Disconnected Diagrams: Progress, Methods and Prospects

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with

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Abstract

Computation of disconnected or non-valence quark loops presents a computationally challenging problem.

Absent these contributions operators with zero flavor quantum numbers can not be studied.

This is a report on progress to measure the strange quark contributions to nucleon form factors.

The increasing importance of GPU architectures and multigrid methods will be discussed, as will prospects for increased accuracy as these methods are further exploited.

Overview

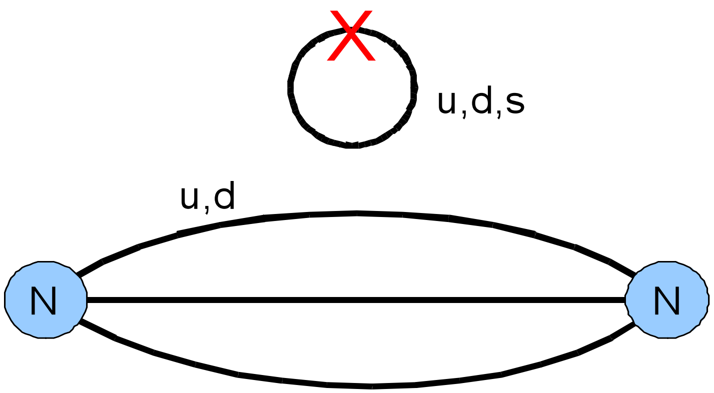
- Physics Goals
- Basic Disconnected Diagram Methods
- Multigrid and Variance Reduction
- Future: DW + Multigrid + GPUs

  ➔ O(100) times speed ?

Full QCD ➔ Nucleon Disconnected diagrams

- E.G.

$$\sum_x \bar{q}(x)\Gamma q(x) = \mathrm{Tr}(\Gamma D^{-1})$$



- REQUIRED FOR
- Few % electro-magnetic form factors et al
- Flavor-singlet spectroscopy.
- Dominant Higgs nucleon coupling for direct detection of neutralino Dark Matter candidate

$$f_{T_s} = \frac{m_s \langle N|\bar{s}s|N\rangle}{\langle N|H|N\rangle} \simeq 0.51(8)(3) \text{ bare neglecting mixing!}$$

Stochastic Trace calculation

- Standard stochastic method:

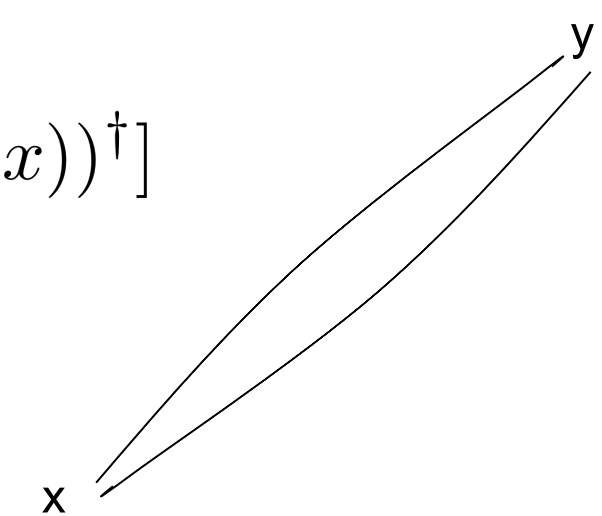
$$\mathrm{Tr}(\Gamma D^{-1}) \approx \frac{1}{N}\sum_{i=1}^{N} \eta_i^\dagger \Gamma D^{-1} \eta_i, \quad \langle \eta_{(x)}^\dagger \eta_{(y)} \rangle = \delta_{x,y}$$

- Unitary noise vectors in elements Z_2, Z_4, U(1), SU(3) etc.

$$\eta_{(x)}^\dagger \eta_{(x)} = 1$$

- Variance is a meson susceptibility

$$\mathrm{var} = \sum_{x \neq y} Tr[\Gamma D^{-1}(x,y)(\Gamma D^{-1}(y,x))^\dagger]$$



- Variance worsens at lighter quark masses.

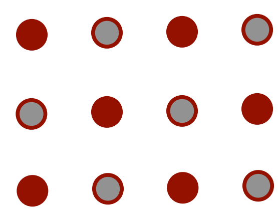
Variance reduction

- **Short-distance:**
  - Dilution in spin/color/space
  - Unbiased subtraction of a truncated hopping parameter expansion (Thron et al., 1998; Dong et al., 2000)
- **Long-distance/general:**
  - Eigenmode subtraction (Neff et al., Giusti et al., Bali et al., Wilcox et al., others)
  - Truncated solver (Collins et al., 2007)
  - Multigrid subtraction (this talk)
- **Best to combine short distance & long distance methods**

Variance Reduction by dilution:

- Dropping near by off-diagonals in stochastic trace.
- Explicitly sum over subsets to get full trace (if necessary)

e.g. even/odd:



(Foley et al., 2005;
generalization of color/spin
“partitioning” used earlier)

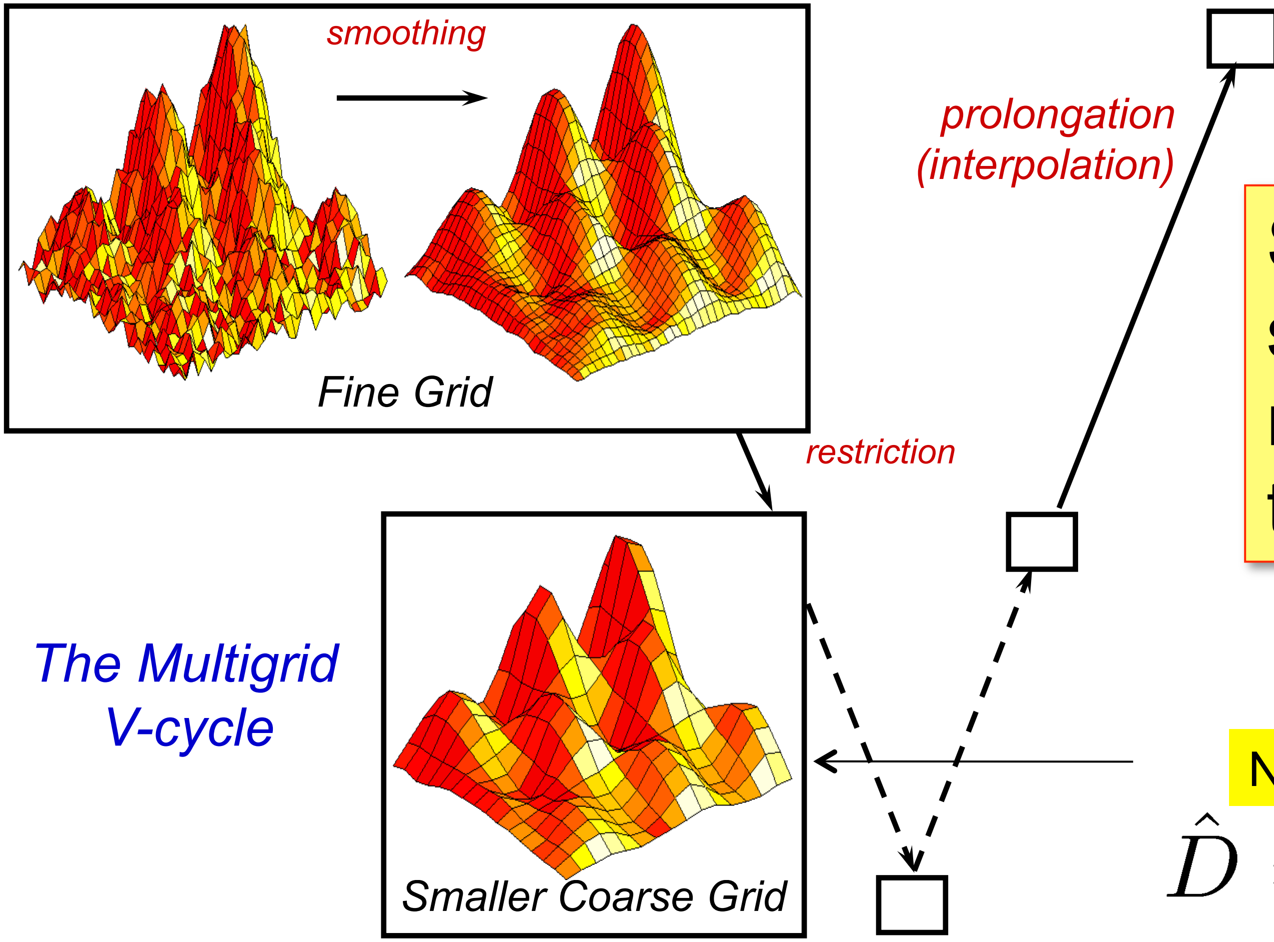
$$\mathrm{Tr}(\Gamma D^{-1}) \approx \frac{1}{N}\sum_{i=1}^{N} \eta_i^{(e)\dagger}\Gamma D^{-1}\eta_i^{(e)} + \frac{1}{N}\sum_{i=1}^{N} \eta_i^{(o)\dagger}\Gamma D^{-1}\eta_i^{(o)}$$

THIS IS A SEMI-STOCHASTIC METHOD:
COMPLETE DILUTION IS EXPLICIT TRACE!

MultiGrid Acceleration and Variance Reduction

Adaptively find near null space of Dirac operator

$D: \mathcal{S} \simeq 0$

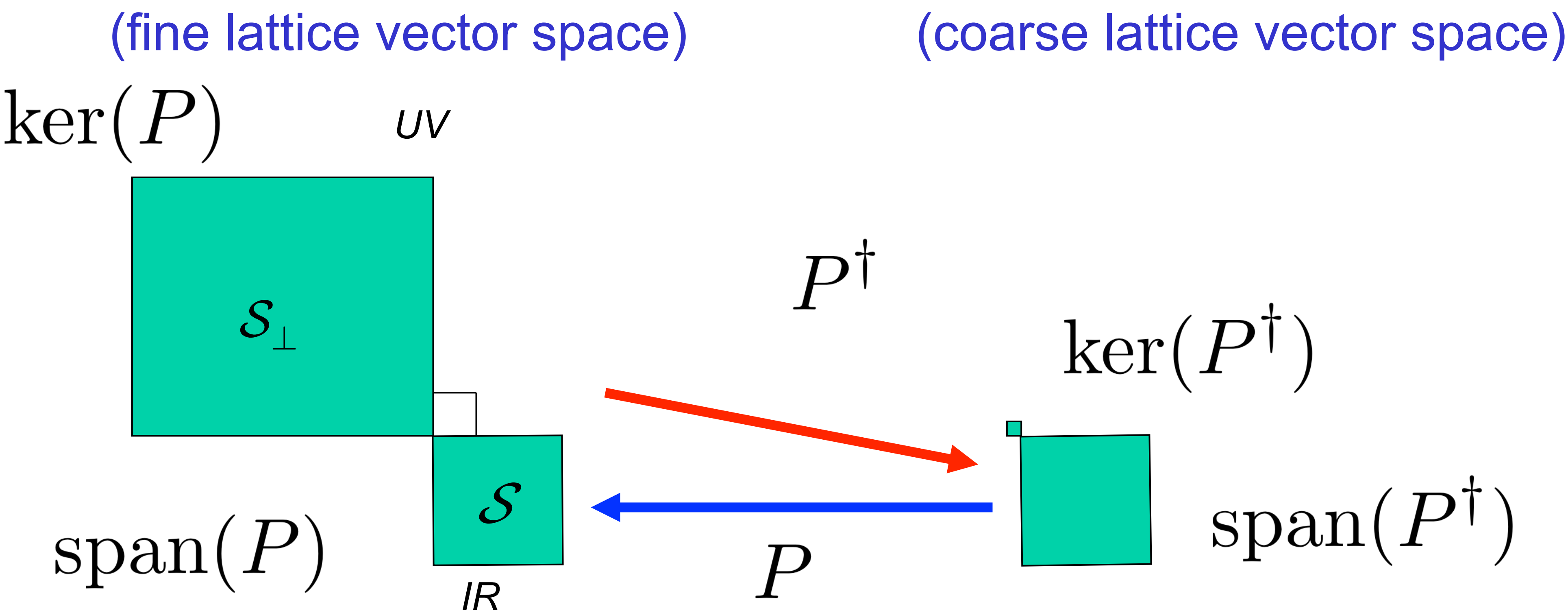


Spilt the vector space into near null space $\mathcal{S}$ and the complement $\mathcal{S}_\perp$

Near null space Dirac op

$$\hat{D} = D_{cc} = P^\dagger D P$$

non-square "Restriction" matrix, $P^\dagger$ *:fine* $\rightarrow$ *coarse*



$$P^\dagger P = 1_{cc} \text{ so } \ker(P) = 0$$

To preserves gamma-5 Hermicity: $P^\dagger \gamma_5 P = \sigma_3$

† *See Front cover of Gilbert Strang's undergraduate text !*

Multigrid preconditioning step

MULTIGRID SOLVER RETURNS RESIDUE

$$r' = [1 - DP\frac{1}{P^\dagger DP}P^\dagger]r$$

$$P^\dagger r' = 0$$

ORTHOGONAL
TO NEAR NULL SPACE

$$\mathcal{P} = 1 - DP\frac{1}{P^\dagger DP}P^\dagger$$

LEFT PETROV-GALLERKIN (oblique)
PROJECTION OPERATOR

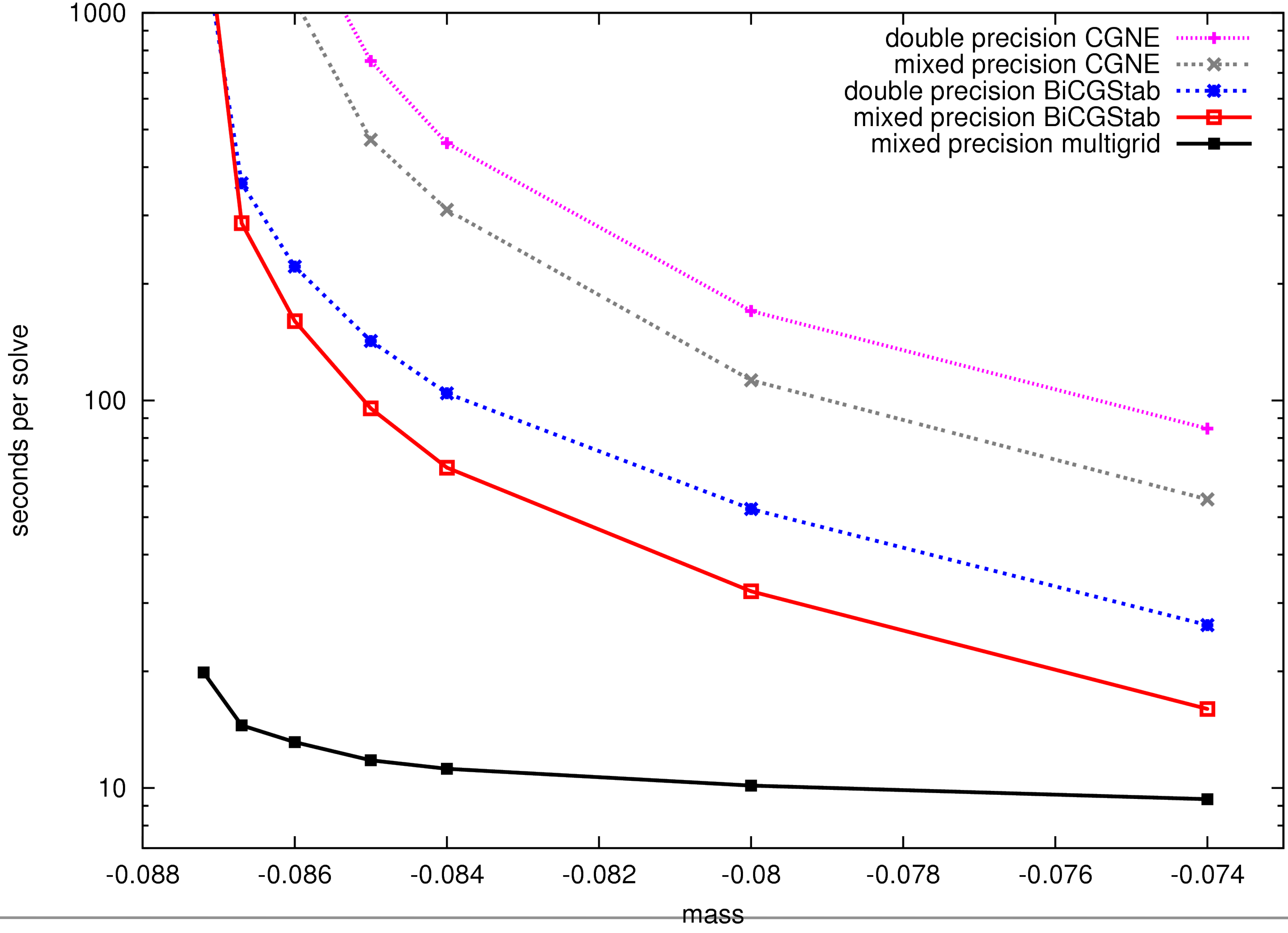
THE COARSE (little) DIRAC OPERATOR:

$$\hat{D} = D_{cc} = P^\dagger DP$$

SCHUR COMPLEMENT:

$$D_{Schur} = D - DP\frac{1}{P^\dagger DP}P^\dagger D$$

Results on 32^3x256 asymmetric clover Wilson lattice



Multigrid (recursive) variance reduction

$$Tr[\Gamma D^{-1}] = Tr[\Gamma(D^{-1} - PD_c^{-1}P^\dagger)] + Tr[\Gamma PD_c^{-1}P^\dagger]$$

(do by stochastic estimator)

Cyclic trace: $$Tr[\Gamma PD_c^{-1}P^\dagger)] = Tr[(P^\dagger\Gamma P)D_c^{-1}]$$

(do by stochastic estimator)

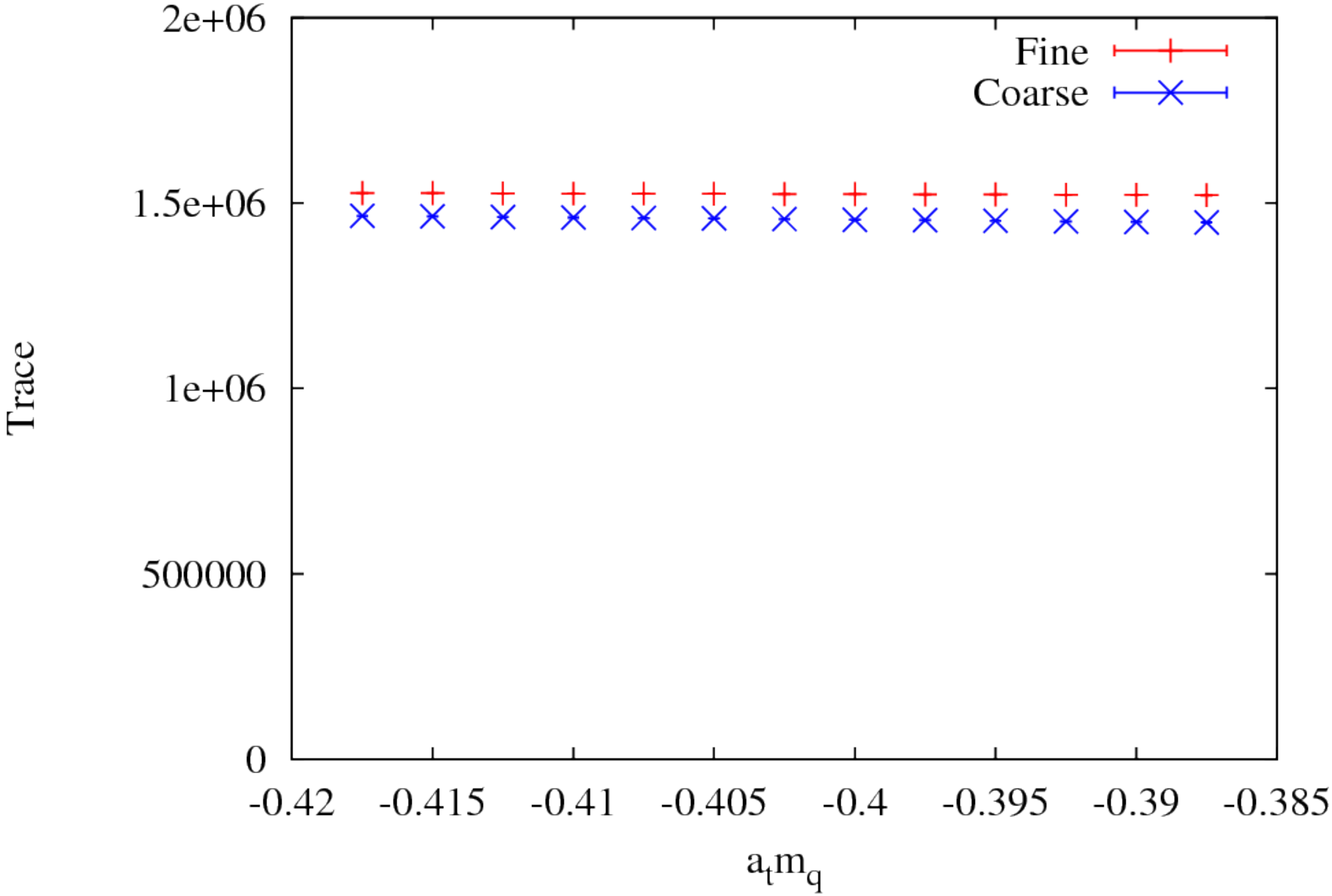
- Inverse on the first coarse level is 50 times cheaper than (MG accelerated) fine level.
- Recursively application cost falls by a factor:

  $O(1/4^4) = O(1/256)$ per level
- Solver coarsest level exactly.
- Trace believe to be O(N log N) at fixed accuracy?

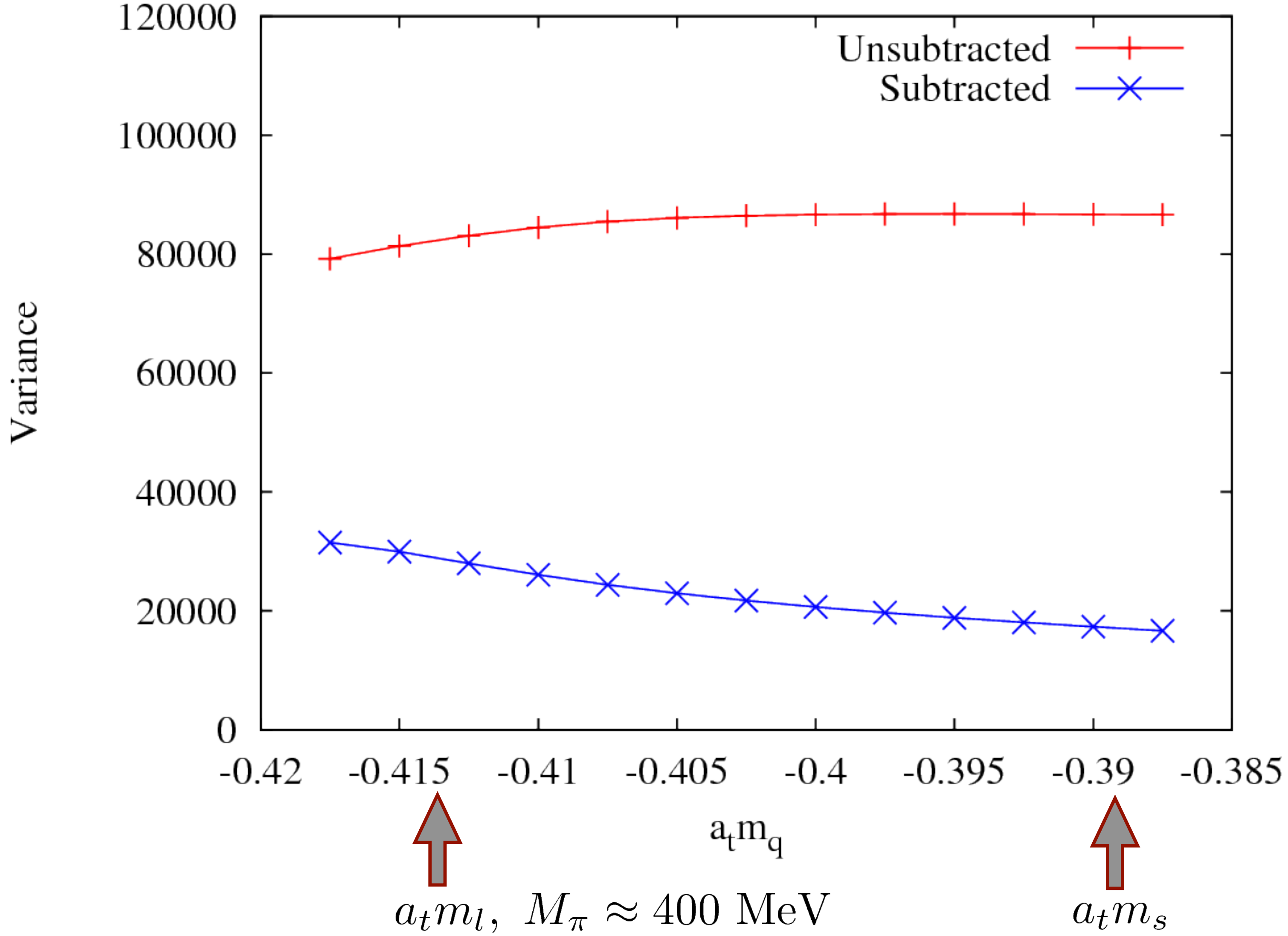
Preliminary results

- Anisotropic Wilson lattice. $16^3 \times 64$
- 3-level Multigrid with 4^4 blocking.
- First level MG variance subtraction
- Dilution in spin, color, even/odd (24 inverses per source).
- Masses: strange quark down to critical mass.
- 20 sources (per dilution subset).

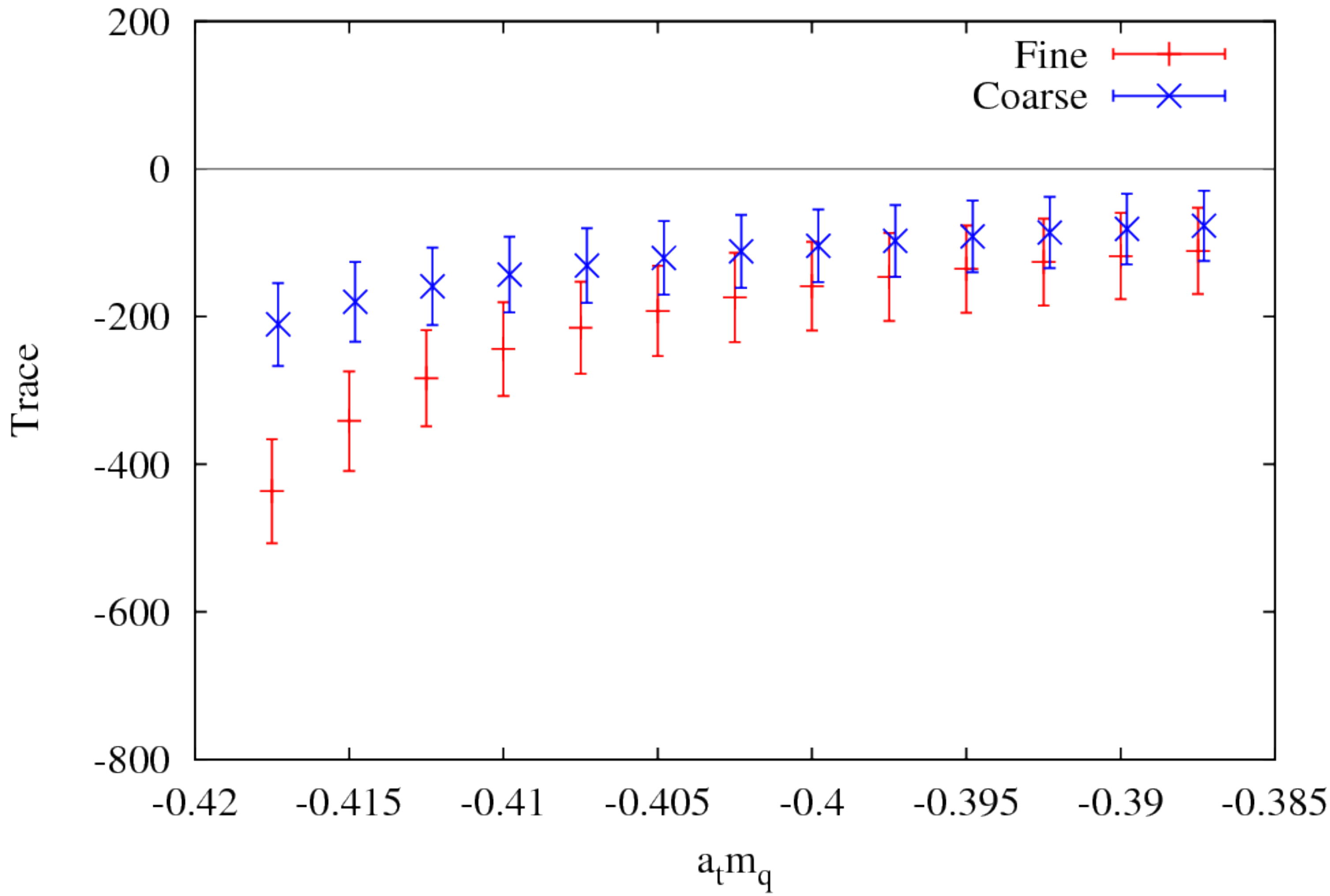
Scalar density



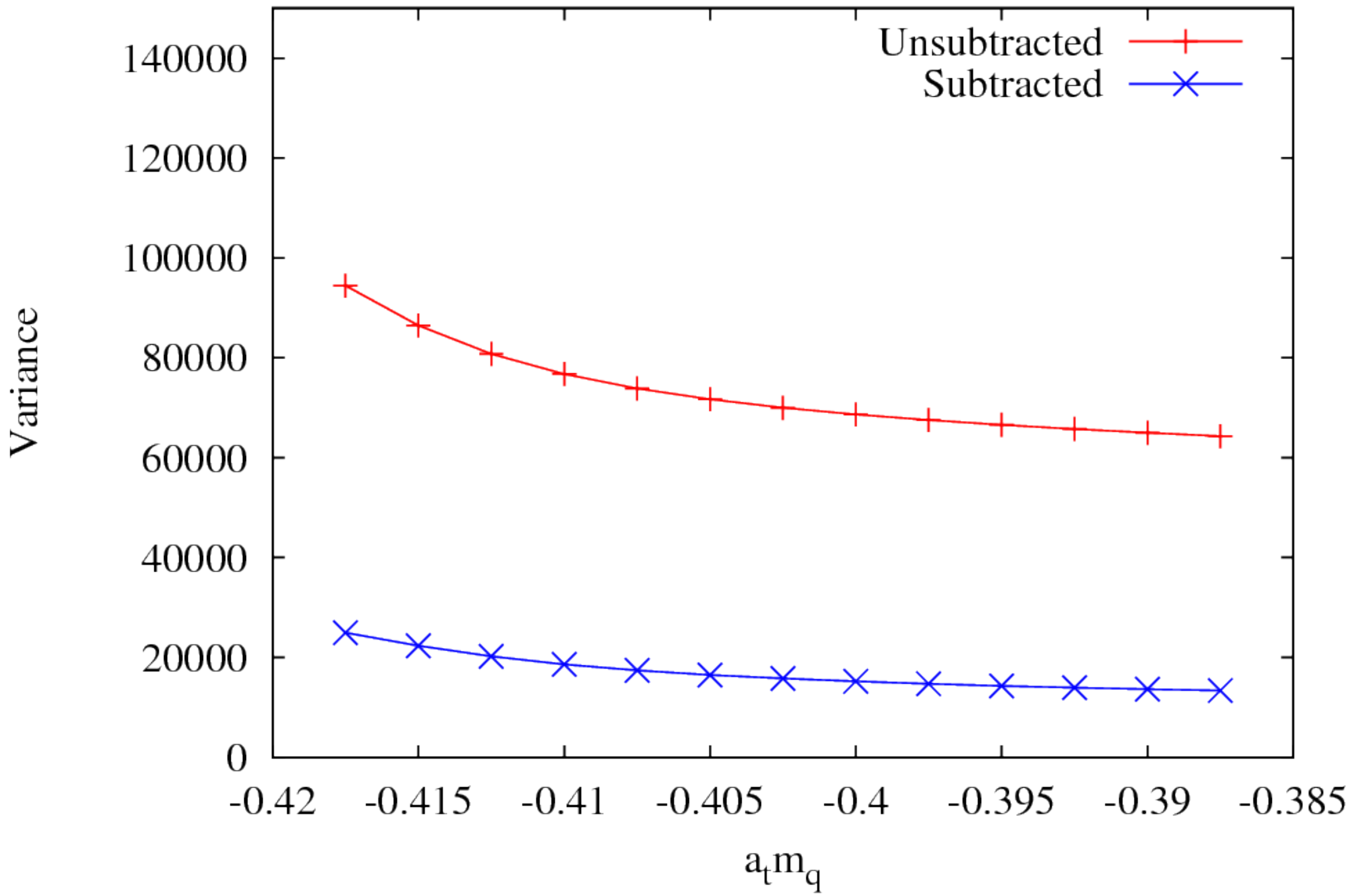
Scalar density: Variance



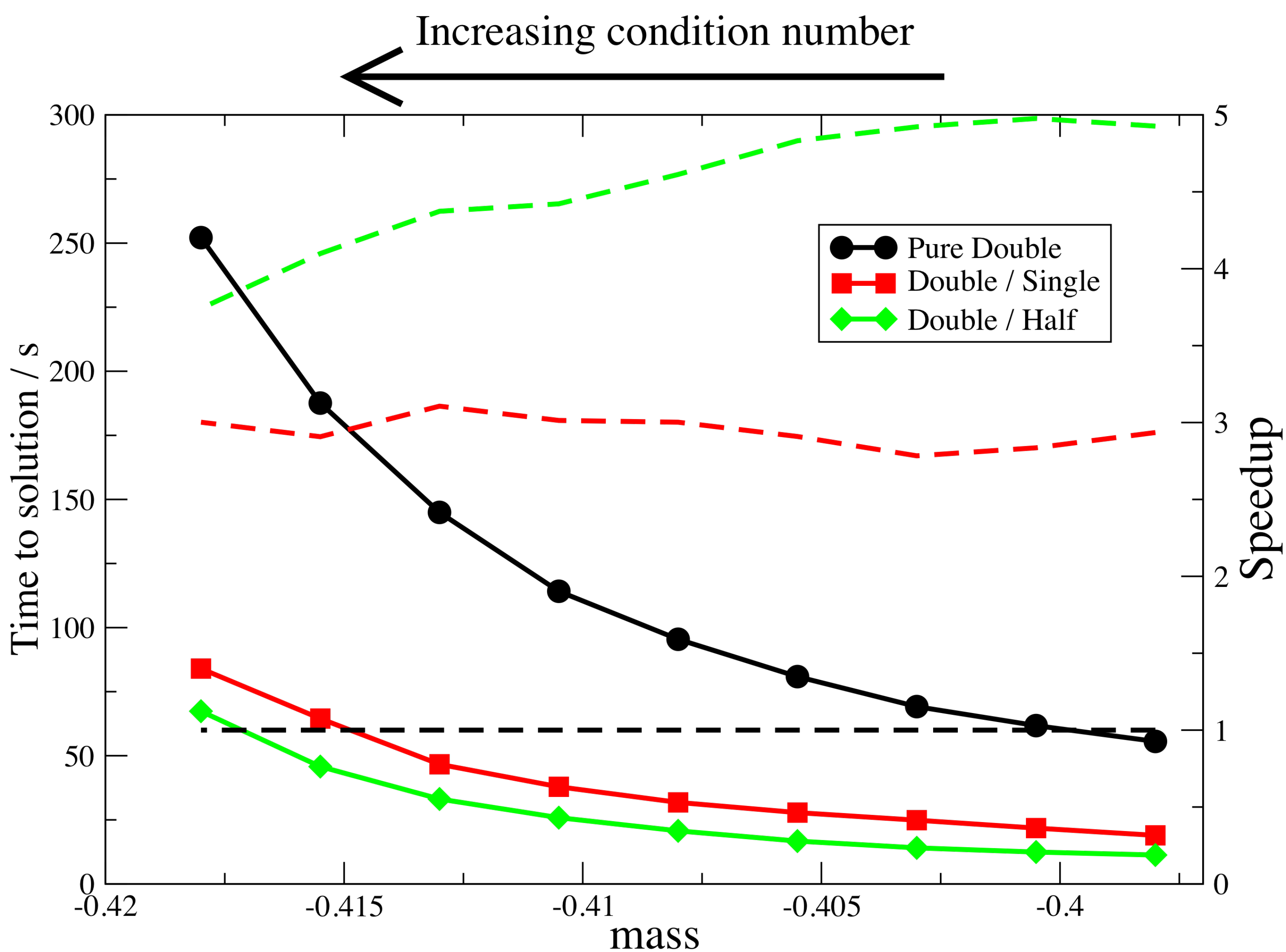
Pseudoscalar



Pseudoscalar: Variance



100 Gflop/s Inverter on Nvidia GeForce GTX 280



Clark, Babich, Barros, Brower and Rebbi "Solving Lattice QCD systems of equations using mixed precision solvers on GPUs"

Concluding Hopes and Plans!

- Combine Wilson Multigrid codes with GPUs to give

$$O(20) \times O(5) \implies O(100)$$

speed up!

- Generalize Multigrid and GPU code to Domain Wall Fermions

- Compute Higgs coupling to Nucleon to few percent!

$$f_{T_s} = \frac{m_s \langle N|\bar{s}s|N\rangle}{\langle N|H|N\rangle}$$

- New era of chiral quark Disconnected Diagrams.