Non-perturbative running of the coupling from four-flavour lattice QCD with staggered quarks.

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Outline

Introduction

- QCD is believed to describe the strong interactions at all scales.
- FIX THE THE FREE PARAMETERS AT A SCALE ⇒ MAKE PREDICTIONS

High energy QCD		Low energy QCD
Asymptotic freedom	⇔	Confinement?
Perturbation theory		Non-perturbative approach
⇓		⇓

A way of connecting these two regimes is thus desired.

- REQUISITE: Non-perturbative techniques ⇒ **LQCD**
- PROBLEM: Large scale diferences involved,

$$a^{-1} >> \mu_{\rm pert} >> \mu_{\rm had} >> L^{-1}.$$

- POSSIBLE SOLUTION: Finite size techniques [Lüscher et al. '91],

1. **Finite volume**: the volume plays the role of a scale, $L \sim 1/\mu$.
2. **Define a coupling**: $\bar{g}(L)$ (runs with the scale).
3. **Low energies**: match with hadronic quantities
4. **Evolution**: discretised RGE + simulations at different volumes.
5. **High energies**: match with a perturbative scheme.

Schrödinger Functional and running coupling

- Non perturbative definition $\bar{g}$ needed with: easy perturbative expansion, easy to calculate numerically, small discretisation errors.
- Schrödinger Functional, (SF): $\mathcal{Z}[C, C'] = \int \mathcal{D}[A] e^{-S[A]}$.

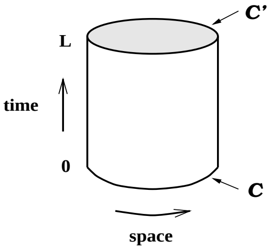
- Boundary conditions:

$$A_k(x)\big|_{x_0=0} = C_k, \quad A_k(x)\big|_{x_0=T} = C'_k.$$

[Narayanan et al. '92]

- Effective action, with B the background field: $\Gamma[B] = -\ln \mathcal{Z}[C, C']$.
- Consider a one parameter family, $B(\eta)$, $C, C' \sim L^{-1}$, $T = L$. One scale.
- Weak coupling expansion:

$$\Gamma[B] \overset{g_0 \to 0}{\longrightarrow} \frac{1}{g_0^2}\Gamma_0[B] + \Gamma_1[B] + g_0^2 \Gamma_2[B] + \ldots$$



(LxLxL box with periodic b.c.)

- **Definition of** $\bar{g}$: $\bar{g}^2(L) = \frac{\Gamma'_0}{\Gamma'}$, $\Gamma' = \partial_\eta \Gamma[B(\eta)]\big|_{\eta=0}$.

Staggered fermions on the SF set up

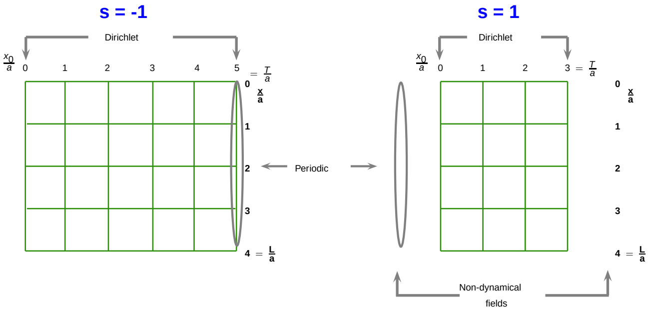
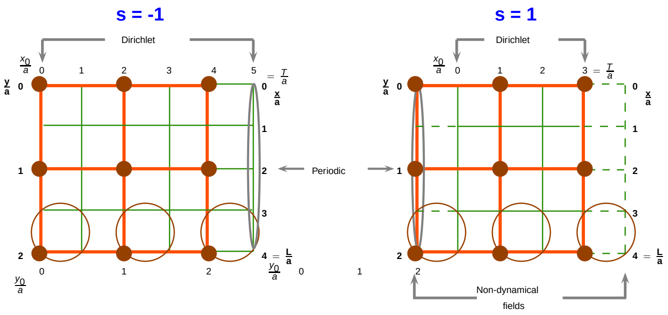


Figure: SF with staggered fermions

- Technical problem with staggered fermions (Miyazaki and Kikukawa '94 & Heller '97): T/a **odd** and L/a **even**.
- Modified conventions: take the continuum limit at $T' = L$. $T' = T + sa$ is the extent of the "effective" lattice ($s = \pm 1$).
- This modifies the $\mathrm{O}(a)$ effects in the pure gauge theory even at tree level.

Staggered fermions on the SF set up



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Symanzik improvement

In the SF framework $\mathrm{O}(a)$ effects are of two types,

BULK:

- **Pure gauge**: No $\mathrm{O}(a)$ effects.
- **Staggered fermions**: No $\mathrm{O}(a)$ effects. SUBTLE $\Rightarrow$Improved staggered fermions (time direction) in space + chiral twist (spatial directions).

BOUNDARIES:

- **Pure gauge**: $c_t \mathrm{tr}\{F_{0k}F_{0k}\}$, $c_t = c_t^{(0)} + g_0^2(c_t^{(1,0)} + N_f c_t^{(1,1)}) + \mathrm{O}(g_0 4)$,

$$c_t^{(0)} = \frac{2}{2+s}, \qquad \begin{array}{ll} c_t^{(1,0)_1} = 0.0274(2), & c_t^{(1,1)_1} = 0.0077856(4), \\ c_t^{(1,0)_{-1}} = -0.4636(6), & c_t^{(1,1)_{-1}} = -0.0266(8). \end{array}$$

- **Staggered fermions**: $(d_{1s}-1)[\hat{\mathcal{O}}_1 + \hat{\mathcal{O}'}_1]$. If $P_\pm = \frac{1}{2}(1 \pm \gamma_0)$,

$$\hat{\mathcal{O}}_1 = \bar{\psi}(0,\mathbf{y})P_+\gamma_k\mathcal{D}_k\psi(0,\mathbf{y}), \quad d_{1s}^{(0)} = 1 + \frac{s}{4}.$$

Finite size techniques

- The step scaling function is defined in the continuum,

$$\sigma(u) = \bar{g}^2(2L)|_{u=\bar{g}^2(L)}.$$

- On the lattice, it can be obtained from a sequence of pairs of lattices with sizes L/a and $2L/a$,

$$\sigma(u) = \lim_{a\to 0} \Sigma(u, a/L).$$

- Repeat the procedure for a range of u values in $[\bar{g}^2(L_{\min}), \bar{g}^2(L_{\max})]$,

$$u_0 = \bar{g}^2(L_{\min}) \quad u_k = \sigma(u_{k-1}) = \bar{g}^2(2^k L_{\min}), \quad k = 1, 2, \ldots$$

- After 7,8 steps, scale differences of $\mathrm{O}(100)$ are bridged.
- LOW ENERGIES:Connection with physical units: Compute $F_\pi L_{\max}$
- HIGH ENERGIES: Connection with PT, $\bar{g}^2_{\overline{\mathrm{MS}}} = \bar{g}^2 \left(1 + C_g^{(1)}\bar{g}^2 + \ldots\right).$

Numerical details

- Algorithm used: HMC. Customised version of the MILC code, [MILC Collab. '02] implementing the $\mathrm{O}(a)$ improvement.
- Lattices with $L/a = 4, 6, 8, 12, 16$, and $s = \pm 1$.
- Statistics ranging $\sim 60000 - 160000$.
- Data analysis: `Uwerr.m`, [U. Wolff, '04].
- No tuning made beforehand. Interpolation of the data. [T. Appelquist et al, '09].

Extrapolation of $\Sigma(u, a/L)$

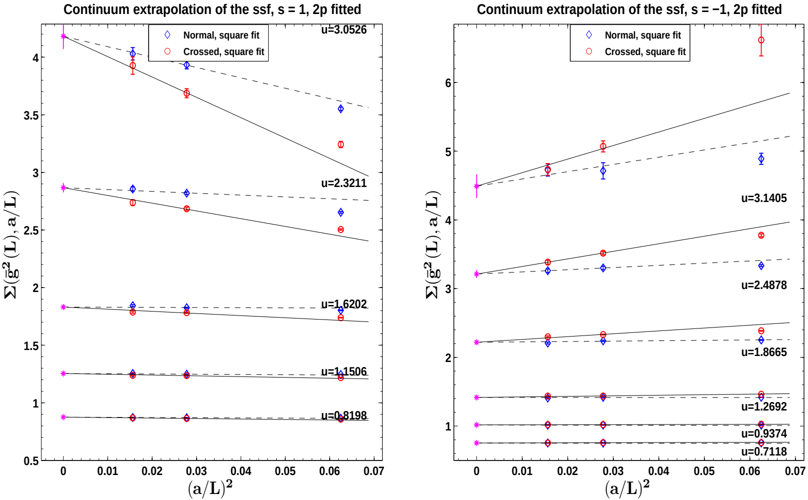


Figure: Continuum limit extrapolation of the lattice step scaling function

$\sigma(u)$ vs u and Λ - parameter

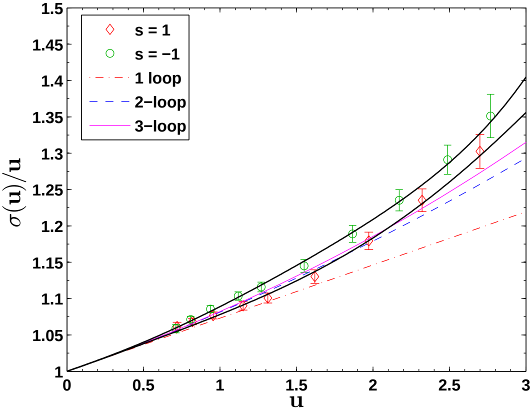


Figure: Evolution of $\sigma(u)/u$ vs u

Conclusions and outlook

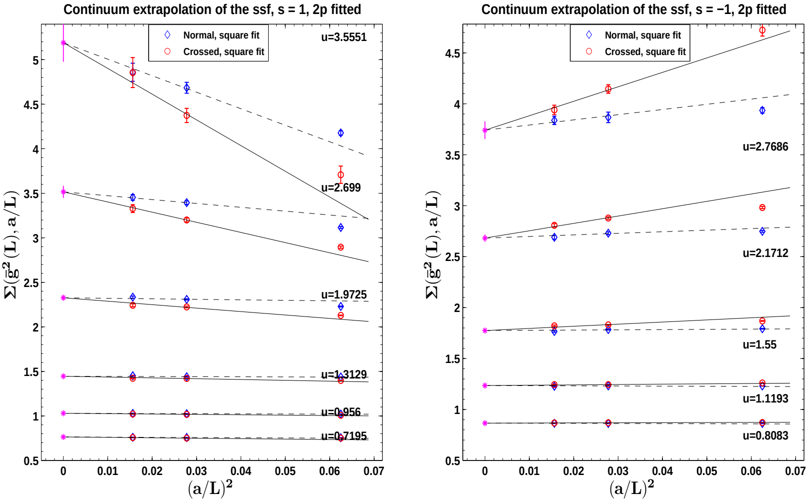
- Running of the renormalised coupling constant for QCD with 4 flavours of staggered quarks has been computed.
- Requisites:

  1. Formulation of SF with staggered quarks.
  2. Implementation of the $\mathrm{O}(a)$ improvement.
  3. Simulations and data analysis.

OUTLOOK

- Connection with physical units still has to be performed.
- Comparative analysis with the work just presented as a test of universality, [R.Sommer, F.Tekin, U.Wolff, 10'].
- $\bar{g}$ under control $\Rightarrow$ Computation of other observables is possible.

Extrapolation of $\Sigma(u, a/L)$, (II)



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