Nucleon Matrix Elements with $N_f = 2 + 1 + 1$ Maximally Twisted Fermions

Simon Dinter
NIC, DESY Zeuthen

in collaboration with C. Alexandrou, M. Constantinou, V. Drach, K. Jansen, D. Renner

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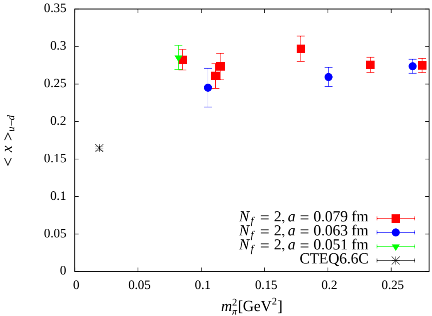
Motivation

- ETMC: successful research program for $N_f = 2$:
  - 4 lattice spacings 0.051 fm $\lesssim a \lesssim$ 0.1 fm
  - $m_\pi \gtrsim$ 280 MeV
  - $m_\pi L > 3.5$
- Research project: $\langle x \rangle_{u-d}$, benchmark for lattice calculation



- Puzzle: too high
- also problem for other collaborations [Lattice 2009 review, arxiv:1002.0925 [hep-lat]]

$\Rightarrow$ need to explore 140 MeV $\lesssim m_\pi \lesssim$ 300 MeV

Plan of ETMC: go down to $m_\pi = 160$ MeV with a minimal lattice spacing of $a \approx 0.06$ fm in the $N_f = 2+1+1$ runs

- realistic physical setup
- including charm natural for twisted mass
- $N_f = 4$ runs started for renormalization

First calculation of $\langle x \rangle_{u-d}$ for $2+1+1$

First moment of parton distribution function (PDF) $q(x, \mu^2)$:

$$\langle x \rangle_{q,\mu^2} = \int_{-1}^{1} dx\ xq(x,\mu^2) = \int_{0}^{1} dx\ x \left\{ q(x,\mu^2) + \overline{q}(x,\mu^2) \right\}$$

Moments of PDFs related to local operators $\Rightarrow$ calculable on the lattice

We concentrate on $\langle x \rangle_{u-d}$

$$\langle p, s | \underbrace{\overline{q} \gamma^{\{\mu} iD^{\nu\}} \tau^3 q}_{O^{\mu\nu}} | p, s \rangle \Big|_{\mu^2} = 2 \langle x \rangle_{u-d,\mu^2} p^{\{\mu} p^{\nu\}}$$

On the lattice:

- 3-point functions of nucleon with $O_{\mu\nu}$ calculated with sequential method
- source-sink separation gives rise to systematic error

$N_f = 2+1+1$ twisted mass fermions

- dynamical up, down, strange and charm
- fermionic action

$$S^{\mathrm{l}}_{\mathrm{tm}} = a^4 \sum_x \bar{\chi}_{\mathrm{l}}(x) \left[D_W[U] + m_0 + i\gamma_5\tau_3\mu_{\mathrm{l}} \right] \chi_{\mathrm{l}}(x)$$

$$S^{\mathrm{h}}_{\mathrm{tm}} = a^4 \sum_x \bar{\chi}_{\mathrm{h}}(x) \left[D_W[U] + m_0 + i\gamma_5\tau_1\mu_\sigma + \mu_\delta\tau_3 \right] \chi_{\mathrm{h}}(x)$$

- strange and charm masses

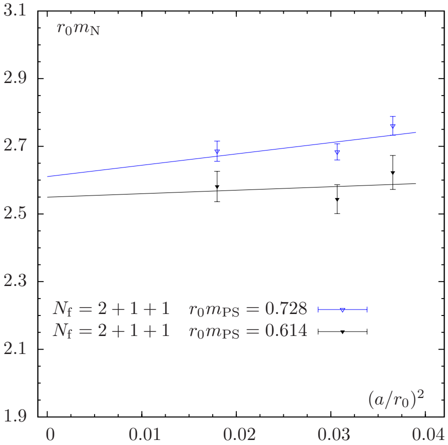
$$(m_s)_{\mathrm{R}} = Z_{\mathrm{PS}}^{-1} \left(\mu_\sigma - Z_{\mathrm{PS}}/Z_{\mathrm{S}}\, \mu_\delta \right),$$

$$(m_c)_{\mathrm{R}} = Z_{\mathrm{PS}}^{-1} \left(\mu_\sigma + Z_{\mathrm{PS}}/Z_{\mathrm{S}}\, \mu_\delta \right)$$

- automatic $\mathcal{O}(a)$ improvement of physical observables [Frezzotti, Rossi, 2003]
- no operator improvement necessary for $\langle x \rangle_{u-d}$
- for more details → talk by G. Herdoiza

Scaling behaviour

- $\mathcal{O}(a)$ corrections absent
- good scaling in $\mathcal{O}(a^2)$, shown here for nucleon mass
  $\rightarrow$ talk by V. Drach
- expect good scaling in $\mathcal{O}(a^2)$ also for $\langle x \rangle_{u-d}$



Renormalization

- want massless scheme ⇒ need $N_f = 4$ simulations
- calculation started for biliniears, not yet available for $\langle x \rangle$
- temporary solution: use $N_f = 2 + 1 + 1$ configurations
- momentum source method → talk by M. Constantinou

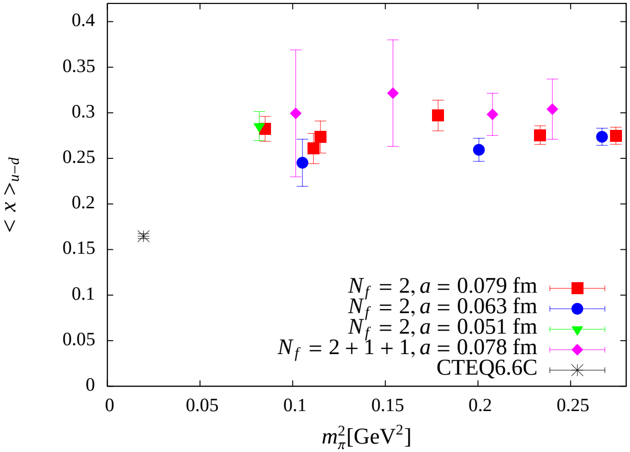
- calculated in RI-MOM scheme at $\mu = 2$ GeV, matched to $\overline{\text{MS}}$
- practically no light sea quark mass dependence observed, but so far only a few masses
- repeat calculation with $N_f = 4$
- for $\langle x \rangle_{u-d}$: $Z_{O_{44}}^{\beta=1.95}(\mu = 2\text{ GeV}) = 1.073(34)$ (preliminary)

Lattice details

$2 + 1 + 1$ dynamical flavor configurations from ETM collaboration
$\rightarrow$ talk by S. Reker
one lattice spacing: $a \approx 0.078$ fm
pion masses from 320 MeV to 490 MeV, $m_\pi L > 4$

Contractions performed with parallel contraction code "ahmidas"
A. Deuzemann, S. Reker, SD $\rightarrow$ http://code.google.com/p/ahmidas

Results



$N_f = 2$ and $N_f = 2+1+1$ results for $\langle x \rangle_{u-d}$

Summary and Prospects

- first calculation of $\langle x \rangle_{u-d}$ with $2+1+1$ dynamical flavours
- non-perturbative Z-factor
- agreement with $N_f = 2$ data

future goals:

- second lattice spacing ($a \approx 0.06$ fm)
  $\rightarrow$ reach physical point
- more observables: g_A, higher derivatives, form factors
- test of a new method