# Charm Current-Current Correlators in Twisted Mass Lattice QCD

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## Content

# Outline

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# Outline

## Motivation

- heavy quark masses and strong coupling constant: fundamental parameters of the Standard Model, essential input parameters for processes involving heavy quarks
- non-perturbative determination with high precision from moments of electromagnetic current of the charm quark at zero momentum: (Kühn 1001.5173 [hep-ph])
  $\rightarrow$ dispersion relations using $e^+e^- \rightarrow \gamma^* \rightarrow$ hadrons cross section measurements
  $\rightarrow$ comparison of perturbative calculation with experiment
- ab initio calculations in LQCD provide control over non-perturbative effects of strong interaction
  $\Rightarrow$ alternative approach with less experimental input
- sub-percent level precision reached with HISQ discretised fermion action (HPQCD 1004.4285 [hep-lat])

Aim: combination of pQCD and Twisted Mass LQCD to extract fundamental Standard Model parameters

## Low momentum expansion of Π in pQCD

- hadronic contributions to vacuum polarisation functions from charm quark currents

$$q^2\,\Pi^\kappa = i\int d^4x e^{iqx}\langle 0|\,T\{J^\kappa(x)\,J^\kappa(0)\}\,|0\rangle$$

$$\left(-q^2 g_{\mu\nu} + q_\mu q_\nu\right)\Pi^\delta + q_\mu q_\nu \Pi^\delta_L = i\int d^4x\, e^{iqx}\langle 0|\,T\{J^\delta_\mu(x)\,J^\delta_\nu(0)\}|0\rangle\,,$$

with $\delta = v, a$, $\kappa = p, s$, $J^p = \bar\psi\gamma_5\psi$, $J^s = \bar\psi\psi$, $J^v_\mu = \bar\psi\gamma_\mu\psi$, $J^a_\mu = \bar\psi\gamma_\mu\gamma_5\psi$

- low momentum region: expansion of $\Pi^{\kappa,\,\delta}$ in $z = \frac{q^2}{4m_c^2(\mu)}$ in $\overline{MS}$ scheme

$$\Pi^{\kappa,\delta}(q^2) = \frac{3}{16\pi^2}\sum_{k\geq -1}\bar C_k^{\kappa,\delta} z^k\,,\quad \bar C_k = \sum_{m\geq 0}\left(\frac{\alpha_s}{\pi}\right)^m \bar C_k^{(m)}\left(\log\left(\frac{m_c^2(\mu)}{\mu^2}\right)\right)$$

- coefficients for (axial) vector and (pseudo-)scalar correlator available up to third order in $\alpha_s$ (cf. e.g. Maier et al. 0907:2117 [hep-ph])

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# Twisted Mass Lattice QCD (JHEP 0108:058,2001)

- Wilson-type fermion discretisation for $n_f = 2$ mass degenerate quark flavours *up*, *down*:

$$\mathcal{S}_{tm} = a^4 \sum_x \bar{\psi}(x) \left[ D_W + m_0 + i\mu_0\gamma_5\tau^3 \right] \psi(x) \,;$$

- $m_0$ bare (untwisted) quark mass, $\mu_0$ twisted mass parameter, $\tau^3$ 3rd Pauli matrix acting in flavour space
- automatic $\mathcal{O}(a)$ improvement of physical observables, if $m_0 \to m_{cr}$ $\Leftrightarrow$ "maximal twist" (JHEP 0108:058,2001)
- partial quenching: no strange, charm quark in the sea, but heavy charm doublet added in valence sector

## Moments of current correlators in tmLQCD

- renormalised moments from charmed currents at $\vec{p} = 0$

$$C_0^\delta(t) = a^6 \frac{1}{L^3} \sum_{\vec{x}} \langle J_c^\delta(\vec{x}, t) J_c^\delta(\vec{0}, 0) \rangle$$

- at maximal twist $C_R^p = Z_S^2 C_0^p$ and $C_R^s = Z_p^2 C_0^s$; $\mu_{c\,R} = \mu_{c\,0}/Z_P$

$$G_n^p = \left(\frac{Z_S}{Z_P}\right)^2 \sum_{t/a=-N_t/2+1}^{N_t/2-1} \left(\frac{t}{a}\right)^n (a\mu_{c\,0})^2 C^p(t)$$

$$G_n^s = \sum_{t/a=-N_t/2+1}^{N_t/2-1} \left(\frac{t}{a}\right)^n (a\mu_{c\,0})^2 C^s(t)$$

$$G_n^\delta = Z_\delta^2 \sum_{t/a=-N_t/2+1}^{N_t/2-1} \left(\frac{t}{a}\right)^n C^\delta(t), \qquad \delta = v,\, a$$

## Moments of current correlators in tmLQCD

- dimensional analysis implies

$$G_n^\kappa = \frac{g_n^\kappa(\alpha_s(\mu), m_c(\mu)/\mu)}{(am_c(\mu))^{n-4}} + \mathcal{O}((a\, m_c)^m)$$

$$G_n^\delta = \frac{g_n^\delta(\alpha_s(\mu), m_c(\mu)/\mu)}{(am_c(\mu))^{n-2}} + \mathcal{O}((a\, m_c)^m)$$

  for $\kappa = p,\ s,\ \delta = v,\ a$ and $m_c(\mu)$ and $\alpha_s(\mu)$ renormalized in $\overline{MS}$-scheme

- comparison of left-hand side to expansion of the right-hand side in powers of $\alpha_s(\mu)$, $\log(m_c(\mu)/\mu)$ in the continuum limit

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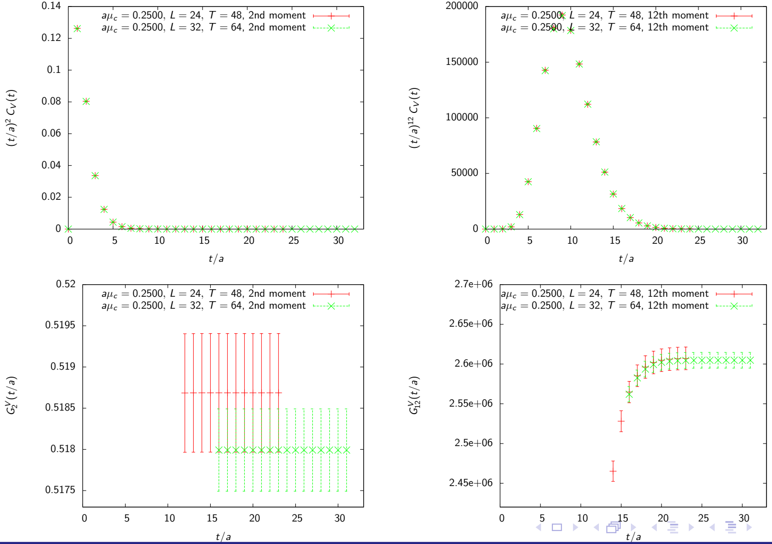
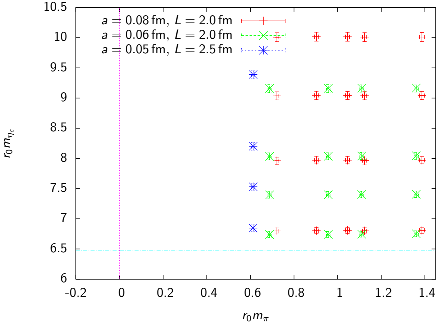
# Finite T Cut-off and Volume Effects on $G_n$



# Investigation of light and heavy quark mass dependence

- parametrisation of light quark mass dependence in terms of $r_0 m_\pi$, heavy quark mass dependence in terms of $r_0 m_{\eta_c}$
- interpolation to common reference masses, continuum extrapolation for fixed $(r_0 m_\pi^{\text{ref}}, r_0 m_{\eta_c}^{\text{ref}})$, finally extrapolation to zero pion mass and $r_0 m_{\eta_c} = r_0 m_{\eta_c}^{\text{phys}}$




- $270\,\text{MeV} \lesssim m_\pi \lesssim 600\,\text{MeV}$
- $3.2\,\text{GeV} \lesssim m_{\eta_c} \lesssim 4.6\,\text{GeV}$

## Comparison of Lattice Vector Moments with Experiment

- moments of the charm vector current accessible in experiment via measurement of $R_c(s)$:

$$M_n = \int \frac{ds}{s^{n+1}} R_c(s)\,, \qquad R_c(s) = \sigma(e^+e^- \to c\bar{c})/\sigma_{pt}$$

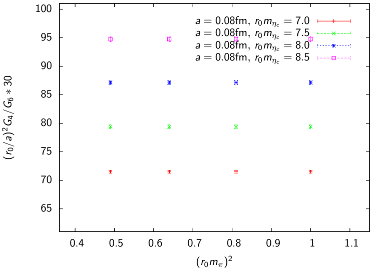
- related to derivatives of the charm vacuum polarization function via dispersion relations

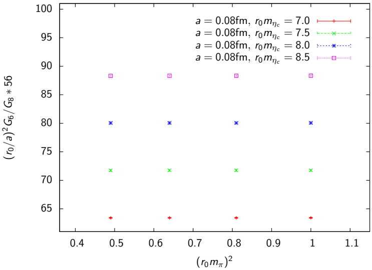
$$M_n = \frac{12\pi^2}{n!}\left(\frac{d}{dq^2}\right)^n \Pi_c(q^2)\Big|_{q^2=0} = \frac{9}{4} Q_c^2 \frac{\bar{C}_n}{(2m_c)^{2n}}$$

$$G_{2n+2} \propto \left(\frac{d}{dq_0}\right)^{2n+2} \left(q^2\Pi_c(q^2)\right)\Big|_{\vec{q}=0,q_0=0} \propto (2n)!\frac{\bar{C}_n}{(2m_c)^{2n}}$$

$$R_n = r_0^2 \frac{M_n}{M_{n+1}} = \left(\frac{r_0}{a}\right)^2 \frac{G_{2n+2}}{G_{2n+4}}(2n+4)(2n+3) = \frac{\bar{C}_n}{\bar{C}_{n+1}}(2r_0 m_c)^2$$

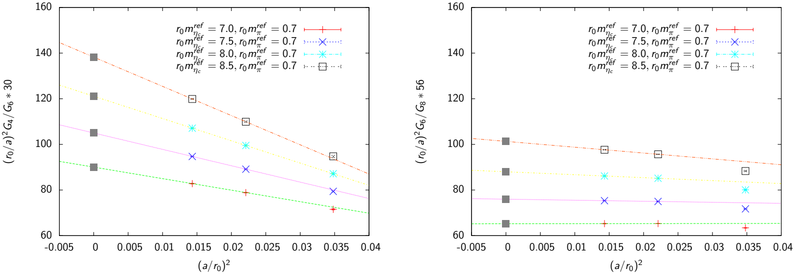
# (continued) - Pion Mass Dependence



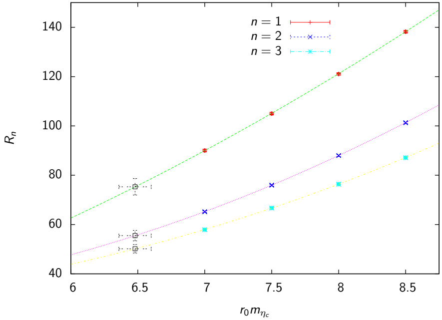

No significant dependence on the reference pion masses.

# (continued) - Continuum Extrapolation




Continuum extrapolation with two smallest lattice spacings.

## (continued) - Extrapolation to the Physical Point




parametrisation of $(r_0 m_\pi, r_0 m_{\eta_c})$ dependence:

$$R_n(r_0 m_\pi, r_0 m_{\eta_c}) = \sum_{i=0}^{N} \left(a_{i0} + a_{i1}(r_0 m_\pi)^2\right) (r_0 m_{\eta_c})^i,\ N = 2,\, 3$$

## (continued) - Current Status

Comparison with experimental values (Kühn et al. hep-ph/0109084, hep-ph/0702103)

| $n$ | $R_n^{exp}$ | $R_n^{lat}$ | $m_c(3\mathrm{GeV})$ |
|---|---|---|---|
| 1 | 68.4(1.6)(2.6) | 75.3(3.3) | |
| 2 | 53.9(1.5)(2.0) | 55.5(2.1) | 1.019(26)(17)(19) |
| 3 | 49.6(1.5)(1.8) | 50.2(1.7) | 1.011(23)(21)(19) |
| 4 | 47.7(4.6)(1.8) | 48.0(1.6) | 1.012(23)(22)(19) |
| 5 | 46.8(4.6)(1.7) | 47.0(1.6) | 1.016(24)(23)(19) |
| 6 | 46.2(4.6)(1.7) | 46.4(1.6) | 1.017(24)(25)(19) |

- $m_c$ obtained as solution of $r_0 m_c(\mu) = \frac{1}{2}\sqrt{(R_n^{lat})}/\sqrt{(\bar{C}_n/\bar{C}_{n+1})}$ and $\sqrt{(\bar{C}_n/\bar{C}_{n+1})} = \sum_{i=0}^{3} c_i(\log(m_c(\mu)/\mu))\,(\alpha_s(\mu)/\pi)^i$
- $\alpha_s(3\mathrm{GeV}, n_f = 4) = 0.252(10)$ used to extract $m_c$
- errors on $m_c$ are given by variation of (1) $R_n^{lat}$, (2) $\alpha_s$ and (3) $r_0$

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# Summary and Outlook

- tmLQCD allows for the calculation of renormalized moments for (pseudo) scalar and (axial) vector currents
- ratios of consecutive moments of the charm vector current are compatible with experimental values for all but the lowest $n = 1$
- but lowest moments are necessary for small error bands (HPQCD 0805.2999 [hep-lat])
- dependence on light sea quark mass or $r_0 m_\pi$ is very weak
- dependence on heavy quark mass or $r_0 m_{\eta_c}$ is crucial: necessitates refined tuning, additional data close to and below $r_0 m_{\eta_c}^{phys}$
- once these issues are resolved: proceed with analysis of also (pseudo) scalar and axial vector currents
- repeat the analysis on ETMC's $N_f = 2 + 1 + 1$ configurations

Thank you very much for your attention.