Lattice calculation for unitary fermions in a finite box

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Motivation

- Challenge : Study many strongly interacting non-relativistic fermions from microscopic theory

  i.e. Unitary fermions, nuclei, neutron star, ...

  $\Rightarrow$ Lattice calculation with effective field theory (EFT)

- Systematic errors are under control (without trap) :

  discretization errors
  finite volume effects
  continuum limit
  } by perfect dispersion relation and tuning parameters (2-body sector)

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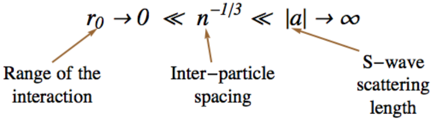
- *Is that enough to simulate many strongly interacting non-relativistic fermions without external trap?*

Outline

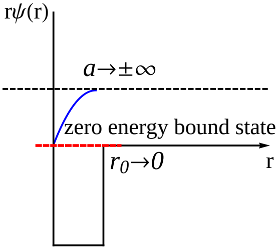
1. Lattice approach for unitary fermions
2. Pairing source for the calculation of many particles up to $N = 38$
3. Preliminary numerical results : ξ and Δ
4. Challenges and future work

What is unitary fermion?

- Unitary fermions : Spin 1/2 fermions with attractive interactions



$$\Longleftrightarrow \quad p\cot\delta_0 = 0 \quad (\text{or } \delta_0 = \pi/2)$$



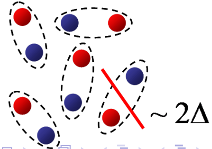
  - ▲ Universal
  - ▲ Strongly interacting
  - ▲ Non-relativistic conformal

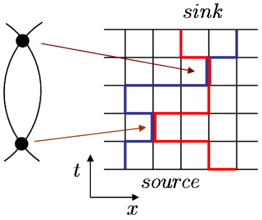
- No intrinsic scale except density (n)
  - Universal constant ξ (Bertsch parameter)

$$E(n) = \xi E_{free}(n)$$

  - Pairing gap : energy cost to break a pair

$$\frac{\Delta(n)}{\mu_{free}(n)} = constant$$





- *Zero range interaction* : Time-like four-fermi contact interaction with Z_2 auxiliary field, $\phi = \pm 1$ (open B.C. in t and periodic B.C. in $\mathbf{x}$)

$$\mathcal{T} = D^{-1/2}(1 - \kappa\phi)D^{-1/2}$$

$\Rightarrow$ **Quenched simulation is exact**

- *Infinite scattering length* : Couplings are tuned to reproduce the Luscher's lower energy eigenvalues in a finite volume for $p \cot \delta = 0$ in continuum theory (2-body sector)

(lattice, finite)		(continuum,finite)		(continuum, infinite)
eigenvalues of the 2-particles transfer matrix	$\Longleftrightarrow$	**Luscher's energy eigenvalues**	$\Longleftrightarrow$	$p \cot \delta = 0$

Overlap problem in the simulation of many unitary fermions

- Measurement : N-body correlation function by Slater-determinent

$$C_{(N,N)}(\tau) = \langle \det S(\tau) \det S(\tau) \rangle_\phi, \quad \text{where } S_{i,j}(\tau) = \langle i | \mathcal{T}^\tau | j \rangle$$
$$\sim z e^{-E_{(N,N)}\tau}, \quad \text{in the large } \tau \text{ limit}$$

True in principle

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True in principle, but NOT Practically

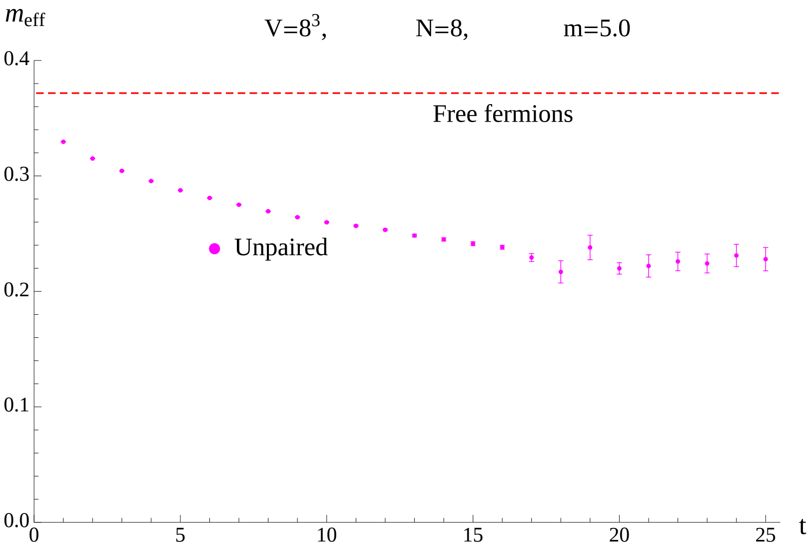
- Overlap of the Slater-determinent of single particle source with the true ground state exponentially decreases with N

- 2-particle wave functions for free and unitary fermions :

$$\psi_{free}(r) \sim 1, \qquad \psi_{unitary}(r) \sim \frac{1}{r}$$

$\Rightarrow$ *Very small overlap*

Effective mass plot : unpaired vs paired



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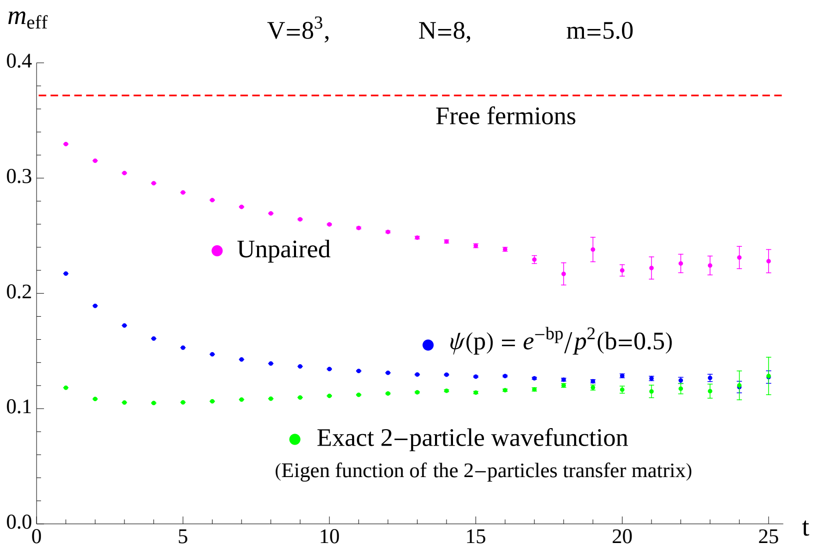


N-particle correlation function with pairing source

- Trial pairing source

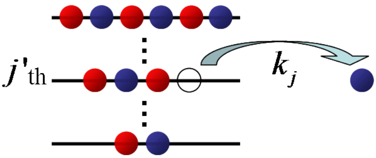
$$\psi(r) = \frac{1}{\sqrt{r^2+b^2}}, \qquad \tilde{\psi}(p) = \frac{e^{-bp}}{p^2}$$

- N-body correlation function for unpolarized 2-component fermions ($N_\uparrow = N_\downarrow = N/2$) with pairing source ψ

$$G_{(N_\uparrow,N_\downarrow)}(\tau) = \begin{vmatrix} \langle\psi|\mathcal{T}_\uparrow^\tau \mathcal{T}_\downarrow^\tau|11\rangle & \langle\psi|\mathcal{T}_\uparrow^\tau \mathcal{T}_\downarrow^\tau|12\rangle & \dots & \langle\psi|\mathcal{T}_\uparrow^\tau \mathcal{T}_\downarrow^\tau|1N_\downarrow\rangle \\ \langle\psi|\mathcal{T}_\uparrow^\tau \mathcal{T}_\downarrow^\tau|21\rangle & \langle\psi|\mathcal{T}_\uparrow^\tau \mathcal{T}_\downarrow^\tau|22\rangle & \dots & \langle\psi|\mathcal{T}_\uparrow^\tau \mathcal{T}_\downarrow^\tau|2N_\downarrow\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle\psi|\mathcal{T}_\uparrow^\tau \mathcal{T}_\downarrow^\tau|N_\uparrow 1\rangle & \langle\psi|\mathcal{T}_\uparrow^\tau \mathcal{T}_\downarrow^\tau|N_\uparrow 2\rangle & \dots & \langle\psi|\mathcal{T}_\uparrow^\tau \mathcal{T}_\downarrow^\tau|N_\uparrow N_\downarrow\rangle \end{vmatrix}$$

- For $N_\uparrow = N_\downarrow - 1$, replace j'th row by

$$\langle j|\mathcal{T}_\downarrow^\tau|1\rangle \quad \langle j|\mathcal{T}_\downarrow^\tau|2\rangle \quad \dots \quad \langle j|\mathcal{T}_\downarrow^\tau|N_2\rangle$$

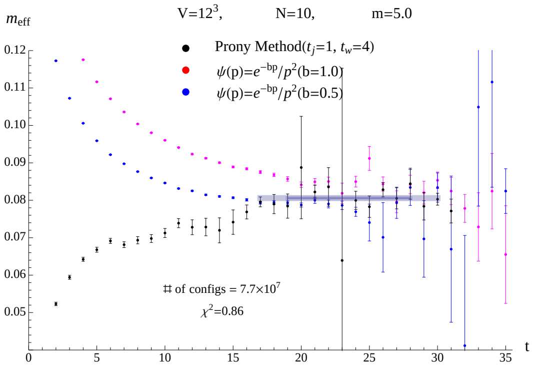


Data analysis

- Extract ground state energy from the N-body correlation functions

$$\begin{aligned} m_{eff} &\equiv \log[G_N(\tau+1)/G_N(\tau)] \\ &\sim m_0, \qquad \text{for large } \tau \end{aligned}$$

- Analysis method
  - Jackknife error estimate
  - Prony Method
  - correlated χ^2 fit



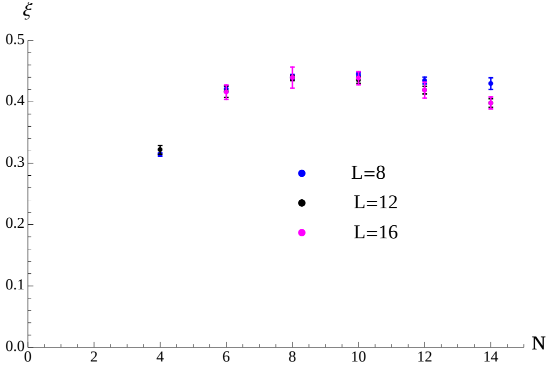
- Not free from signal-to-noise (S/N) problem

Finite volume effects

- Calculate ξ (Bertsch parameter) from the ground state energy of unpolarized unitary fermions

$$\xi = \frac{E(N)}{E_{free}(N)}$$

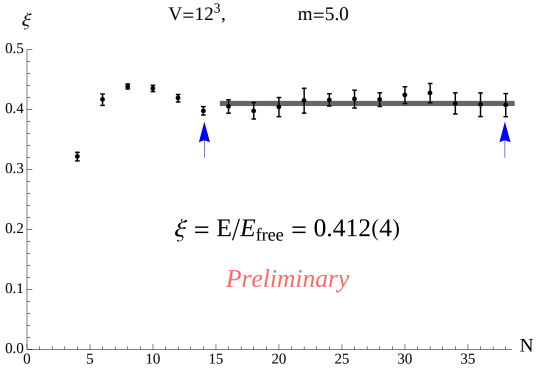
- Tuning lattice parameters $\Rightarrow$ No volume dependence, only if $r_i p < 1$



- Finite volume effects are negligible except $N \geq 12$ for $L = 8$

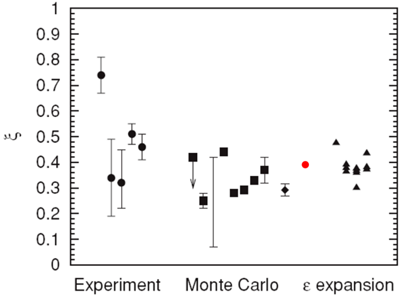
Results : Extract ξ

- ξ (Bertsch parameter) for N up to 38



- The ground state energy of unitary fermions has clear shell structure
- Have not reached to the thermodynamic limit yet

Comparison : ξ from Experiment, QMC, and ϵ-expansion



Our Work : $\xi = 0.412(4)$
(preliminary)

T.Abe and R. Seki,
Phys. Rev. C 79, 054003(2009)

- Recent calculations of ξ

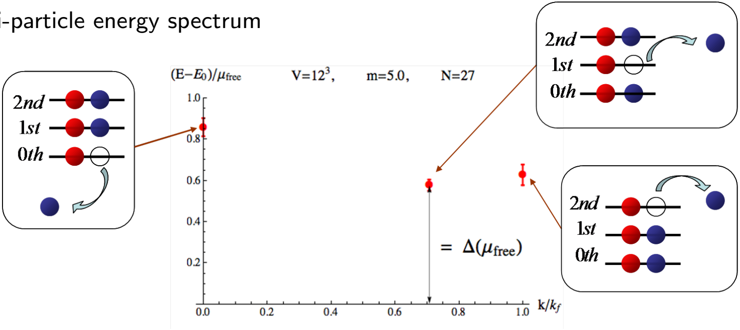
Method	Authors/year	ξ
Cold Atomic Exp	J. Kinast et al(2005)	0.46(5)
Cold Atomic Exp	G. B. Partridge et al(2006)	0.51(4)
GFMC	A. Gezerlis and J. Carlson (2008)	$\leq 0.40(1)$
QMC	A. Bulgac et al(2008)	0.37(5)
Lattice EFT	Dean Lee(2008)	0.329(5)
ϵ-expansion	D. Son and Y. Nishida(2007)	0.365(10)

Results : Quasi-particle energy spectrum

- Pairing gap

$$\Delta = E(N) - E_0(N), \qquad E_0 = \frac{E(N+1) + E(N-1)}{2} \text{ and } N \text{ is odd,}$$

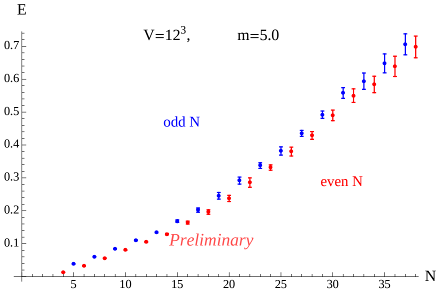
- Quasi-particle energy spectrum



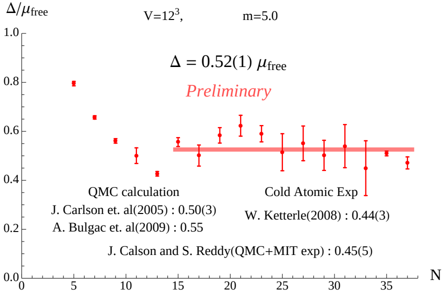
- For the pairing gap (Δ) calculation, use E of odd N unitary fermions by taking out quasi-particle from the first shell

Results : Even-odd staggering and pairing gap

- Ground state energies for unpolarized and slightly polarized $(N/2, N/2-1)$ unitary fermions



- Pairing gap (Δ)



Challenges and future work

- Further improvements and challenges to go to higher N
  - Improve the tuning procedure
  - Improve pairing source
  - Signal-to-noise problem

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- Lesson in simulating many strongly interacting particles in a finite box
  - *Good source* : necessary to see true ground state
- Future directions
  - Exploring polarized system
  - Study neutron matter with finite scattering length and finite range of interaction

Acknowledgements

- Most of these simulations were performed on New York Blue (BG/L) at Brookhaven National Laboratory