^{-1}$$

then we have

$$\Lambda_{\pi^0\gamma^*\gamma^*} = 1 - \frac{2}{f_\pi^2}\Delta_\pi - \frac{256\pi^2}{3}m_\pi^2 w_3 + \frac{1}{2}\Big[\frac{P_1^2}{P_1^2 + M^2(P_1^2, m_\pi^2)} + \frac{P_2^2}{P_2^2 + M^2(P_2^2, m_\pi^2)}\Big],$$

$$M(P^2, m^2) = \Big[-\frac{I(P^2, m^2)}{2P^2 f_\pi^2} - \frac{32\pi^2}{3}w_1\Big]^{-1/2}$$

Last term is same form as VMD ansatz

New fit ansatz - based on CHPT

- Fitting at

$(aP_1^2) = 0.154$ (lowest)

$(aP_2^2) = 0.038,\ 0.154$

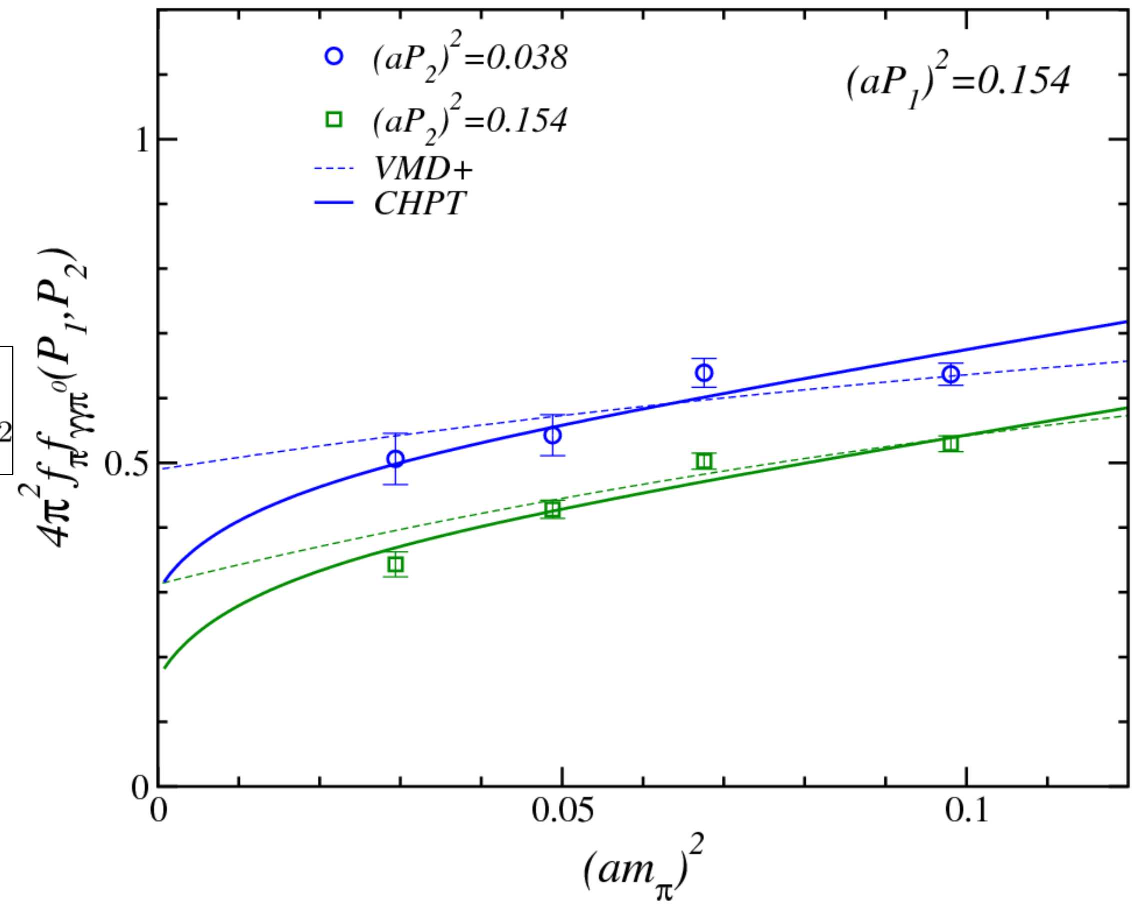
and four quark masses

LECs

$$w_1^{\chi PT} = -0.0271(37)\,\mathrm{GeV}^{-2}$$
$$w_3^{\chi PT} = -0.00120(12)\,\mathrm{GeV}^{-2}$$

Cf. Phenomenology (VMD):

$$w_1^{\mathrm{ph}} = -0.0055\,\mathrm{GeV}^{-2}$$
$$w_3^{\mathrm{ph}} = 0.00131\,\mathrm{GeV}^{-2}$$



New fit ansatz - combined with VMD

- One-loop CHPT + VMD

$$\Lambda_{\pi^0\gamma^*\gamma^*} = 1 - \frac{2}{f_\pi^2}\Delta_\pi - \frac{256\pi^2}{3}m_\pi^2 w_3 + \frac{1}{f_\pi^2}\Big[I(P_1^2, m_\pi^2) + I(P_2^2, m_\pi^2)\Big]$$
$$+ \frac{1}{2}\left(\frac{P_1^2}{m_v^2 + P_1^2} + \frac{P_2^2}{m_v^2 + P_2^2}\right)\frac{z_1}{m_v^2} + \frac{P_1^2}{m_v^2 + P_1^2}\frac{P_2^2}{m_v^2 + P_2^2}z_2$$

w_3 : LECs at $O(p^6)$, $z_{1,2}$:VMD coupling

New fit ansatz - combined with VMD

- Fit results

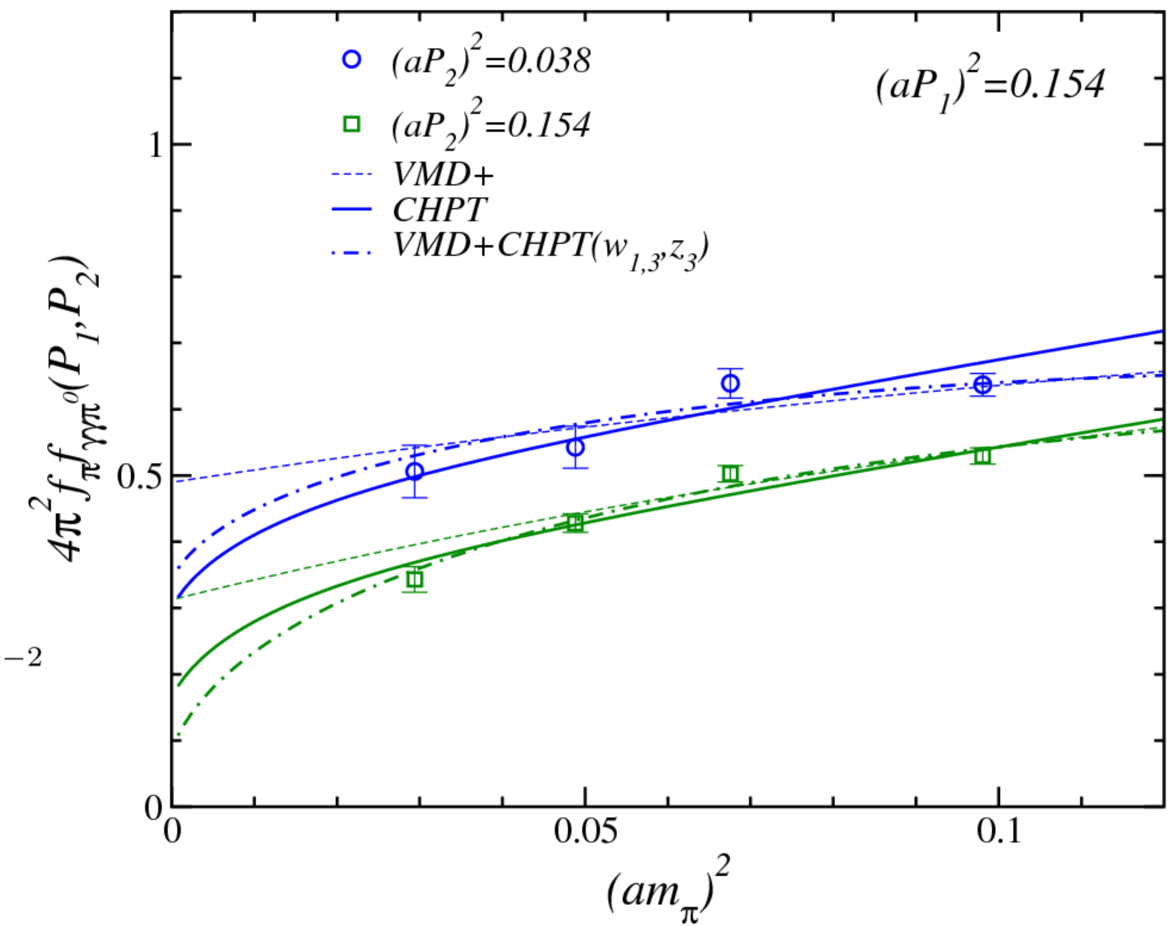
$$w_3 = 0.00048(27)\,\mathrm{GeV}^{-2}$$
$$z_1 = -0.0039(9)\,\mathrm{GeV}^{-2}$$
$$z_2 = 0.58(70)$$

LECs values are similar to phenomenology,

$$w_3^{\mathrm{ph}} = 0.00131\,\mathrm{GeV}^{-2}$$

but different from re-summed CHPT:

$$w_3^{\chi PT} = -0.00120(12)\,\mathrm{GeV}^{-2}$$

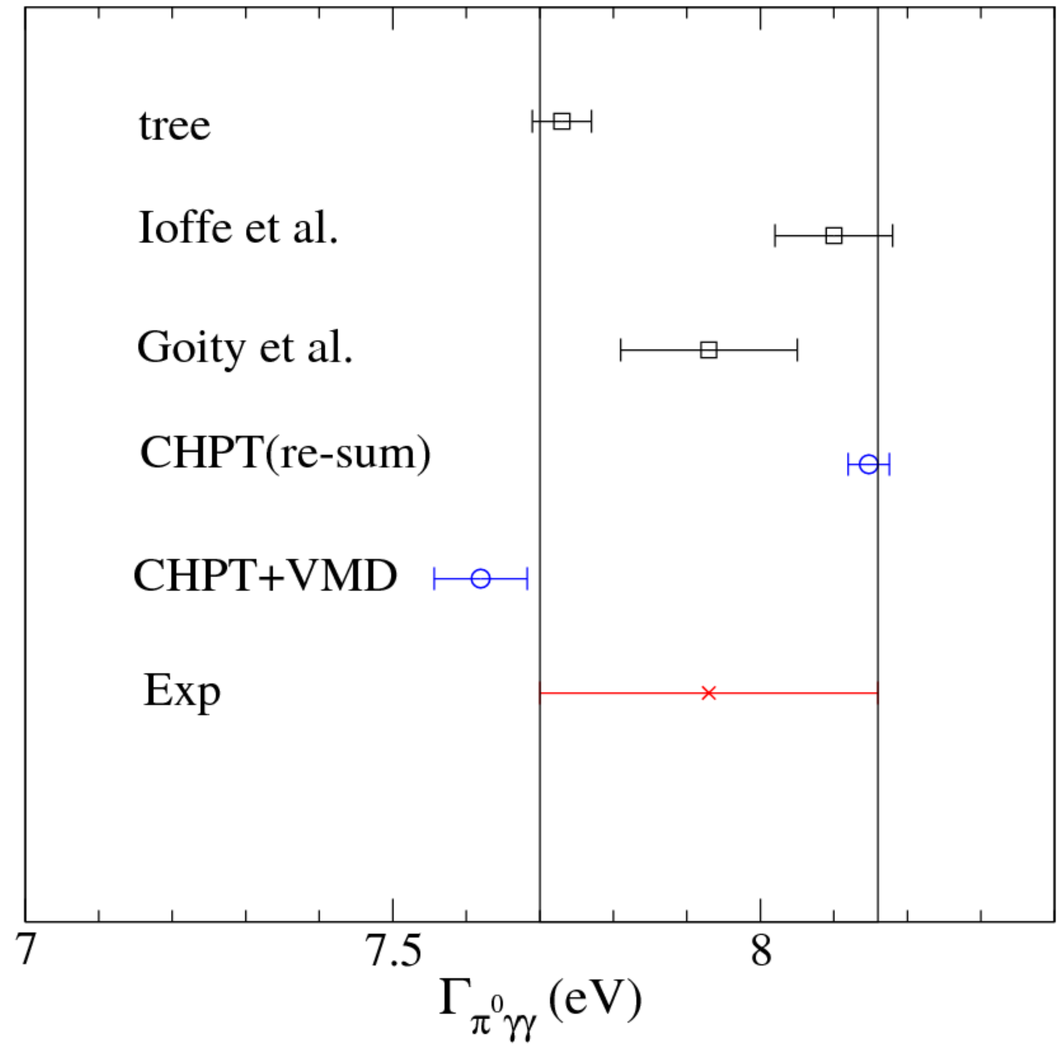


Decay width

$$\Gamma_{\pi^0\gamma\gamma} = \frac{m_\pi^3 \alpha_e^2}{64\pi^3 f_\pi^2} \Lambda^2_{\pi^0\gamma\gamma}$$

$= 8.15(3)\,\mathrm{eV}\ :\ \mathrm{CHPT}$

$= 7.62(6)\,\mathrm{eV}\ :\ \mathrm{CHPT+VMD}$



Summary

- CHPT analysis has been done with π^0 decay to 2 photon
- Both re-summed CHPT and CHPT + VMD fit function well describes lattice data
- Decay width of π^0

$$\Gamma_{\pi^0\gamma\gamma} = 7.89(3)(\pm 37)\,\mathrm{eV}$$

which is consistent with experiment, but remains large fit uncertainties in our fitting region

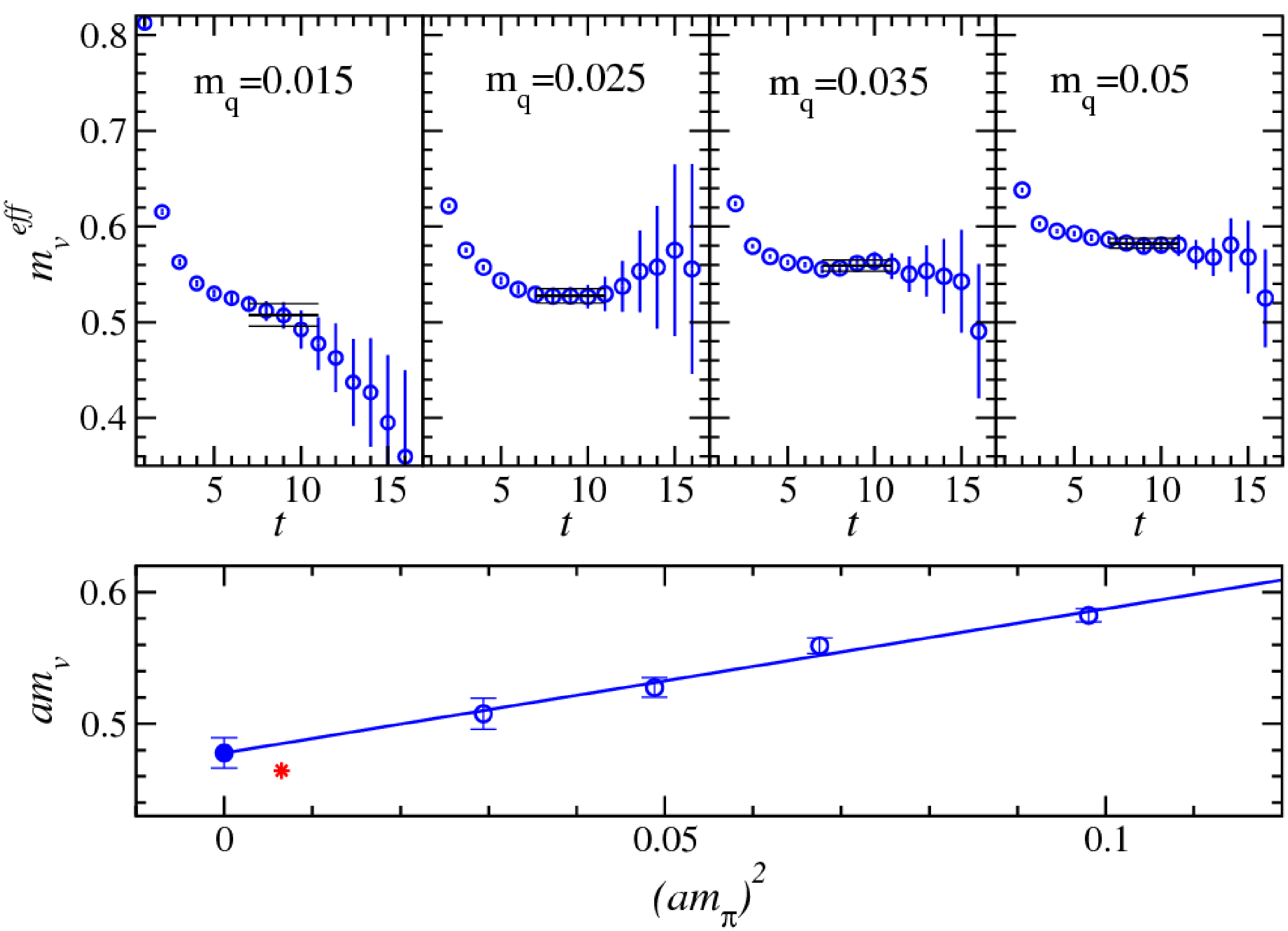
- Future
  - To reduce sys. error, need small momentum → large lattice, $24^3 \times 48$
  - extend to higher momentum as 3 or 4 GeV^2

→ cross check of CLEO, BaBar result

→ provides suggestive information for NLO hadronic contribution to muon g-2

Backup

Vector meson mass



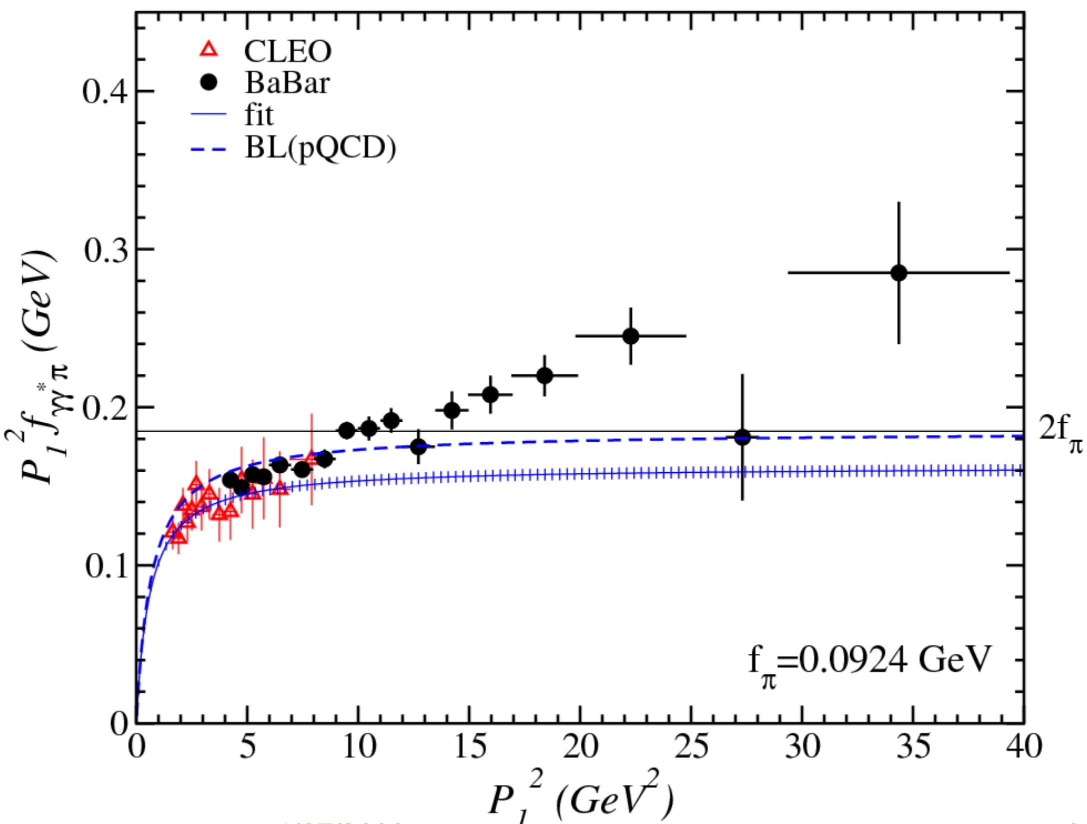
Comparison with experiment (preliminary)

- $\gamma\gamma^* \to \pi^0$ transition form factor
  - Perturbative QCD prediction:

Brodsky, Lepage (1980)

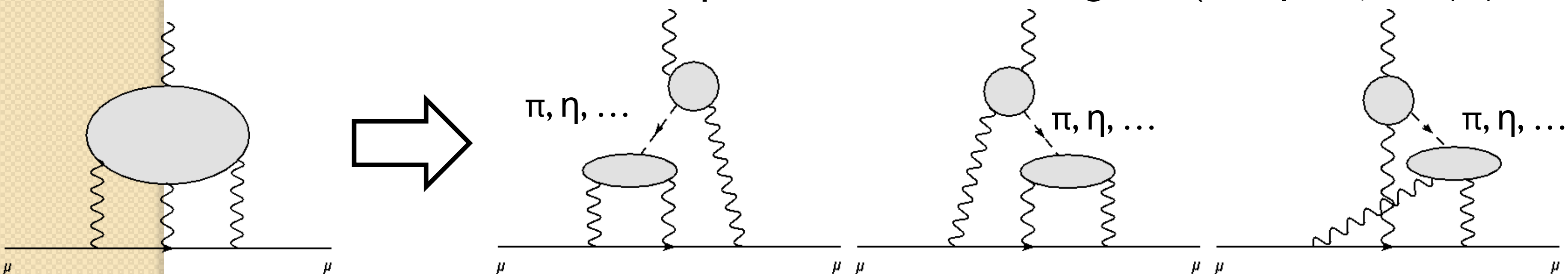
$$f_{\pi^0\gamma\gamma^*}(0,Q^2) = \frac{1}{4\pi^2 f_\pi}\frac{1}{1+Q^2/(8\pi^2 f_\pi^2)} \to \frac{2f_\pi}{Q^2}\,(Q^2 \gg \Lambda^2)$$

- pQCD:
  division from BaBar
  at $p_1^2 > 15\,\mathrm{GeV}^2$
- Our fit result is consistent with both CLEO and BaBar, but still large error.



Hadronic contribution: NLO

- Hadronic light-by-light scattering diagram (3 off-shell, 1 on-shell)
  - Direct calculation: hard task !
  - Indirect calculation: decomposition into two diagram $(\pi^0, \eta^0, \ldots) \rightarrow \gamma^*\gamma^*$



  - Pion pole dominance model

Focus on pion decay, other meson decay is sub-leading:

M. Knecht, A. Nyffeler (2002)

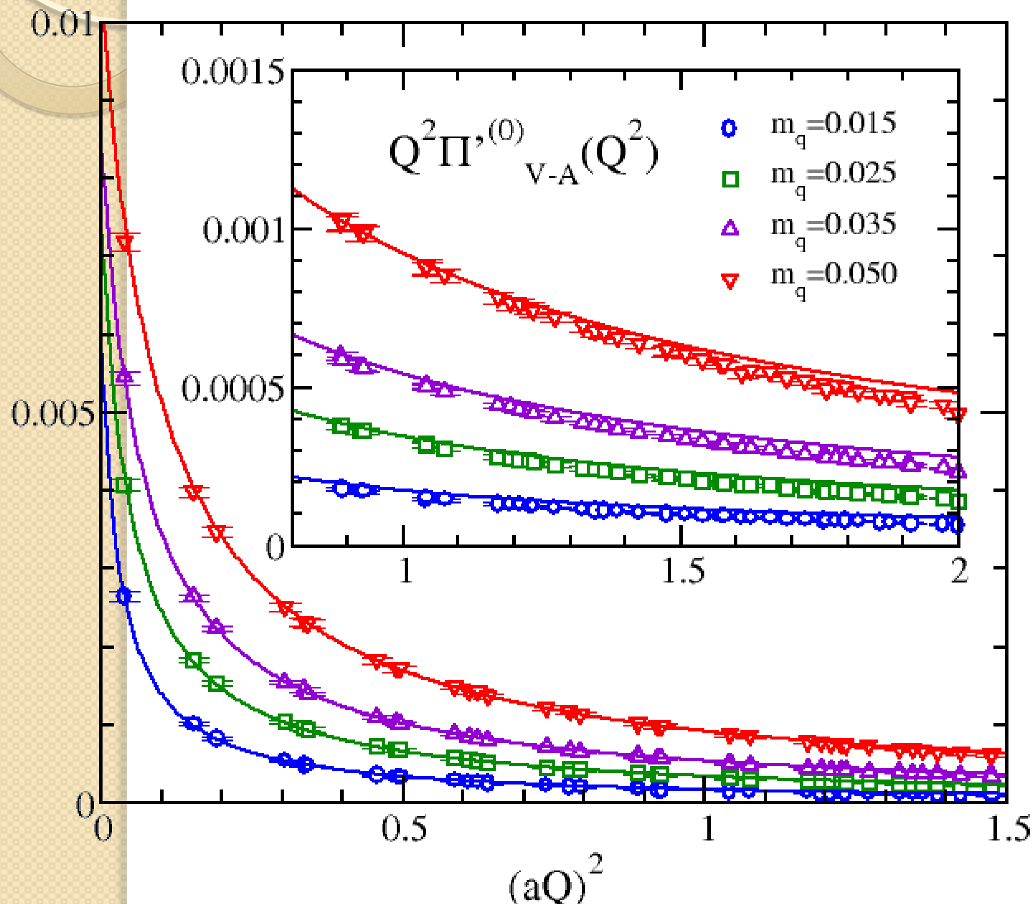
$$a_\mu^{\rm LbyL} = \int_{p_1}\int_{p_2} [p_1^2 p_2^2 (p_1+p_2)^2]^{-1} [((p+p_1)^2 - m_\mu^2)((p-p_2)^2 - m_\mu^2]^{-1} \Pi_{\pi^0\gamma^*\gamma^*}(p_1,p_2,p)$$

$$\Pi_{\pi^0\gamma^*\gamma^*}(p_1,p_2,p) \sim \left\{ \frac{f_{\pi^0\gamma^*\gamma^*}(q_1^2,(q_1+q_2)^2) f_{\pi^0\gamma^*\gamma}(q_2^2,0)}{p_2^2 - m_\pi^2}, \frac{f_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) f_{\pi^0\gamma^*\gamma}((q_1+q_2)^2,0)}{(p_1+p_2)^2 - m_\pi^2} \right\}$$

Since we have assumed some resonance models (e.g. VMD) in LbyL diagram, then non-perturbative form plays important role.

Comparison with CHPT

- $\Pi_{V\text{-}A}^{(0)}$



CHPT prediction:

$$Q^2\Pi^{(0)}_{V-A}(Q^2) = \frac{f_\pi^2 m_\pi^2}{Q^2 + m_\pi^2}$$

well describe lattice data

⇩

other lattice artifact including $\Pi_{V\text{-}A}$ term is small ($(aQ)^2 < 2$)