

# Finite temperature QCD at fixed $Q$ with overlap fermions

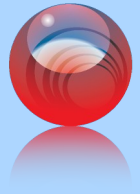
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In collaboration with JLQCD members

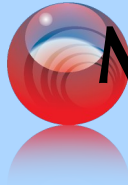
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# Outline

- ✓ Motivation
- ✓ Simulating overlap fermions (Fukaya term)
- ✓ Fixing topology: pure gauge simulations
- ✓ Eigenvalues distribution and correlators
- ✓ Preliminary results on Full QCD
- ✓ Estimate of the  $\eta'$  mass (Very preliminary)
- ✓ Outlook



# Motivation


In the last years JLQCD group published several works on simulations with dynamical overlap fermions at zero temperature around the chiral point.

We decided to start a similar project on finite temperature simulations.

This is the first report of our work in progress.

Since the algorithm fixes the topological sector (next slides) we started with exploratory simulations of pure gauge theory.

I will present some preliminary results on the topological susceptibility at the transition and also the first data on full QCD with two flavors of degenerate fermions.



# Simulations with overlap fermions - I

Overlap operator has a discontinuity along the boundary between different topological sectors.


This implies a large numerical cost in changing topology.

In order to avoid this, the JLQCD group introduced a topology fixing term (aka Fukaya term) with extra Wilson fermions:

$$\det \left[ \frac{H_W(m_0)^2}{H_W(m_0)^2 + \mu^2} \right]$$

With this action the exact zero mode of the Hermitian Wilson operator cannot appear and near zero modes are strongly suppressed.

$\mu$  is a parameter that must be tuned by simulations (see papers by JLQCD collaboration)



## Simulations with overlap fermions - II

Physical quantities like the topological susceptibility can be extracted by using formulas that express deviation of the fixed  $Q$  simulation from the fixed  $\theta$  case.

See reference: *Aoki et al Phys. Rev. D 76, 054508(2007)*

This method was used, for example, to measure the behavior of the topological susceptibility at zero temperature in the chiral limit by measuring the constant term of a time-correlation of the two point function of a singlet pseudoscalar meson.

Ref: *JLQCD & TWQCD collaborations, Phys. Lett. B 665 (2008) 294*

We applied the same methods at finite temperature.



# Pure gauge simulations - I

In order to investigate numerically the validity of the zero temperature methods even at finite temperature we simulated pure gauge theories plus the Fukaya term.

Pure gauge theories are very well studied so we can check possible discrepancies with our simulations at fixed topological charge.

We simulated SU(3) (Iwasaki action) + Topology fixing term around the phase transition which was estimated by using the Polyakov Loop behavior.

## *Tech. Details:*

2 volumes  $16^3 \times 6$ ,  $24^3 \times 6$

Several betas in range  $[2.40-2.55]$  corresponding to  $[(0.93-1.20) * T_c]$

Transition point at  $\beta \sim 2.445$

Sectors  $Q=0,2$

# Pure gauge simulations - II

By calculating the first 50 eigenvalues of the overlap operator  $D_{\text{ov}} + m$  and assuming low mode dominance, we estimated the correlator:

$$C(t) = \sum_{\vec{x}} m_q^2 \langle P^0(x) P^0(0) \rangle$$

where  $P^0(x) = \bar{\psi}(x) \gamma_5 \psi(x)$

Disconnected contributions are averaged over source points.

The constant behavior of the correlator is related to the topological susceptibility by the formula:

$$A = \frac{1}{m_q^2 N_t} \left( \frac{Q_t^2}{\Omega} - \chi_t \right)$$

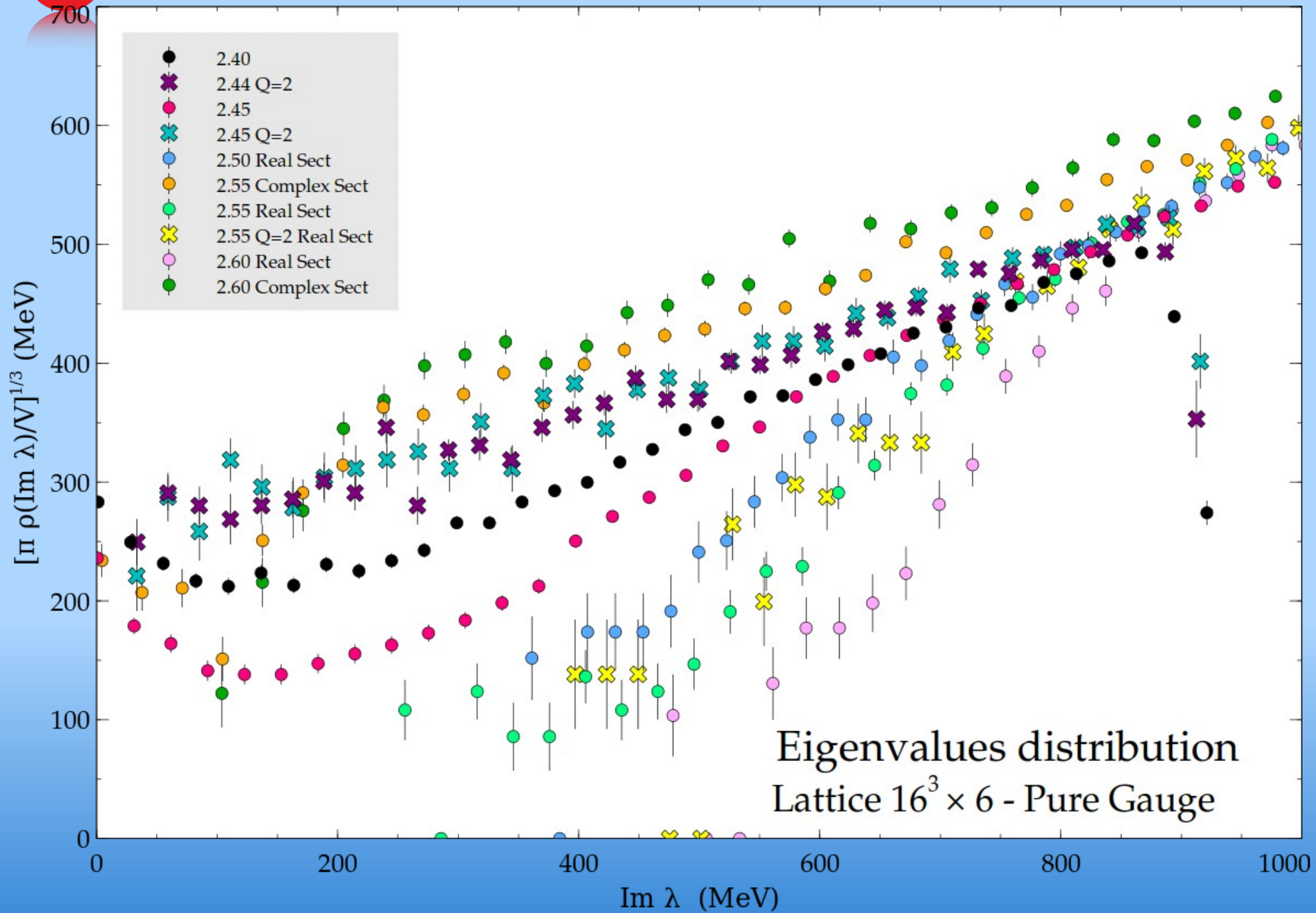
Plus terms that we assume to be small at this stage.

Fit function:  $A + B \cdot (e^{-Ct} + e^{-C(N_t-t)})$

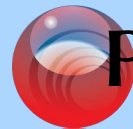
Decay of this correlator in the full QCD case gives the  $\eta'$  mass  
In pure gauge is just a double pole.



# Pure gauge simulations - Eigenvalues







# Pure gauge simulations - Eigenvalues

The distribution of eigenvalues is in perfect agreement with results in literature, calculated without fixing topology.

Even the presence of small eigenvalues accumulating above the transition region is found at  $Q=0$ .

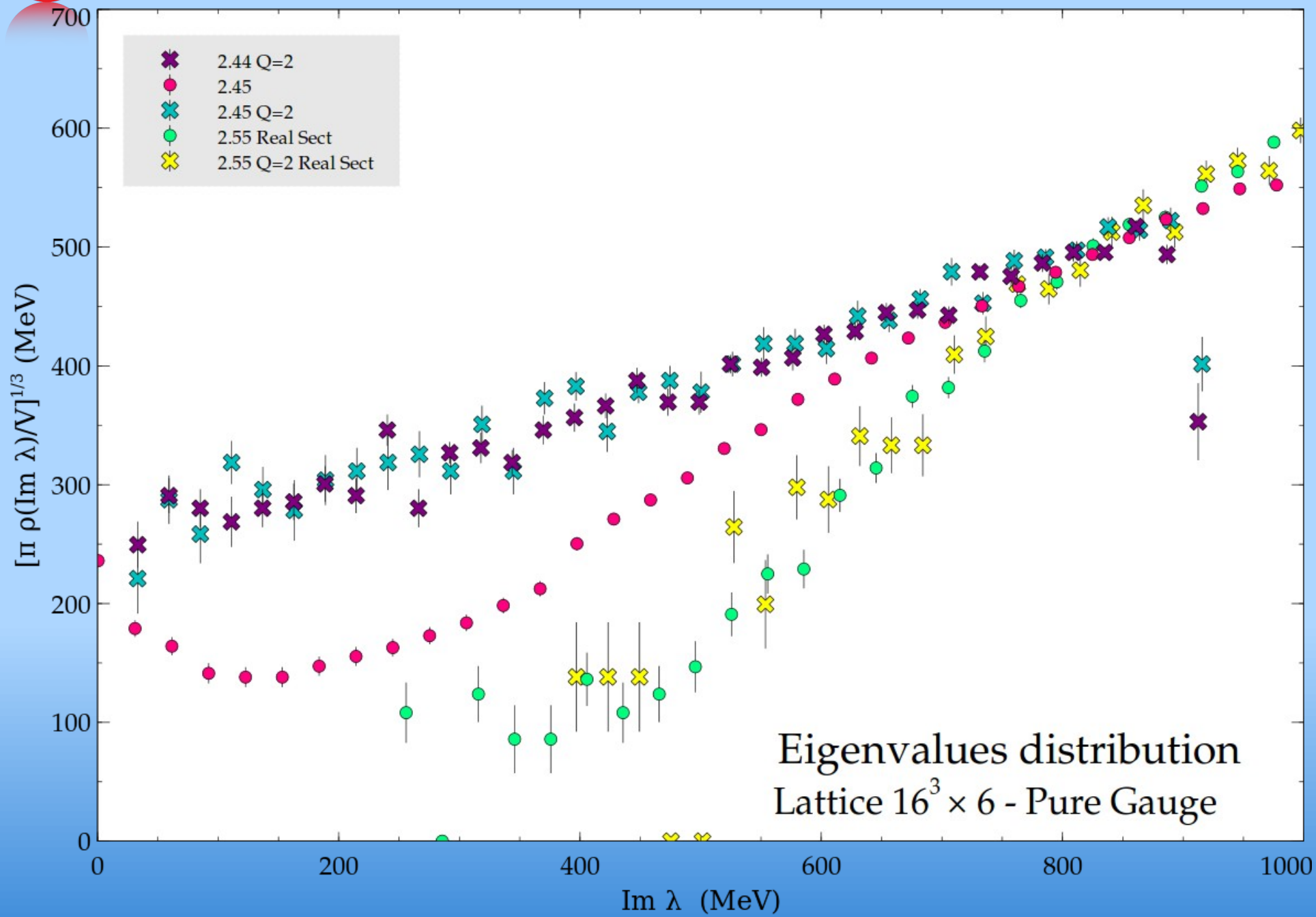
They were associated in *Heller et al. Phys. Rev. D 61, 074504 (2000)* to the presence of an instanton-antiinstanton dilute gas.

So, at the level of Dirac Eigenvalues, we do not find any discrepancy.

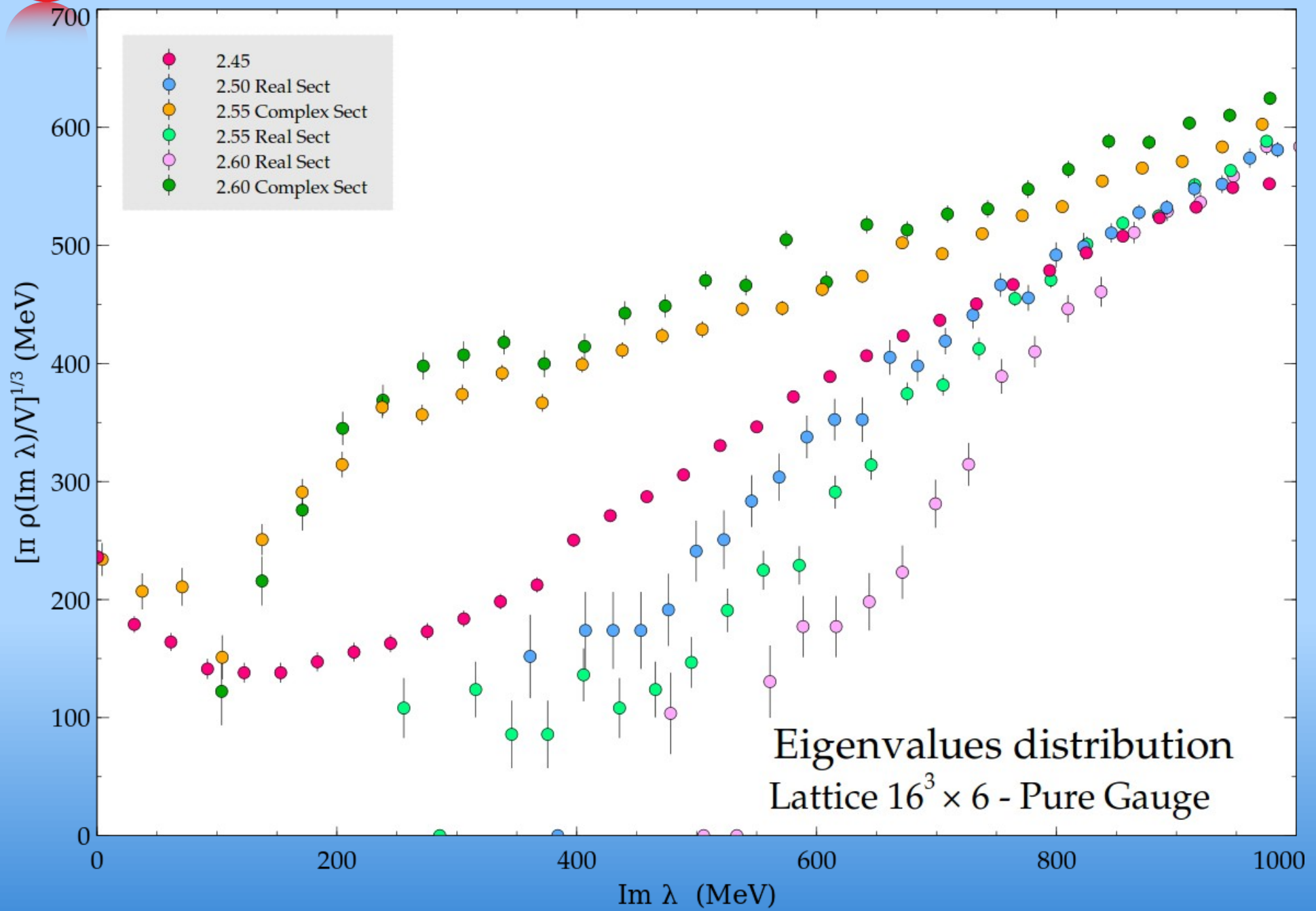
We also checked the behavior in the different “Polyakov Loop sectors” where the gap is expected to become smaller in the “complex” sector (next slides).

Again we agree with known results.

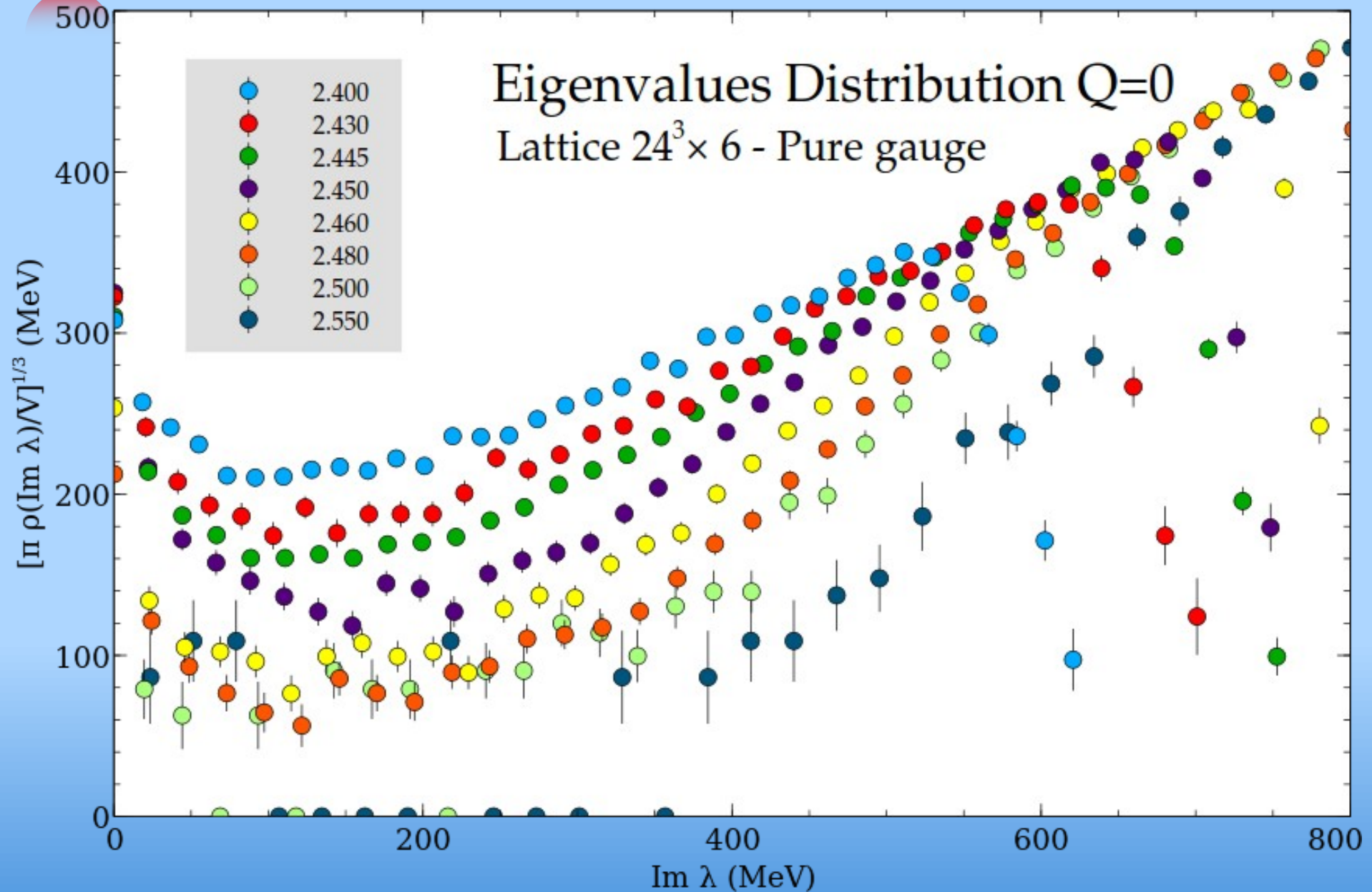
# Pure gauge simulations – Eigenvalues at Q=0,2



# Pure gauge simulations – Eigenvalues in PL sectors

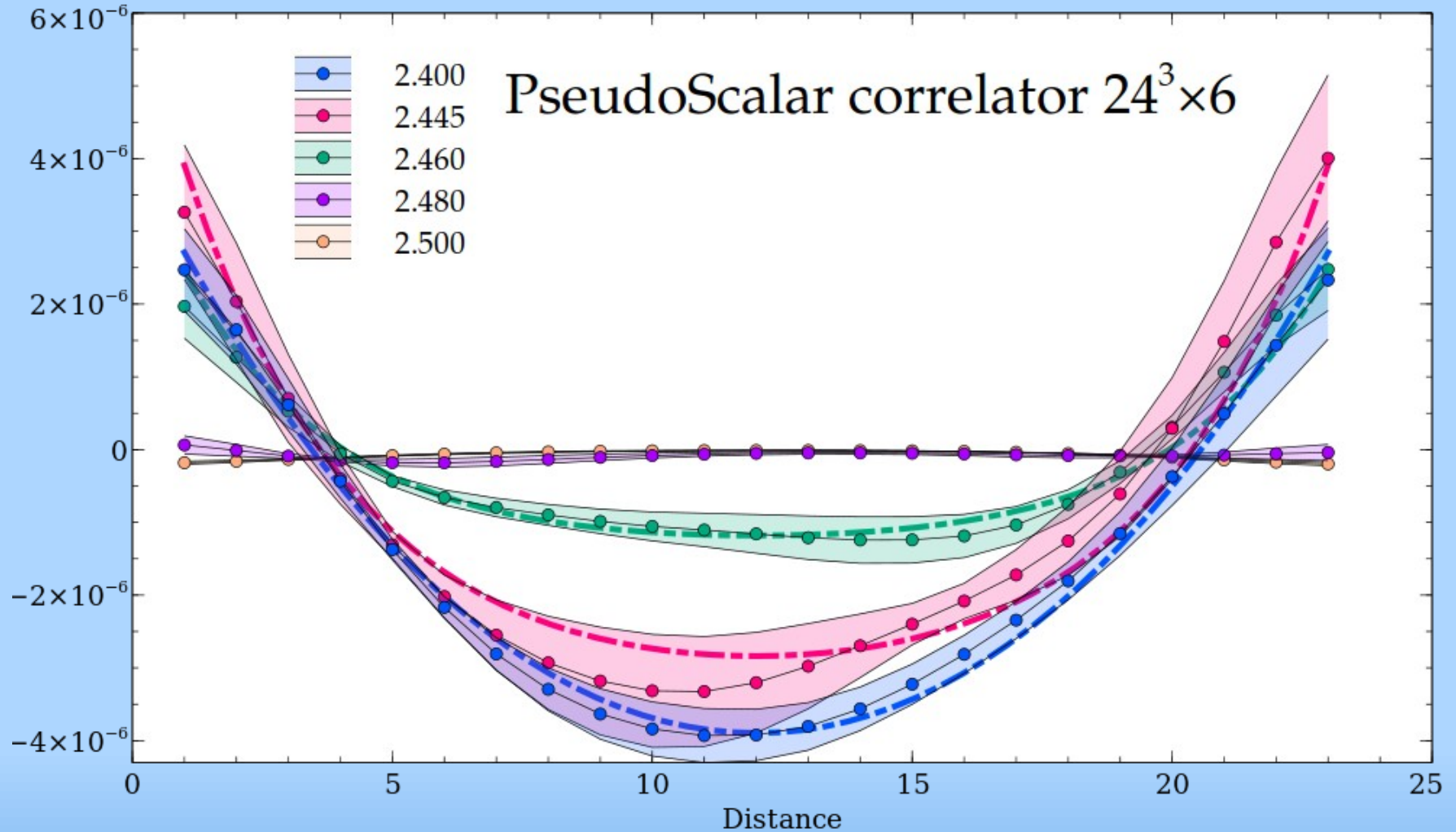


# Pure gauge simulations - Eigenvalues





# Pure gauge simulations - Correlator



Dash-dotted lines are fits. All data points are in the  $Q=0$  sector.

Topological susceptibility clearly decreases with temperature.

We can reproduce quantitatively the results of:

*Alles et al. Nucl. Phys. B 494, 281 (1997)* and

*Gattringer et al. Phys. Lett. B 535, 358 (2002)*



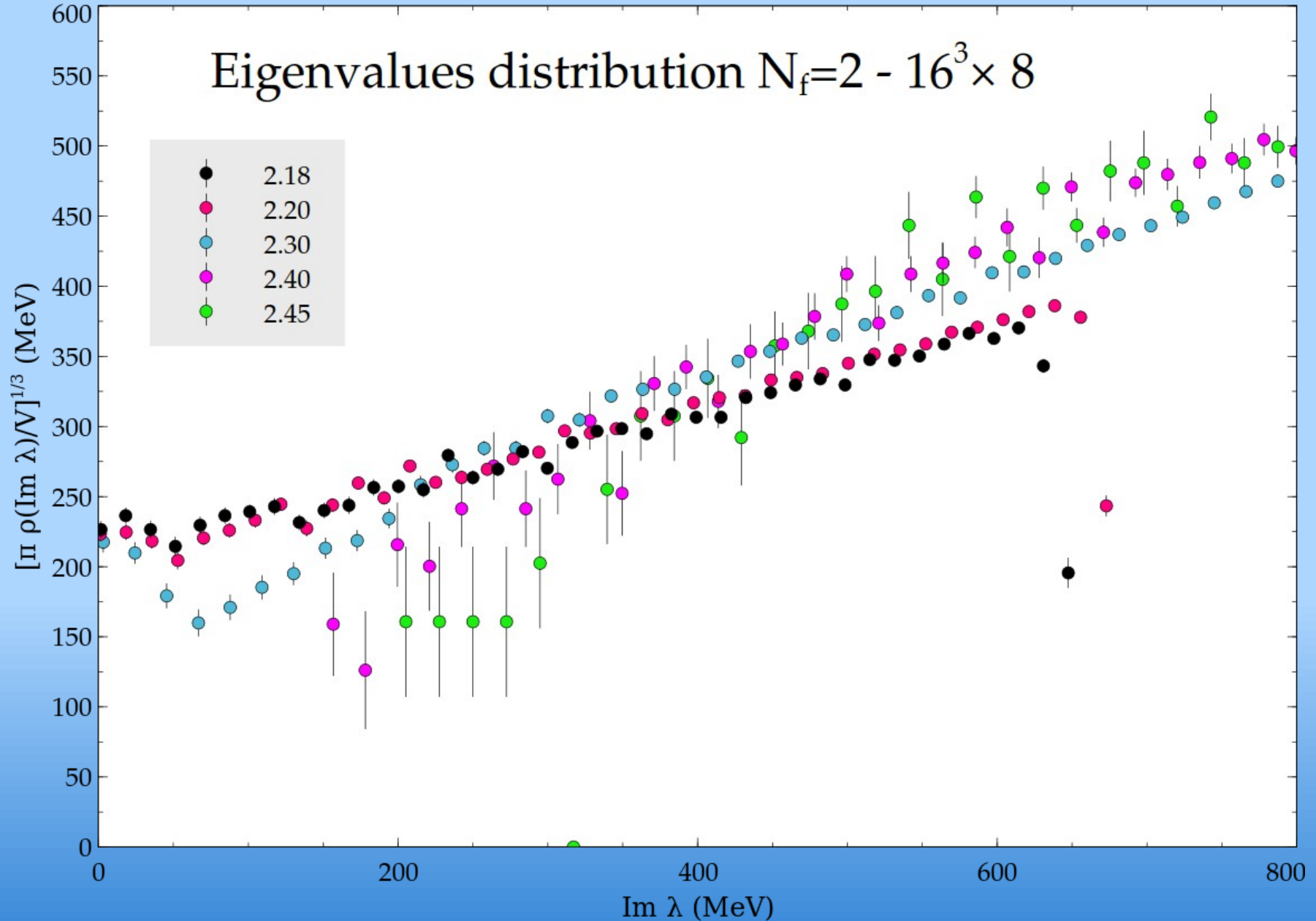
# Full QCD simulations

Brief summary of run parameters

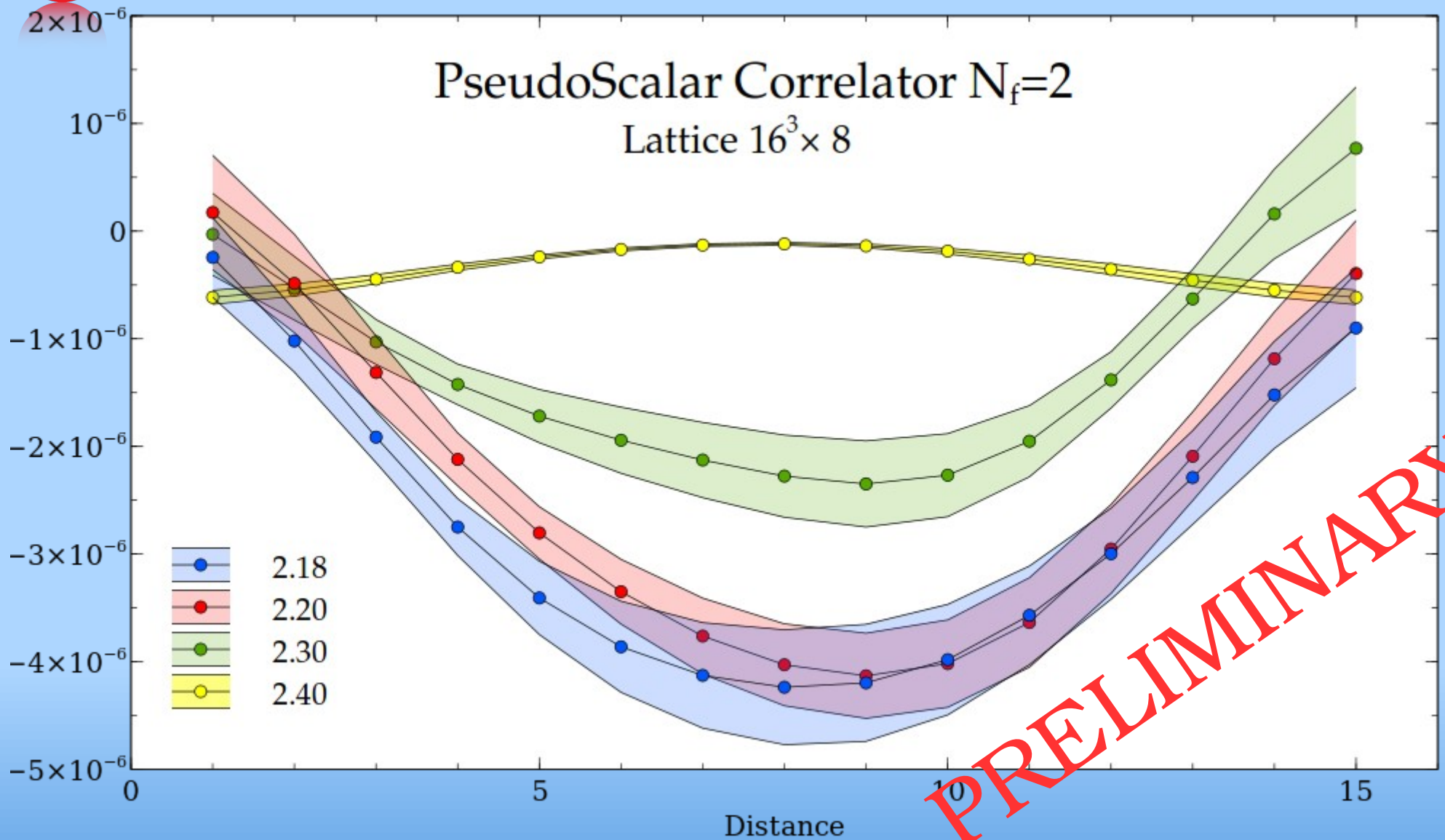
- HMC algorithm
- Iwasaki action + 2 Flavors of overlap fermions + Fukaya term
- $Q = 0$
- $\beta = 2.18, 2.20, 2.30, 2.40$
- Lattice size  $16^3 \times 8$  –  $N_t = 8$  to ensure that configurations are “smooth” enough
- Mass = 0.025
- Temperature range [171-243] MeV
- $T/(T_c = 190 \text{ MeV}) = 0.90, 0.93, 1.09, 1.27$

# Full QCD simulations - Eigenvalues

Temperature range [171-243 MeV]



# Full QCD simulations - Correlator



**PRELIMINARY**

Lattice small but already we see the qualitative behavior of topological susceptibility and of the pseudoscalar mass (still few configs)





# Outlook

I have reported the status of our recent project on finite temperature with overlap fermions

I showed some phenomenology of eigenvalues distributions where our results obtained with the topology fixing algorithm are in agreement with literature (pure gauge simulations)

(!) Eigenvalue saturation to check

In the next months we will concentrate on Full QCD simulations investigating the mass and topological sector dependence of  $\chi_t$  and  $\eta'$  mass at the transition (also we must check finite volume effects)

Studying the critical behavior of the theory by Finite Size Scaling is still far from our current resources.