Magnetic Moment of Negative-Parity Baryons from Lattice QCD

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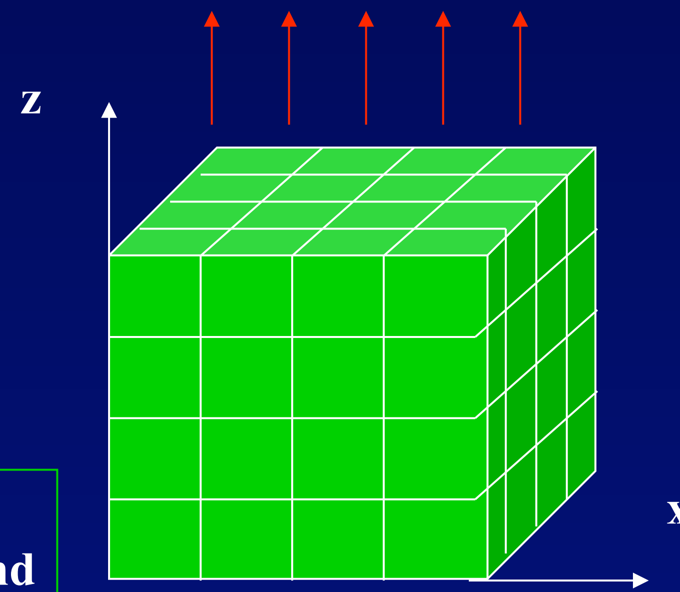
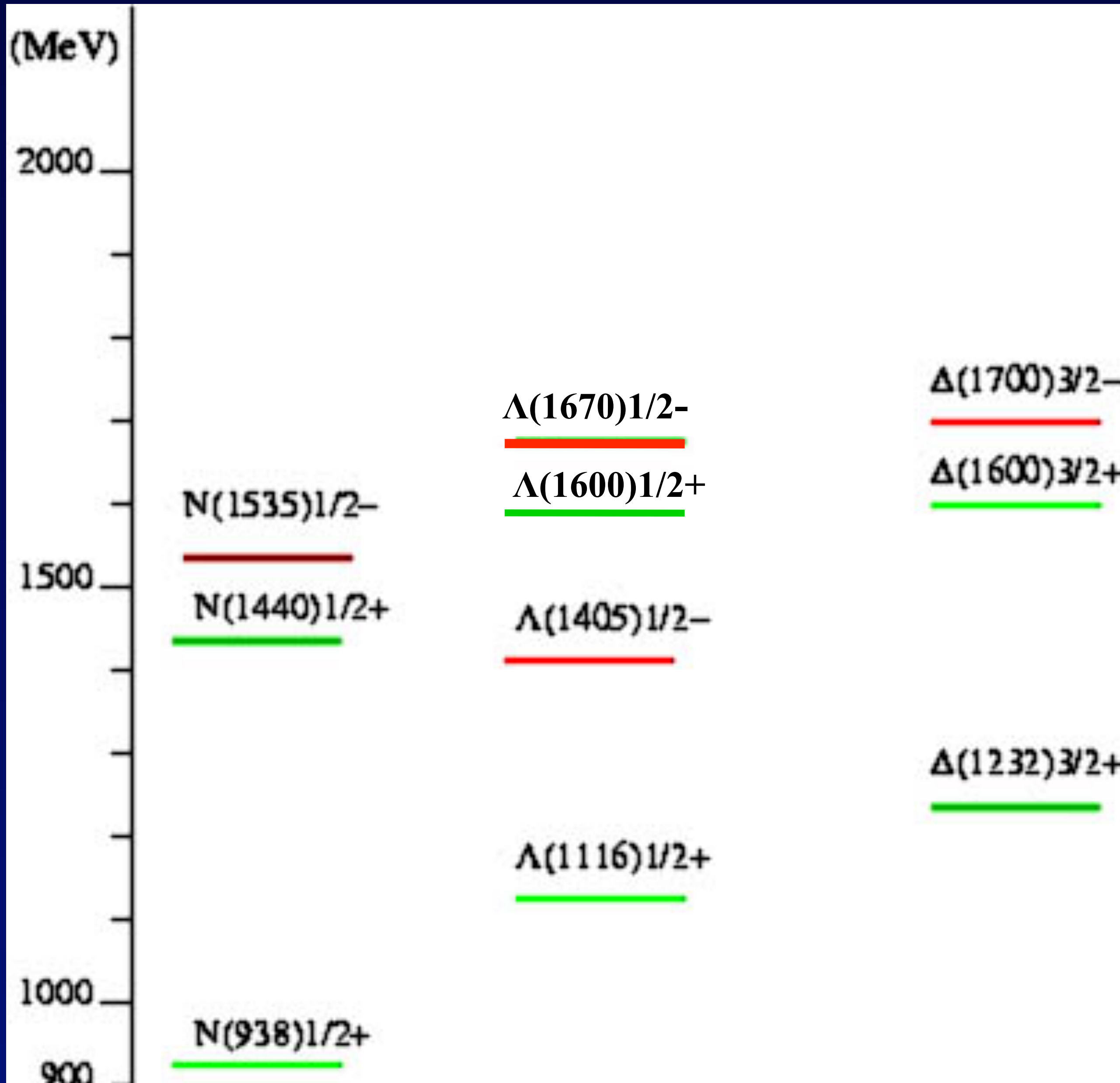
- Physics motivation
- Background field method
- Some results



Thanks: U.S. Department of Energy, National Science Foundation, and computing resources from NERSC and USQCD

Excitations of the Nucleon



Octet Baryons

State (spin-parity)	Mass (MeV)	μ (Expt) (μ_N)
p (1/2 +)	N (938)	2.79
p* (1/2 -)	S^1_{11}(1535)	
n (1/2 +)		- 1.91
n* (1/2 -)	S^0_{11}(1535)	
Λ_O (1/2 +)	Λ(1115)	- 0.61
Λ^*_O (1/2 -)	Λ(1670)	
Λ_S (1/2 -)	Λ(1405)	
Λ^*_S (1/2 +)	Λ (?)	
Σ^+ (1/2 +)	Σ (1190)	2.9
Σ^{+*} (1/2 -)		
Σ^0 (1/2 +)		0.8
Σ^{0*} (1/2 -)		
Σ^- (1/2 +)		- 1.5
Σ^{-*} (1/2 -)		

Hadron Structure via Background Fields

Interaction energy of a hadron in the presence of external electromagnetic fields:

$$H = -\vec{\mu}\cdot\vec{B} - \frac{1}{2}\alpha E^2 - \frac{1}{2}\beta B^2$$
$$-\frac{1}{2}\gamma_{E1}\sigma\cdot\vec{E}\times\dot{\vec{E}} - \frac{1}{2}\gamma_{M1}\sigma\cdot\vec{B}\times\dot{\vec{B}}$$
$$+\gamma_{E2}\sigma_i E_{ij} B_j - \gamma_{M2}\sigma_i B_{ij} E_j$$
$$-\frac{1}{12}\alpha_{E2}E_{ij}^2 - \frac{1}{12}\beta_{M2}B_{ij}^2 + \cdots$$

Time and spatial derivatives: $\dot{E} = \frac{\partial E}{\partial t},\ E_{ij} = \frac{1}{2}(\nabla_i E_j + \nabla_j E_i)$, etc

Probe of internal structure of the system in increasingly finer detail.

μ, α, β:

static bulk response

others :

spatial and time resolution

Mass shifts: $\delta m(B) = m(B) - m(0) = c_1 B + c_2 B^2 + c_3 B^3 + c_4 B^4 + \cdots$

Introduction of an external electromagnetic field on the lattice

- Minimal coupling in the QCD covariant derivative in Euclidean space

$$D_\mu \rightarrow \partial_\mu + gG_\mu + qA_\mu$$

- Recall that SU(3) gauge field is introduced by the link variables

$$U_\mu(x) = \exp(iagG_\mu)$$

- It suggests multiplying a U(1) phase factor to the links

$$U'_\mu(x) = \exp(iaqA_\mu)U_\mu$$

- This should be done in two places where the Dirac operator appears: both in the dynamical gauge generation and quark propagator generation

For Example

$$\vec{B} = \nabla \times \vec{A}$$

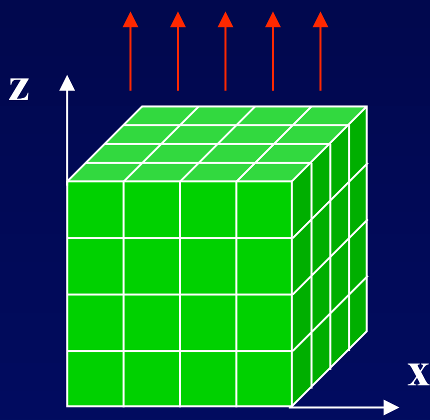
$$\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}$$

- To apply magnetic field **B** in the z-direction, one can choose the 4-vector potential

$$A_\mu \equiv (\phi, \vec{A}) = (0,0,Bx,0)$$

then the y-link is modified by a x-dependent phase factor

$$U_y \rightarrow \exp(iqaBx)U_y$$



- To apply electric field **E** in the x-direction, one can choose the 4-vector potential

$$A_\mu = (0, Et,0,0)$$

then the x-link is modified by a t-dependent phase factor

$$U_x \rightarrow \exp(iqaEt)U_x$$

Computational Demands

$$U_y \rightarrow \exp(iqaBx)U_y$$

- Consider quark propagator generation

$$\frac{\int DG_\mu \det(\not{D} + m_q)\, e^{-S_G} (\not{D} + m_q)^{-1}}{\int DG_\mu \det(\not{D} + m_q)\; e^{-S_G}}$$

$$D_\mu \rightarrow \partial_\mu + gG_\mu + qA_\mu$$

- Fully dynamical: For each value of external field, a new dynamical ensemble is needed that couples to u-quark (q=1/3), d- and s-quark (q=-2/3) in the sea. Valence quark propagator is then computed on the ensembles with matching values.
- Re-weighting: Perturbative expansion of action in terms of external field. Can use existing dynamical ensembles.
- U(1) quenched: no charging the sea, only coupling to the valence on:
  - Dynamical QCD ensembles
  - Quenched QCD ensembles

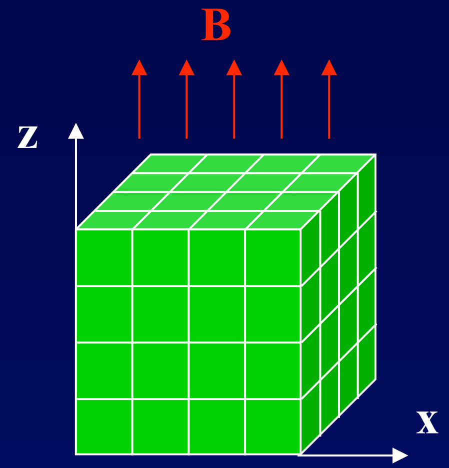
What about boundary conditions?

$$U_y \to \exp(iqaBx)U_y$$

- On a finite lattice with periodic boundary conditions, to get a constant magnetic field, B has to be quantized

$$qBa^2 = \frac{2\pi n}{N_x N_y}, \quad n = 1,2,3,\cdots$$

to ensure that the magnetic flux through the plaquettes in the x-y plane is constant.



- But, for $N_x = N_y = 24$ and $1/a = 2$ GeV, the quantized field values are too strong for small-field expansion. So we have to abandon the quantization condition, and work with much smaller fields.

- To minimize the boundary effects, we work with fixed b.c. in x-direction, so that quarks originating in the middle of the lattice has little chance of propagating to the edge.

Magnetic moment in background field

- For a particle of spin s and mass m in small fields,

$$E_{\pm} = m \pm \mu B$$

where upper sign means spin-up and lower sign spin-down, and

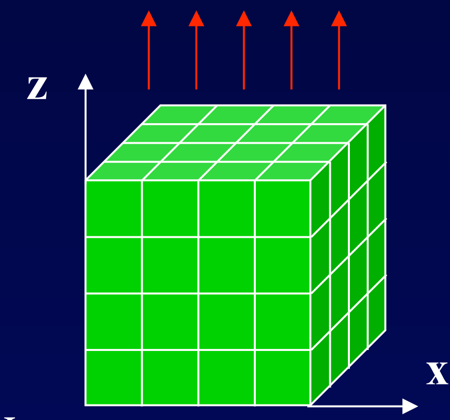
$$\mu = g \frac{e}{2m} s$$

- g factor (magnetic moment in natural magnetons) is extracted from

$$g = m \frac{(E_{+} - m) - (E_{-} - m)}{eBs}$$

Lattice details

$$U_y \to \exp[iqaB(x-x_0)]U_y$$



- Standard Wilson gauge action
  - 24^3x48 lattice, β=6.0 (or a ≈ 0.1 fm)
  - 990 configurations
- Standard Wilson fermion action
  - Set 1: Pion mass about 500, 646, 782, 894, 1010, 1434 MeV
  - Set 2: Pion mass about 338, 362, 384, 405, 444, 693 MeV
  - Boundary conditions: Dirichlet in x, y and t, periodic in z
  - Quark source location (t,x,y,z)=(0,12,12,12)
  - Polyakov loop origin x_0=12
- The following 5 dimensionless numbers $\eta \equiv qBa^2$ =+0.00036, -0.00072, +0.00144, -0.00288, +0.00576 correspond to 4 small B fields

  eBa^2 = -0.00108, 0.00216, -0.00432, 0.00864 for both u and d (or s) quarks.
  - Small in the sense that the mass shift is only a fraction of the proton mass: μB/m ~ 1 to 5% at the smallest pion mass. In physical units, B ~ 10^{13} Tesla.

Baryon Two-point Correlation Function

$$G(t) = \sum_{\vec{x}} \langle vac \,|\, T\left[\chi_1(x)\overline{\chi_1}(0)\right] vac\rangle$$

$$= (1+\gamma_4)[A_+ e^{-m_+(t-t_0)} + bA\_e^{-m_-(N_t+t_0-t)}]$$

$$+ (1-\gamma_4)[bA_+ e^{-m_+(N_t+t_0-t)} + A\_e^{-m_-(t-t_0)}]$$

b = 0 fixed (Dirichlet)
b = 1 periodic
b = -1 anto-periodic

$$\chi_1 = \varepsilon_{abc}(u^{aT} C\gamma_5 d^b)u^c$$

Effective mass plots for **N(1/2+) and N*(1/2-)** states

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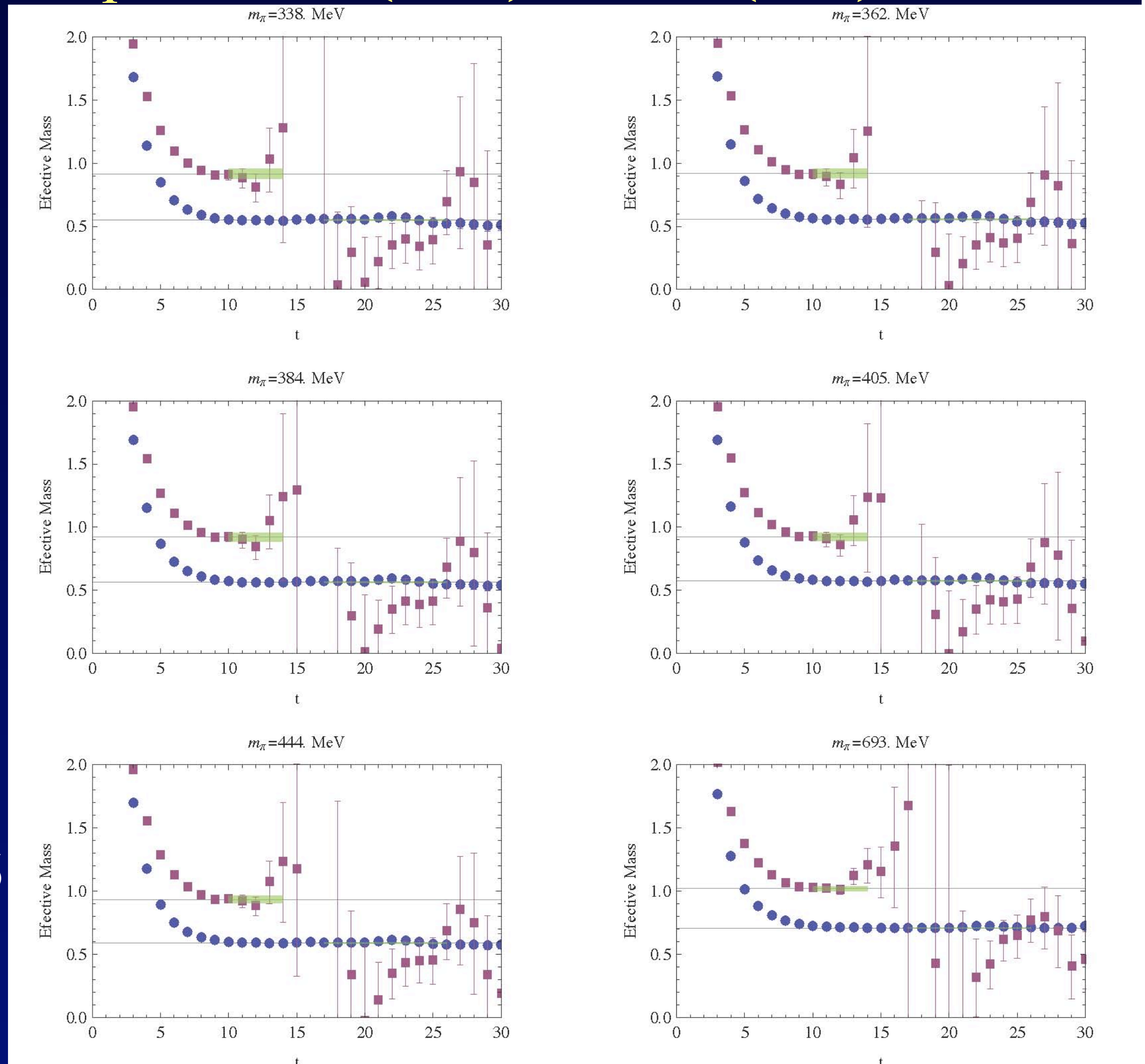
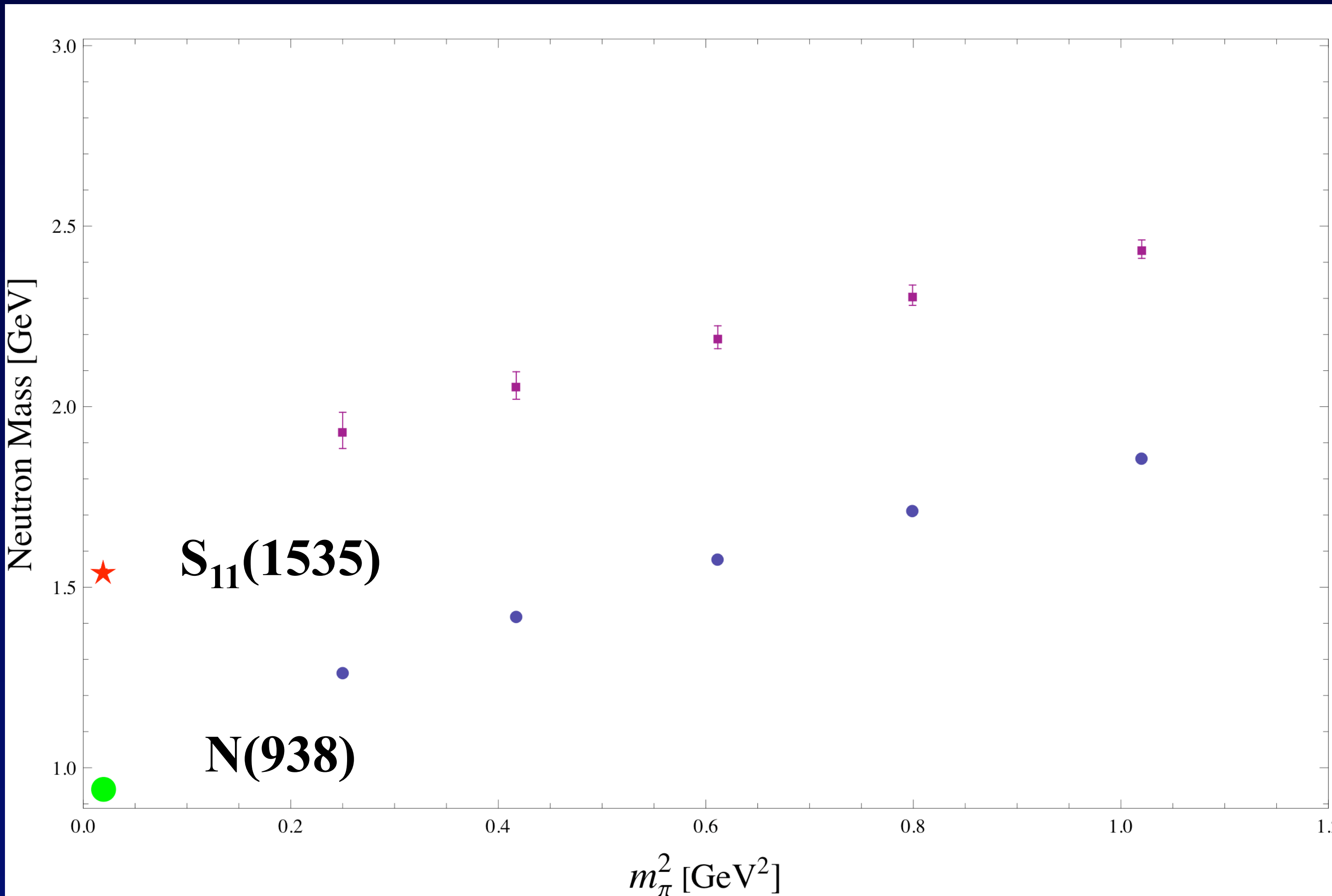
[redacted]



- Good signal for N(1/2+): fit 17-26
- Noisy signal for N(1/2-): fit 10-14

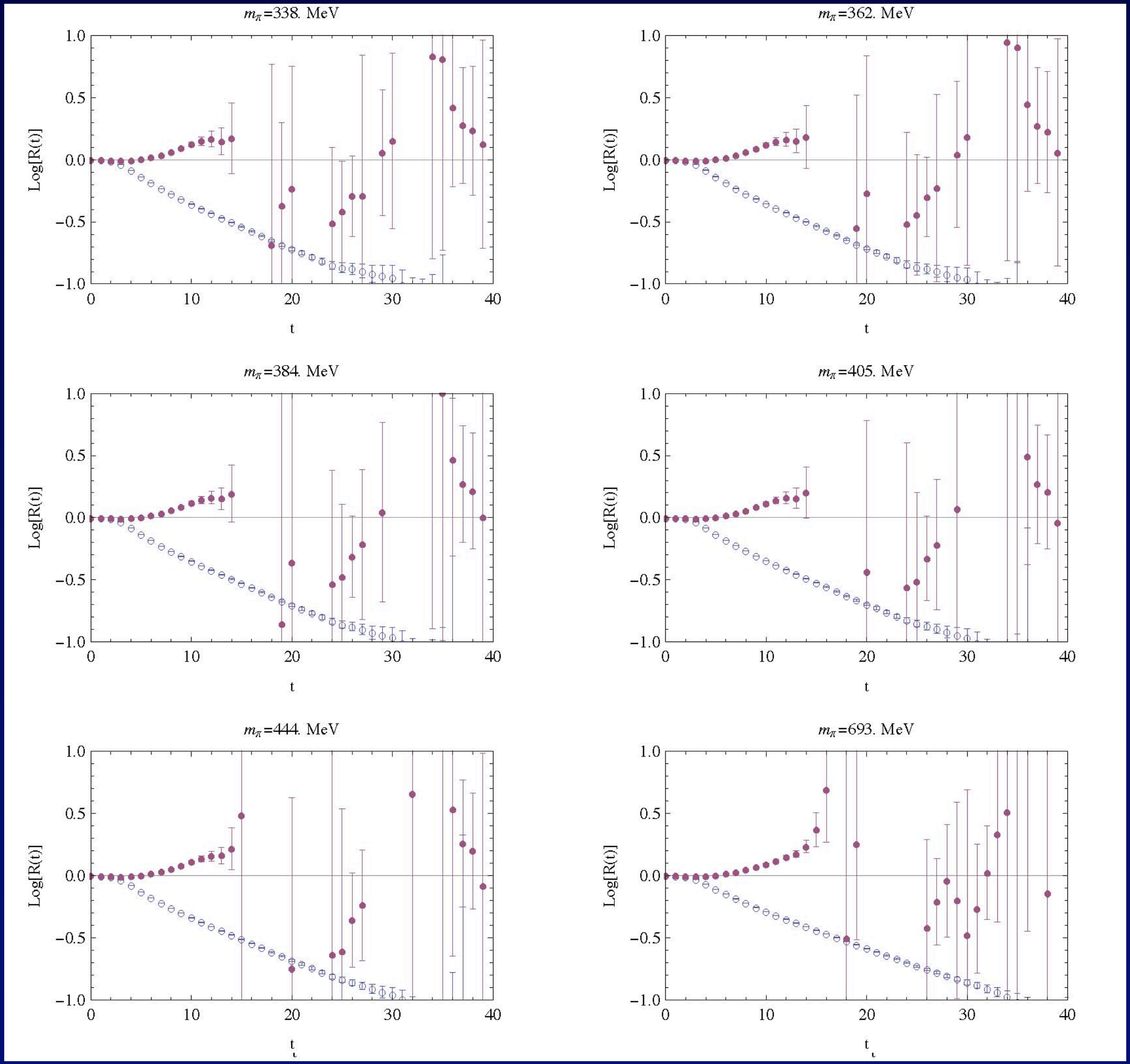
Masses for **N(1/2+) and N*(1/2-)** states



Ratio of correlation functions for p(1/2+) and p*(1/2-) (slope is related to g factor)

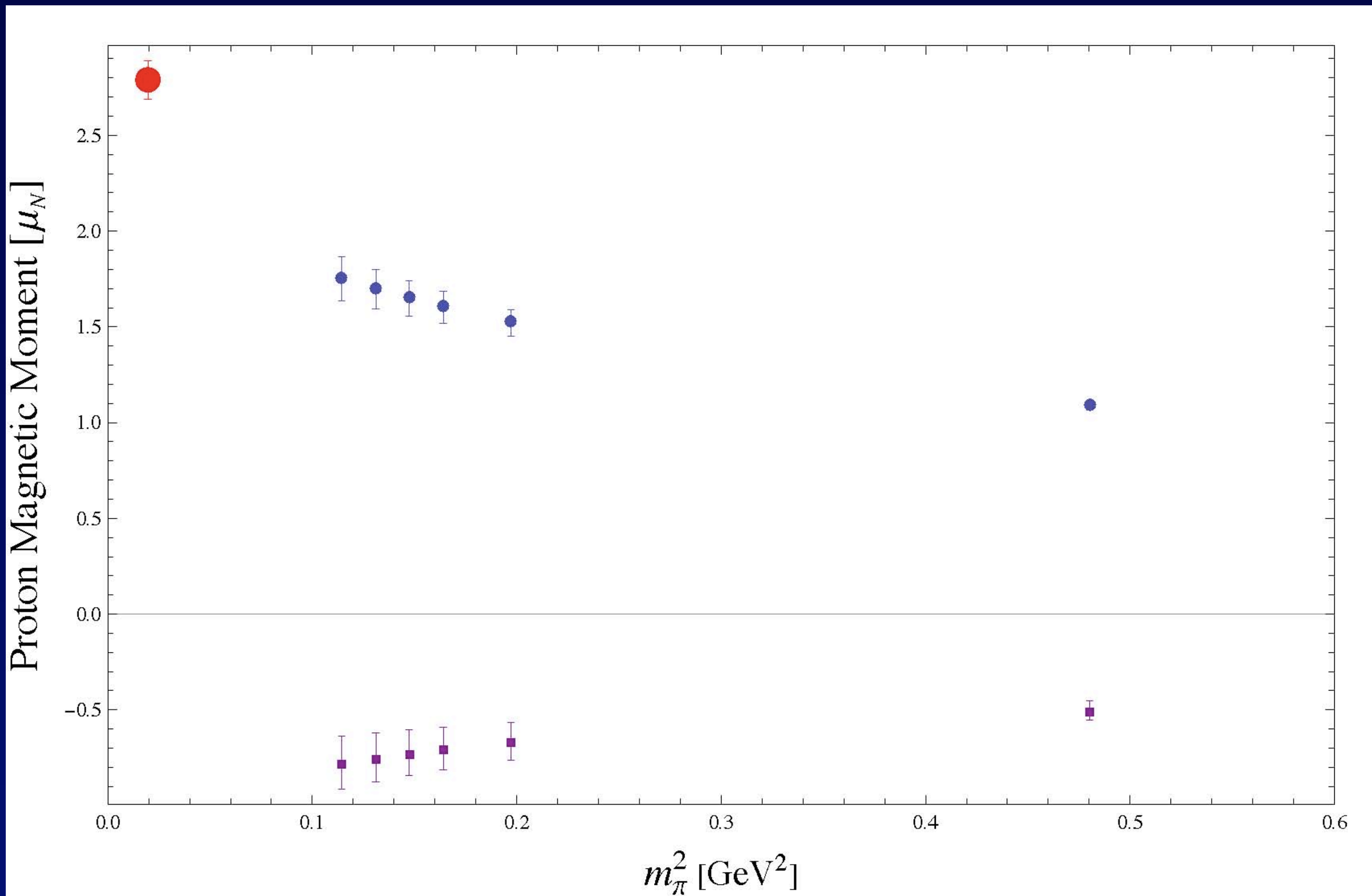
$$R(t) = \frac{G_+(B)}{G_-(B)} \Big/ \frac{G_+(-B)}{G_-(-B)}$$
$$\propto e^{-2\Delta m t}$$

$$\Delta m = [E_+(B) - E_-(B)]$$
$$-[E_+(-B) - E_-(-B)]$$
$$= g\frac{eBs}{m}$$

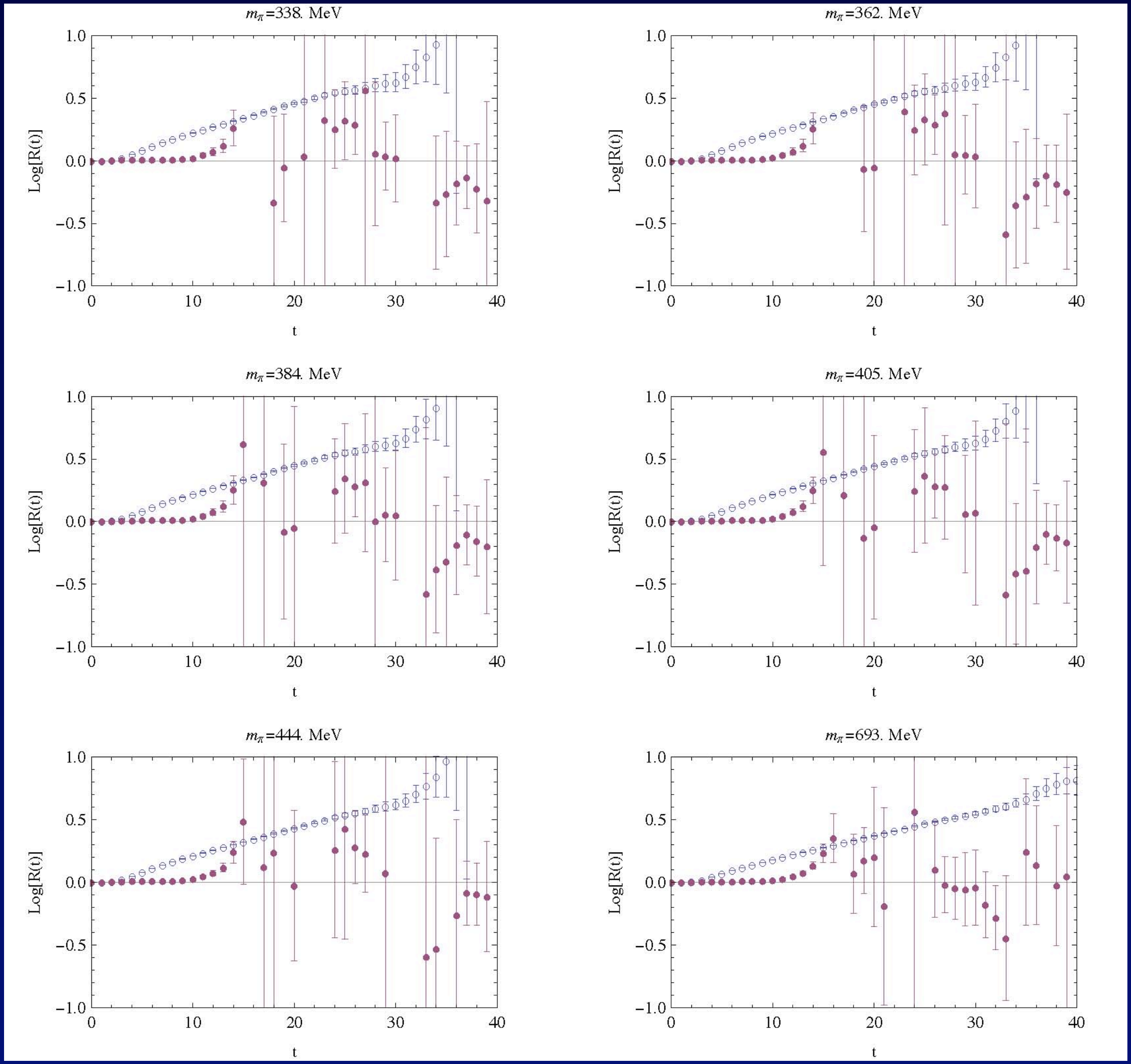


Opposite sign

Magnetic Moments for **p(1/2+) and p*(1/2-)** states

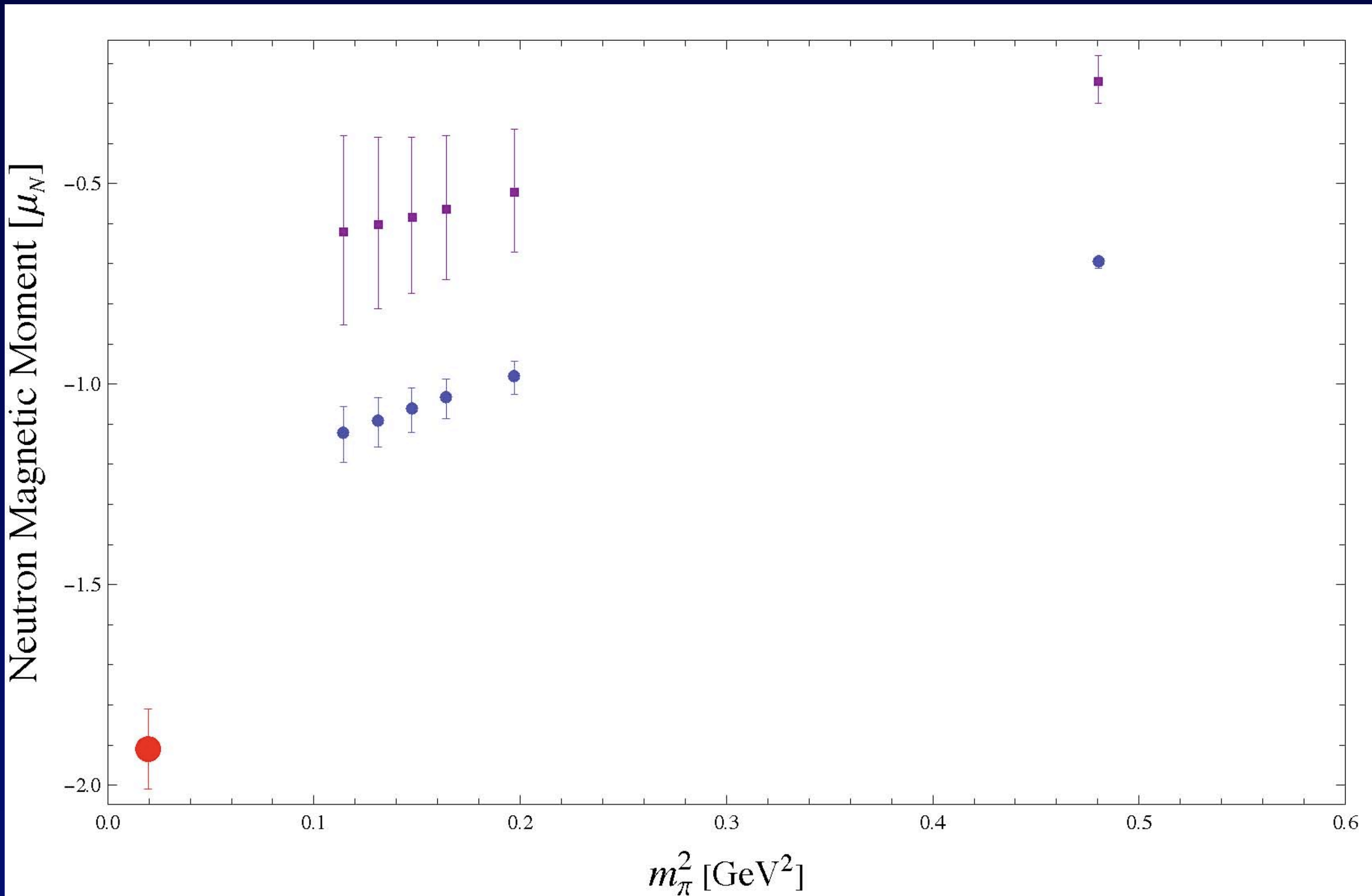


Ratio of correlation functions for n(1/2+) and n*(1/2-) (slope is related to g factor)

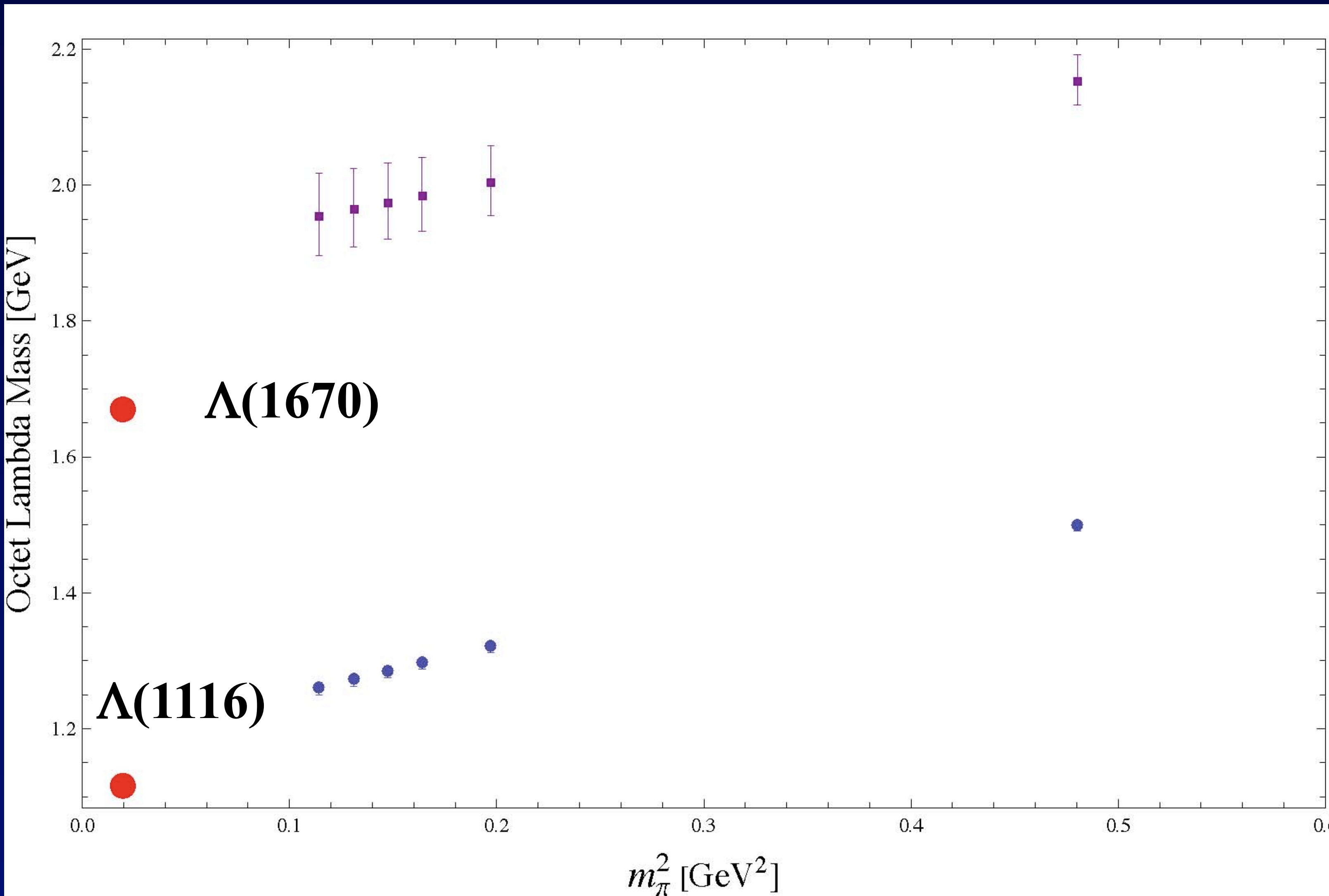


Same sign

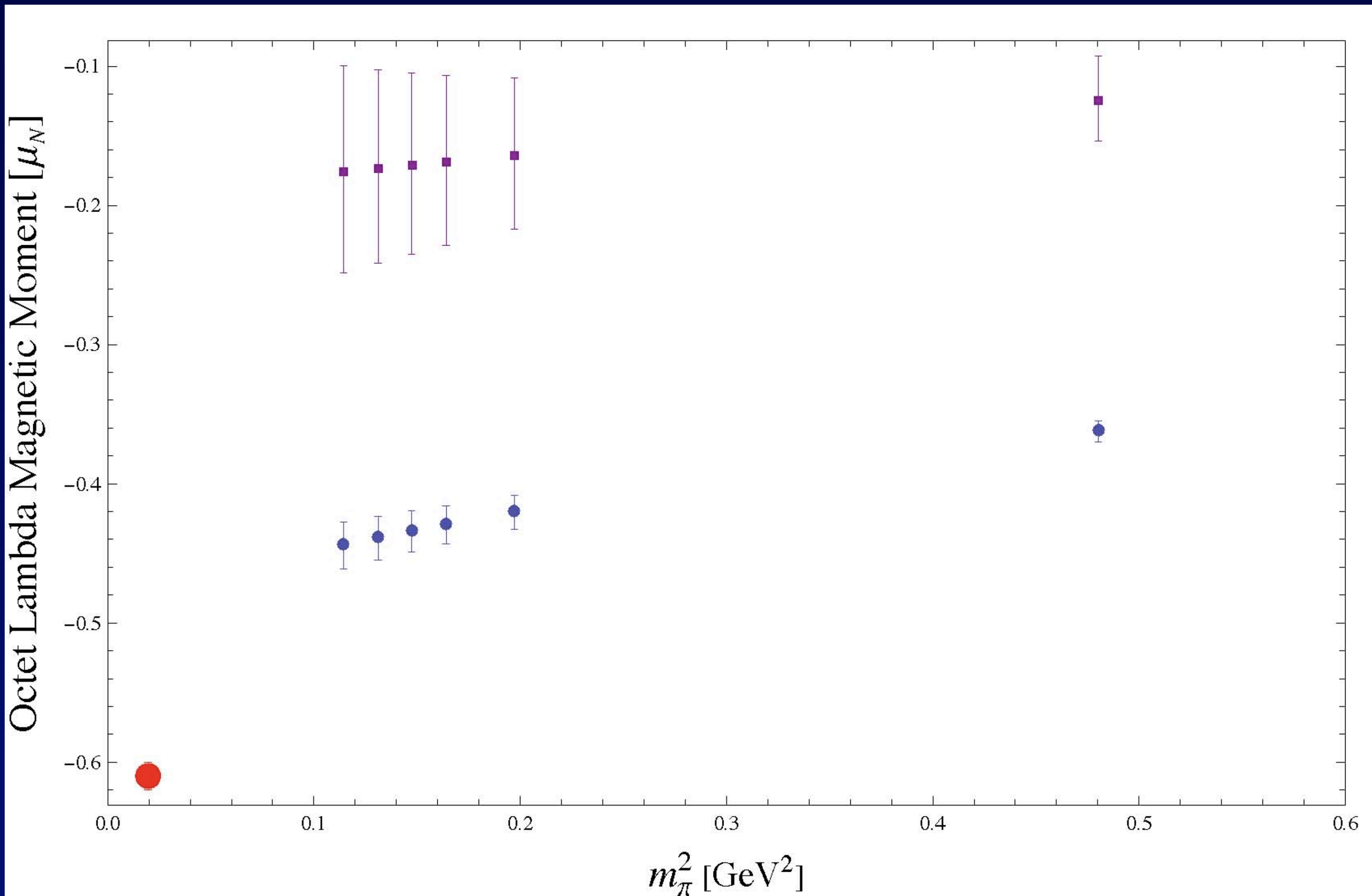
Magnetic moments for **n(1/2+) and n*(1/2-)** states



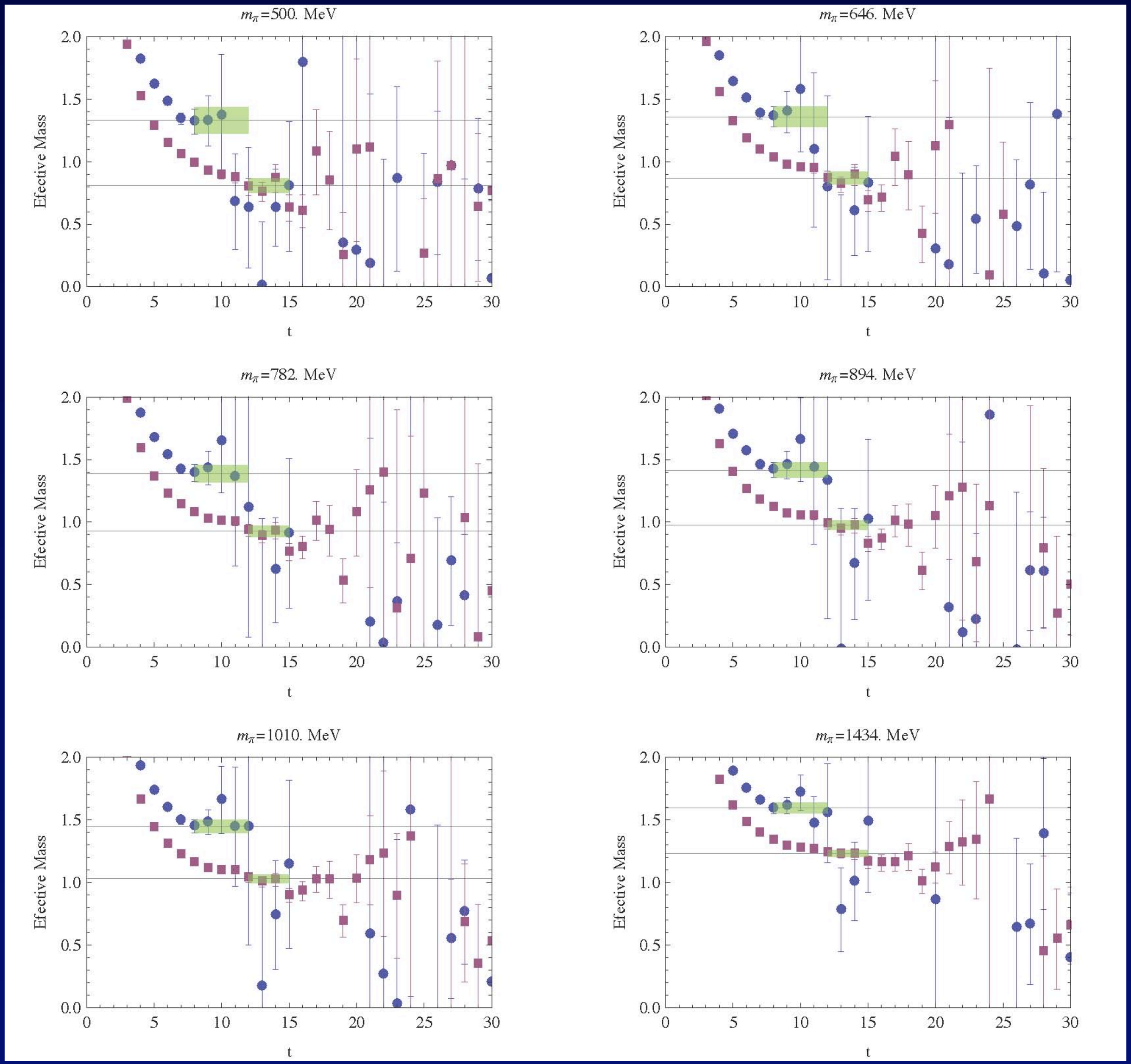
Masses for Octet Λ_O(1/2+) and Λ^*_O(1/2-) states



Magnetic Moments for Octet Λ_O(1/2+) and Λ^*_O(1/2-)

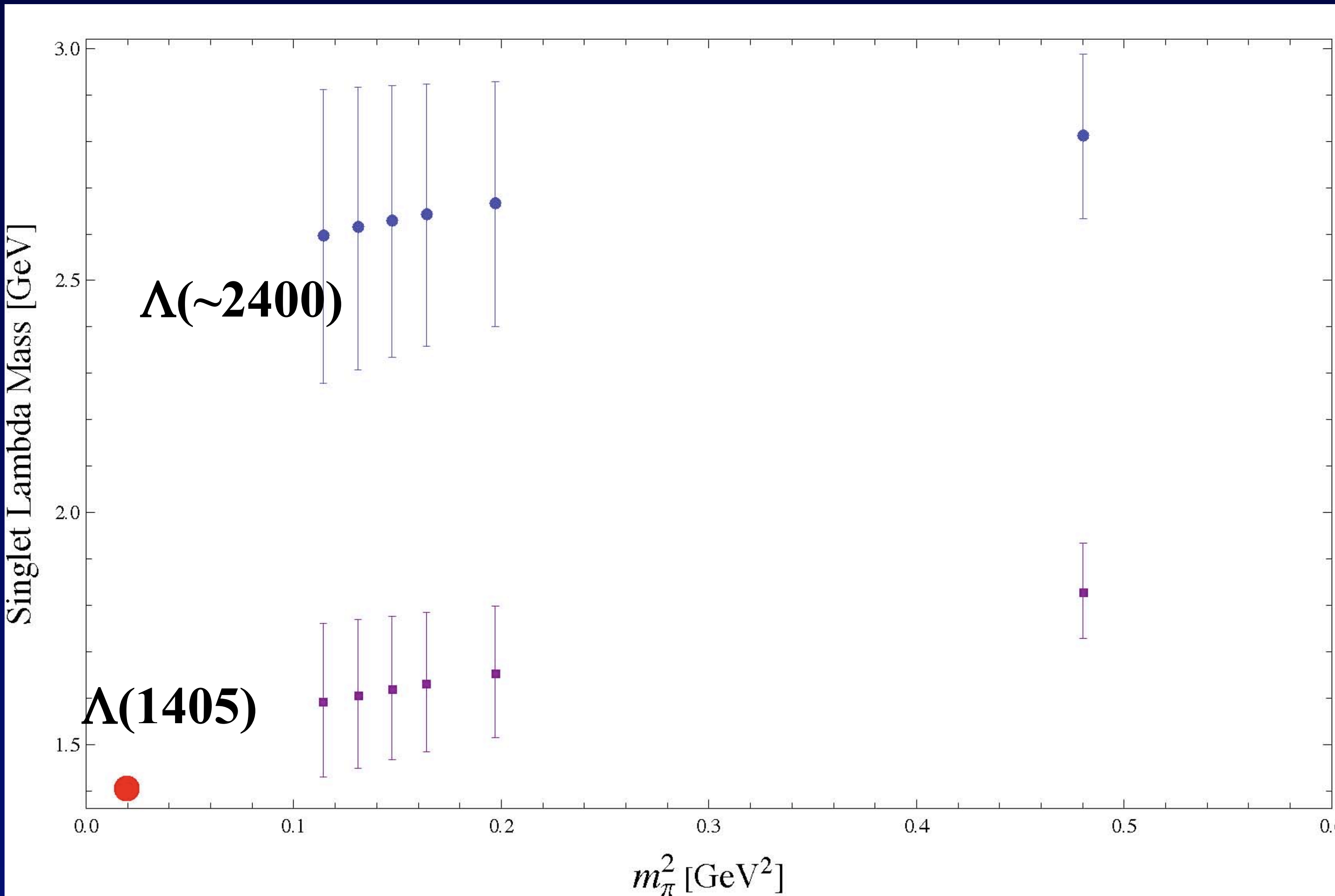


Effective mass plots for **Λ_S(1/2-) and Λ^*_S(1/2+)**

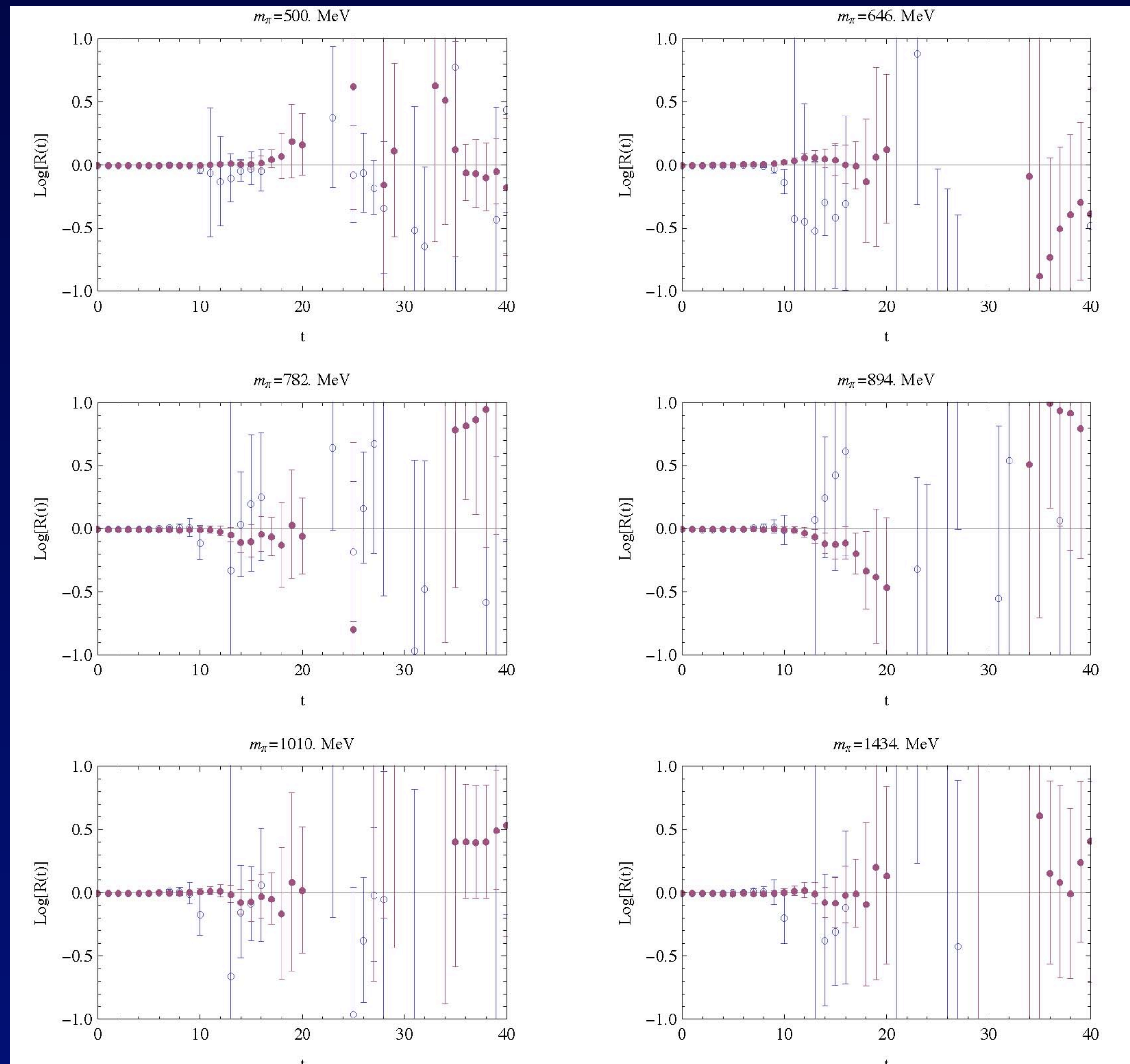


- Good signal for **Λ_S(1/2-)** : fit 12-15
- Noisy signal for **Λ^*_S(1/2+)** : fit 8-12

Masses for singlet Λ_S(1/2-) and Λ^*_S(1/2+) states



Ratio of correlation functions for Λ_S(1/2-) and Λ^*_S(1/2+)



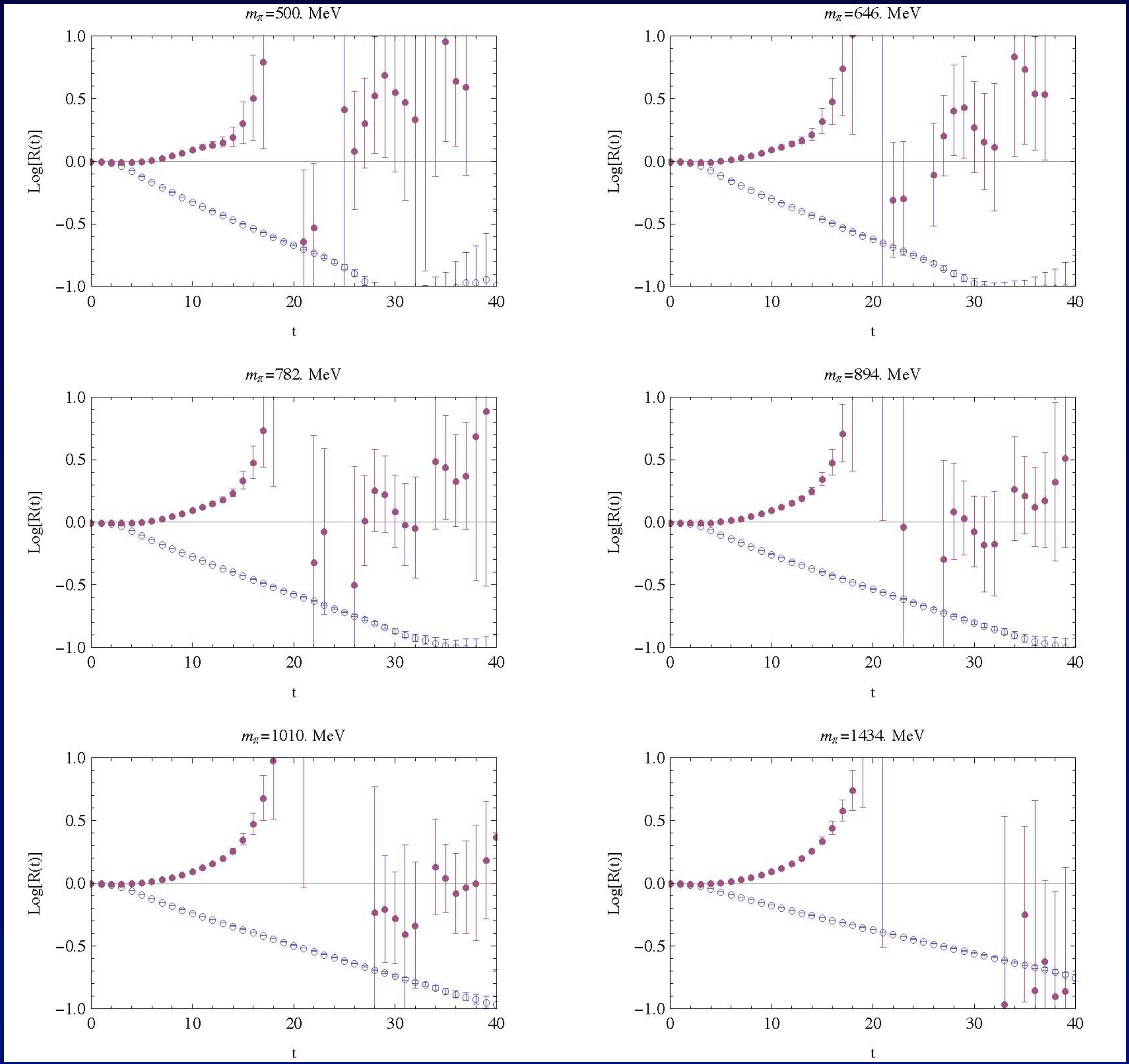
Λ_S(1/2-) ~ zero

Λ^*_S(1/2+) ~ zero but noisy

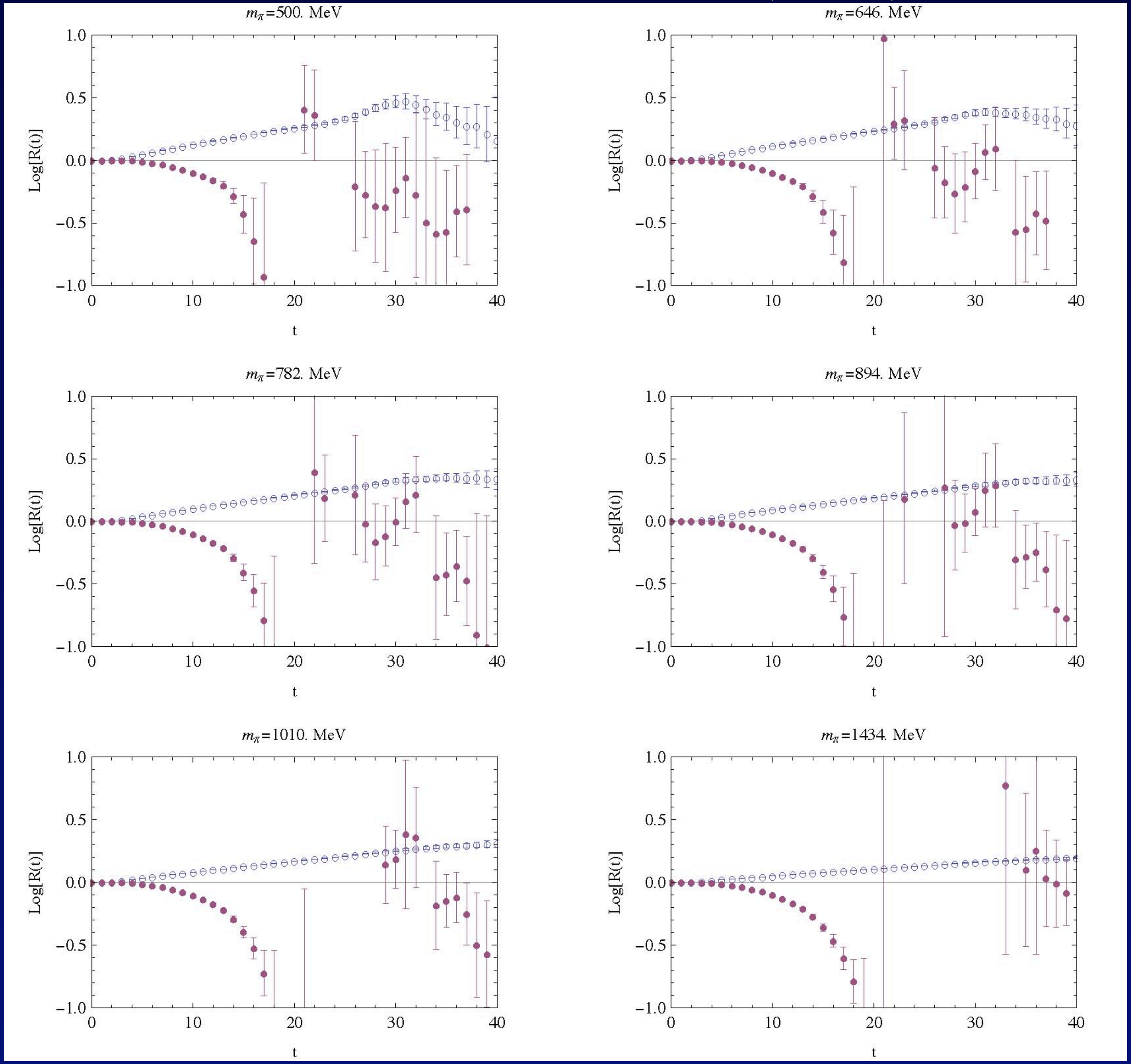
Ratio of correlation functions for Σ^{+}(1/2+) and Σ^{+*}(1/2-)

Non-linear behavior :

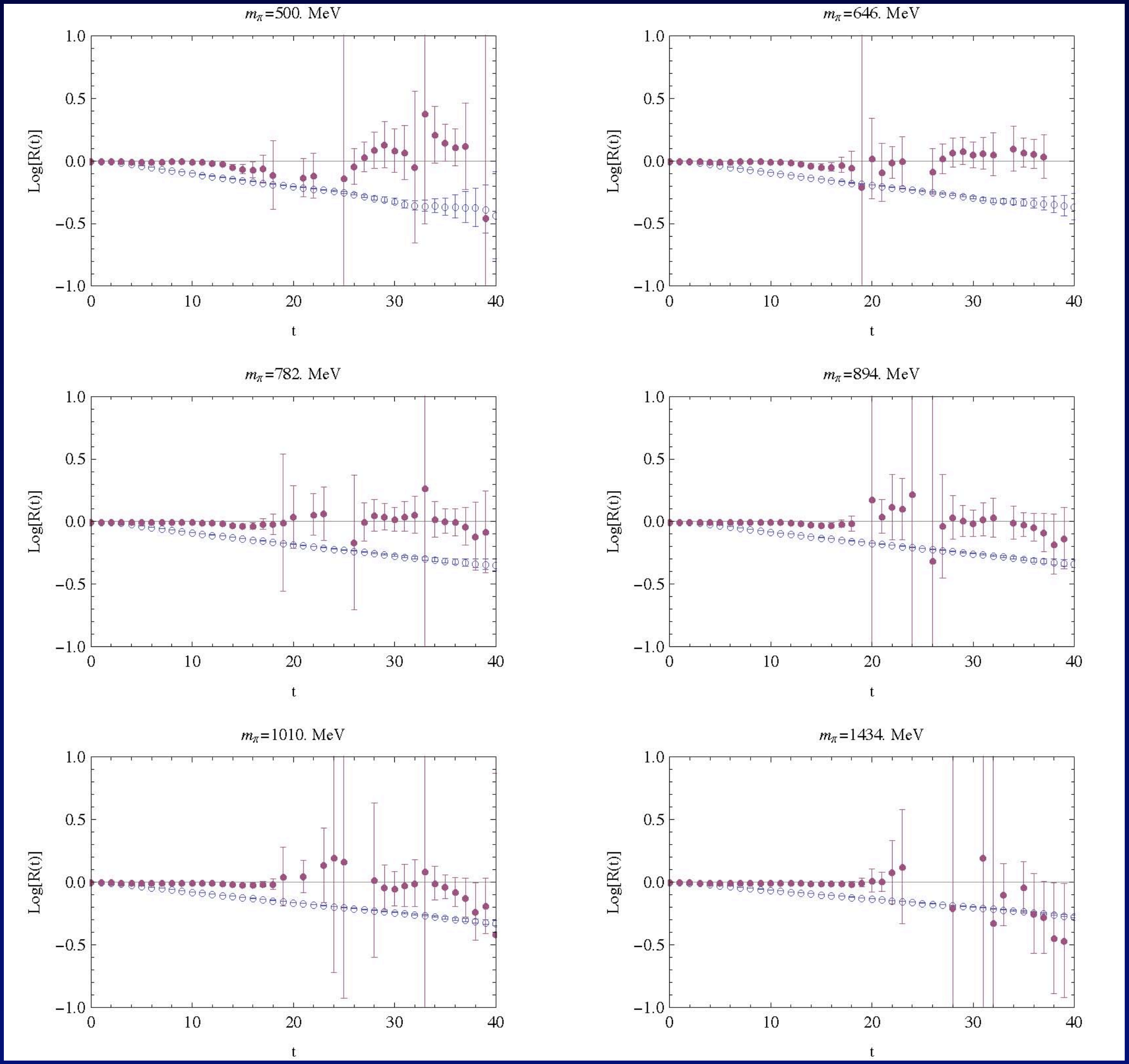
$$R(t) \propto e^{-2\left(\Delta m t + E_1 t^3\right)}$$



Ratio of correlation functions for $\Sigma^{-}(1/2+)$ and $\Sigma^{-*}(1/2-)$



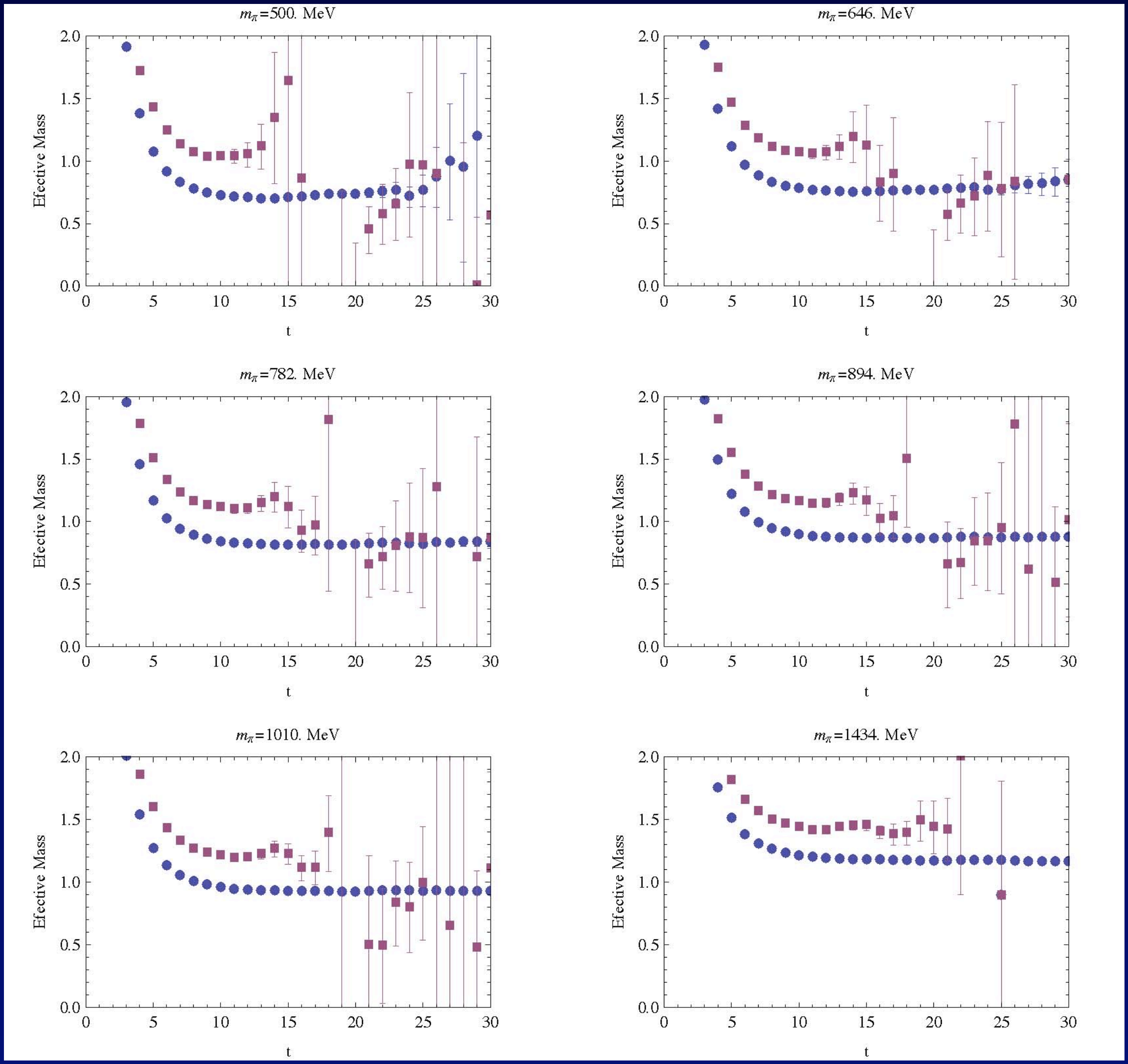
Ratio of correlation functions for Σ^0(1/2+) and Σ^{0*}(1/2-)



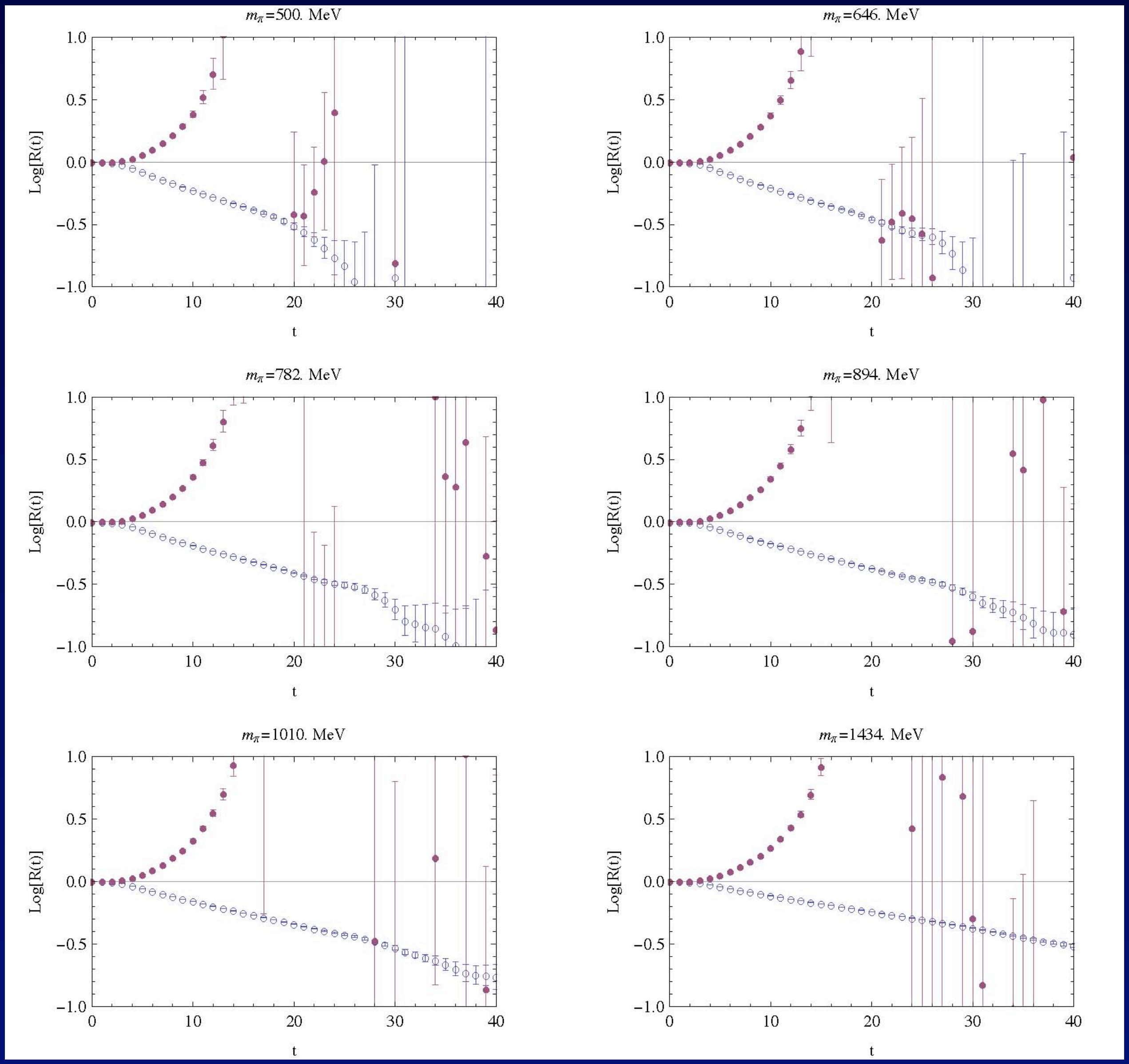
Octet Baryons

State (spin-parity)	Mass (MeV)	μ (Expt) (μ_N)	μ (Lattice QCD)	μ (Unitary χPT)	μ (Quark Model)
p (1/2 +)	N (938)	+ 2.79	~ + 2.8		
p* (1/2 -)	S^1_{11}(1535)		~ - 1.0	+ 1.1	+ 1.9
n (1/2 +)	N (938)	- 1.91	~ - 1.9		
n* (1/2 -)	S^0_{11}(1535)		~ - 0.5	- 0.25	- 1.2
Λ_O (1/2 +)	Λ(1115)	- 0.61	~ - 0.6		
Λ^*_O (1/2 -)	Λ(1670)		~ - 0.3	- 0.29	+ 0.28
Λ_S (1/2 -)	Λ(1405)		~ 0	0.24 to 0.45	+ 0.04
Λ^*_S (1/2 +)	Λ(~2400)		~ 0 (noisy)		
Σ^+ (1/2 +)	Σ (1119)	+ 2.45	~ + 2.9		
Σ^{+*} (1/2 -)			-		
Σ^0 (1/2 +)		+ 0.65	~ + 0.8		
Σ^{0*} (1/2 -)			~ - 0.5		
Σ^- (1/2 +)		- 1.16	~ - 1.5		
Σ^{-*} (1/2 -)			negative		

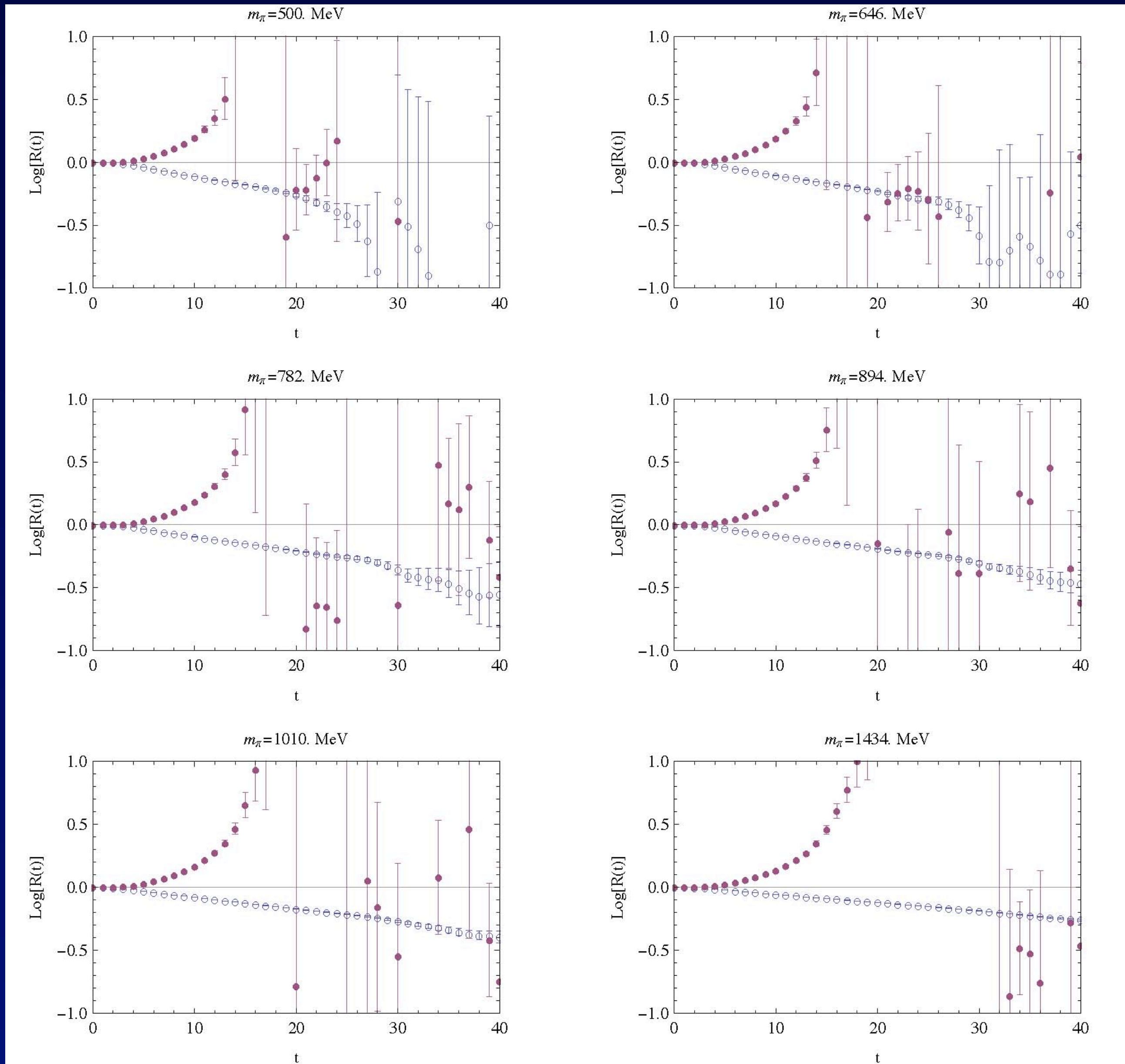
Effective mass plots for Δ(3/2+) and Δ*(3/2-)



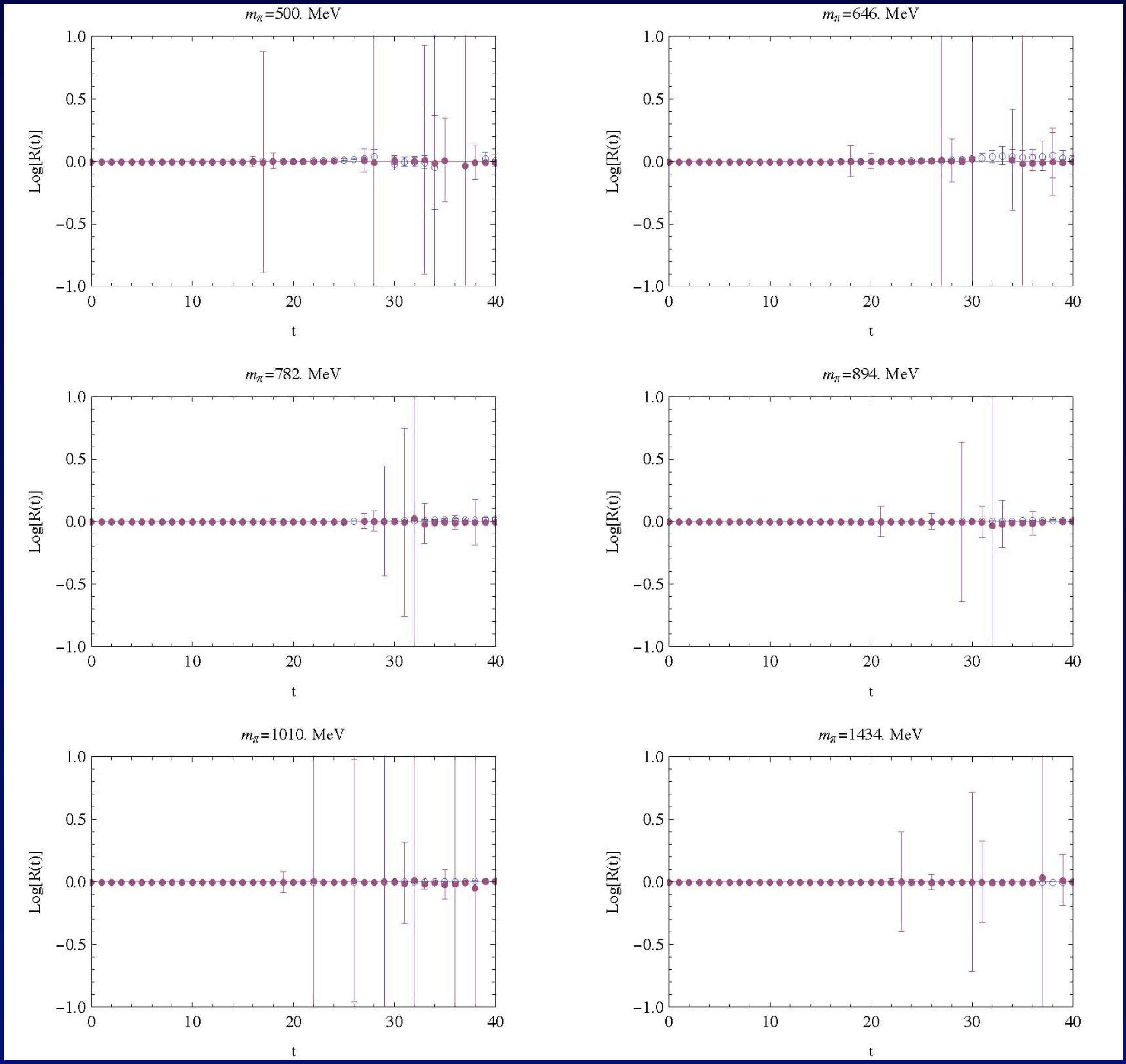
Ratio of correlation functions for Δ^{++}(3/2+) and Δ^{++*}(3/2-)



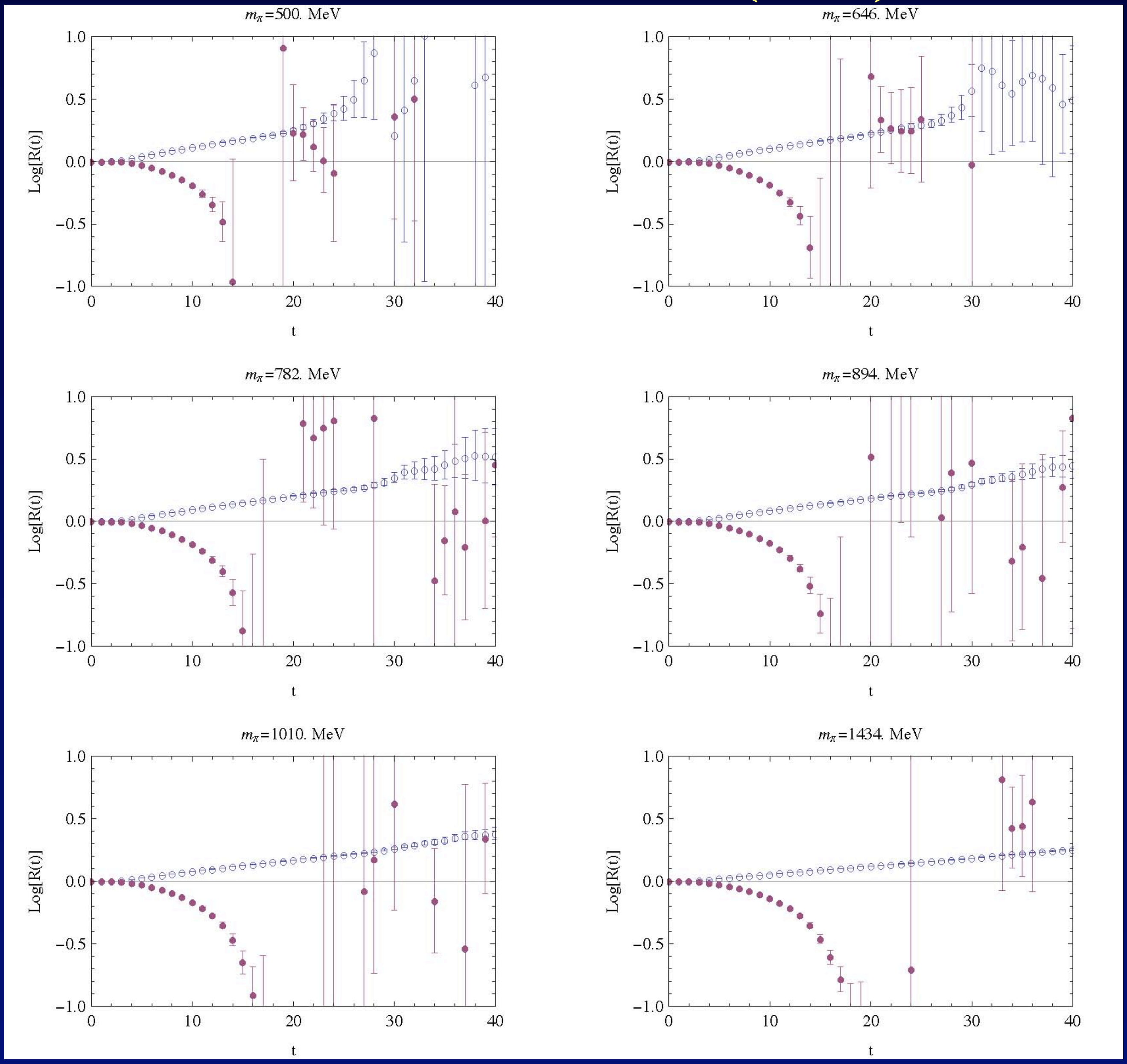
Ratio of correlation functions for Δ^{+}(3/2+) and Δ^{+*}(3/2-)



Ratio of correlation functions for $\Delta^0(3/2+)$ and $\Delta^{0*}(3/2-)$



Ratio of correlation functions for $\Delta^-(3/2+)$ and $\Delta^{-*}(3/2-)$



Delta Baryons

State (spin-parity)	Mass (MeV)	μ (Expt) (μ_N)	μ (Lattice QCD)
Δ^{++} (3/2 +)	Δ (1232)	2.5 to 5.5	~ + 5.0
Δ^{++} * (3/2 -)	Δ (1700)		negative
Δ^{+} (3/2 +)			~ + 2.5
Δ^{+} * (3/2 -)			negative
Δ^{0} (3/2 +)			zero
Δ^{0} * (3/2 -)			zero
Δ^{-} (3/2 +)			~ - 3.0
Δ^{-} * (3/2 -)			positive

Conclusion

- The background field method is a robust probe of hadron internal structure.
- Comparison study of magnetic moments for positive- and negative-parity states offers interesting insight into underlying quark-gluon dynamics
    - Good signal for positive-parity baryon states
    - Non-linear behavior is observed for negative-parity counterparts.
- Better isolation of negative-parity signals
    - smearing, anisotropic lattice, etc
    - use of chiral quarks (overlap, DW) for small pion masses
    - finite volume effects

Reserve Slides

Baryon Interpolating Fields

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$: $\chi_1 = \varepsilon_{abc}(u^{aT} C\gamma_5 d^b)u^c$ $\chi_2 = \varepsilon_{abc}(u^{aT} C d^b)\gamma_5 u^c$

Negative parity (multiply by γ_5): $\chi_1^- = \gamma_5 \chi_1$, $\chi_2^- = \gamma_5 \chi_2$

Non-relativistic limit:

$\chi_1 \to (\text{big - big - big}) \to O(1)$ (couples to nucleon)

$\chi_2 \to (\text{big - small - small}) \to O(p^2/E^2)$ (couples to ?)

$\chi_1^- \to (\text{big - big - small}) \to O(p/E)$ (couples to $\frac{1}{2}^-$ state)

$\chi_2^- \to (\text{big - small - big}) \to O(p/E)$ (couples to $\frac{1}{2}^-$ state)

In the spectrum: $N^*(1535)\frac{1}{2}^-$ and $N^*(1650)\frac{1}{2}^-$.

Caution: Near the chiral limit, the upper and lower components become equally important.