Continuum Thermodynamics of the SU(N_c) Gauge Theory

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Deconfinement in SU(N_c)

- $SU(N_c)$ gauge theory simplifies in the limit of large N_c

  't Hooft '73

  $\lambda = g^2 N_c$ fixed as $N_c \to \infty$: subclass of diagrams important
  Qualitative understanding of many features of hadron physics
- Deconfinement transition: some features of the 3-color theory can be understood from the large N_c theory

  Svetitsky & Yaffe; Pisarski; McLerran & Pisarski; ...
- Deconfinement transition in SU(4)

  Gavai; Ohta & Wingate; '01-'02
- Lucini, Teper, Wenger '02-'07 Lattice study of deconfinement transition for N_c = 4,6,8
- We studied deconfinement in SU(4)(6) for N_τ = 6-12 (10)
  scaling requires (small) correction to 2-loop β fn
  in terms of *renormalized coupling*

  S. Datta & S. Gupta, Lattice 2009, PRD 80 ('09) 114504

Thermodynamics of $SU(N_c)$ Gluon Plasma

- Thermodynamics near T_c ($< 2T_c$) for SU(4-8) ($N_\tau = 5$)

  Bringoltz & Teper, PL B 628(2005) 113.

- Bulk thermodynamic quantities of the gluon plasma
  - SU(4-8), coarse ($N_t = 5$) lattices

    M. Panero, Lattice 2009, PRL 103 ('09) 232001
  - $N_t = 6$ and 8 lattices, SU(4-6)

    S. Datta & S. Gupta, QM 2009, Lattice 2009

- New Results:
  - Detailed study of latent heat
  - Non-perturbative β fn for thermodynamics
  - Equation of state: large volumes to go to $\sim 4T_c$ with $N_t = 8$
- Approach to conformality in SU(N_c) plasma
- Investigate 't Hooft (scaling with λ) vs. strong N_c scaling.

S. Datta and S. Gupta, arXiv:1006.0938

Formalism

$$(\epsilon - 3p)/T^4 = 6N_\tau^4 \, a\frac{\partial\beta}{\partial a} \left(P(T) - P(T=0)\right)$$

$$\boxed{a\frac{\partial\beta}{\partial a} \;=\; -\frac{\partial\beta}{\partial g_R^2(k/a)} \;\cdot\; 2g_R\,\beta(g_R)}$$

where g_R is calculated in Lepage-Mckenzie V-scheme,
and $\beta(g_R)$ includes a correction to β_{2-loop} to get scaling of T_c.

Datta & Gupta, PR D 80 ('09) 114504.

Pressure calculated using the "integral method"

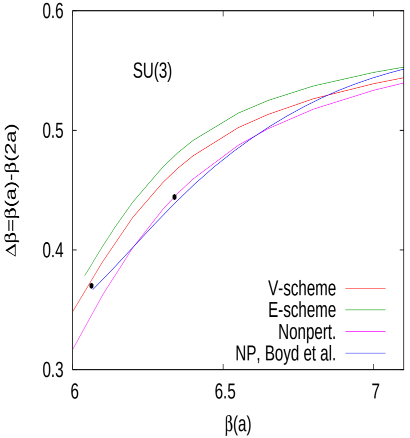
Boyd et al., Nucl.Phys. B 469('96) 419

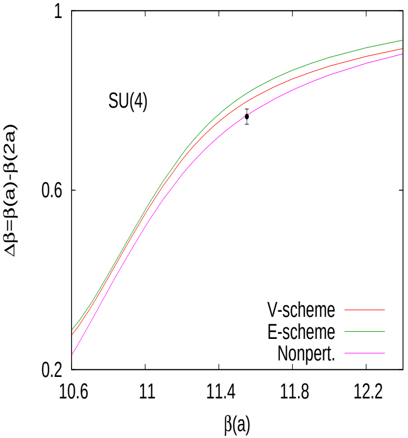
$$\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = 6N_\tau^4 \int_{\beta_0}^{\beta} d\beta \left(P(\beta,T) - P(\beta,T=0)\right)$$

For free gas, $\frac{\epsilon_{SB}}{T^4} = 3\frac{p_{SB}}{T^4} = (N_c^2 - 1)\frac{\pi^2}{15}R(N_\tau)$
Here $R(N_\tau)$ discretization error $= 1 + \frac{8}{21}(\frac{\pi}{N_t})^2 + \ldots$

Engels et al., Nucl.Phys. B 205('82) 545

β function

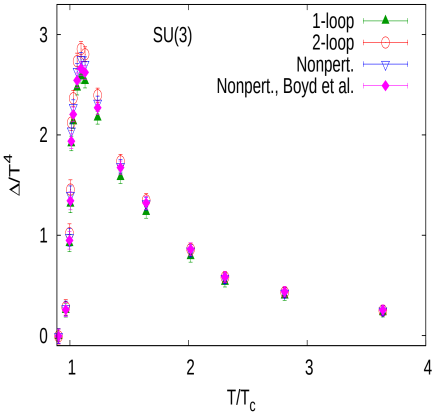




SU(3) data from Boyd et al ('96)

Data points are from $\beta_c(N_t)$ for deconfinement transition.

β function and $(\epsilon - 3p)/T^4$



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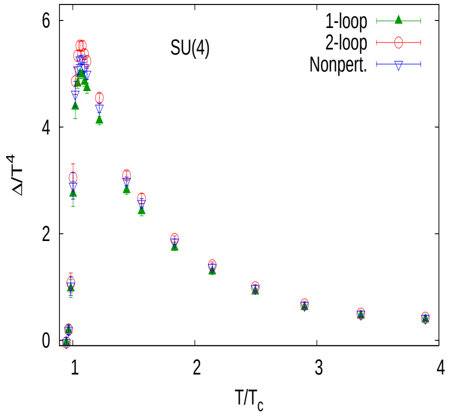

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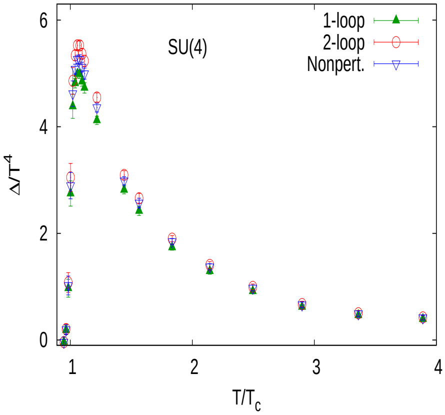
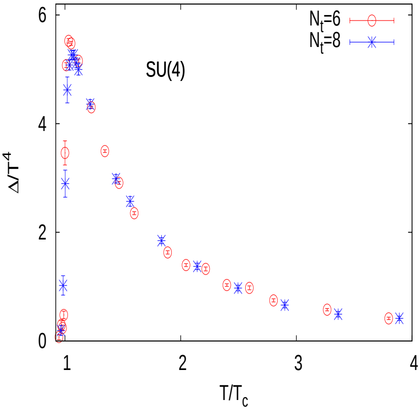


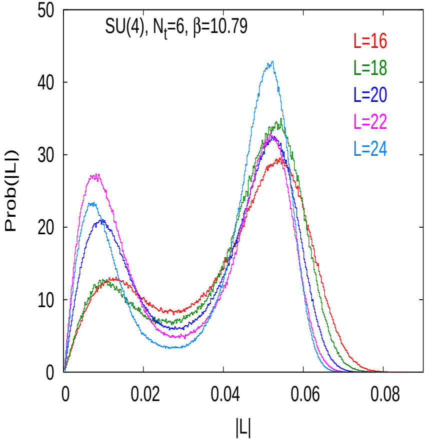


Very little cutoff dependence, except probably very close to T_c
We take the $N_\tau = 8$ result as the continuum result.

Latent Heat: Method

L_h/T_c^4 obtained from the discontinuity of $(\epsilon - 3p)/T^4$ at T_c

- Put $|L|$ cut to identify confined and deconfined phase at T_c



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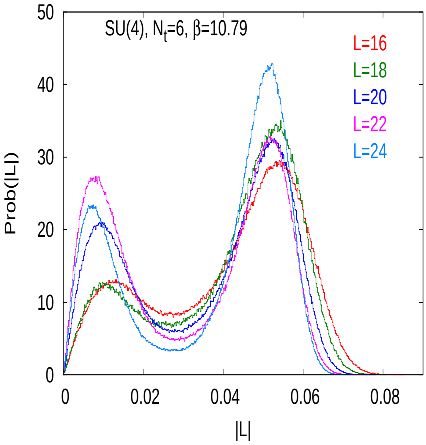
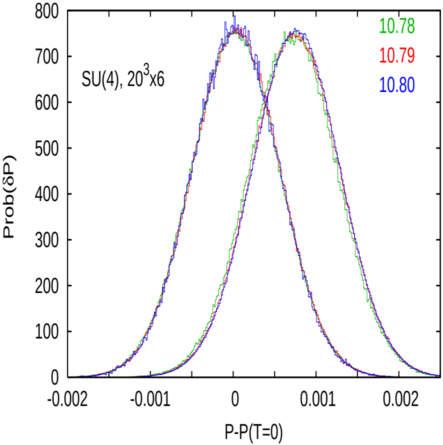


- Procedure stable in the metastability regime



Latent Heat: Results

- Results from $N_\tau = 8$ lattices:

N_c	β	L_h/T_c^4	$L_h/\Delta_{\rm max}$
3	6.0609	1.67(4)(4)	0.68(3)
4	11.08	4.32(6)(6)	0.82(2)
6	25.46	11.93(34)(5)	0.90(3)

- In excellent agreement with the $N_t = 8$ results of Teper et al. (at smaller volume, lower statistics)

  Lucini, Teper, Wenger, JHEP02, 033 ('05)

- A larger value obtained for SU(4) by Gavai. Uses bare coupling.

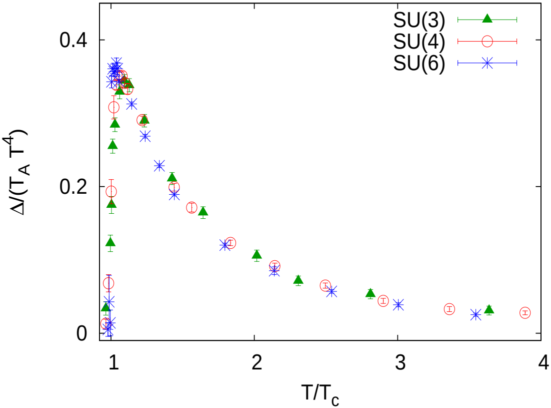
  R. Gavai, Nucl. Phys. B 633, 127 ('02)

$$\frac{L_h}{T_A T_c^4} = 0.388(3) - \frac{1.61(4)}{N_c^2}$$

(statistical error only)

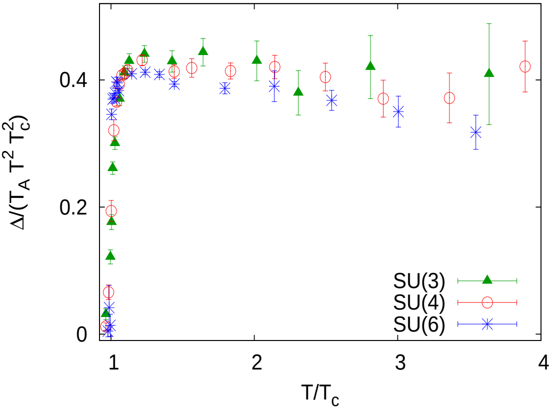
correction considerable at $N_c = 3$

$(\epsilon - 3p)/T^4$



- Good scaling with $T_A = N_c^2 - 1$ except very close to T_c
- Peak moves towards T_c with increasing N_c, requires better accuracy to quantify the movement.
- Substantial conformal symmetry breaking at $2T_c$: $\Delta^{1/4} \sim T$

$\epsilon - 3p \;\sim T^2$?

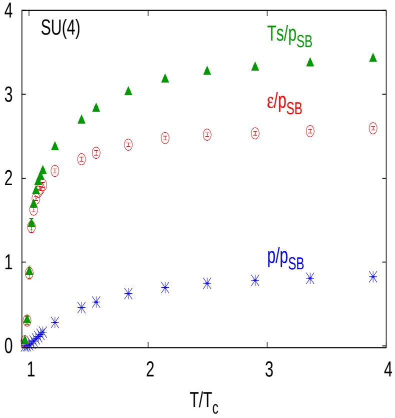


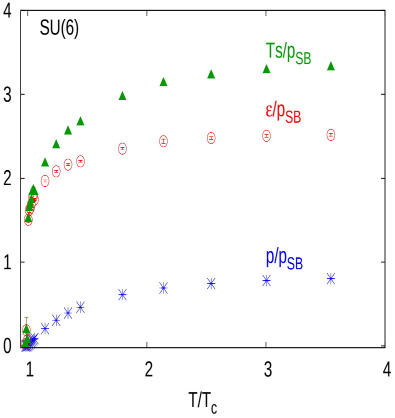
- $e - 3p \sim T^2$ observed for SU(3) over large T range.

  Meisinger, Miller, Ogilvie '02; Pisarski '07

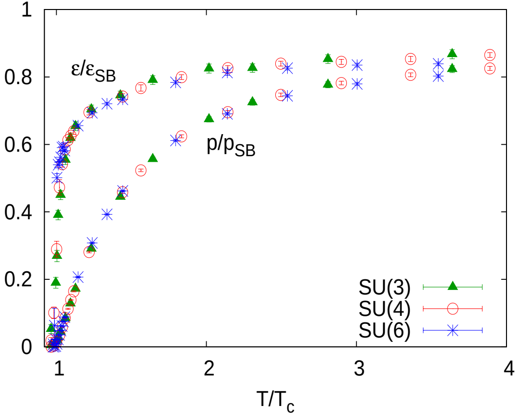
- Similar behavior observed for $N_c = 4{,}6$, subleading term also contribute at SU(6)

Bulk Thermodynamic Quantities

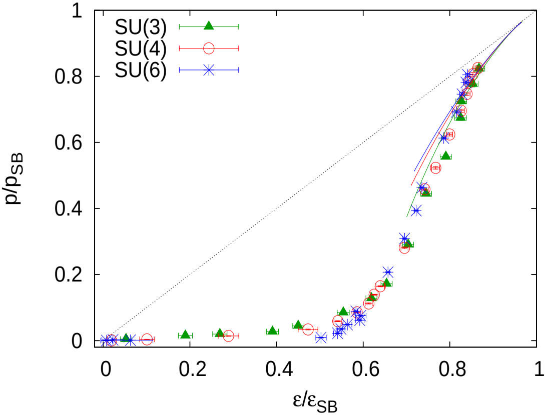




N_c scaling



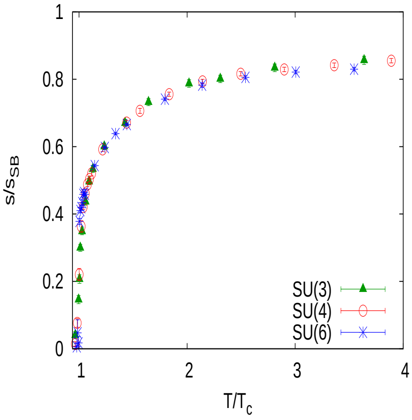
Approach to conformality

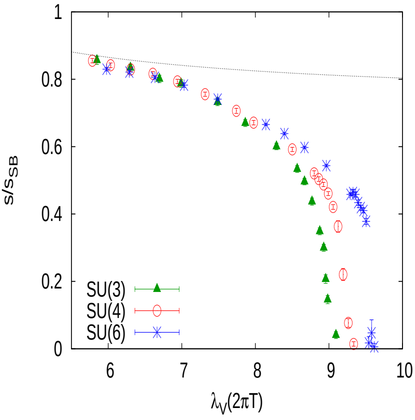


Closer to weak coupling theory (Laine & Schroeder, PR D 73('06))
than to conformal theory.

No evidence of a strongly coupled, near-conformal phase.

Entropy and 't Hooft scaling



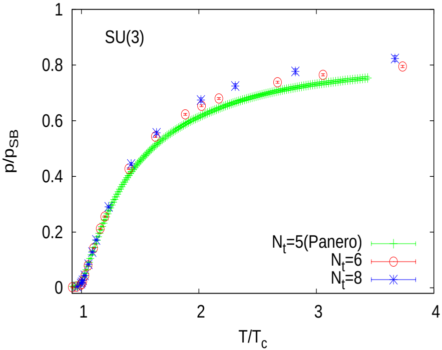


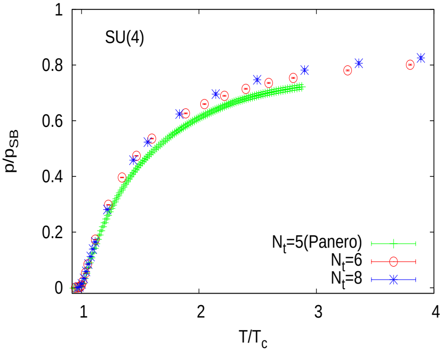
Strong N_c scaling is better than scaling with the 't Hooft coupling, which holds only at the higher temperatures.
The line is the result for $\mathcal{N} = 4$ SYM (Klebanov et al. '02)

Summary

- The thermodynamics of $SU(N_c)$ gauge theory is studied for $N_c \leq 6$, with emphasis on thermodynamic and continuum limit.
- The latent heat of the deconfinement transition is obtained. The transition for 3-color theory is found to be much weaker than that in $N_c > 3$.
- $(\epsilon - 3p)/T^4$ and other bulk thermodynamic quantities scale nicely with $T_A = N_c^2 - 1$, except very close to T_c, indicating that the correction to the leading N_c behavior small.
- The scaling of thermal quantities with N_c is much better than the scaling with the 't Hooft coupling $g^2(2\pi T)N_c$.
- The high temperature theory stays closer to the weak coupling prediction than conformality: no window for a strongly coupled, near-conformal phase in the $SU(N_c)$ gluon plasma.

Cutoff dependence for $N_t = 5$-6





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$N_t = 5$ results from Marco Panero (Thanks!)

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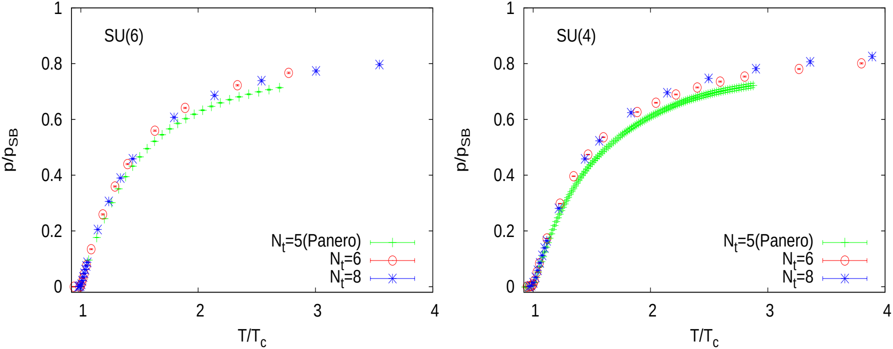

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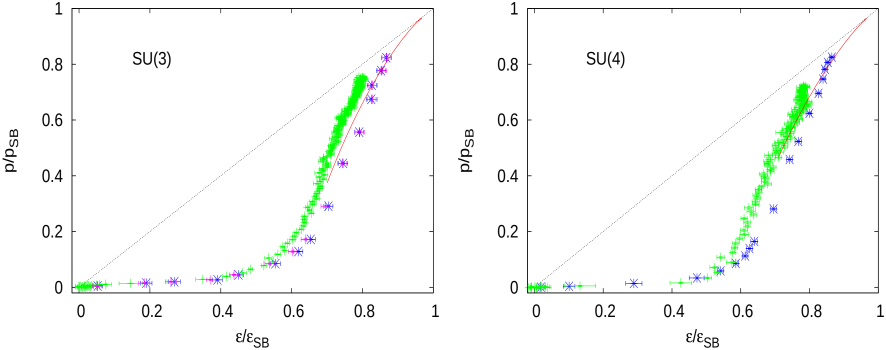
Approach to conformality: Cutoff effects



Green points are $N_t = 5$ results from Marco Panero

The others are $N_t = 8$

Red lines are weak coupling (Laine et al.)

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Red points in SU(3) use the beta function in Boyd et al.

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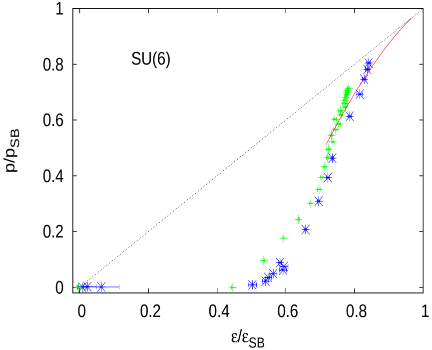

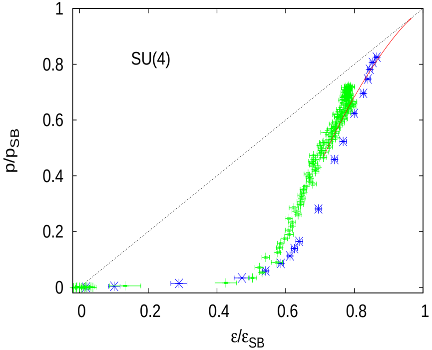

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