Effective Polyakov-loop theory for pure Yang-Mills from strong coupling expansion: analytical details

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Introduction

Introduction

- Goal: Study (Lattice) QCD at finite temperature and density
- Svetitsky-Yaffe conjecture: $(d+1)$ dimensional SU(N) thermal phase transition is described by an effective spin model in d dimensions with **short-ranged** interactions
- Possible approach: Strong coupling expansions; Leading order effective action derived in [Polonyi, Szlachanyi 1981]
- Since then: various generalizations, e.g. [Green, Karsch 1984], [Ogilvie 1984], [Wipf et al. 2004, 2007], ...
- Common simplification: Neglect of spatial plaquettes (Leading correction for SU(2) computed in [Caselle et al. 1996])
- *Here*: Explore what accuracy can be achieved by inclusion of spatial plaquettes

General strategy

- Start with the partition function of (3+1) dimensional lattice gauge field theory at finite temperature
- Integrate out degrees of freedom in order to have an effective action in terms of the order parameter (*here*: Polyakov loop)

$$-S_{eff} = \lambda_1 S_1 + \lambda_2 S_2 + \lambda_3 S_3 + \ldots$$

- S_n depend only on Polyakov loops
- Find critical parameters $\lambda_{n,crit}$ and relate back to critical lattice couplings β_{crit} for different N_τ

$\longrightarrow$ Crucial to know mappings $\lambda_n(N_\tau, \beta)$

Calculational details for SU(2)

SU(2) technical details

- Partition function with Wilson's gauge action

$$Z = \int [dU] \exp\left[\frac{\beta}{2}\sum_p \mathrm{tr}\, U_p\right]$$

- Split temporal and spatial link integration and use character expansion ($a_r(\beta)$: expansion parameter of representation r)

$$\begin{aligned} Z &= \int [dW] \exp\left\{\ln \int [dU_i] \prod_p \left[1 + \sum_{r\neq 0} d_r a_r(\beta)\chi_r(U_p)\right]\right\} \\ &\equiv \int [dW] \exp\left[-S_{eff}\right] \qquad W(\vec{x}) = \prod_{\tau=1}^{N_\tau} U_0(\tau, \vec{x}) \end{aligned}$$

- Allows for a systematic strong coupling expansion of S_{eff}

Interlude: Spatial strong coupling limit

- Neglect of spatial plaquettes:

$$Z = \int [dW] \sum_{\langle ij \rangle} \left[1 + \sum_r a_r^{N_\tau} \chi_r(W_i) \chi_r(W_j) \right]$$

- Exponential function disappears in this limit
- Next-to-nearest-neighbour interactions are an effect that depends on the inclusion of spatial plaquettes

- Leading order graph in case of $N_\tau = 4$:

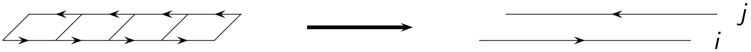


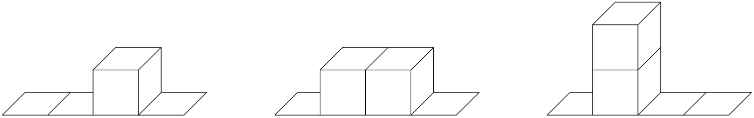
Figure: 4 plaquettes in fundamental representation lead to a 2 Polyakov loop interaction term

- Integration of spatial link variables leads to

$$-S_1 = u^{N_\tau} \sum_{\langle ij \rangle} \mathrm{tr}\, W_i \,\mathrm{tr}\, W_j$$

- $u \equiv a_f(\beta)$: Faster convergence, relation analytically known
- Possible generalizations: larger distance, higher dimensional representations, larger number of loops involved, ...
- *Here*: Decorate LO graph with additional spatial and temporal plaquettes

Higher order graphs



- Higher order graphs exponentiate (i.e. resumming graphs):

$$\longrightarrow \qquad -S_1 = \lambda_1(N_\tau, u) \sum_{\langle ij \rangle} \mathrm{tr}\, W_i \,\mathrm{tr}\, W_j$$

$$\lambda_1(N_\tau, u) = u^{N_\tau} \exp\left[N_\tau P(N_\tau, u) \right]$$

- Polynomials $P(N_\tau, u)$ known up to $\mathcal{O}(u^{10})$: Crucial to relate λ_1 of effective spin model to β of full gauge theory
- **One** determination of $\lambda_{1,crit}$ gives all $\beta_{crit}(N_\tau)$

Integration

- Z(2) symmetric 3 dimensional partition function

$$Z = \int [dW] \exp\left[\lambda_1 \sum_{\langle ij \rangle} \mathrm{tr}\, W_i \, \mathrm{tr}\, W_j\right]$$

- Can be further simplified by using $L \equiv \mathrm{tr}\, W$ as degrees of freedom: ordinary integration instead of group integration
- Introduces potential term: $V_{SU(2)} = \frac{1}{2} \sum_i \ln\left[4 - L_i^2\right]$

$$Z = \int [dL] \exp\left[\lambda_1 \sum_{\langle ij \rangle} L_i L_j + \frac{1}{2} \sum_i \ln\left[4 - L_i^2\right]\right]$$

Higher order interaction terms

- Subclass of higher order interaction terms (Powers of the leading order term) arrange schematically as

$$-S_{eff} = \lambda_1 (LL) - \frac{\lambda_1^2}{2} (LL)^2 + \frac{\lambda_1^3}{3} (LL)^3 - \ldots = \ln \left[1 + \lambda_1 (LL) \right]$$

- SU(2) effective theory to be simulated

$$Z = \int [dL] \prod_i \sqrt{4 - L_i^2} \prod_{\langle ij \rangle} \left[1 + \lambda_1 L_i L_j \right]$$

- Critical coupling: $\lambda_{1,crit} = 0.2142(1)$

Results and comparison with Monte Carlo

N_τ	$\beta_c(S_{eff})$	$\beta_c(MC)$
1	0.884	0.873
2	1.898	1.873
3	2.213	2.177
4	2.335	2.299
5	2.409	2.373
6	2.454	2.427
8	2.505	2.510
12	2.551	2.636
16	2.573	2.731

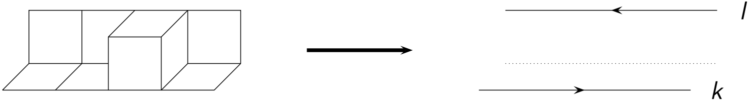
- Good results up to $N_\tau = 8$

Systematic errors

- Truncation of series expansion in u
  - Good approximation for small N_τ (i.e. small β_{crit}) as $\beta_{crit}(N_\tau)$ increases with N_τ
  - No much room left to increase the strong coupling expansion to higher orders
- Neglect of higher order interactions terms
  - Good approximation for larger N_τ, since corrections come at least with an additional power u^{N_τ} (and $0 \leq u \leq 1$)
  - $\longrightarrow$ Next task

Next-to-nearest-neighbour interactions

- Leading order graphs:



- Investigate effect of L-shaped next-to-nearest neighbours: Two-coupling theory with partition function

$$Z = \int [dL] \exp\left[-S_1 + V_{SU(2)}\right] \prod_{[kl]} \left[1 + \lambda_2 L_k L_l\right)\right]$$

$$\lambda_2 = N_\tau (N_\tau - 1) u^{2N_\tau + 2} \left(1 + \mathcal{O}\left(u^2\right)\right)$$

- Straight next-to-nearest-neighbours are of $\mathcal{O}\left(u^{2N_\tau+6}\right)$

Generalization to SU(3)

- SU(3) straightforward, but: Now also with anti-fundamental representation (i.e. L_i are complex)

$$
\begin{aligned}
Z &= \int [dL] \exp\left[-S_1 + V_{SU(3)}\right] \\
&= \int [dL] \prod_{\langle ij \rangle} \left[1 + 2\lambda_1 \mathrm{Re}\Big(L_i L_j^*\Big)\right] * \\
&\quad * \prod_i \sqrt{27 - 18|L_i|^2 + 8\mathrm{Re}L_i^3 - |L_i|^4}
\end{aligned}
$$

- Functional form of $\lambda_1(N_\tau, u)$ and next-to-nearest-neighbour effects are analogous to SU(2)

Conclusion: Part I

- Derived different effective spin models for finite temperature lattice gauge theories
- Computed up to 10 more orders in the strong coupling expansion of effective theory couplings
- Investigate the effect of spatial plaquettes and next-to-nearest-neighbour couplings
- See S. Lottini's talk for numerical results and final conclusions

Backup slides

Character expansion

- Expand plaquette action in terms of characters $\chi_r(U_p)$

$$\exp\left[\frac{\beta}{2}\mathrm{tr}\, U_p\right] = c_0(\beta)\left[1 + \sum_{r\neq 0} d_r a_r(\beta)\chi_r(U_p)\right]$$

- Expansion parameter $a_r(\beta)$ are certain combinations of modified Bessel functions for SU(N) gauge groups
- In contrast to a direct exponentiation, each plaquette variable is now only allowed to contribute once in a given representation