Study of the scaling properties in SU(2) gauge theory with eight flavors

Hiroshi Ohki (Osaka University)

@Lattice 2010

Collaborators

- Tatsumi Aoyama [a]
- Etsuko Itou [b]
- Masafumi Kurachi [c]
- C. –J. David Lin [d]
- Hideo Matsufuru [e]
- Tetsuya Onogi [g]
- Eigo Shintani [f]
- Takeshi Yamazaki [g]

Numerical simulation was carried out on the vector supercomputer

NEC SX-8 in YITP, Kyoto University

and RCNP, Osaka University

SR and BlueGene in KEK

a : Nagoya University

b: Osaka University

c : Tohoku University

d : National Chio-Tung University, and National Center for Theoretical Science

e: KEK

f: RIKEN

g: Tsukuba University

Introduction

- **We don't know much about the physics of the conformal gauge theory.**

Theoretical interest

Phase structure of the non-SUSY gauge theories

(gauge group, representation, # of flavor)

Phenomenology

Test of the models of physics beyond the SM

(ex. Walking technicolor model)

-> SU(2) gauge theory is very important.

Outline

- **Introduction**
- **Method for the running coupling**
- **Quenched test**
- **Nf=8 flavor study**
- **Summary**

Our work

- **We use the methods for extracting the gauge coupling with twisted boundary conditions (alternative method without O(a) error) .**

- **We study the scaling properties of SU(2) with 8 flavor gauge theory.**

The 2-loop perturbative results suggest that the SU(2) with 8 flavor theory is near the conformal window.

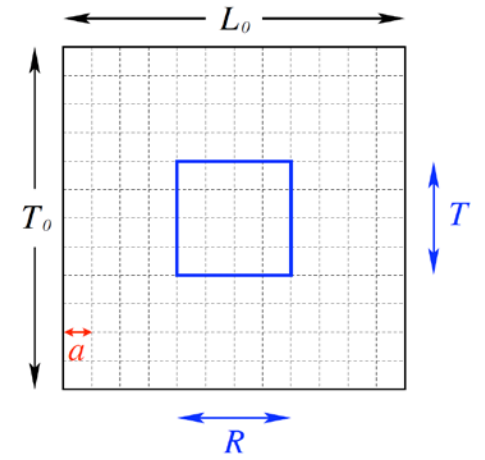
Therefore this model may be a candidate for walking technicolor scenario.

(twisted) Wilson loop scheme (1)

[Bilgici et. al. Phys.Rev.D80:034507,2009]

1. Definition of the gauge coupling in finite volume

$$k g_w^2 \equiv k g_0^2 + \mathcal{O}(g_0^4) = -R^2 \frac{\partial^2}{\partial R \partial T} \log \langle W(R,T) \rangle |_{T=R}$$



In lattice we use the Creutz ratio

$$\chi(R+1/2, L/a) = -\log \left(\frac{W(R+1,T+1;L/a)W(R,T;L/a)}{W(R+1,T;L/a)W(R,T+1,L/a)} \right) \Big|_{T=R}$$

$$g_w^2 \left(L, r, \frac{a}{L} \right) = r^2 \chi(r; L/a)/k$$

$$r \equiv (R + a/2)/L$$

where k is the tree level matching factor.

2. We employ twisted boundary condition

- Eliminate degenerate Z_N vacua
- Kill the zero mode contributions
- No need of boundary term

$$A_\mu(x + L\hat{\mu}) = \Omega_\nu A_\mu(x) \Omega_\nu^\dagger$$

$$\Omega_0 = \Omega_3 = 1 \qquad \Omega_1 \Omega_2 = e^{2\pi i/N} \Omega_2 \Omega_1$$

$$\Omega_\mu^N = 1 \qquad \Omega_\mu \Omega_\mu^\dagger = 1$$

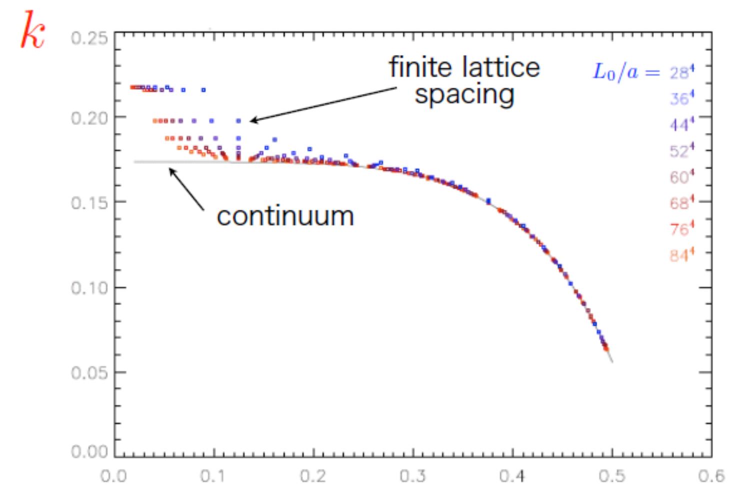
We measure the Wilson loop of untwisted-untwisted direction.

(twisted) Wilson loop scheme (2)

3. **Improvement by using discretized** $k(r, L/a)$

$$g_w^2\left(L, r, \frac{a}{L}\right) = r^2\chi(r; L/a)/k(r, L/a)$$

present work -> finite lattice k
(previous work -> continuum k)



- **Cancellation of the huge discretization error between the finite lattice calculation of the Creutz ratio and k(r).**
- **g^2(r) becomes smooth function of r.**

k(r) has large discrepancy between discrete and continuum one.

4. **Fixing the scheme**

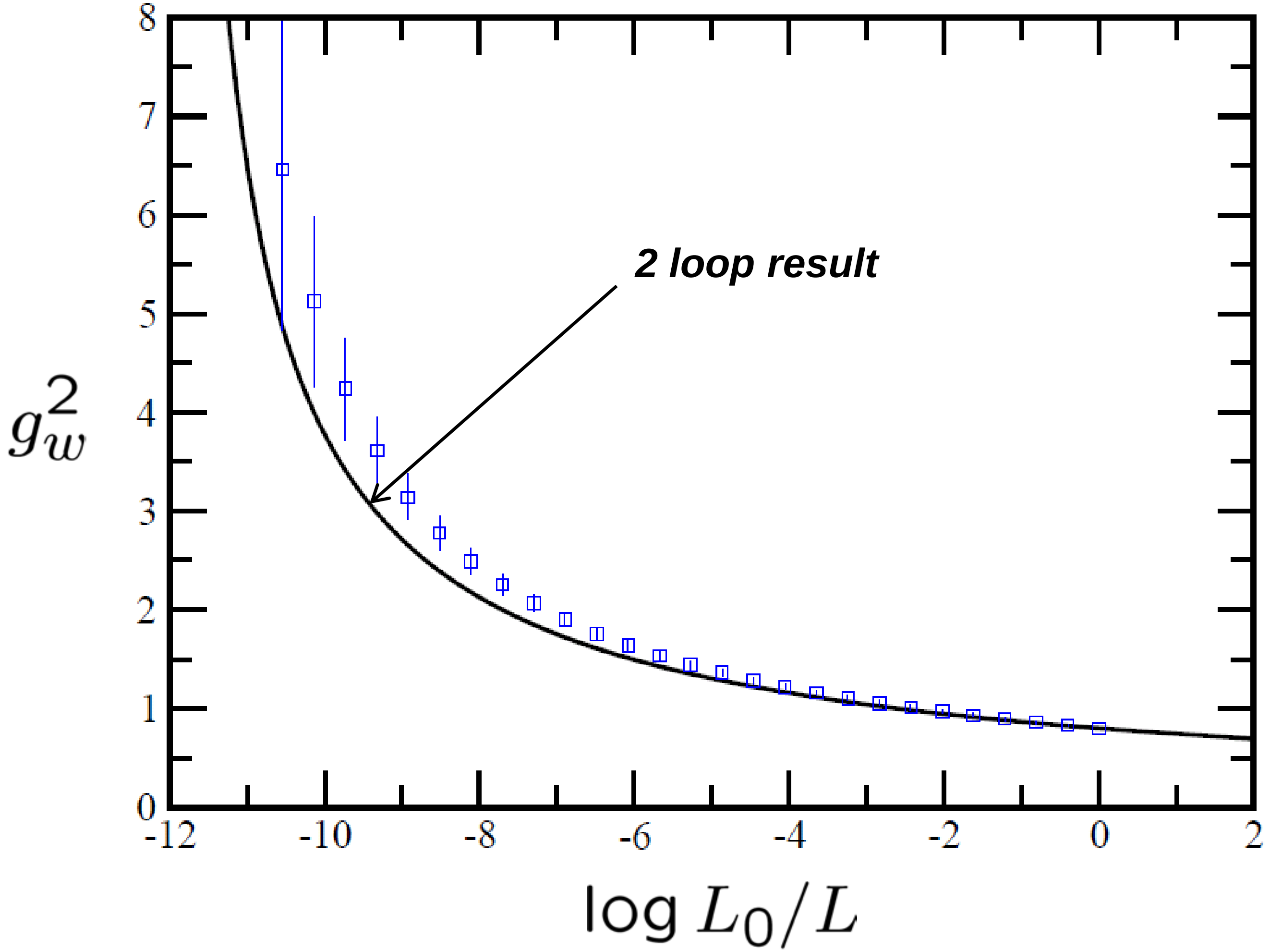
We take r= 0.25, (R=T)

5. **Running coupling by finite scaling method**

-> Quenched test is succeeded.

Pure SU(2) results

Good agreement with perturbative 2 loop result



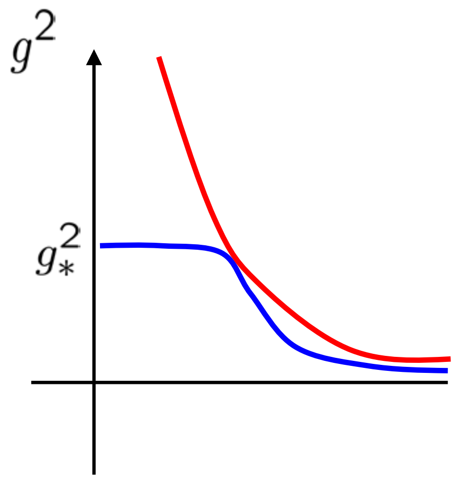
8 flavor SU(2) case

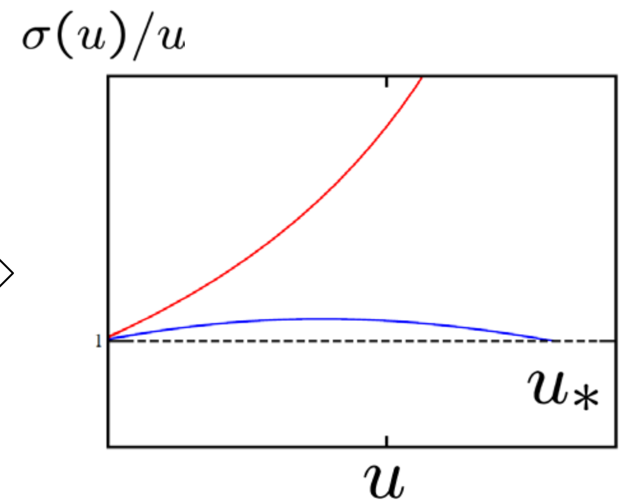
Theoretical expectation of the conformal fixed point

- study the scaling properties of SU(2) with 8 flavor gauge theory.
- We use following functions

the relative step scaling function $\sigma(u)/u$

$u = g_w^2(L) \qquad \sigma(u) = g_w^2(sL)$ **S=1.5**





2 loop perturbative prediction

- In the SU(2) with 8flavors theory, it has a IR fixed point u*~ 15.8.
- Then a inflection point is around u~7.9.

We explore such a behavior beyond perturbation.

simulation setup

- **Wilson gauge action**
- **Staggerd fermion with twisted boundary condition**

Introduce "smell" $\rightarrow \psi^a_\alpha(x)$: $N_c \times N_s (= N_c)$ matrix

Parisi, 1983(Unpublished)

$$\psi^a_\alpha(x + \hat{\nu}L/a) \;=\; e^{i\pi/3}\Omega^{ab}_\nu \psi^b_\beta(x)(\Omega^\dagger_\nu)_{\beta\alpha} \quad (\nu = 1, 2)$$

Smell degree is extra flavor degree

Staggered fermion requires $N_f = 4 \times N_s$

- **HMC (Omelyan integrator)**
- **Every sweep measurement of Wilson loop.**
- **Numerical calculation by NEC SX-8@YITP, @RCNP**
  KEKSR-11000@KEK
- **Simulation parameters**
  L =6,8,10,12,14,16,18, Beta= 1.375 ~ 20

  #config 10000~50000 for L=6~12,
  2000~ 6000 for L=14, 16, 18

analysis step

1. **Calculation of the gauge coupling by Wilson loop of each L, beta.**
   -> global fit of g^2 as a function of beta for each L

2. **Interpolation to odd lattice for step scaling**

3. **Step scaling by continuum extrapolation**

$$u = g_w^2(L)$$

$$\sigma(u) = g_w^2(sL) = \lim_{a \to 0} g_w^2(\beta, \frac{a}{sL})$$

4. **Studying the running of the gauge coupling we use the relative step scaling function**

$$\sigma(u)/u$$

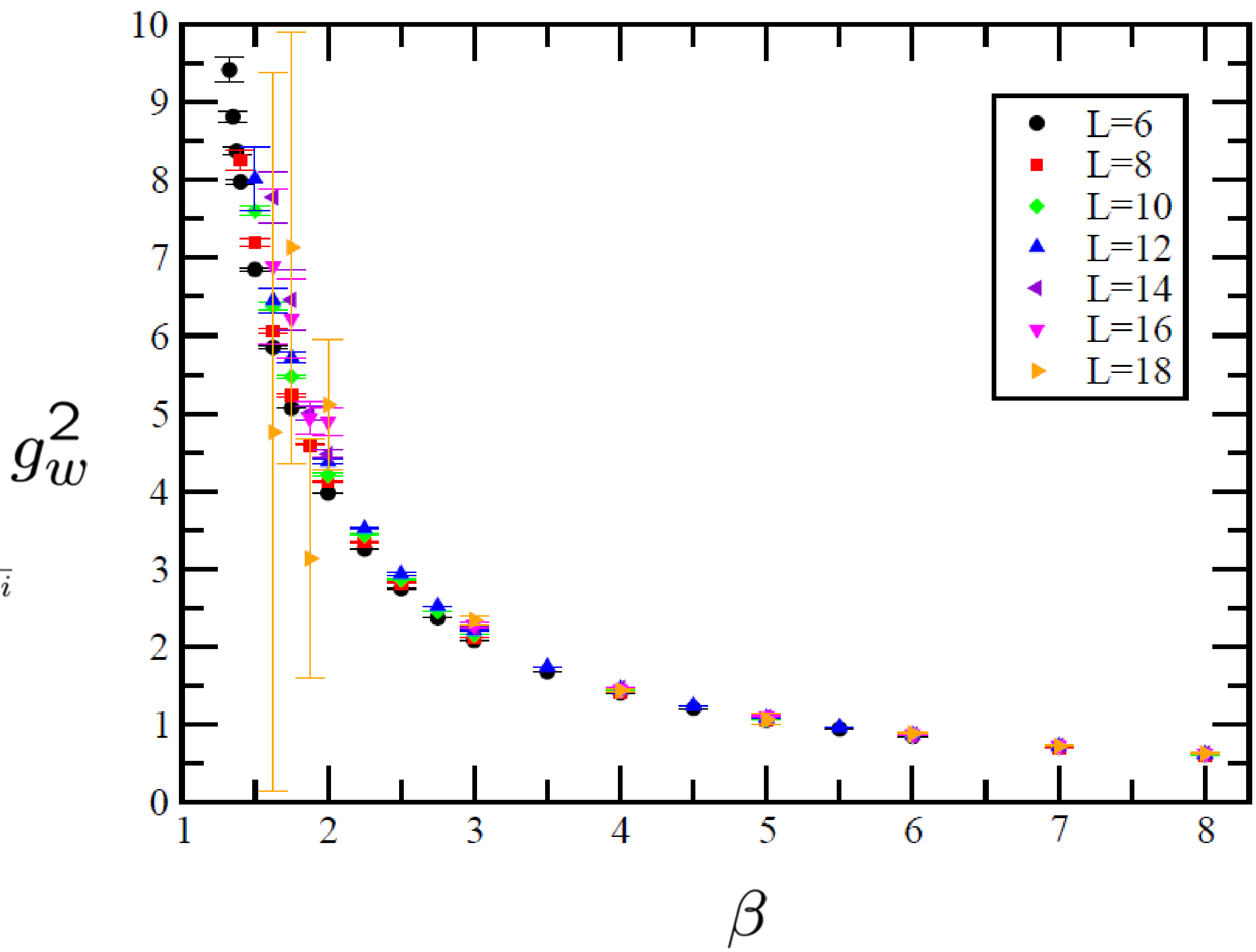
results

stage 1. Plot of the coupling for L=6~18, beta=1.375~15

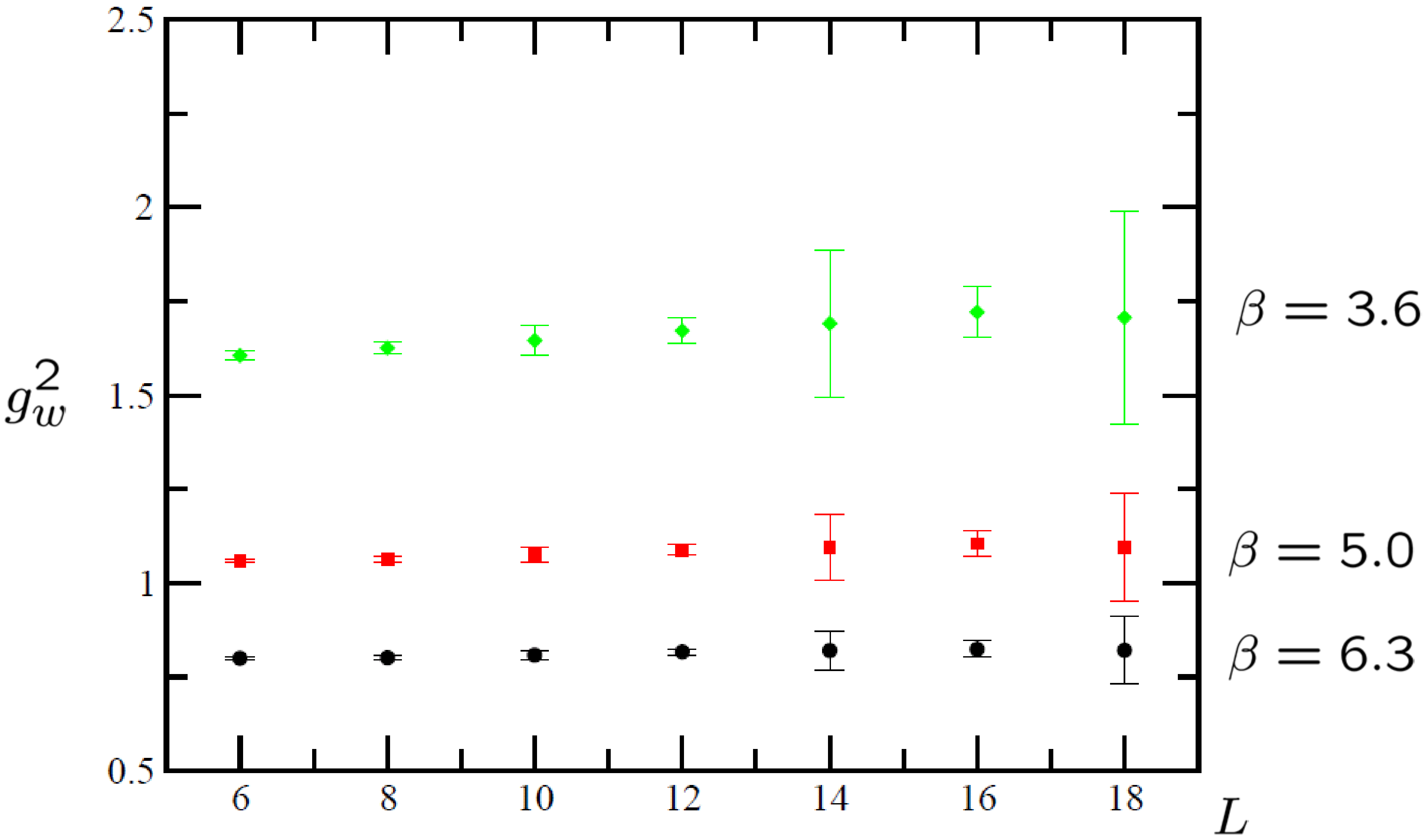
Interpolation function

$$g_w^2(\beta) = \sum_{i=1}^{n} \frac{A_i}{(\beta - B)^i}$$

n=3,4



stage2, L dependence of the fit result



Smooth volume dependence

possible to interpolate to odd number by linear fit functions

L/a=9 -> linear interpolation using L/a=6,8,10,12

L/a=15 -> linear interpolation using L/a=12,14,16,18

stage3, Step Scaling

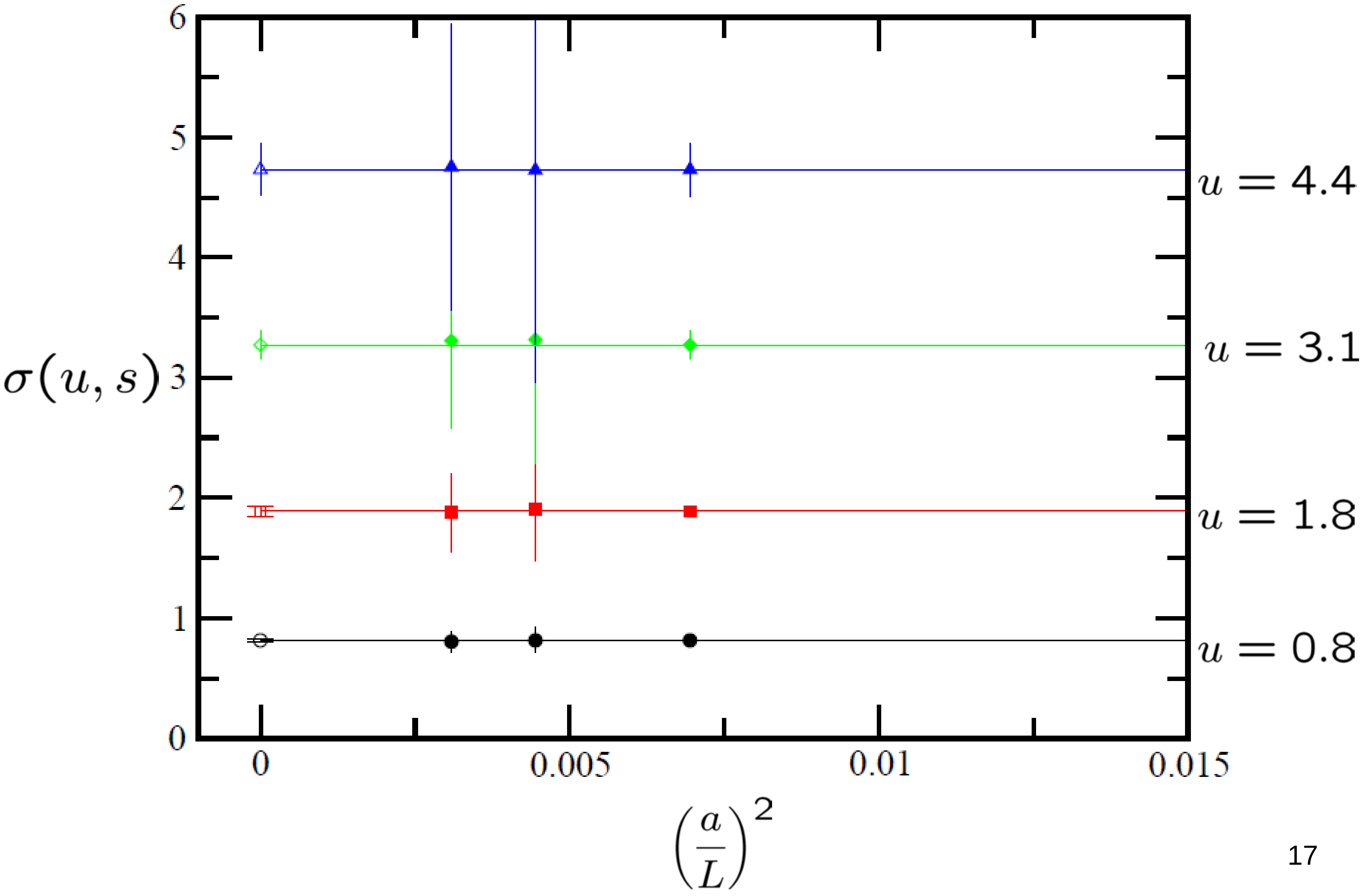
Running coupling : finite step scaling method

Scaling step s=1.5

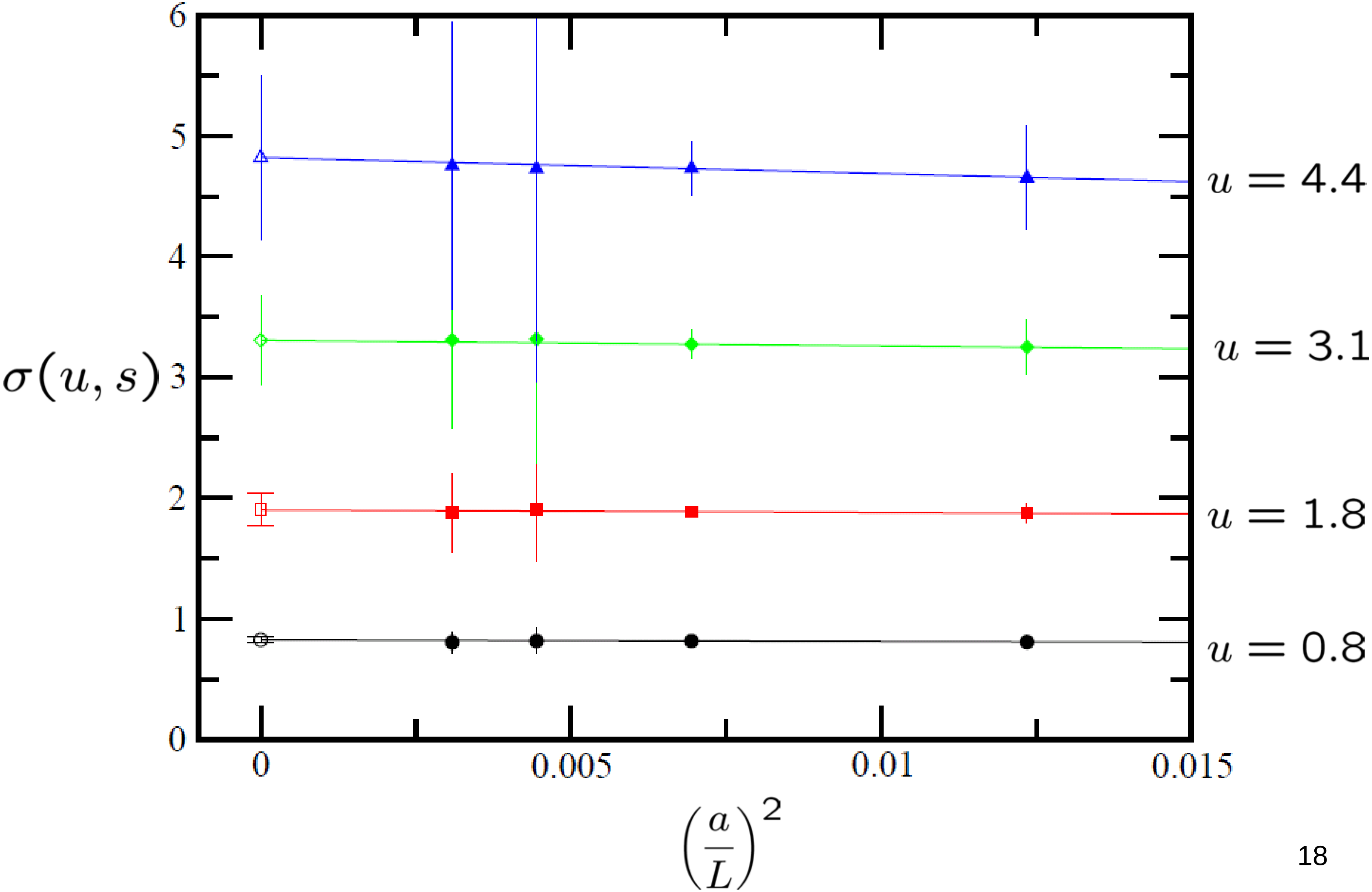
We use constant and linear of (a/L)^2 continuum extrapolation for the estimation of the systematic uncertainty of discretization error.

Constant extrap. (3pt) L/a= 8, 10, 12 -> sL/a= 12, 15, 18
Linear extrap. (4pt) L/a=6, 8, 10, 12 -> sL/a=9, 12, 15, 18

Constant Continuum limit (3pt.)

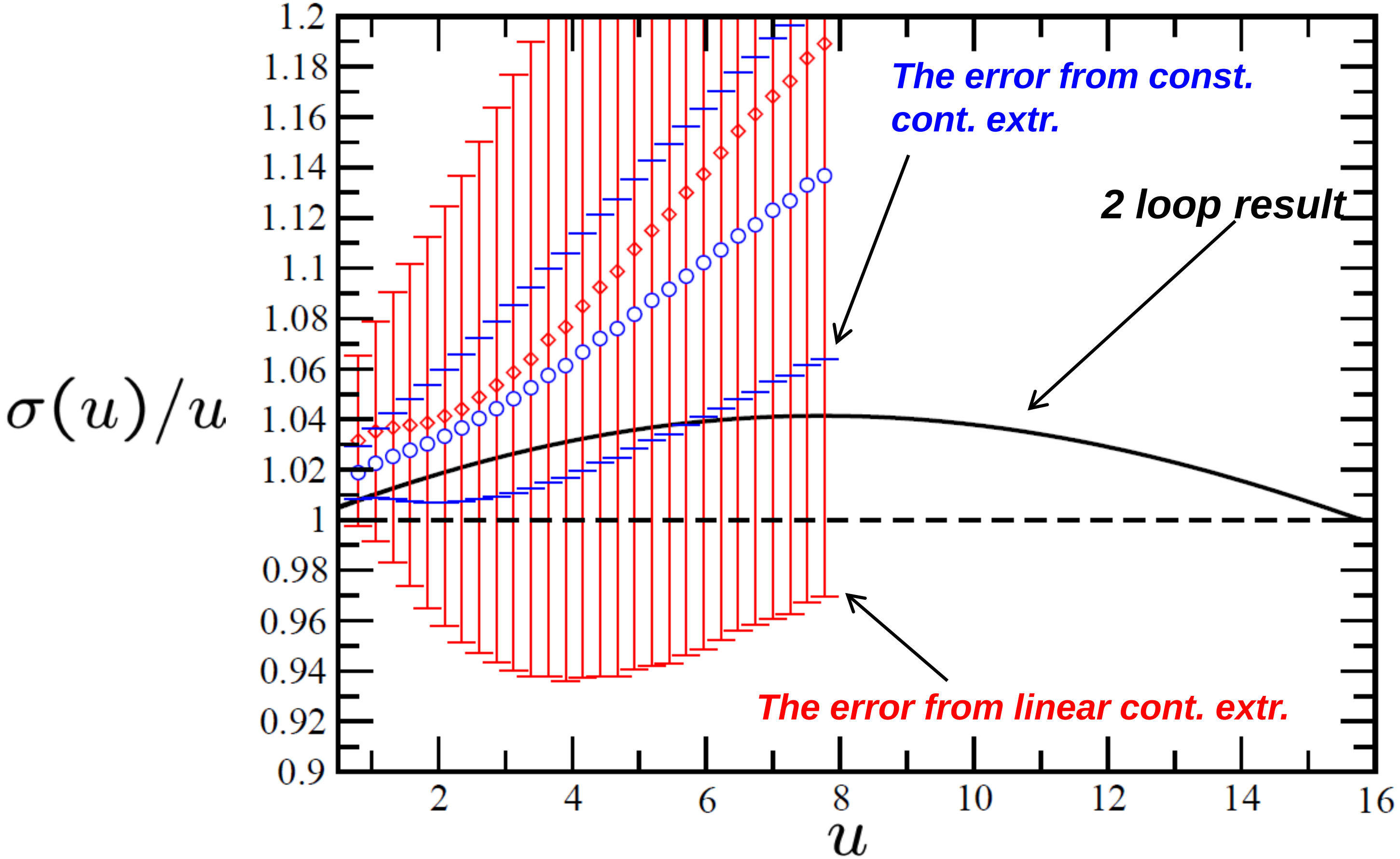


linear Continuum limit (4pt.)



Stage4. The relative step scaling function $\sigma(u)/u$

$u = g_w^2(L) \qquad \sigma(u) = g_w^2(sL)$ **S=1.5**



The error is only statistical.
Large systematic uncertainty from the continuum extrapolation.
Constant extr. -> $\sigma(u)/u > 1 \quad (0 < u < 8)$
Linear extr. has large error, more statistics are needed.

Summary

- **We have calculated the running coupling in SU(2) 8 flavors**
- **All the results are very preliminary.**
- **There is no signal of the IFTP up to g^2~8.**
- **We need more statistics for larger lattice and have to take continuum limit carefully.**

Future prospects

- **We survey the running of the coupling in the large coupling region**
- **Measurement of the anomalous dimension at the IR point (universality check)**
- **Other theory is possible to simulate in this method (adjoint representaion)**

End