Quantum entanglement and KPZ relations

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Foreword

- This talk is based on
  - M. Caraglio, FG, *Entanglement Entropy and twist fields,* JHEP 11(2008) 076 [arXiv:0808.4094].
  - FG, L. Tagliacozzo, *Entanglement Entropy and the complex plane of replicas* J. Stat. Mech. (2010) P01002 [arXiv:09103003].
  - FG,*2D quantum gravity from quantum entanglement*, to appear
- it deals with a new approach to 2D quantum gravity based on quantum entanglement

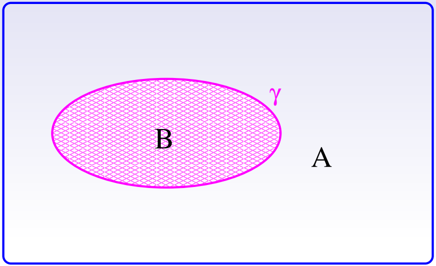
Plan of the talk

1. Quantum entanglement
2. Replica approach
3. The back-reaction of the accessible subsystems
4. Numerical simulations
5. Colnclusions

Quantum entanglement

- In extended quantum systems S with many degrees of freedom a complete description of the information available to an observer who has access to a subsystem A is the reduced density matrix ρ_A. Principal ingredients:
  - $|\Psi\rangle$= pure quantum state (e.g. the ground state)
  - S can be subdivided into two complementary subsystems A and B $S = A \cup B$
  - the reduced density matrix is $\rho_A = \mathrm{tr}_B\, |\Psi\rangle\langle\Psi|$
  - The von Neumann or Entanglement Entropy is $S_A \equiv -\mathrm{tr}\rho_A \ln \rho_A = -\mathrm{tr}\rho_B \ln \rho_B \equiv S_B$
  - Other useful probes of quantum entanglement are the Rényi entropies $R_A(n) = \log \mathrm{tr}\rho_A^n/(1-n)$ and the Tsallis entropies $T_A(n) = (\mathrm{tr}\rho_A^n - 1)/(1-n)$
  - ⇨ $S_A = \lim_{n\to 1} R_A(n) = \lim_{n\to 1} T_A(n)$

Entanglement Entropy and Black Holes



$$S_A = S_B \Longrightarrow S_A \text{ non extensive}$$

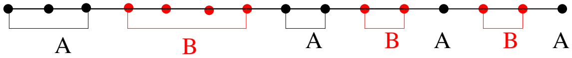
$$S_A = C\,|\gamma|$$

- S_A is the entropy for an observer who is accessible only to the subsystem A and cannot receive any signals from B
- B is analogous to the inside of a black hole

$$S_{BH} = \frac{\text{area of horizon}}{4G_N}$$

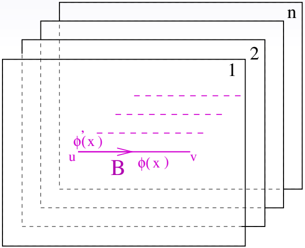
- The entanglement (or geometric) entropy is deeply related to the physics of the black holes ('t Hooft 1984, Srednicki 1993, Callan & Wilczek 1994, Ryu & Takayanagi 2006)

- The entanglement entropy has been also extensively studied in low dimensional quantum systems as a new tool to investigate the nature of quantum criticality (1D quantum system= 2D Euclidean system in the functional integral approach)
- In the following we consider a bipartite quantum spin chain in which the unobserved subsystem B is an arbitrary set of disjoint intervals
- A = accessible subsystem, B = inaccessible subsystem



Replica approach

- we may compute $\mathrm{tr}\rho_A^n$ by cutting the system along the unobserved subsystem B, making n copies of the system and sewing them together cyclically along the cut so that $\phi(x)_k = \phi(x)'_{k+1} \quad x \in B$ ($\phi(x)$ and $\phi(x)'$ fields evaluated in the lower and upper border of the cut)



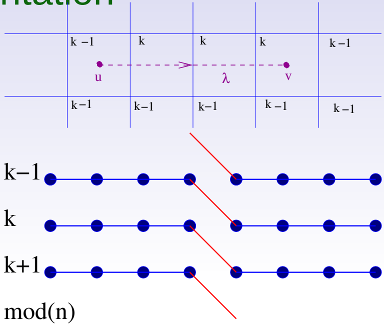
n replicas

$\Rightarrow \quad \mathrm{tr}\rho_A^n = \frac{Z_n(A)}{Z^n}$

$Z =$ partition function of the unperturbed system

$Z_n(A) =$ partition function of the system defined on the Riemann surface made with n sheets

Lattice implementation



Action for the coupled system of n sheets

$$S_A[\phi^{(1)},\dots,\phi^{(n)}]=\sum_{k=1}^{n}\sum_{\langle xy\rangle}S^{(k)}_{\langle xy\rangle}$$

$$S^{(k)}_{\langle xy\rangle}=\begin{cases}S[\phi^{(k)}_x,\phi^{(k+1)}_y] & x\in B\\ S[\phi^{(k)}_x,\phi^{(k)}_y] & x\notin B\end{cases}$$

Partition function of the coupled system

$$Z_n(A) = \int \prod_{k=1}^{n} \mathcal{D}[\phi_k] e^{-S_A[\phi^{(1)},\dots,\phi^{(n)}]}$$

- $\mathrm{tr}\rho_A^n = \frac{Z_n(A)}{Z^n}$
- we can then use this coupled system to study the quantum entanglement effects produced by the accessible system A

Back-reacting subsystems

- In all previous studies the accessible subsystem A is chosen to be fixed
- We treat instead A as a back-reacting, dynamical subsystem whose position, form and extension is determined by its interaction with the system.
- We implement it by “summing over all histories”, i.e. by putting the system in equilibrium with the Gibbs ensemble $\{A\}$ of all the possible subsystems.

⇨
$$Z_n = \sum_{\{A\}} Z_n(A)$$

⇨ in numerical simulations the dynamical coupling to A is obtained in a straightforward way by updating it with a heat-bath method

- the only elements of A having an intrinsic geometrical -and physical- meaning are the end points of the cuts, i. e. the branch points of the Riemann surface.
- They correspond to conical singularities with deficit angle $2\pi(n-1)$.
- the n-sheeted covering of the plane with N branch points is a Riemann surface of genus $g=(n-1)(N-2)/2$
- ⇨ summing over all accessible subsystems corresponds, to a double sum over genera and moduli of these Riemann surfaces.
- This is the first indication that this issue is related to 2D quantum gravity.

- 1D critical quantum spin chain $\leftrightarrow$ 2D conformal field theory
- ➪ the dynamical effects of the back-reaction of the accessible subsystems A in a quantum spin chain are intimately related to a 2D CFT in thermal equilibrium with a gas of conical singularities
- Knizhnik (1987):
  The conical singularities correspond to primary fields $\Phi_n(z,\bar{z})$ of scaling dimensions

$$\Delta_n = \bar{\Delta}_n = \frac{c}{24}\left(1 - \frac{1}{n^2}\right) ,$$

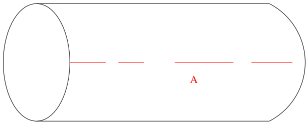
c = central charge of the system at the critical point.

- ➪ $\Phi_n(z,\bar{z})$ is a relevant operator ➪ drives the system away form the critical point
- ➪ The new stable fixed point has $c = 0$

Numerical simulations

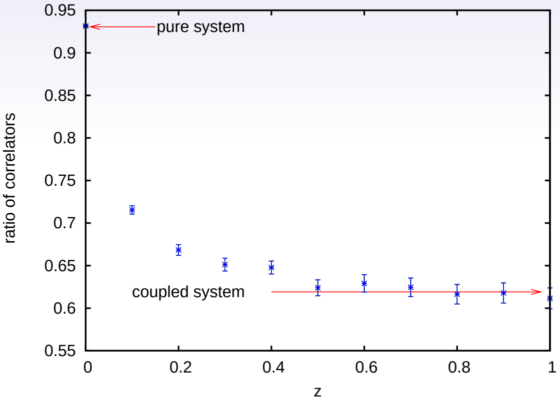
We investigated the nature of the fixed point with Monte Carlo calculations on the spin-$\frac{1}{2}$ quantum chain coupled to a transverse magnetic field h $\widehat{H} = -\lambda \sum_i \sigma^z_i \sigma^z_{i+1} - h \sum_i \sigma^x_i$

- ❋ It has a $T = 0$ phase transition described by a 2D CFT with $c = \frac{1}{2}$ (i.e.a critical Ising model)
- ❋ In a first series of numerical experiments we considered a 1D setting for the ensemble $\{A\}$

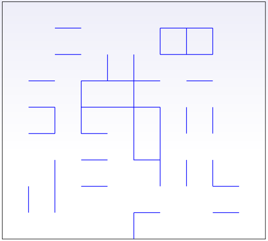


$$C(L,s) = \langle \sigma^z_i \sigma^z_{i+L/s} - \sigma^z_i \sigma^z_{i+L/2} \rangle \;;\;\; C(\lambda L, s) = \lambda^{-x} C(L,s)$$

$\lambda^{-x} = C(\lambda L, 8)/C(L, 8)$ as a function of branch point fugacity z for $L = 96$, $\lambda = \frac{4}{3}$ and two replicas



- In a second series of numerical experiments we considered a truly 2D setting, with no limitations on the location of cuts representing the accessible subsystems.



- the gas of conical singularities is spread in the bulk and drives the system away from the critical point of the pure system
- For a critical value of z the coupled system undergoes a second order phase transition

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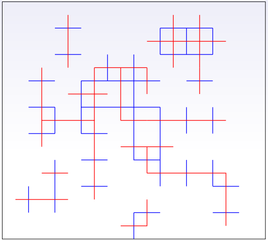

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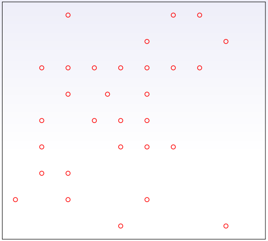

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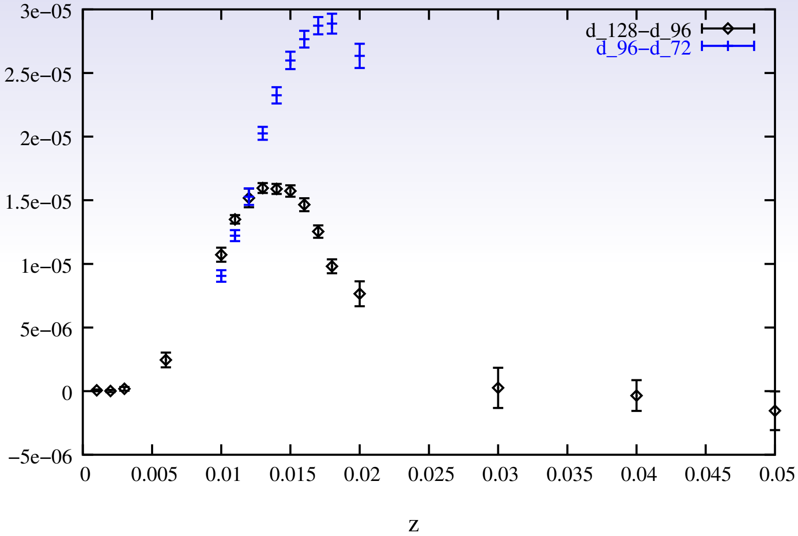
Finite size effects on branch point density against z



➪ $z_c = 0.01127(1)$

Knizhnik-Polyakov- Zamolodchikov relations

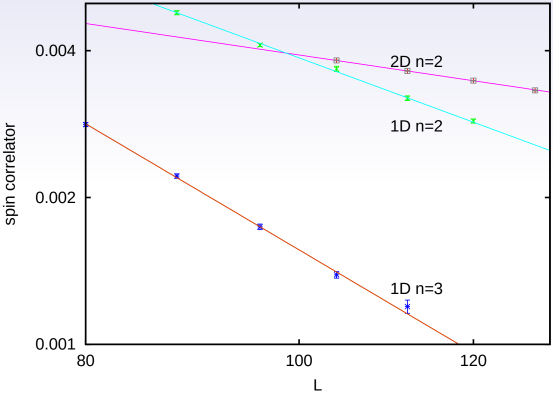
$$\Delta^o = \Delta + \frac{\gamma^2}{4}\Delta(\Delta - 1), \quad \gamma = \sqrt{\frac{25-c}{6}} - \sqrt{\frac{1-c}{6}} \,,$$

relate the scaling dimensions Δ^o of a primary field of a CFT to the scaling dimension Δ of this operator when the theory is coupled to 2D quantum gravity.

In the critical Ising model $c = \frac{1}{2}$

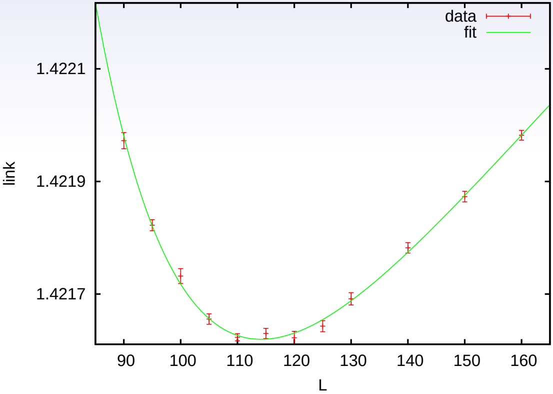
- spin primary field $\Delta^o_\sigma = \frac{1}{16} \Rightarrow \Delta_\sigma = \frac{1}{6}$
- energy primary field $\Delta^o_\epsilon = \frac{1}{2} \Rightarrow \Delta_\epsilon = \frac{2}{3}$

Power law of critical correlators



- 1D $n = 2, n = 3$ setting $C(L, s) = \frac{a}{L^{4\Delta_\sigma + n - 1}}$
- 2D setting: $C(L, s) = \frac{a}{L^{4\Delta_\sigma}}$ $\quad \Delta_\sigma = \frac{1}{6}$

Energy primary field



link= $\langle \sigma^z_x \sigma^z_{x+a} \rangle = e_o + e_1/L^{2\Delta_\epsilon} + e_2/L^{2\Delta_\epsilon+1} \qquad \Delta_\epsilon = \frac{2}{3}$

Conclusions

- We studied the back-reaction of the accessible subsystems of a 1D quantum system.
- the coupling to the Gibbs ensemble of all the possible subsystems is relevant and drives the system into a new fixed point
- numerical experiments on the critical Ising model show that the new critical exponents agree with the KPZ formula of 2D quantum gravity
- Extension to higher dimensions is straightforward