Rho decay from twisted mass fermions

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on behalf of **ETMC**

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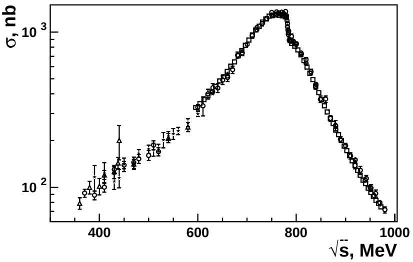
Start from experiment

- ρ resonance appears in cross section $\sigma(e^+e^- \to \pi^+\pi^-)$



[Achasov, 06]

- m_ρ: σ reaches its maximal; Γ_ρ: σ drops to half its peak
- Lüscher's finite size methods $\Rightarrow$ $\pi\pi$ scattering phase $\delta(p)$

$$\delta(p)\Big|_{E=m_\rho} = \frac{\pi}{2} \quad \text{or} \quad \delta(p)\Big|_{E=m_\rho\pm\Gamma_\rho/2} = \frac{\pi}{4}$$

Lattice details

- maximally twisted mass fermions
  - automatically $O(a)$ improved
  - isospin and parity symmetry breaking
- ensemble information

a (fm)	L/a	m_{π^+} (MeV)			
0.079	24			420	480
0.079	32	290	330		
0.063	32		320		

- m_{π^+}: 290–480 MeV $\Rightarrow$ pion mass dependence
- m_{π^+}/m_{ρ^0}: 0.30–0.43 $\Rightarrow$ threshold is open for $\rho^0 \to \pi^+\pi^-$
- two lattice spacings $\Rightarrow$ check for lattice artifacts

Requirement for large L

- in the free case: $\vec{p} = (2\pi/L)\vec{n}$

$$E = 2\sqrt{m_\pi^2 + p^2}$$

- elastic scattering region: $2m_\pi < E < 4m_\pi$
- close to resonance peak: $E \simeq m_\rho$

$$\frac{2\pi}{L} = p \simeq \sqrt{m_\rho^2/4 - m_\pi^2}$$

- small p requires large lattice size L
- large phyiscal volume $\Rightarrow$ large computer resources

Moving frame method

- center-of-mass frame (CMF), e.g.

$$\vec{p}_1 = -\vec{p}_2 = (2\pi/L)\vec{e}_3$$

- moving frame (MF) [Rummukainen & Gottlieb, 95], e.g.

$$\vec{p}_1 = (2\pi/L)\vec{e}_3 \,, \quad \vec{p}_2 = \vec{0}$$

  Lorentz boost to the CMF

$$\vec{p}_1^* = -\vec{p}_2^* = (2\pi/L)\vec{e}_3/(2\gamma) \quad \Rightarrow \quad L^* = L(2\gamma)$$

  smaller p^*, or equivalently larger L^* is available

- a second MF is employed

$$\vec{p}_1 = (2\pi/L)(\vec{e}_1 + \vec{e}_2) \,, \quad \vec{p}_2 = \vec{0}$$

  three Lorentz frames $\Rightarrow$ probe resonance region

Energy of pion-pion system

- correlation matrix to isolate ground and 1st excited state:

$$C(t) = \frac{1}{T}\sum_{t_s=0}^{T-1}\begin{pmatrix} \langle(\pi\pi)(t+t_s)(\pi\pi)^\dagger(t_s)\rangle & \langle(\pi\pi)(t+t_s)\rho^\dagger(t_s)\rangle \\ \langle\rho(t+t_s)(\pi\pi)^\dagger(t_s)\rangle & \langle\rho(t+t_s)\rho^\dagger(t_s)\rangle \end{pmatrix}$$

- place Z_4 stochastic sources on all the time slice

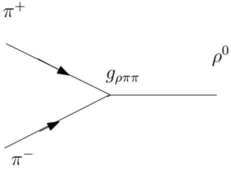
- collect ground state and 1st excited state in three frames
  2×3 energy levels $\Rightarrow$ scattering phase δ

Energy dependence of δ

- effective range formula

$$\tan\delta(p) = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{p^3}{E_{CM}(m_\rho^2 - E_{CM}^2)}\,, \quad p = \sqrt{E_{CM}^2/4 - m_\pi^2}$$

- determine resonance mass m_ρ and coupling constant $g_{\rho\pi\pi}$



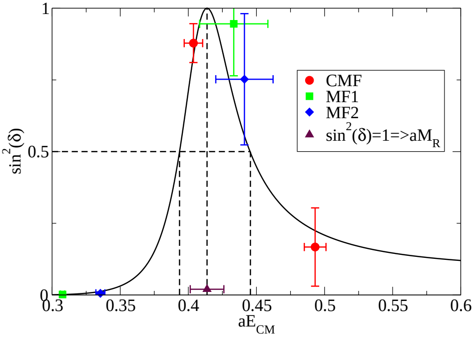
- decay width is largely determined by $g_{\rho\pi\pi}$:

$$\Gamma_\rho = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{p^3}{m_\rho^2}\,, \quad p = \sqrt{m_\rho^2/4 - m_\pi^2}$$

- $g_{\rho\pi\pi}$ is almost m_π independent?? [Hanhart et al, 08]

Results

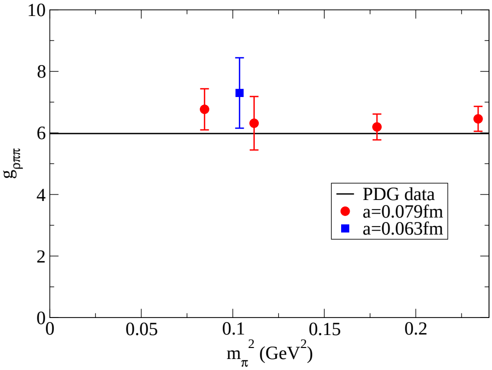
- one example of five:
  $m_{\pi^+} = 330$ MeV, $a = 0.079$ fm, $L/a = 32$



- $m_\rho = 1033(31)$ MeV, $g_{\rho\pi\pi} = 6.31(87)$, $\Gamma_\rho = 123(43)$ MeV

Pion mass dependence

- $g_{\rho\pi\pi}$ as a function of m_π^2



- $g_{\rho\pi\pi}$ is almost independent of the pion mass

Pion mass dependence

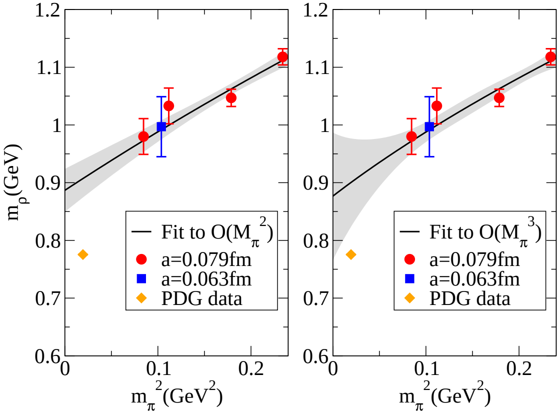
- we follow EFT using complex mass renormalization [Djukanovic et al, 10]
- m_ρ and Γ_ρ are considered as the real and imaginary part of the complex pole Z of the ρ–meson propagator

$$Z = (m_\rho - i\Gamma_\rho/2)^2$$

- study pion mass dependence of Z

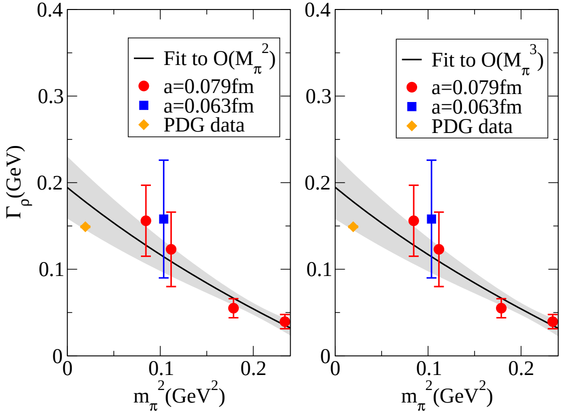
$$Z = Z_\chi + C_\chi M_\pi^2 - \frac{1}{24\pi} g_{\omega\rho\pi}^2 Z_\chi^{1/2} M_\pi^3 + O(M_\pi^4 \ln(M_\pi^2))$$

Pion mass dependence



- large error in fit to $O(M_\pi^3)$; fit to higher order??
- need to explore rho decay at lighter m_π, say $m_\pi < 300$ MeV

Pion mass dependence



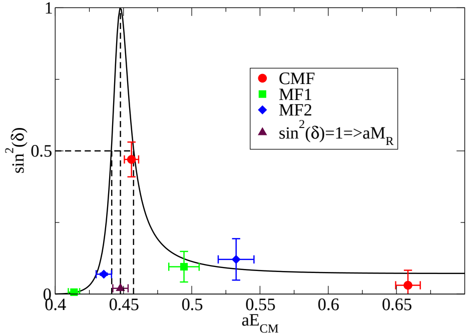
- statistically agree with PDG data
- however, error are relatively large

Conclusion

- we calculate m_ρ, Γ_ρ, $g_{\rho\pi\pi}$ from pion-pion scattering
- three Lorentz frames $\Rightarrow$ δ at six different energy levels
  $\Rightarrow$ map out the resonance region
- require to explore rho decay at lighter m_π region
- check lattice artifacts at two lattice spacings
  $\Rightarrow$ no obvious effects are found
- this work is more of a conceptual nature to understand how resonances can be treated in LQCD

Backup slides

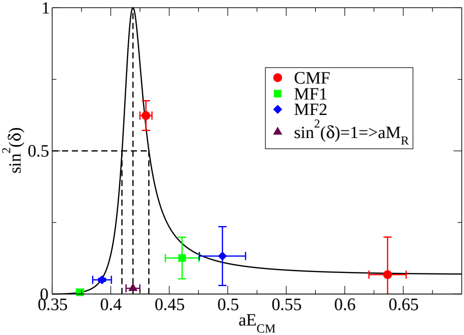
- $m_{\pi^+} = 480$ MeV, $a = 0.079$ fm, $L/a = 24$



- $m_\rho = 1118(14)$ MeV, $g_{\rho\pi\pi} = 6.46(40)$, $\Gamma_\rho = 39.5(8.2)$ MeV

Backup slides

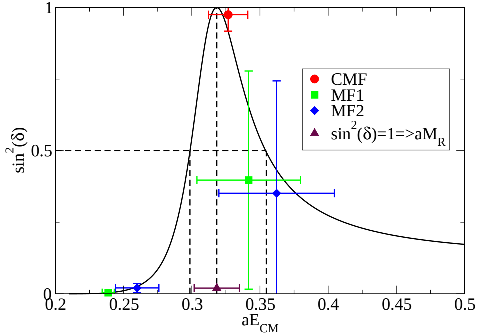
- $m_{\pi^+} = 420$ MeV, $a = 0.079$ fm, $L/a = 24$



- $m_\rho = 1047(15)$ MeV, $g_{\rho\pi\pi} = 6.19(42)$, $\Gamma_\rho = 55(11)$ MeV

Backup slides

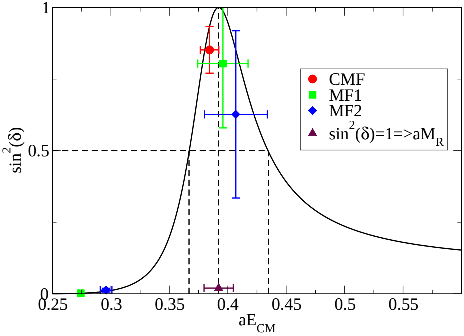
- $m_{\pi^+} = 320$ MeV, $a = 0.063$ fm, $L/a = 32$



- $m_\rho = 997(52)$ MeV, $g_{\rho\pi\pi} = 7.3(1.1)$, $\Gamma_\rho = 158(68)$ MeV

Backup slides

- $m_{\pi^+} = 290$ MeV, $a = 0.079$ fm, $L/a = 32$



- $m_\rho = 980(31)$ MeV, $g_{\rho\pi\pi} = 6.77(67)$, $\Gamma_R = 156(41)$ MeV