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The q^{bar}-q potential from Bethe-Salpeter amplitudes on lattice

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in collaboration with

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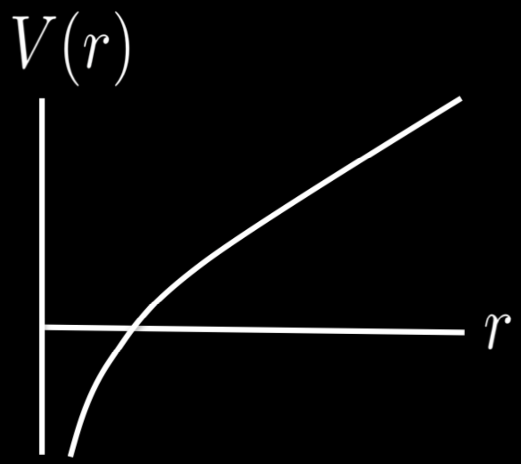
- Introduction : Inter-quark potentials
- Formalism : Wave function & Potential from LQCD
- Numerical results and discussions
- Summary and future plans

✓ Introduction (Why inter-quark potential?)

Understanding of inter-quark potential
→ (quark) confinement mechanism & hadron spectroscopy



Shape of anti-quark – quark potential
→ Linear (confinement) + Coulomb form

$$V(r) = \sigma r - \frac{A}{r} + \epsilon$$

- String tension : Regge slope
- Coulomb coefficient : Mass splitting of heavy quarkonium

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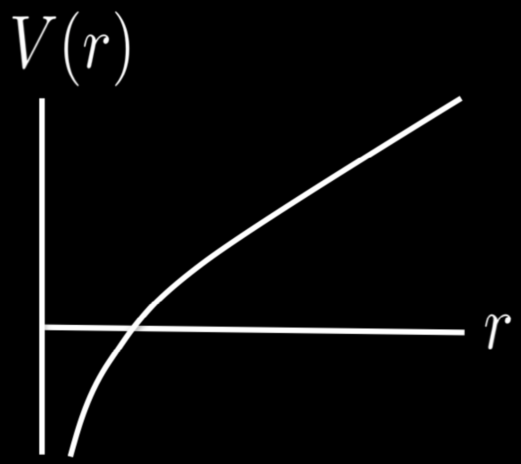
Understanding of inter-quark potential
→ (quark) confinement mechanism & hadron spectroscopy



Shape of anti-quark – quark potential
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Application of Q^{bar}-Q potential

Strongly coupled pNRQCD (EFT of heavy mesons)

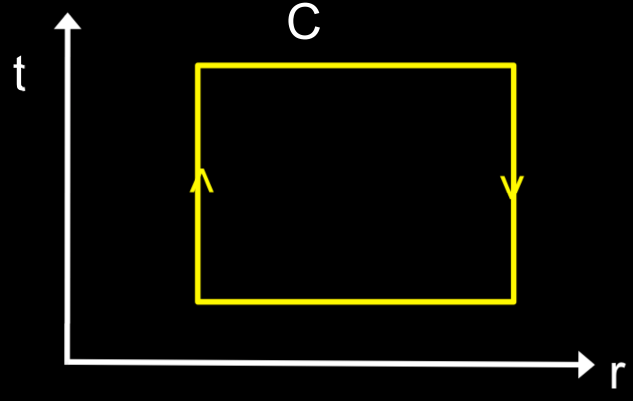
$$\mathcal{L}_{\text{pNRQCD}} = S^\dagger \left(i\partial_0 - \frac{\vec{p}^2}{2\mu} - V_{\bar{q}-q}(\vec{r}) \right) S$$

→ Dynamics of heavy mesons are systematically studied

→ q^{bar}-q potential derived from QCD plays an important role

✓ Q^{bar}-Q potential from LQCD

Static Q^{bar}-Q potential energy can be extracted in LQCD



- Expectation value of Wilson loop

$$< W(C) > = < tr\mathcal{P}e^{ig\int_c dz_\mu A_\mu(z)} > \propto e^{-V(R)T}$$

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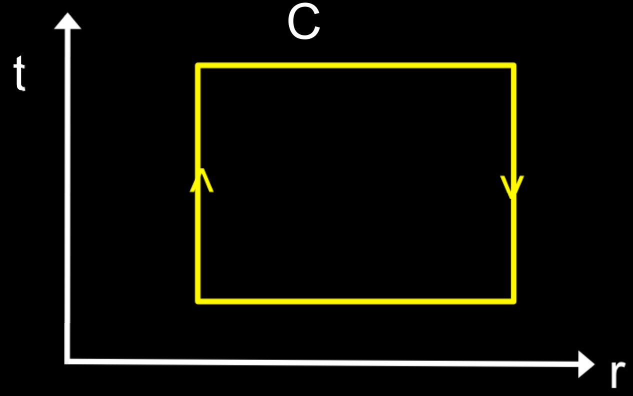

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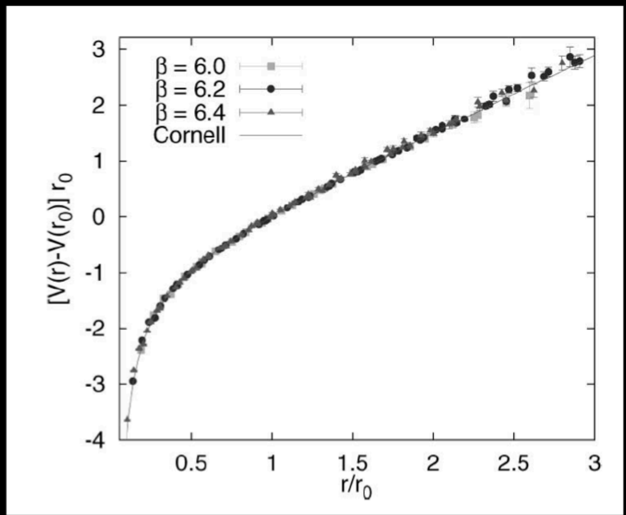
Quenched QCD



Bali, Phys Rep. 343 (2001).

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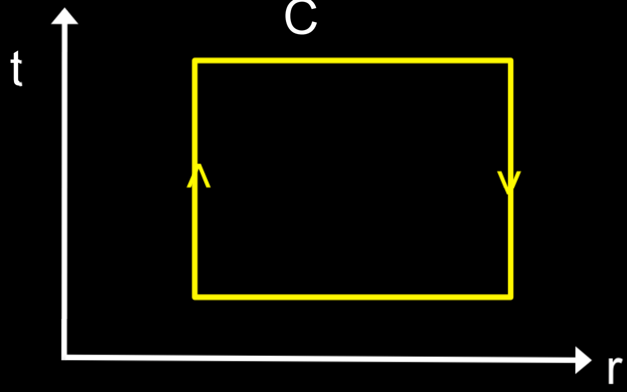

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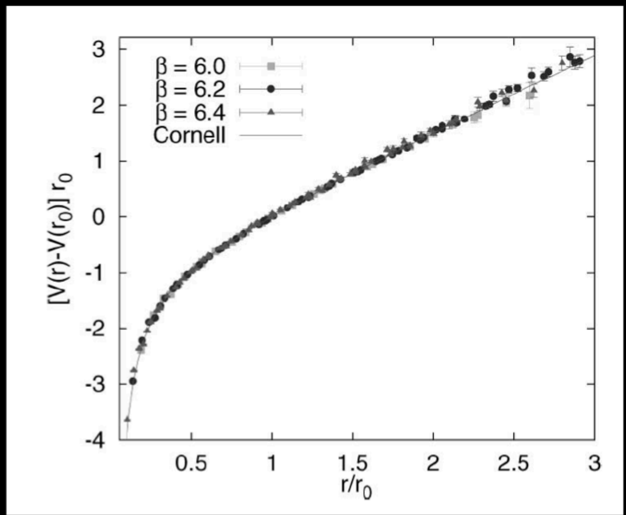
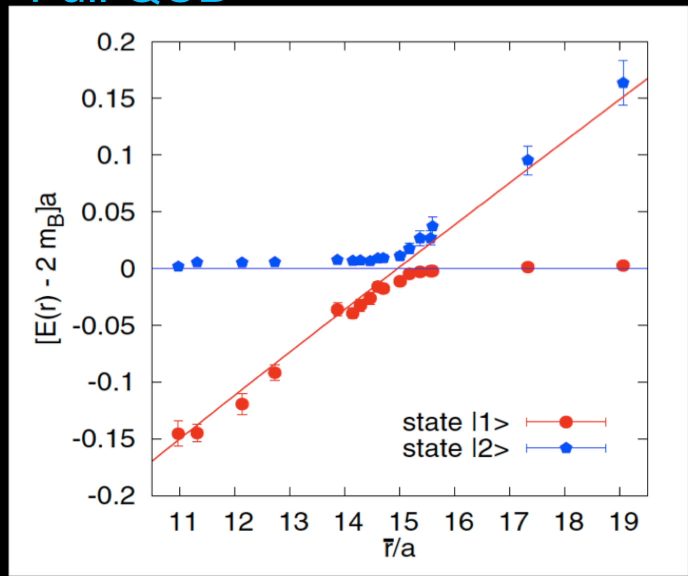
Quenched QCD



Bali, Phys Rep. 343 (2001).

Full QCD



String breaking by dynamical quarks

SESAM Collaboration, PRD**71** (2005).

✓ Relativistic correction to Q^{bar}-Q potential

- Static Q^{bar}-Q potential from Wilson loop

$$V(r) = \sigma r - \frac{A}{r} + \epsilon$$

$\sigma = 0.9$ GeV/fm $A = 0.055$ GeV fm

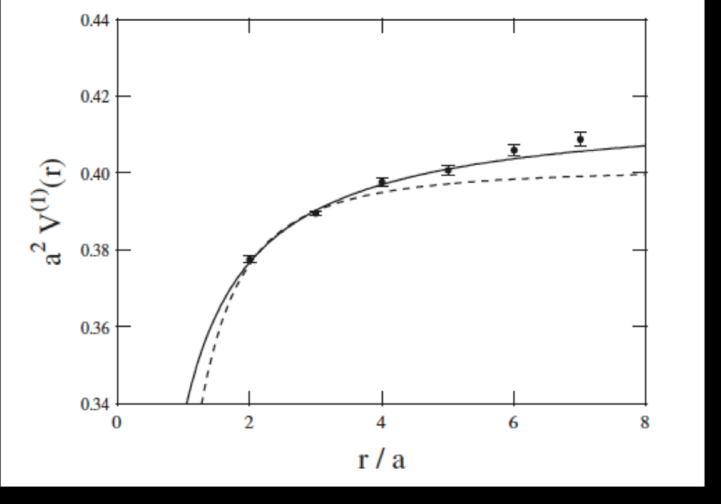
✓ Relativistic correction to Q^{bar}-Q potential

- Static Q^{bar}-Q potential from Wilson loop

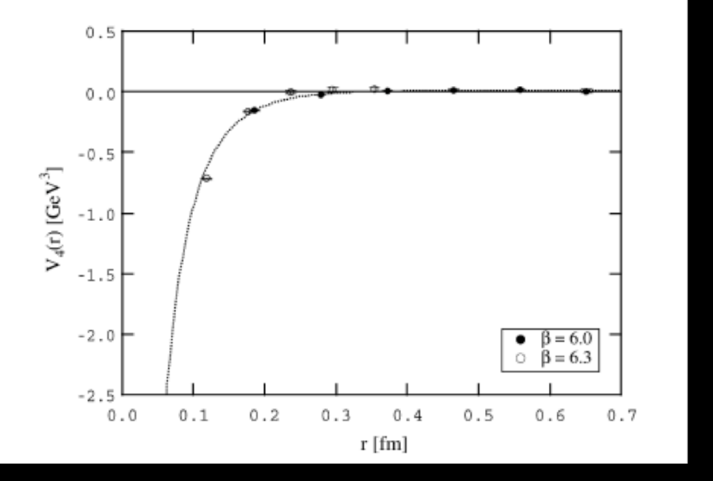
$$V(r) = \sigma r - \frac{A}{r} + \epsilon \qquad \sigma = 0.9 \text{ GeV/fm} \qquad A = 0.055 \text{ GeV fm}$$

- Finite quark mass effects are taken into account through relativistic correction

Bali, Phys Rep. **343** (2001), Koma-Koma, NP**B769** (2007).



Correction from O(1/m)



Correction from $O(1/m^2)$

✓ Our motivation

We study "q^{bar}-q" potential including fully non-perturbative finite quark mass effect

↓

Bethe-Salpeter Wave function → q^{bar}-q potential

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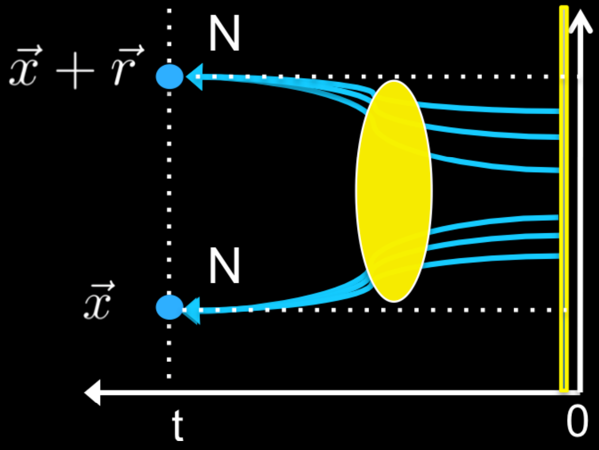
We study "q^bar-q" potential including fully non-perturbative finite quark mass effect

Bethe-Salpeter Wave function → q^bar-q potential

- Nuclear force from BS wave function



$$\phi_{\alpha\beta}(\vec{r},t) = \sum_{\vec{x}} < N_\alpha(\vec{x}+\vec{r},t) N_\beta(\vec{x}) \mathcal{J}^\dagger_{\mathbf{src}} >$$

$$\rightarrow A_0 \phi(\vec{r}) e^{-Wt}$$

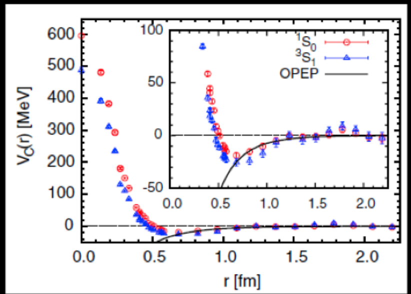
Two-nucleon Bethe-Salpeter wave function, φ(r) is obtained from LQCD

$$U(\vec{r},\vec{r}') = V(\vec{r},\nabla)\delta(\vec{r}-\vec{r}')$$

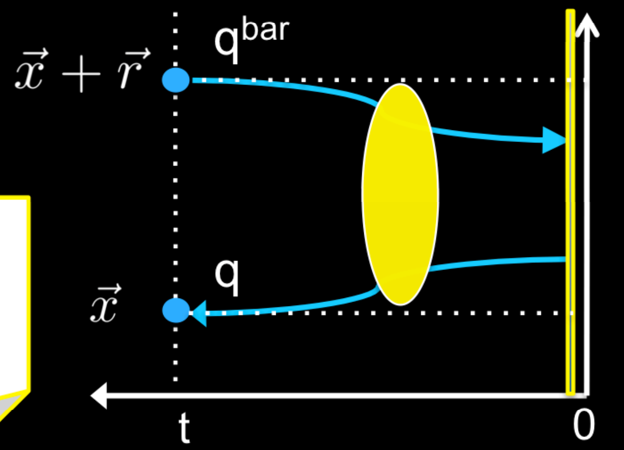
$$V(\vec{r}) = \frac{1}{m_N}\frac{\nabla^2\phi(\vec{r})}{\phi(\vec{r})} + E$$

Ishii, Aoki, Hatsuda, PRL**99**, 022001.

Aoki, Hatsuda, Ishii, PTP**123**, 89.



✓ Formalism



- Starting with equal-time BS amplitudes

$$\chi(\vec{r},t) =< \bar{q}(\vec{x}+\vec{r},t)\Gamma q(\vec{x},t)|\bar{q}q;J^P > \\ \rightarrow A_0\phi(\vec{r})e^{-m_{\rm eff}t}$$

Spatial correlation φ(r) is Bethe-Salpeter wave function for q^{bar}-q system

- S-wave projection

$$\phi^{\rm S-wave}(\vec{r}) = \frac{1}{24}\sum_{\mathcal{R}\in\mathcal{O}}\frac{1}{L^3}\sum_{\vec{x}} < \bar{q}(\vec{x}+\mathcal{R}(\vec{r}))\Gamma q(\vec{x})|\bar{q}q;J^P >$$

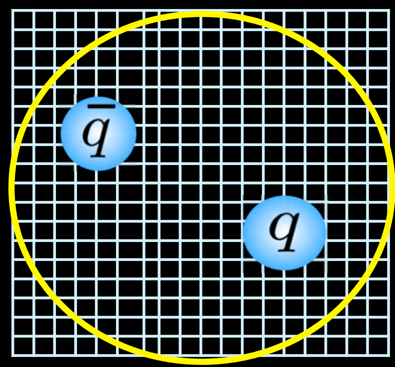
$$\Gamma = \gamma_5\ (0^-),\ \ \gamma_i\ (1^-)$$

- Effective central q^{bar}-q potential

$$V(\vec{r}) = \frac{1}{m_q}\frac{\nabla^2\phi^{\rm S}(\vec{r})}{\phi^{\rm S}(\vec{r})} + E$$

✓ Numerical setup

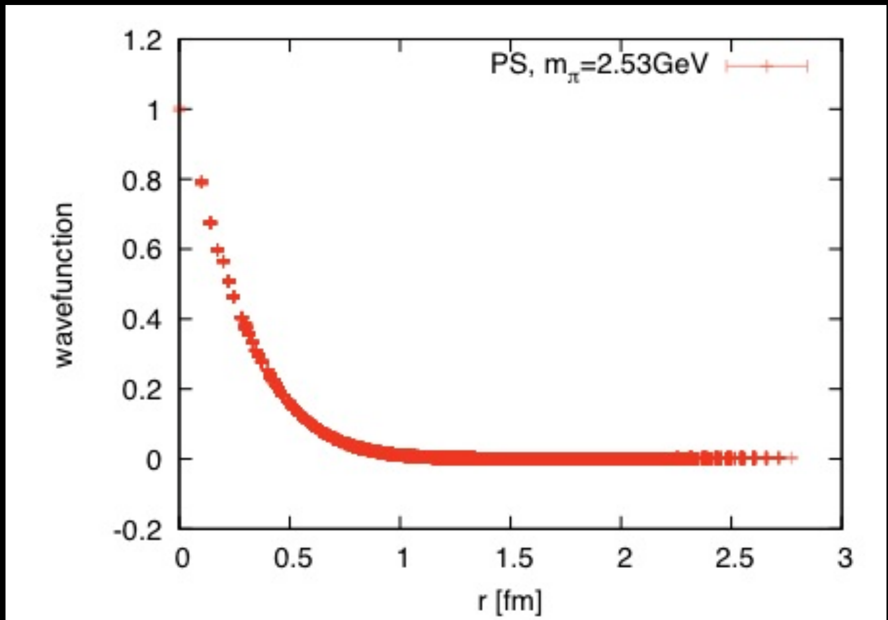
- ✓ Quenched QCD
- ✓ Plaquette gauge action & Standard Wilson quark action
- ✓ β=6.0
- ✓ Lattice spacing : a=0.104 [fm]
- ✓ Size of Lattice : 32^3 X 48 → L=3.3 (fm)
- ✓ Quark mass : m_{PS} = 2.53, 1.77, 1.27, 0.94 (GeV)
  m_{VE} = 2.55, 1.81, 1.35, 1.04 (GeV)
- ✓ # of conf. = 100
- ✓ Flat wall source
- ✓ Periodic boundary condition
- ✓ Gauge fixing : Coulomb gauge



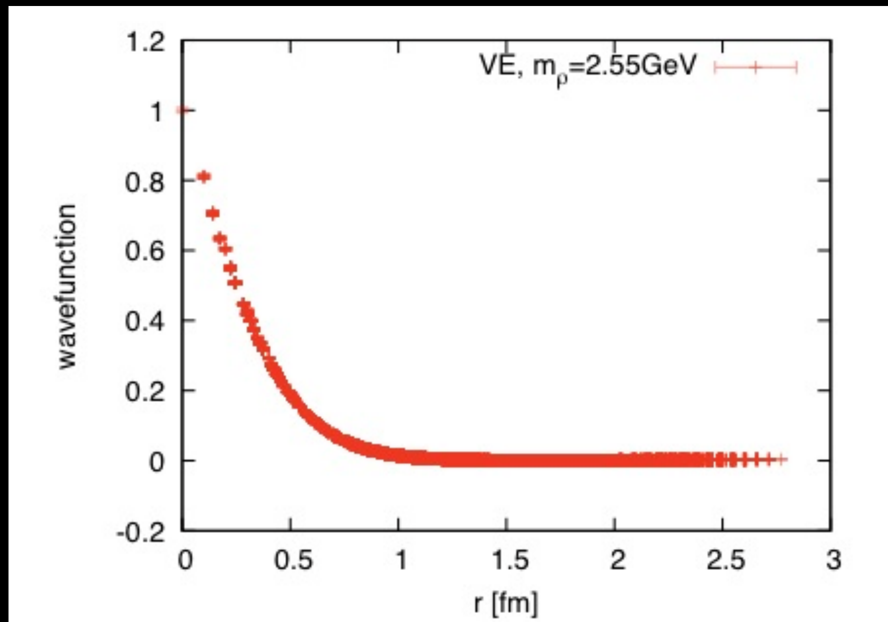
✓ Results (S-wave q^{bar}-q wave function)

- $\phi^s(r)$ at m_{PS}=2.53 and m_{VE}=2.55 (GeV)

Pseudoscalar channel



Vector channel



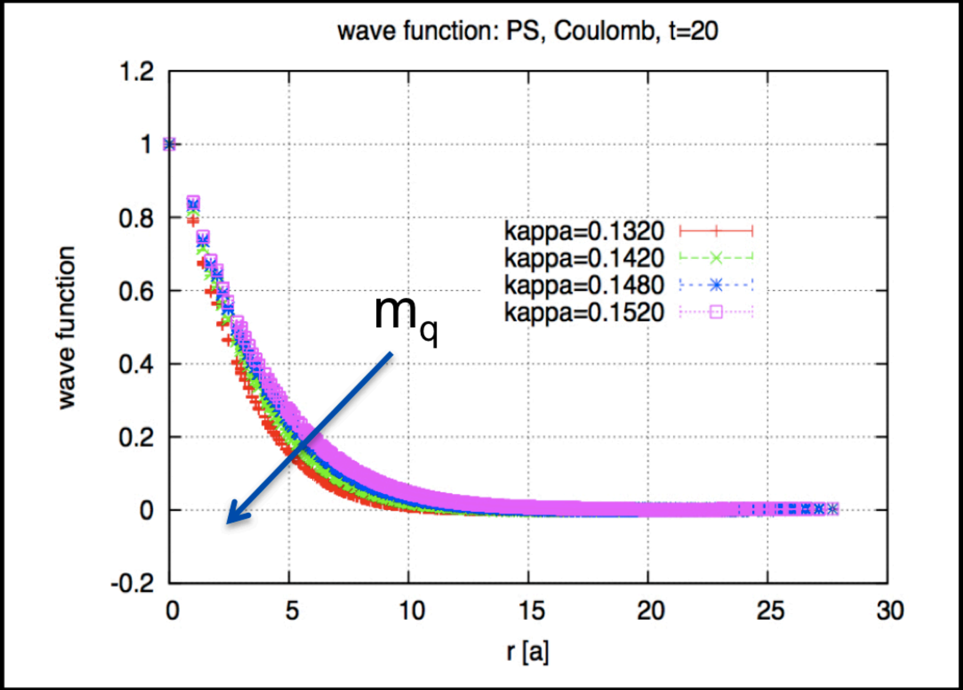
- q^{bar}-q wave function is localized within 1.5 (fm)
  → box size is enough
- There is little dependence between pseudoscalr and vector channels

✓ Results (S-wave q^{bar}-q wave function)

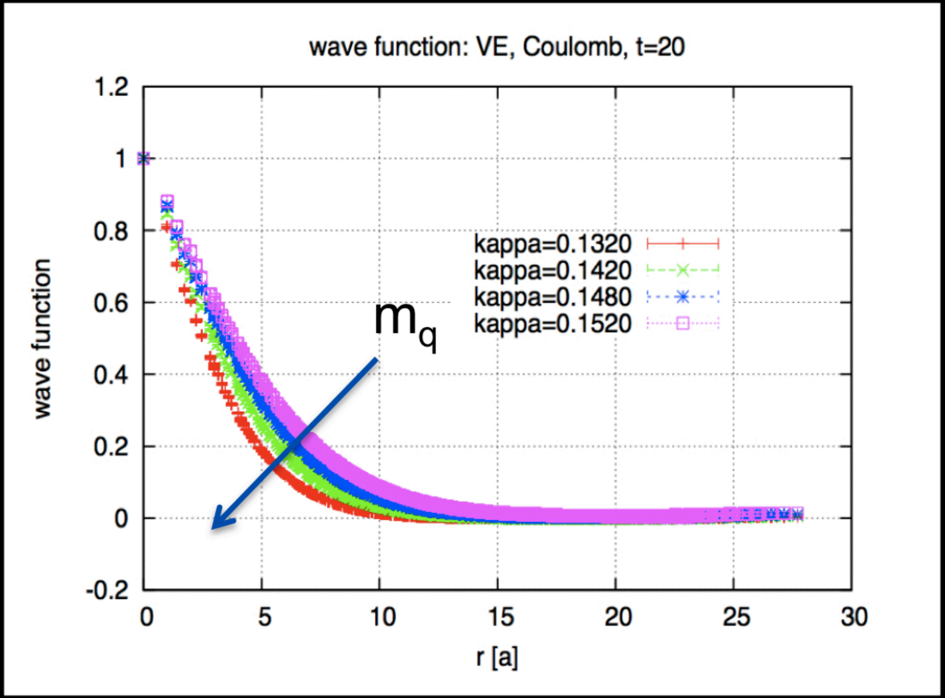
● $\phi^s(r)$ at various quark masses

m_{PS} = 2.53, 1.77, 1.27, 0.94 (GeV)
m_{VE} = 2.55, 1.81, 1.35, 1.04 (GeV)

Pseudoscalar channel



Vector channel

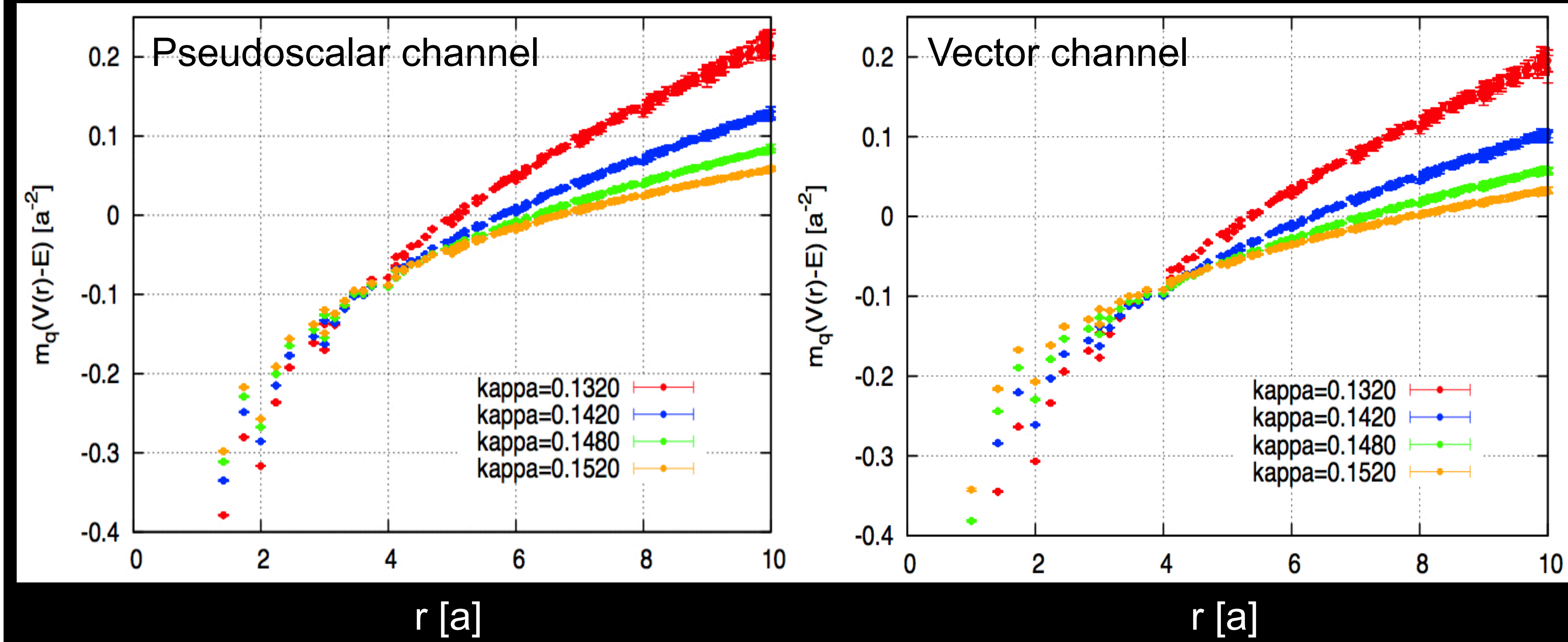


● Size of wave function becomes smaller as increasing m_q
● All the wave functions are localized in this box size

✓ Results (S-wave q^{bar}-q potential)

$$\frac{\nabla^2 \phi(\vec{r})}{\phi(\vec{r})} = m_q(V(\vec{r}) - E)$$

m_{PS} = 2.53, 1.77, 1.27, 0.94 (GeV)
m_{VE} = 2.55, 1.81, 1.35, 1.04 (GeV)



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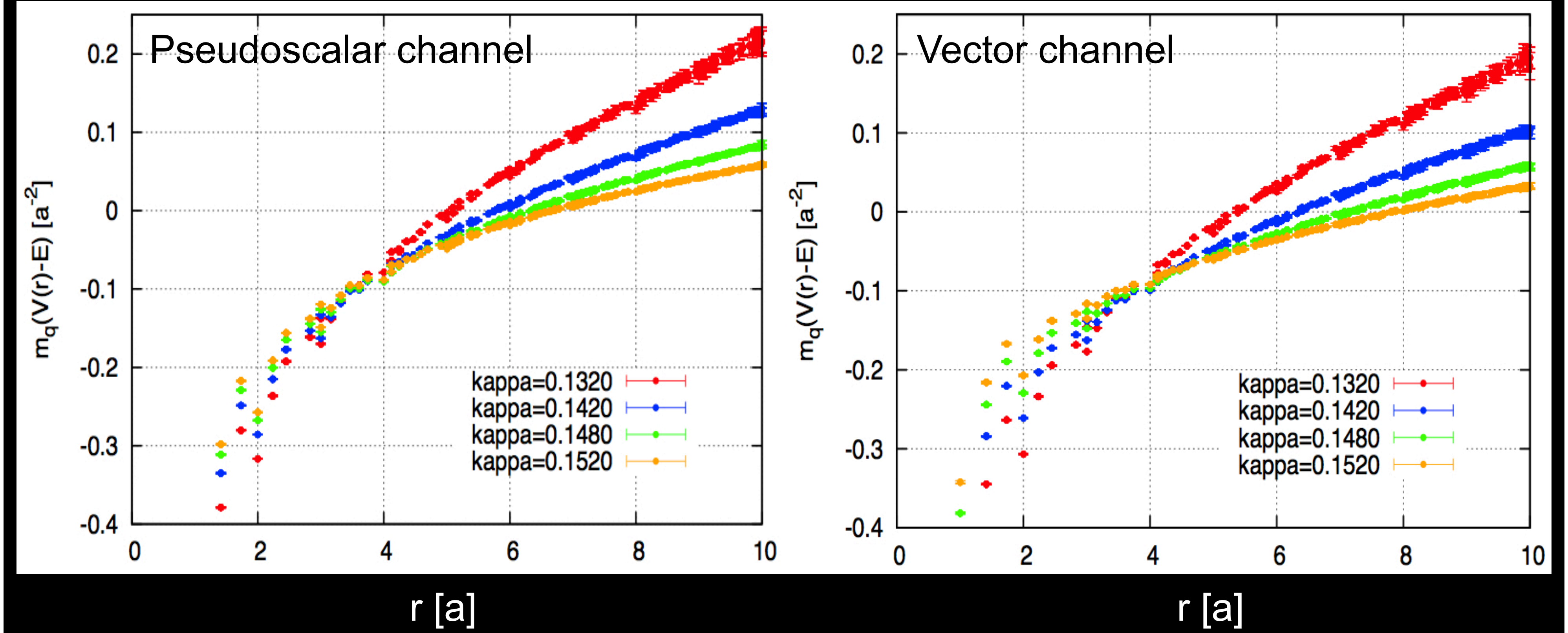
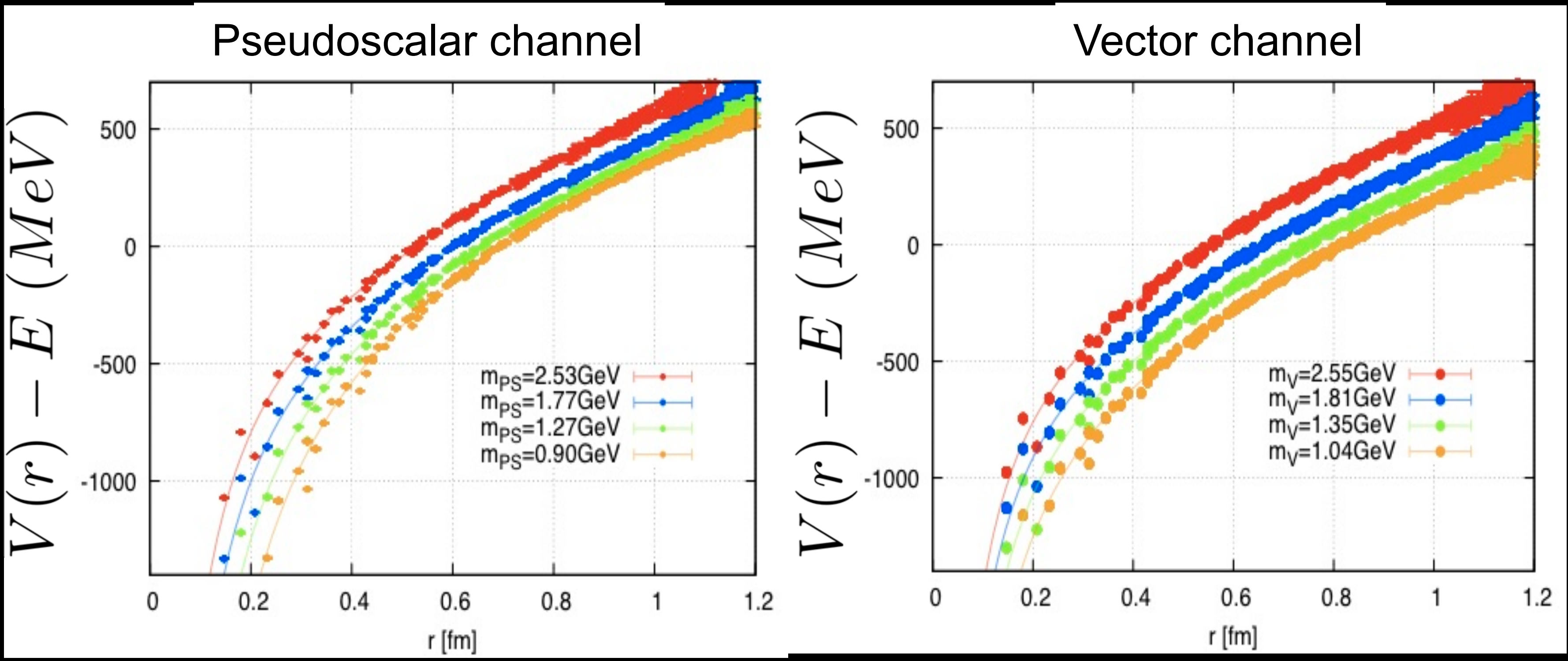


These potentials show Coulomb + linear behavior

✓ Fitting Results

- In order to fit analytic function to LQCD data, we define m_q as $m_{VE}/2$



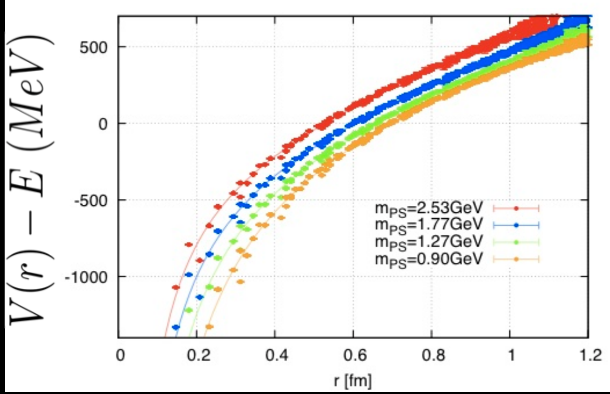
- We choose Coulomb + linear function as a fitting function

$$f(r) = \sigma r - \frac{A}{r} + \epsilon$$

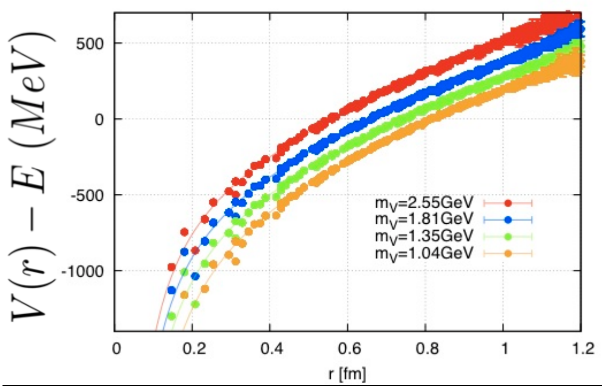
- Fitting works very well

✓ Fitting Results

Pseudoscalar channel



Vector channel



Fitting function

$$f(r) = \sigma r - \frac{A}{r} + \epsilon$$

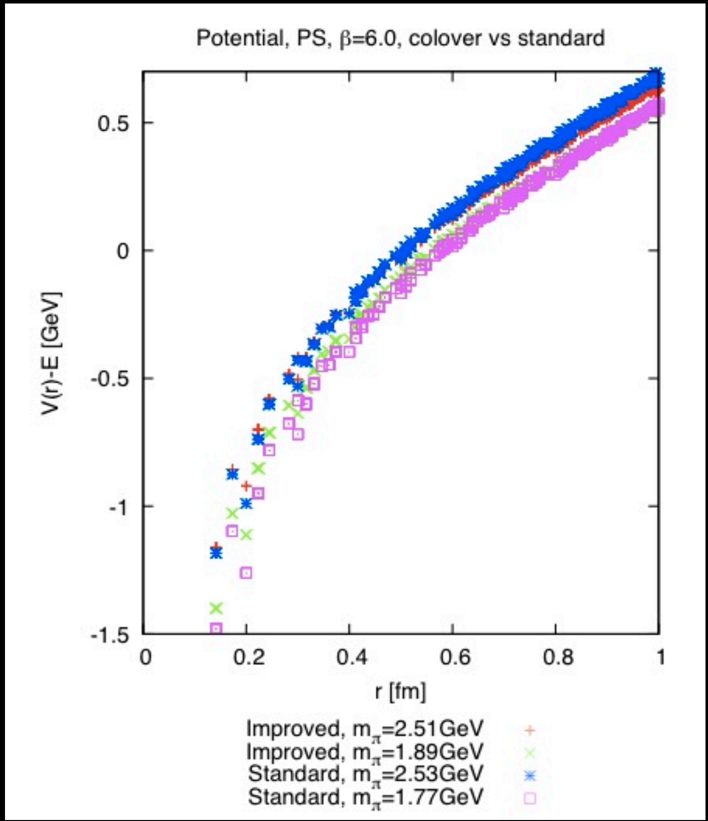
Pseudoscalar			m_q ↑	Vector		
	σ (MeV/fm)	A (MeV·fm)			σ (MeV/fm)	A (MeV·fm)
$m_{PS} = 2.53$	950	155		$m_V = 2.55$	1011	123
$m_{PS} = 1.77$	878	193		$m_V = 1.81$	951	136
$m_{PS} = 1.27$	821	250		$m_V = 1.35$	920	156
$m_{PS} = 0.94$	762	329		$m_V = 1.04$	914	182

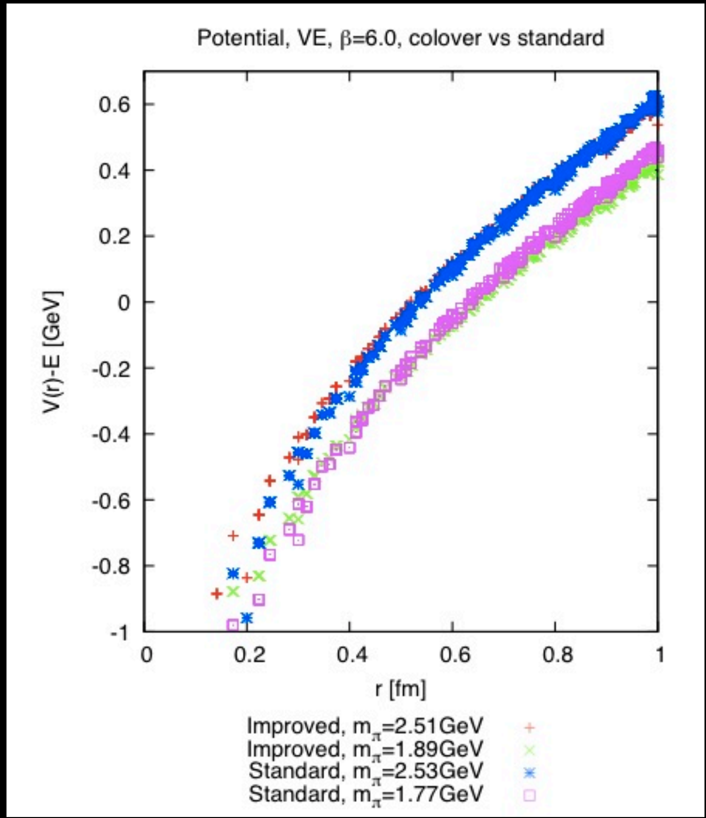
- Roughly reproduce known value of string tension from Wilson loop
- String tension has moderate m_q dependences
- Coulomb coefficient becomes smaller and smaller as increasing m_q

✓ O(a) improvement for quark action

- We study cutoff dependence of the q^{bar}-q potentail
  by adopting O(a)-improved Wilson-clover quark action

We compare Standard Wilson quark action
with O(a) improved action (clover action)



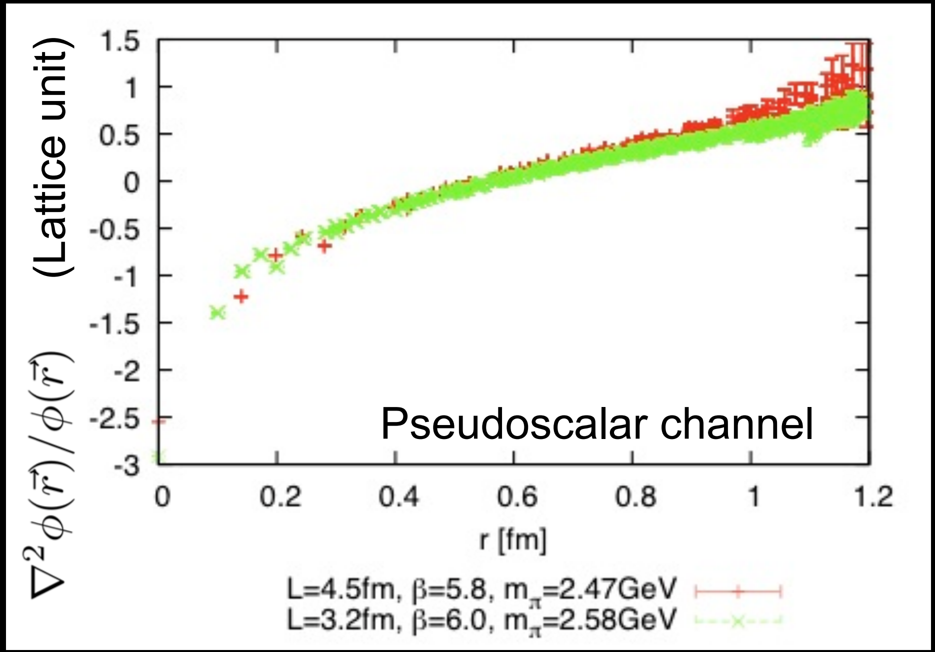


Small difference

✓ Volume dependence

- We study volume dependence of the q^{bar}-q potentail by varying lattice spacing for O(a)-improved Wilson-clover quark action

L=4.5fm (β=5.8, m_{PS}=2.47GeV, clover): red
L=3.2fm (β=6.0, m_{PS}=2.58GeV, standard): green



- Small difference between them … volume is enough

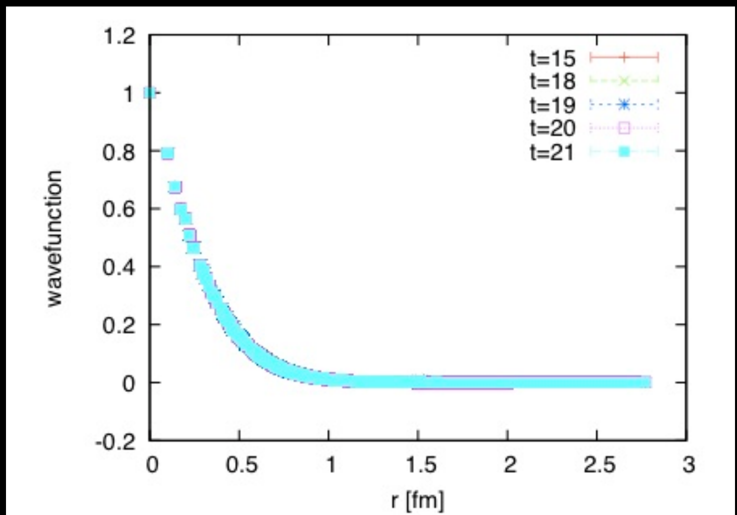
Our setup (β=6.0, a=0.1fm, standard Wilson, $(3.2fm)^3$) seems sufficient for the calculation of q^{bar}-q potential (in quark mass region calculated here)

✓ Summary

- S-wave q^{bar}-q potentials derived from BS wave function are studied
  - → Fully non-perturbative quark mass effect can be taken into account
    - → "Coulomb + linear" behavior for various quark masses
    - → Moderate quark mass dependence for string tension
    - → Strong quark mass dependence for Coulomb coefficient

Future plan :

- Gauge invariant q^{bar}-q potentials
- Three-quark potentials
- Inter-quark potentials in full QCD (channel dependence)
- Simulation at finite temperature


wavefunction
r [fm]
t=15
t=18
t=19
t=20
t=21
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.5
1
1.5
2
2.5
3