Lattice String Field Theory

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Outline

What is String Theory?

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- World-sheet action?
  - Gives "Feynman rules"
  - Perturbative expansion.
  - Not a full non-perturbative definition.

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- World-sheet action?
  - Gives “Feynman rules”
  - Perturbative expansion.
  - Not a full non-perturbative definition.
- Non-perturbative objects: D-branes, ...
- AdS/CFT correspondence?
  - Define string theory in terms of a field theory.
  - Only works on certain backgrounds.

String Field Theory

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What is String Field Theory?

- Several different string field theories.
- Witten's Open String Field Theory:

$$S \equiv S_2 + S_3 = -\int \left(\frac{1}{2\alpha'} \Psi \star Q\Psi + \frac{g_o}{3} \Psi \star \Psi \star \Psi \right).$$

- Ψ is infinite sum of component fields.
- Space of string fields not well understood.

Open String Field Theory

Open String Field Theory reproduces string theory order-by-order in perturbation theory.

- Automatically contains closed strings as well.

Some *classical* non-perturbative solutions have been found — D-branes.

What about full quantum theory?

Lattice String Field Theory

We can try to put String Field Theory on a lattice. This would:

- give a full non-perturbative definition of the theory. *Definition* of string theory, as lattice field theory is definition of QFT.
- allow definition / evaluation of higher loop processes.
- allow us to answer non-perturbative questions, e.g. what is the vacuum?

Problems

Ψ is infinite sum of component fields. So infinite number of degrees of freedom at each space-time point.

- Cut this off using *level truncation*. Keep only fields and terms in action below a certain level.
- $l =$ conformal weight $+$ constant
- Unclear how to treat gauge freedom and ghosts.

String theory requires 26 dimensions to be consistent. Restricts us to very small lattices!

- $2^{26} \approx 91^4$
- Superstring theory needs 10 dimensions. Could reach maybe L=6 ($6^{10} \approx 88^4$).
- But would need to include fermions.

Tachyons if $d > 2$.

$d = 1$ linear dilaton

It is possible to make string theory consistent in fewer dimensions by adding a “linear dilaton” background.

Choose $d = 1$, with dilaton $V = -\sqrt{\frac{25}{6\alpha'}}$. Simple model, but not fully understood.

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Start with level 0.

- Only one field at this level, "tachyon" T.
- No ambiguities about gauge or ghosts.

Express action in terms of $\tau(x) = e^{\frac{V\cdot x}{2}} T(x)$:

$$S = -\frac{1}{2}\int d^d x \left(m_0^2\tau^2 + (\nabla\tau)^2\right) - \frac{g_o K^{3\left(1-\frac{\alpha' V^2}{4}\right)}}{3}\int d^d x\, e^{-\frac{V\cdot x}{2}} (K^{\alpha'\nabla^2}\tau(x))^3.$$

More problems

Action is space-dependent and non-local. Easier to work in momentum space, on a finite interval x_{min} to $x_{min} + L$.

Expand τ in Fourier components τ_n:

$$S = -\frac{1}{2}\sum_{n=1}^{N}\left(\frac{1}{24\alpha'} + \left(\frac{\pi n}{L}\right)^2\right)\tau_n^2 - \frac{g_o K^{3\left(1-\frac{\alpha' V^2}{4}\right)}}{3}\sum_{n_{1,2,3}=1}^{N} K^{-\alpha'\left(\frac{\pi}{L}\right)^2(n_1^2+n_2^2+n_3^2)}\tau_{n_1}\tau_{n_2}\tau_{n_3} f_{n_1 n_2 n_3}.$$

Consistency: $n < \frac{L}{\pi\alpha'}$, otherwise need higher-level fields.

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Another problem:

$$f_{n_1,n_2,n_3} \equiv \left(\frac{2}{L}\right)^{\frac{3}{2}} \int_0^L dx e^{-\frac{Vx}{2}} \sin\left(\frac{\pi n_1 x}{L}\right) \sin\left(\frac{\pi n_2 x}{L}\right) \sin\left(\frac{\pi n_3 x}{L}\right)$$

can have either sign, so there are unstable cubic terms.

ϕ^3 theory

Consider ϕ^3 theory with Lagrangian

$$L = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}m^2\phi^2 + \lambda\phi^3.$$

Unstable for $\phi \to -\infty$.

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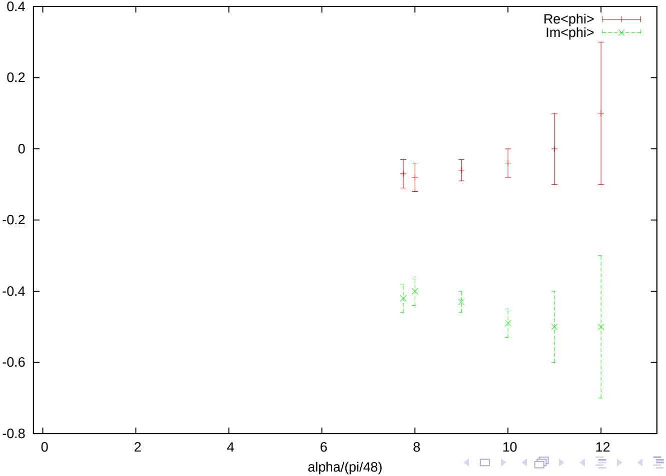
Consider $\int d\phi(x)$ as a contour integral, and rotate away from real axis, $\phi(x) \to \phi(x)e^{i\alpha}$ for $\phi(x) < 0$.

- Becomes stable if $\alpha \geq \frac{\pi}{6}$.
- Action becomes complex. Use reweighting.
- c.f. Witten, arXiv:1001.2933

ϕ^3 theory



Analytic continuation

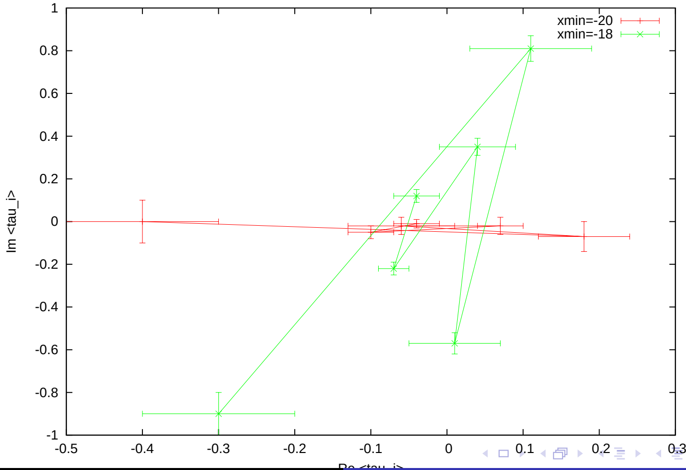
$$S = -\frac{1}{2}\sum_{n=1}^{N}\Big(\frac{1}{24\alpha'} + (\frac{\pi n}{L})^2\Big)\tau_n^2 - \frac{g_o K^{3\left(1-\frac{\alpha' V^2}{4}\right)}}{3}\sum_{n_{1,2,3}=1}^{N} K^{-\alpha'(\frac{\pi}{L})^2(n_1^2+n_2^2+n_3^2)}\tau_{n_1}\tau_{n_2}\tau_{n_3} f_{n_1 n_2 n_3}.$$

Take $\tau_n \longrightarrow \tau_n e^{i\frac{\pi}{6}}$.

- Cubic term becomes pure imaginary, so not unstable.
- Implemented with Metropolis algorithm.
- Investigate behaviour as function of x_{min},L,n,α'...

Results

Most important parameter is x_{min}.



Results

- $Im\langle\tau_i\rangle$ become large around $x_{min} + L = 2$, indicating instability.
- For large $x_{min} + L$, $Im(S)$ becomes large, so simulation becomes exponentially expensive.
- Increasing n increases instability.
- Continuum limit corresponds to increasing α'. Also increases instability

Looks like level-0 theory is indeed unstable for sufficiently large x. Not surprising, since cubic term is large.

- May be stabilised by higher levels?

Conclusions

- Lattice String Field Theory is a possible non-perturbative definition of string theory.
- Implemented lattice version of 1-d linear dilaton, at level 0.
- As expected, theory is unstable

Future: Higher levels, more dimensions, fermions.

Level-1 action

Six new fields at level 1: A,B,C,$\mathcal{A}$,$\mathcal{B}$,$\mathcal{C}$.

$$\begin{aligned} S \quad = \quad & -\int d^d x \left(\frac{m_0^2 T^2 + (\nabla T)^2}{2} + \frac{m_1^2 A^2 + (\partial_\nu A_\mu)^2}{2} + i(m_1^2 BC + \nabla B \cdot \nabla C) \right) \\ & - \frac{g_o \mathcal{N}}{3} \int d^d x \, \tilde{T}^3 e^{-\frac{V \cdot x}{2}} \\ & - \frac{8 g_o \mathcal{N}}{27} \cdot \int d^d x \left(2\tilde{A}^\mu \tilde{A}_\mu \tilde{T} + \tilde{A}_\mu \tilde{A}_\nu \partial^\mu \partial^\nu \tilde{T} + \partial^\mu \tilde{A}_\nu \partial^\nu \tilde{A}_\mu \tilde{T} - 2A_\nu \partial^\nu \tilde{A}_\mu \partial^\mu \tilde{T} \right) e^{-\frac{V \cdot x}{2}} \\ & - \frac{16 i g_o \mathcal{N}}{27} \int d^d x \tilde{B} \tilde{C} \tilde{T} e^{-\frac{V \cdot x}{2}} . \end{aligned}$$

Bonus materials