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Low energy charmonium-nucleon scattering with twisted boundary conditions

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Need detail information of the charmonium-nucleon interaction from lattice QCD

- ✓ Central potential (Kawanai's talk)
- ✓ Scattering length a_0 and effective range r_0 (scattering phase shift)

to explore the possibility of charmonium bound to light nuclei (d, ^{3}He, ...)

Effective-range expansion

- Phase shift at low energies

$$p \cot \delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2 \sum_{i=0}^{\infty} (r_i^2 p^2)^i$$

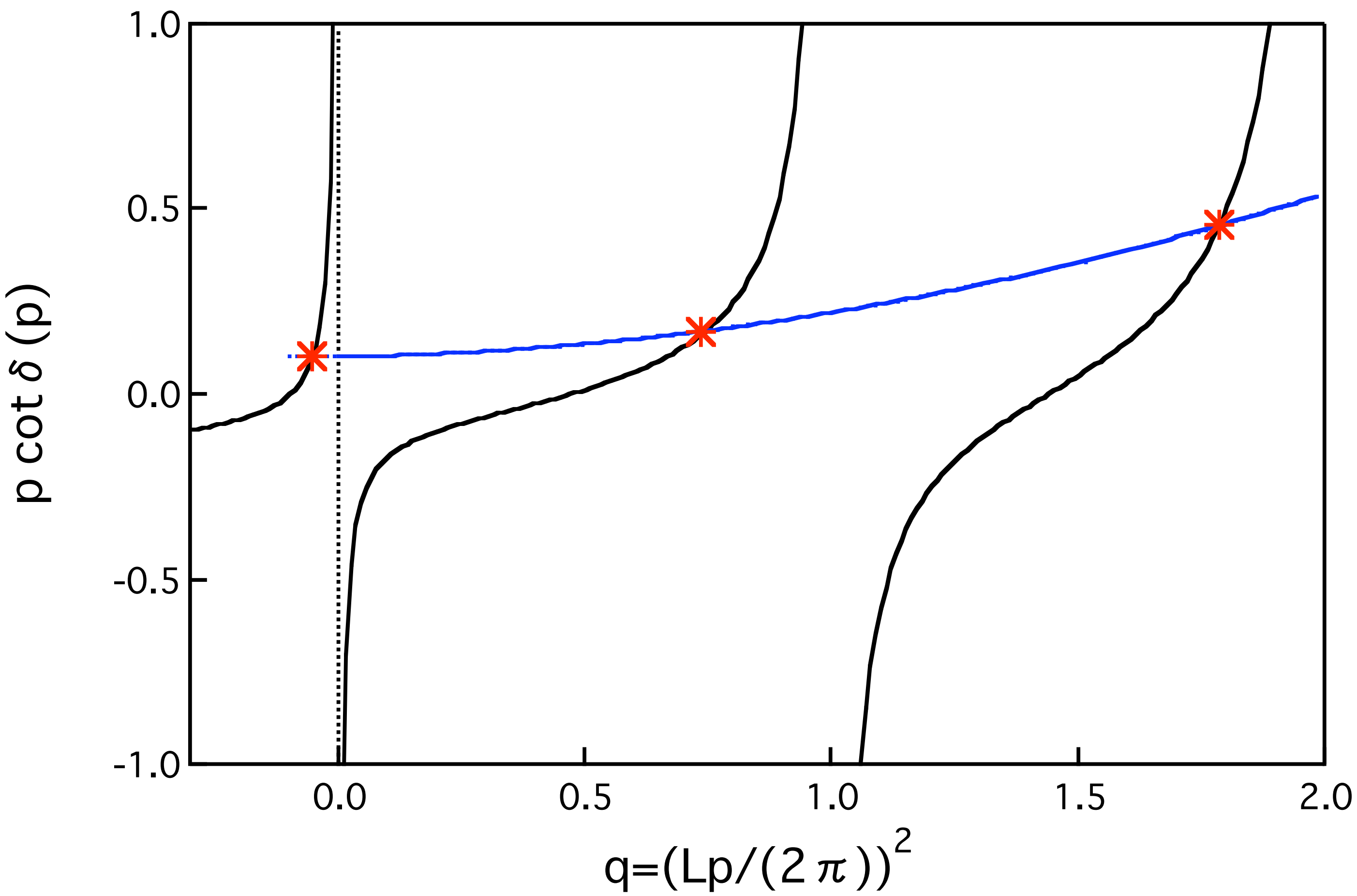
- Model-independent information of the low energy interaction is encoded in a small set of parameters (a_0 and r_0)

- These parameters are associated with the low energy constants in the effective field theory (EFT).

Lüscher's finite-size formalism

- Phase shift can be calculated by

$$p\cot\delta(p) = \frac{\mathcal{Z}_{00}(1,q^2)}{L\pi} \qquad \text{with } q^2 = (Lp/(2\pi))^2$$



$$\mathcal{Z}_{00}(s,q^2) = \sum_{\mathbf{n}\in\mathbf{Z}^3} \frac{1}{(\mathbf{n}^2 - q^2)^s}$$

But, the limited values of the phase shift are only accessible due to the discrete momenta in finite volume

Two difficulties (1)

- We must calculate several values of the phase shift at lower momenta

$$p^2 = \left(\frac{2\pi}{L}\right)^2 \cdot m \quad (m = 0, 1, 2, \cdots)$$

➡ Different momentum modes do mix since the relative momentum is not conserved due to scattering (Maiani-Testa, PLB245, (90) 585)

✓ require the diagonalization method (Michael, Lüscher-Wolff), which is a sophisticated but expensive calculation

Two difficulties (2)

- We must calculate several values of the phase shift at lower momenta

$$p^2 = \left(\frac{2\pi}{L}\right)^2 \cdot m \quad (m = 0, 1, 2, \cdots)$$

➡ Recall the size of non-zero smallest momentum under the periodic b.c.

$$|p_{\rm min}| = \frac{2\pi}{L} \approx 420 \text{ MeV for } L \sim 3 \text{ fm}$$

$$\approx 250 \text{ MeV for } L \sim 5 \text{ fm}$$

which might be beyond the radius of convergence for the effective-range expansion in the attractive interaction case

Our strategy

- Lüscher's method with twisted boundary conditions
  - Benefits:
    - can access any small momentum even in finite volume
    - not necessary to calculate the higher Fourier modes ➡ can focus only on the lowest mode **n**=(0,0,0)
    - can stick to the wall source in order to maintain the translational invariance (4pt function ➡ "2pt" function)

Lüscher finite-size method with twisted boundary conditions

Twisted boundary condition (1)

P.F. Bedaque, PLB593 (04) 82

- Generalized spatial boundary condition (b.c.)

$$\psi(x+L) = e^{i\phi}\psi(x)$$

$\phi = 0$: periodic boundary condition (PBC)
$\phi = \pi$: anti-periodic boundary condition (APBC)

- All momenta are quantized in finite volume as

$$p = \frac{2\pi}{L}\left(n + \frac{\phi}{2\pi}\right) \text{ with integer } n$$

accessible to any small momentum with the angle ϕ

Twisted boundary condition (2)

- Perform the field redefinition of the quark as

$$\psi'(x) = e^{i\vec{\theta}\cdot\vec{x}}\psi(x)$$

  - ✓ The new quark fields ψ' satisfy the PBC if $\theta = \phi$ ／L

- The hopping terms in the action now is transformed as

$$\bar{\psi}'(x)\left[e^{ia\theta_i}U_i(x)(1-\gamma_i)\psi'(x+\hat{i}) + e^{-ia\theta_i}U_i^\dagger(x-\hat{i})(1+\gamma_i)\psi'(x-\hat{i})\right]$$

- This indicates that the twisted BC is easily implemented by replacing the link variables as

$$\{U_i(x)\} \rightarrow \{\, e^{ia\theta_i}U_i(x)\}$$

θ corresponds to the constant U(1) background field

Twisted boundary condition (3)

- The validity of this novel trick has been tested in the dispersion relation of single hadron states

G.M. de Divitiis et al., PLB595 (04) 408
J.M. Flynn et al., PLB632 (06) 313

- It is now widely used in various calculations:
  - the pion electromagnetic form factor

    P.A. Boyle et al., JHEP 05 (07) 016

  - $K \to \pi$ semi-leptonic decay form factor

    P.A. Boyle et al., JHEP 07 (08) 112

  - $\Delta I=3/2$ $K \to \pi\pi$ decay amplitude

    C.H. Kim and C.T. Sachrajda, arXiv:1003.319

Lüscher formula
with twisted boundary conditions

P.F. Bedaque, PLB593 (04) 82

$$p \cot \delta(p) = \frac{\mathcal{Z}_{00}^{\mathrm{d}}(1, q^2)}{L\pi}$$

where $\mathcal{Z}_{00}^{\mathbf{d}}(s, q^2) = \sum_{\mathbf{n} \in \mathbf{Z}^3} \frac{1}{((\mathbf{n} + \mathbf{d})^2 - q^2)^s}$

with $\mathbf{d} = \left(\frac{\phi_1}{2\pi}, \frac{\phi_2}{2\pi}, \frac{\phi_3}{2\pi} \right)$

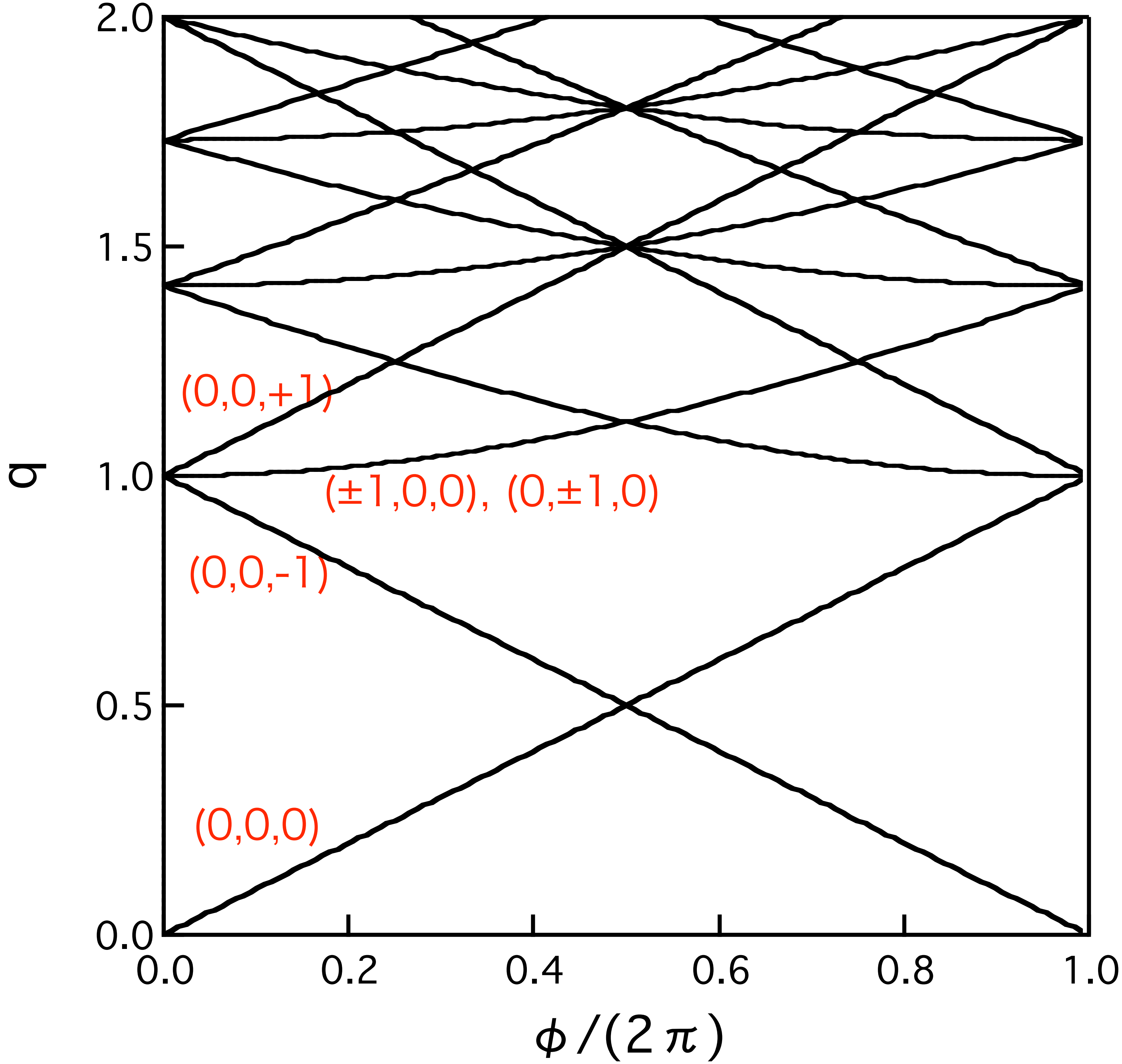
which is defined via analytic continuation from s>3/2 to s=1 pole position: $q^2 = (\mathbf{n} + \mathbf{d})^2 = \mathbf{n}^2 + 2\mathbf{n}\cdot\mathbf{d} + \mathbf{d}^2$

$\mathbf{d} = (0, 0, \frac{\phi}{2\pi})$

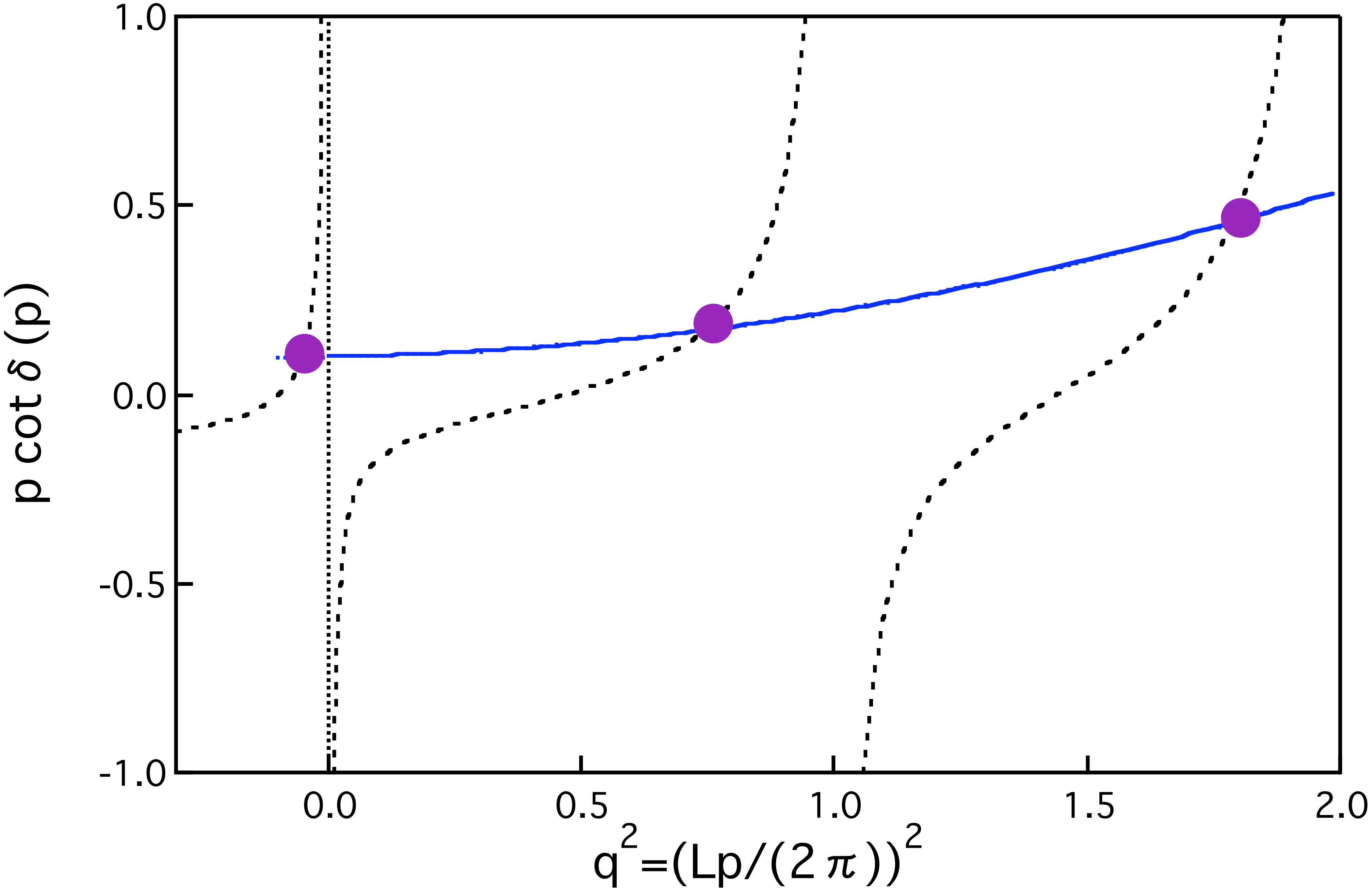
cubic group

↓

point group D4

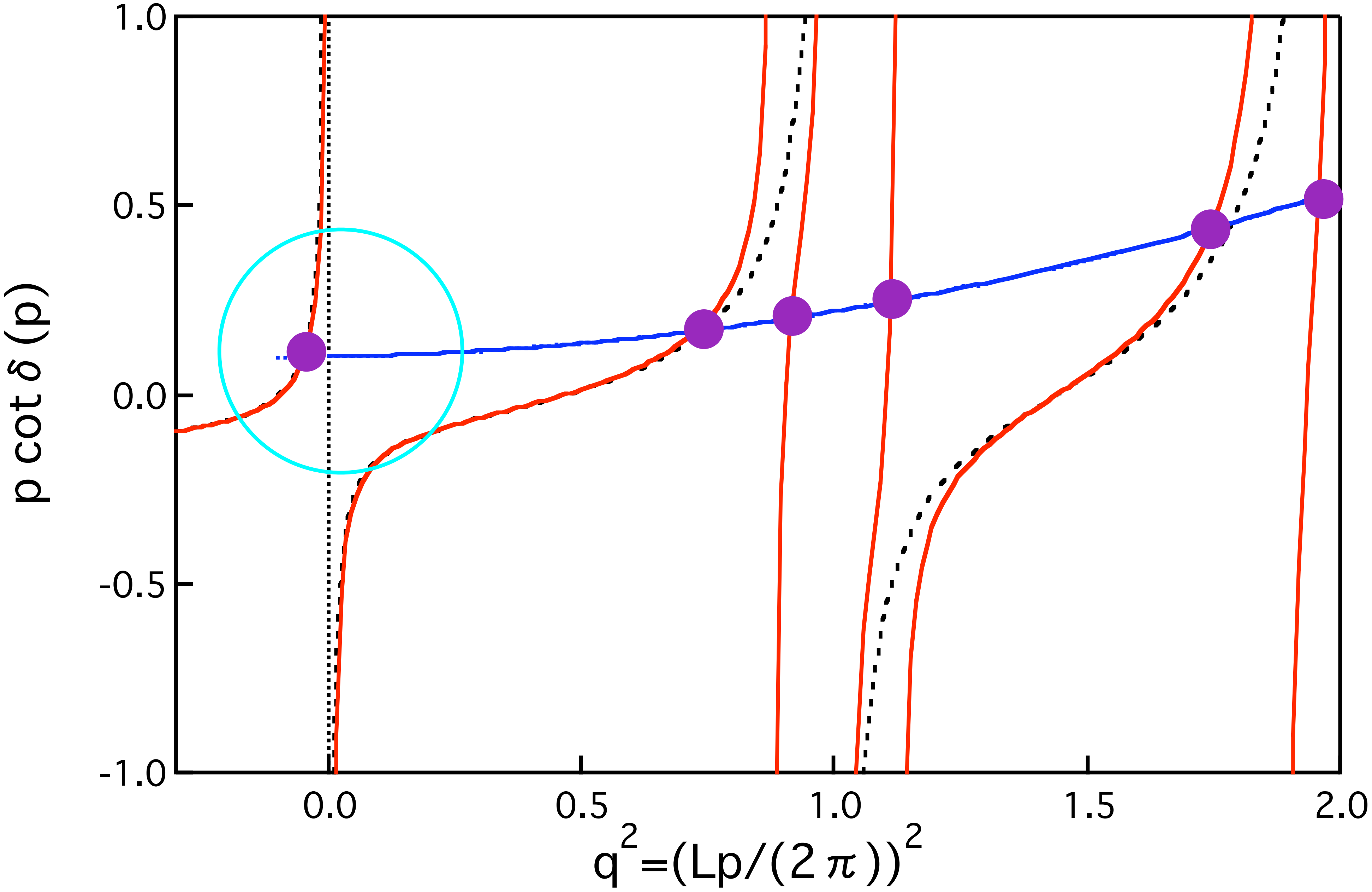


$\phi = 0$



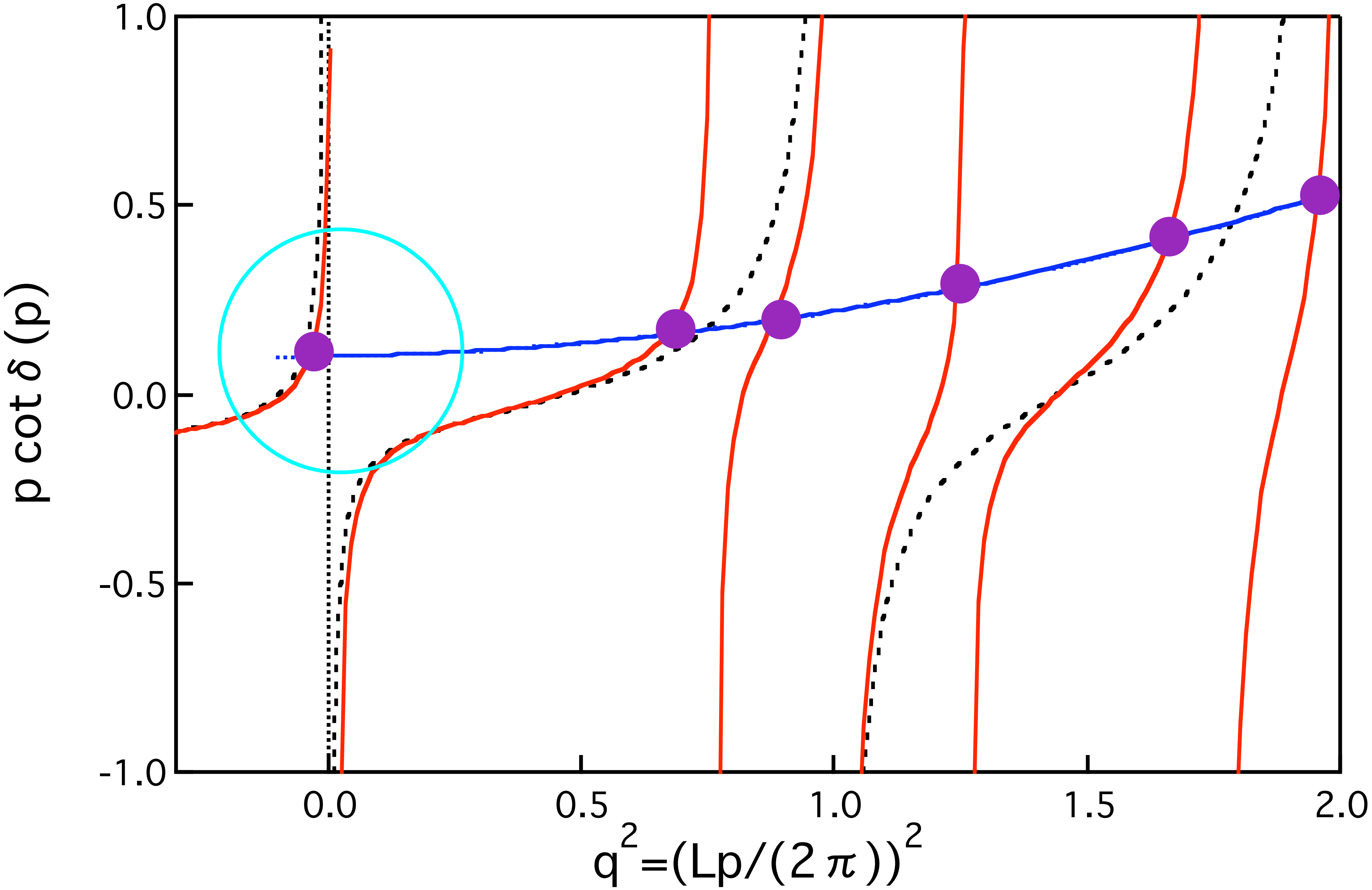
$$\mathbf{d} = (0, 0, \frac{\phi}{2\pi})$$

$\phi = \pi/8$



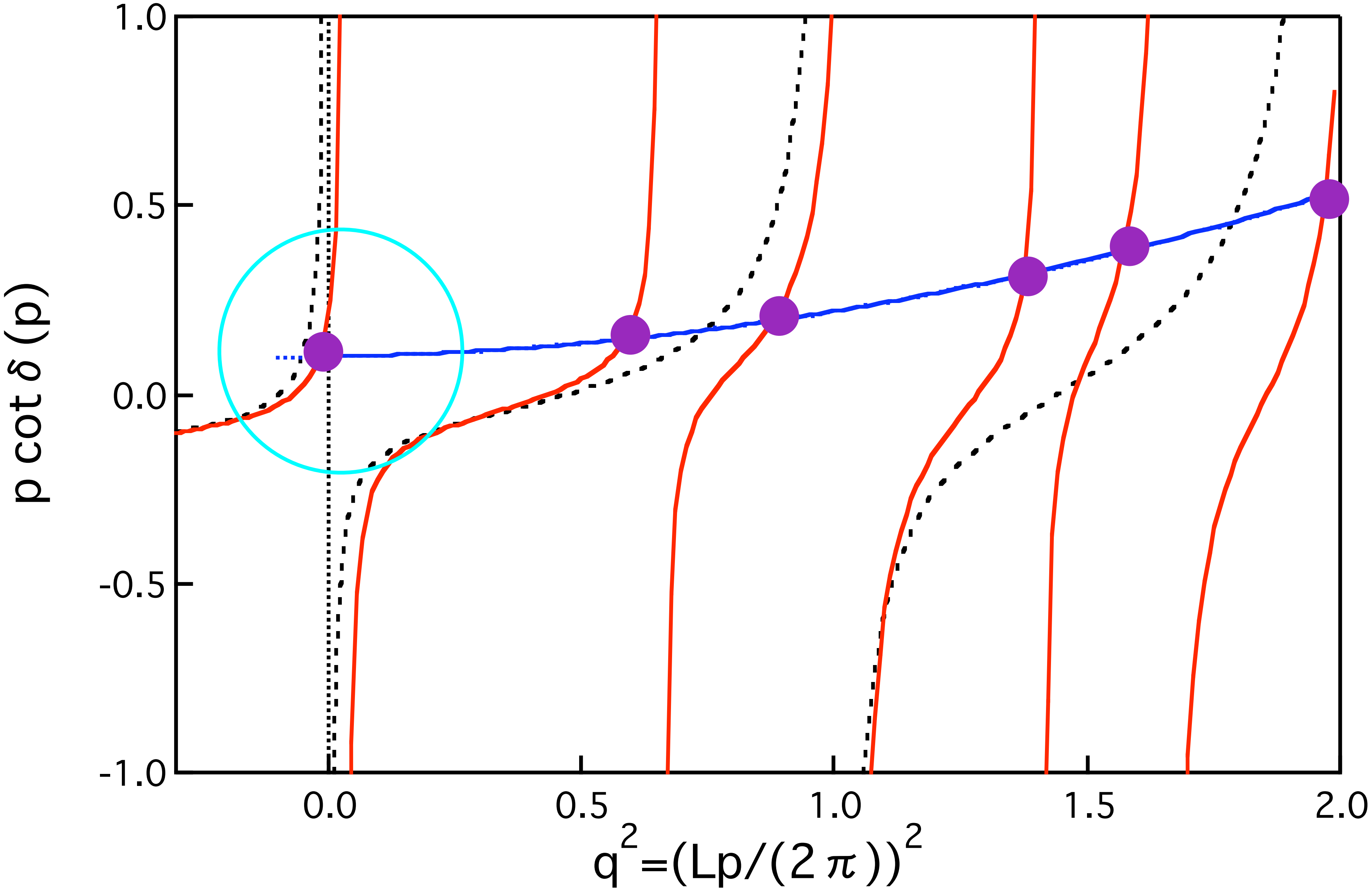
$\mathbf{d} = (0, 0, \frac{\phi}{2\pi})$

$\phi = \pi/4$



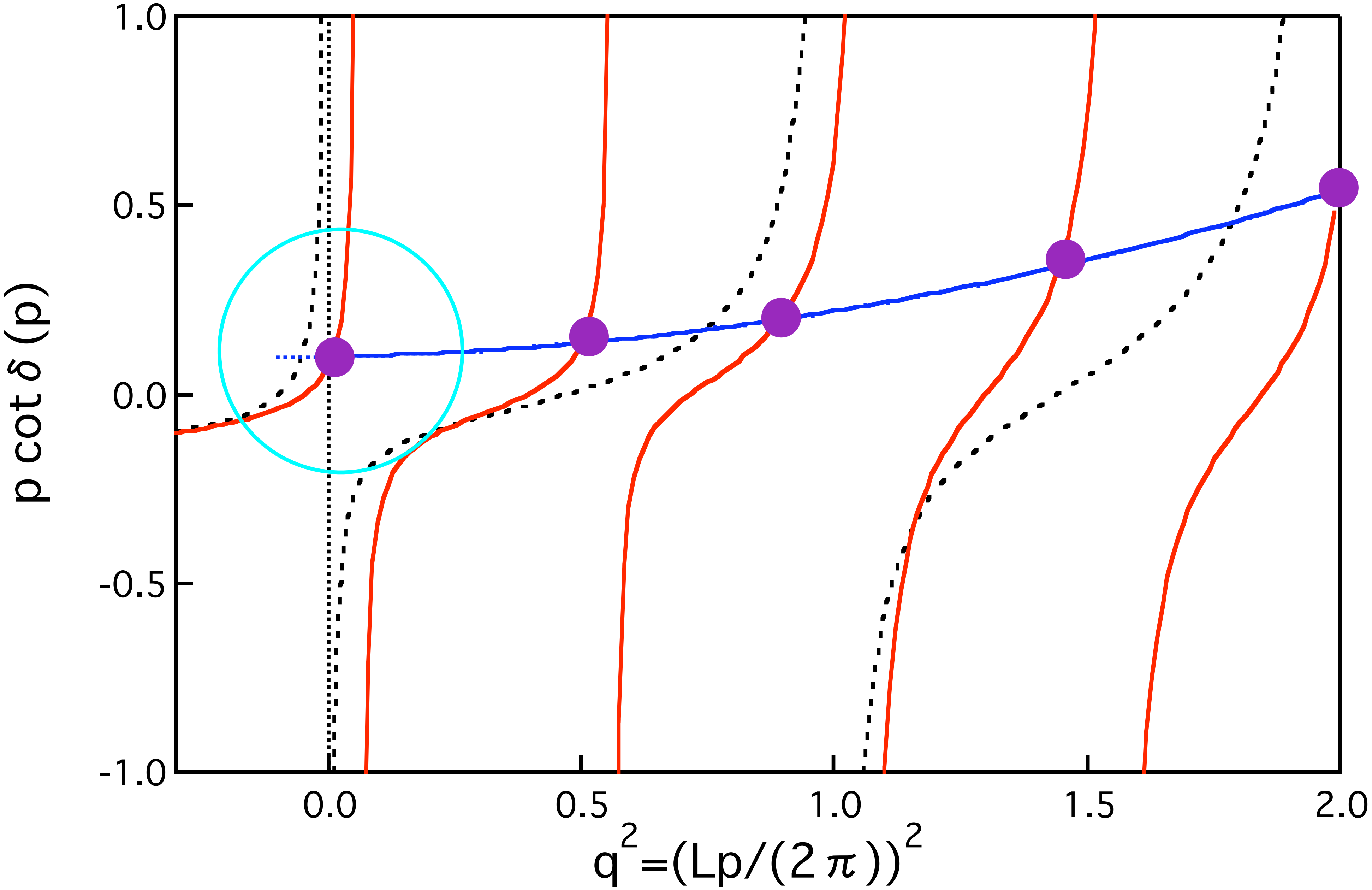
$$\mathbf{d} = (0, 0, \frac{\phi}{2\pi})$$

$\phi = 3\pi/8$



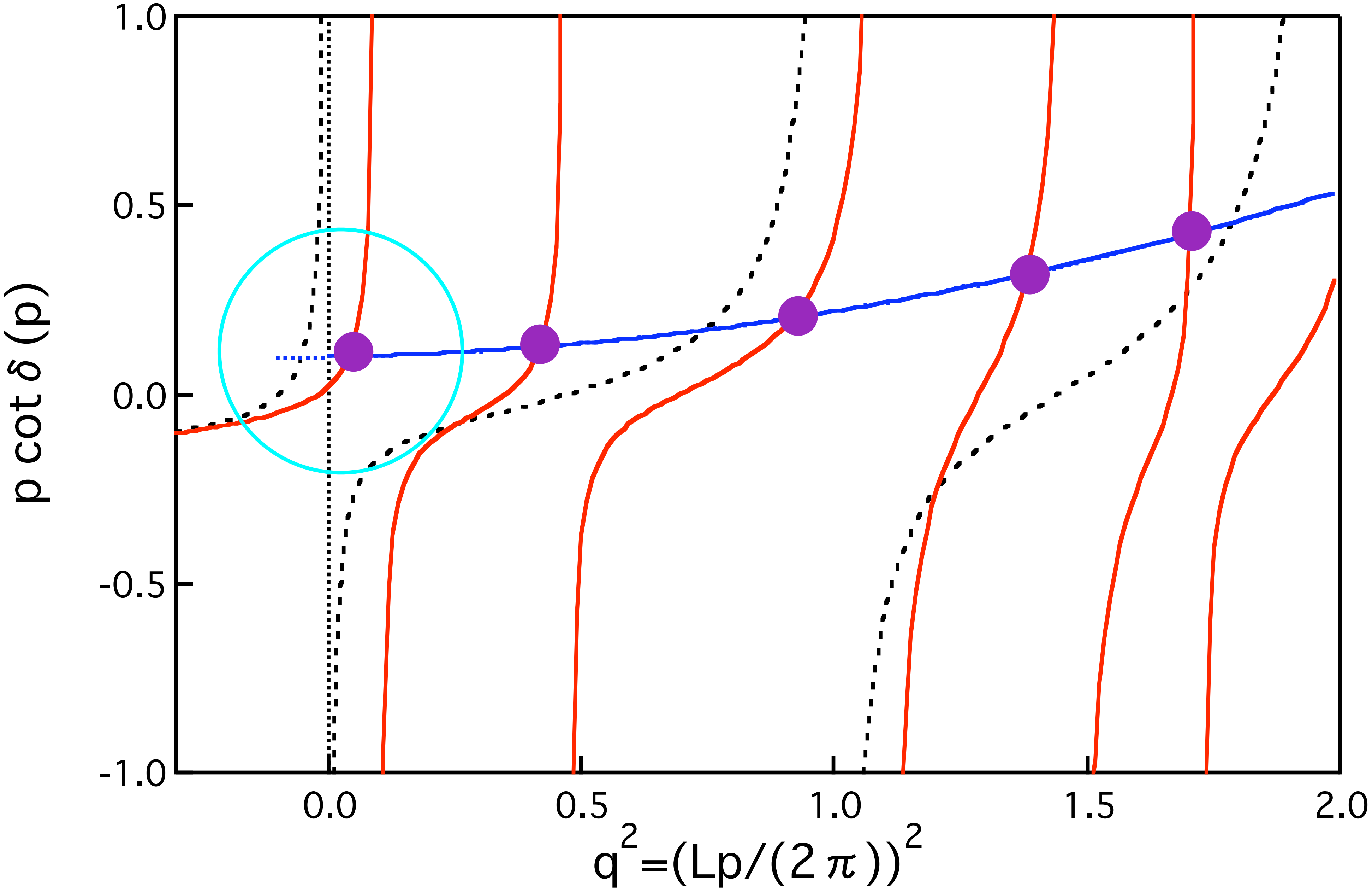
$$\mathbf{d} = (0, 0, \frac{\phi}{2\pi})$$

$\phi = \pi/2$



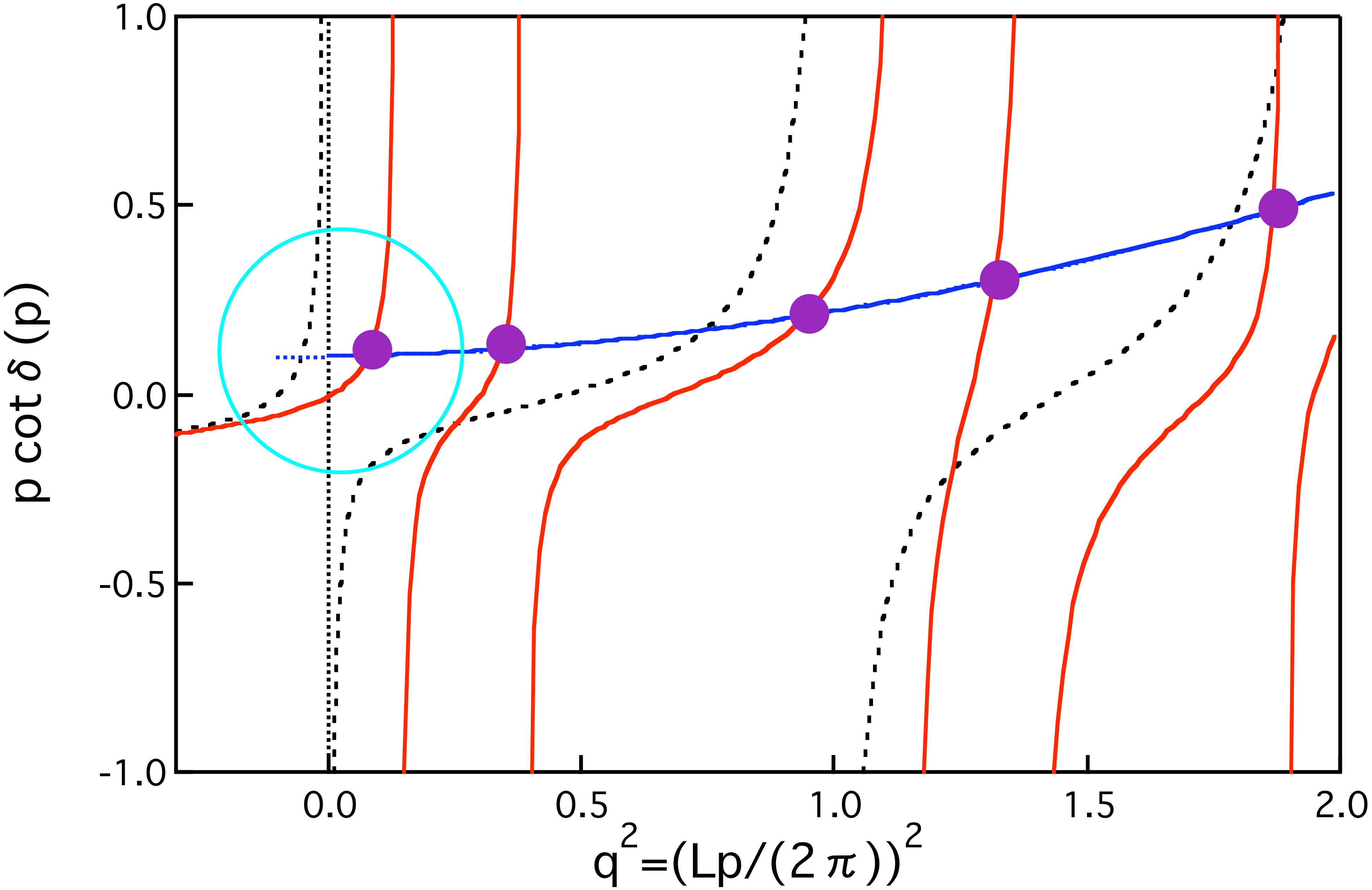
$$\mathbf{d} = (0, 0, \frac{\phi}{2\pi})$$

$\phi = 5\pi/2$



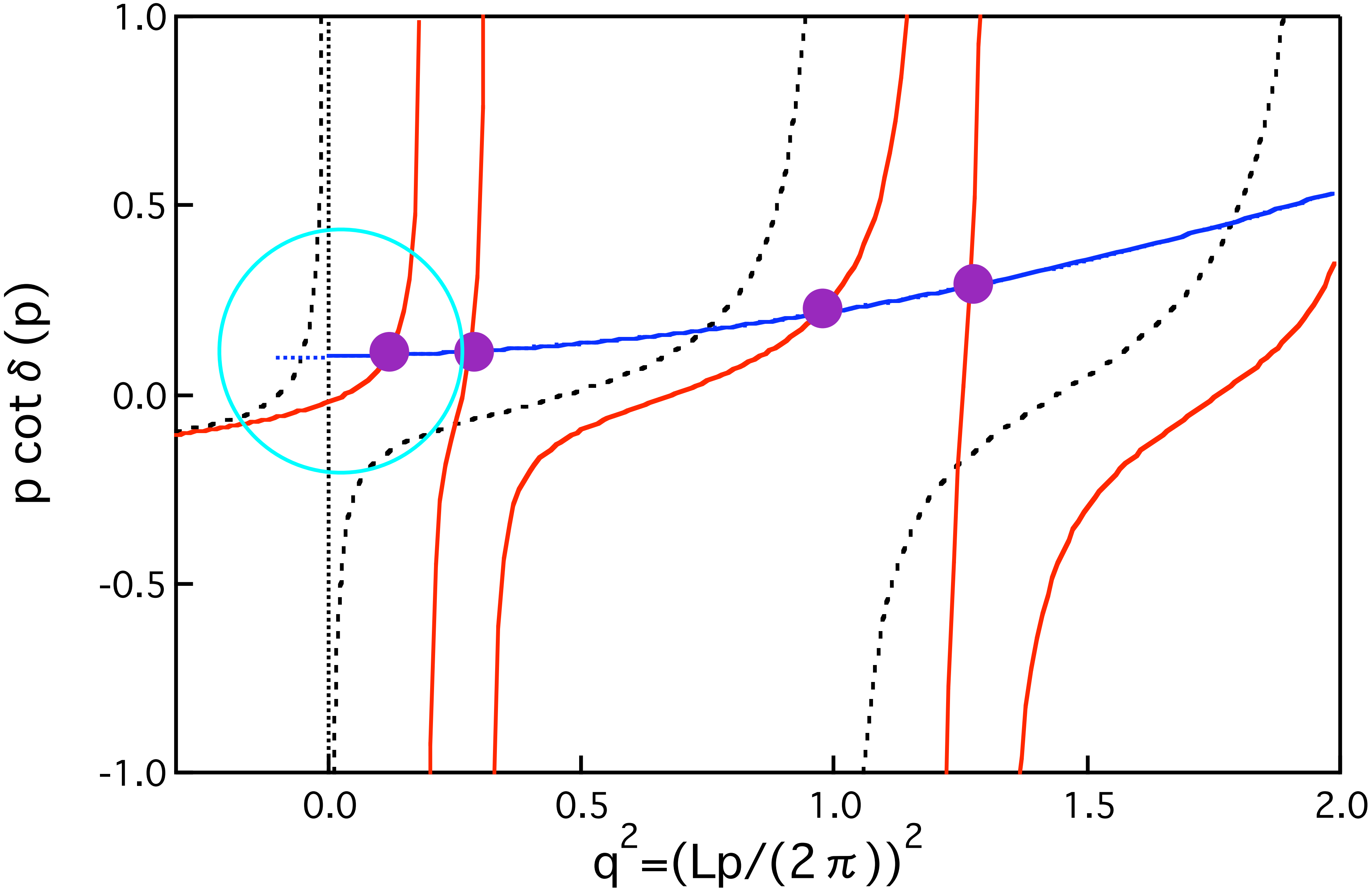
$$\mathbf{d} = (0, 0, \frac{\phi}{2\pi})$$

$\phi = 3\pi/4$



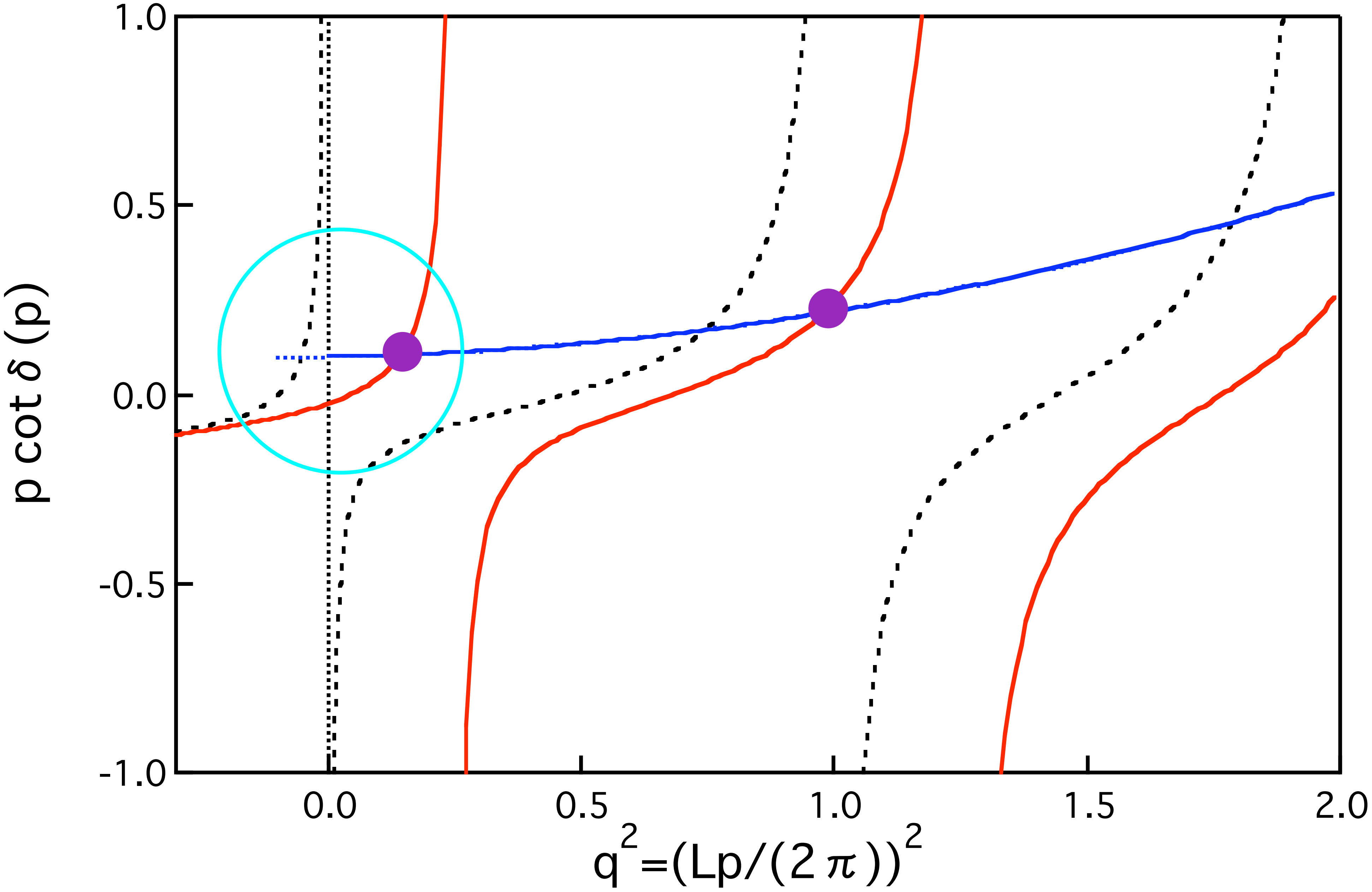
$$\mathbf{d} = (0, 0, \frac{\phi}{2\pi})$$

$\phi = 7\pi/4$



$$\mathbf{d} = (0, 0, \frac{\phi}{2\pi})$$

$\phi = \pi$



$$\mathbf{d} = (0, 0, \frac{\phi}{2\pi})$$

Numerical results

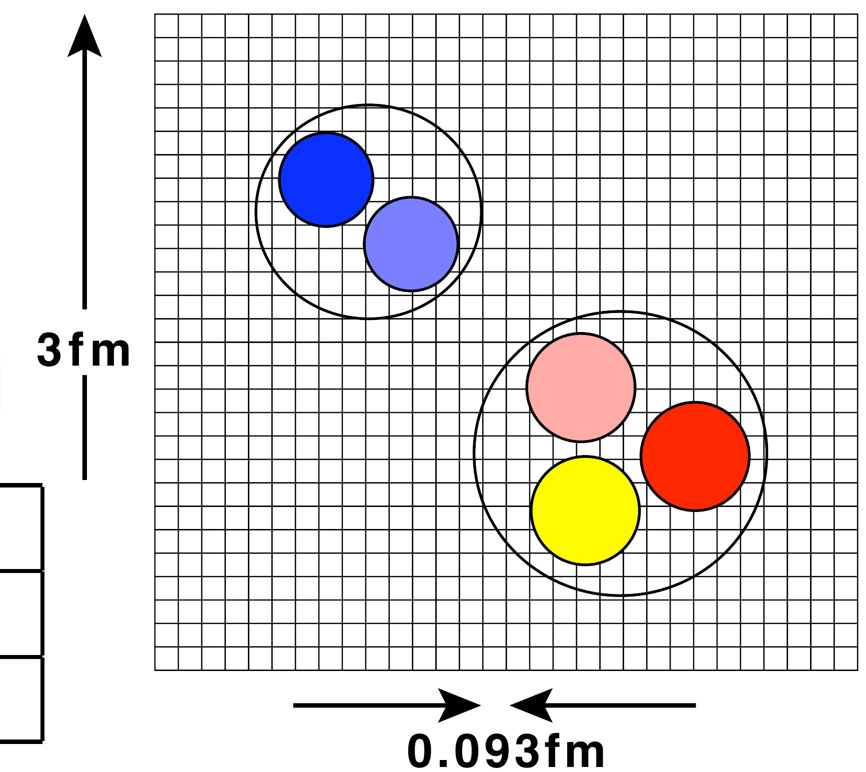
Simulation setup

- Quench approximation
- Lattice size : $L^3 x T = 32^3 x 48$ at $6/g^2 = 6.0$
- Plaquette gauge action

  ＋NP Clover fermions (u,d quarks)

  ＋RHQ action (charm quark)
- total statistics : O(600)
- charm: $\kappa_{\rm charm} = 0.10190$, ($m_{\eta c}$=2.92 GeV)
- light:

κ	0.1342	0.1339	0.1333
m_π [GeV]	0.64	0.73	0.87
m_N [GeV]	1.43	1.52	1.70



Quark propagators

- Twisted b.c. in space

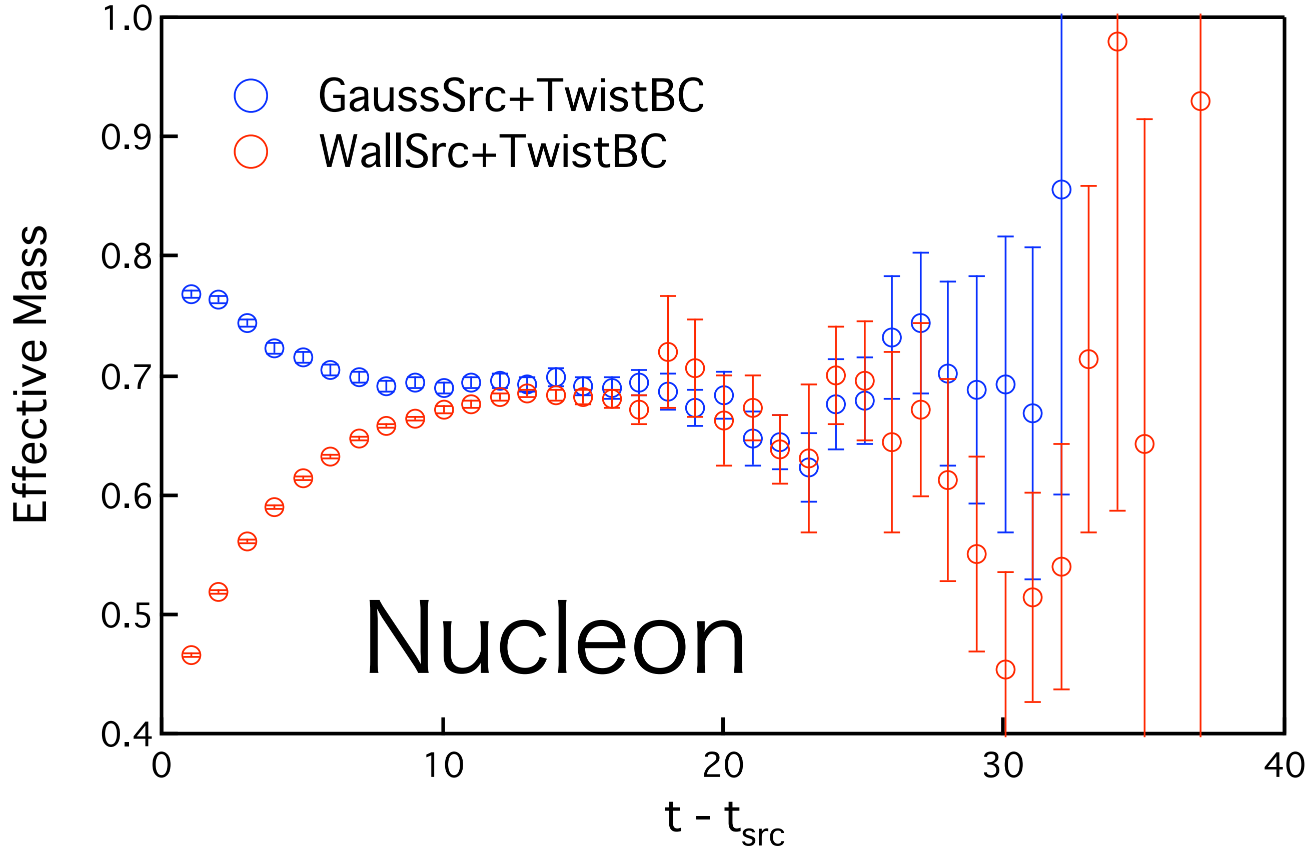
  ✓ $\mathbf{d}=(0,0,\phi/(2\pi))$, $\phi=0,\ \alpha,\ 2\alpha,\ 3\alpha$

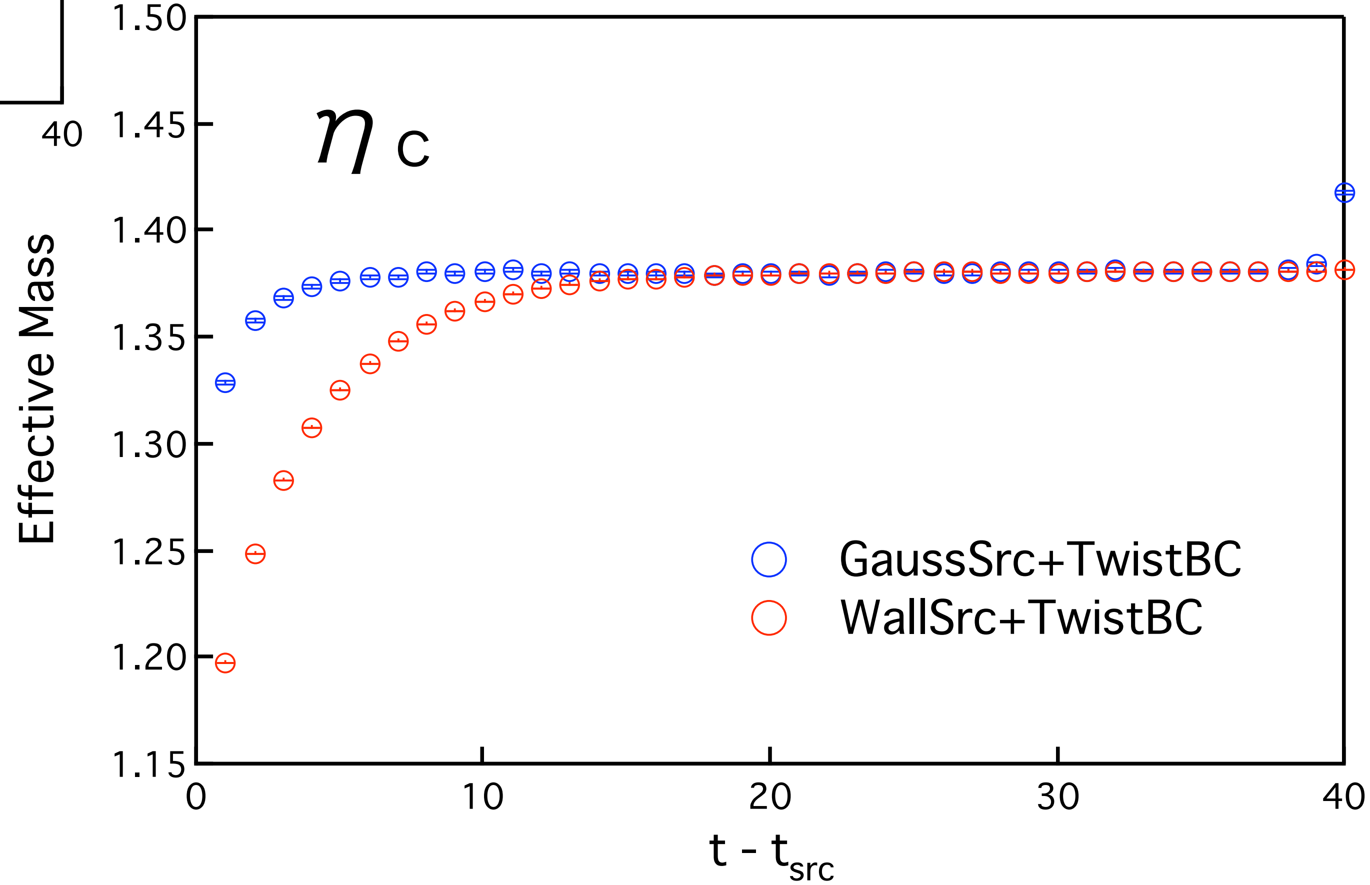
  $\alpha = 0.03\times 32 \approx 3\pi/10$

- Dirichlet b.c. in time
- Wall source with the Coulomb gauge

  (+ Gauge inv. gauss smeared source)

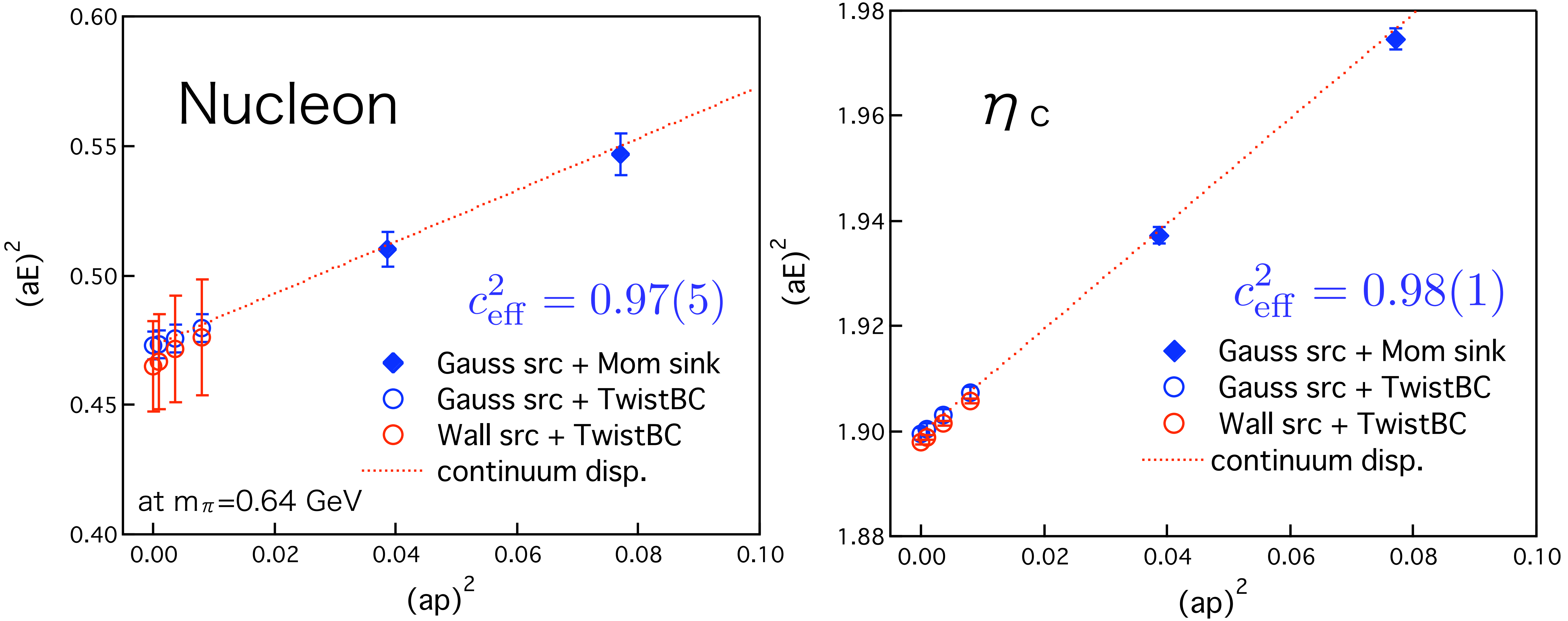
Gauss src vs. Wall src under Twisted B.C.





Dispersion relation of single particles

continuum disp. relation $E^2 = p^2 + M^2 \quad (p = |\vec{p}|)$



$$E^2 = M^2 + c^2_{\rm eff} \cdot p^2$$

charmonium-hadron scattering

4-point correlator of two hadron states

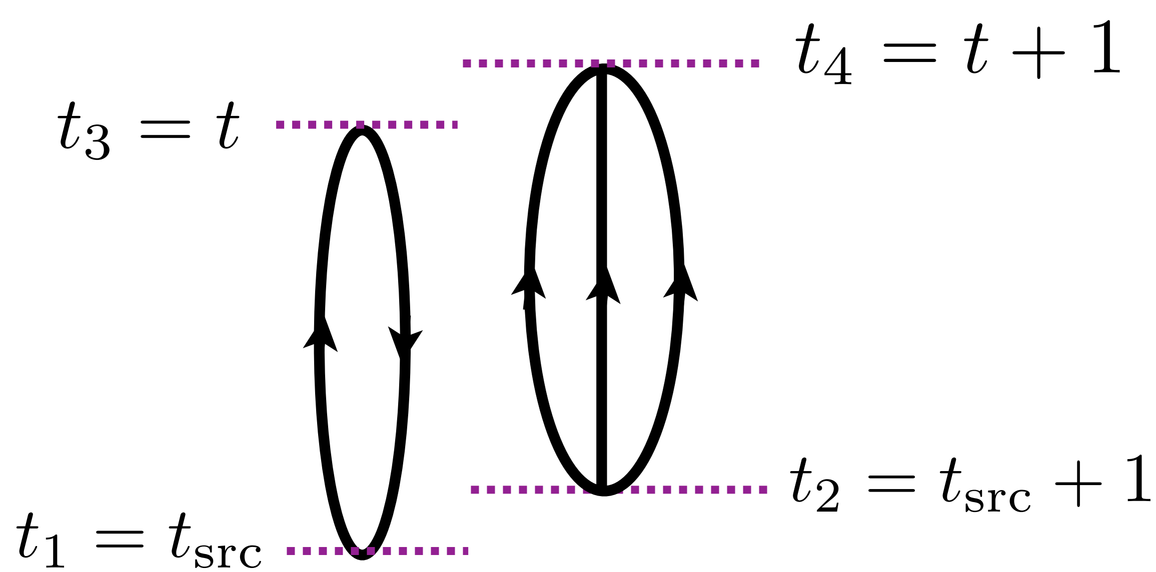
4-point correlator

$$G^{h_1\text{-}h_2}(t_4, t_3; t_2, t_1) = \langle \mathcal{O}^{h_1}(t_4)\mathcal{O}^{h_2}(t_3) \left(\mathcal{O}^{h_1}(t_2)\mathcal{O}^{h_2}(t_1)\right)^\dagger \rangle$$

$$\mathcal{O}^h(t) = \sum_{\mathbf{x}} \mathcal{O}^h(\mathbf{x}, t)$$ projected on the lowest mode

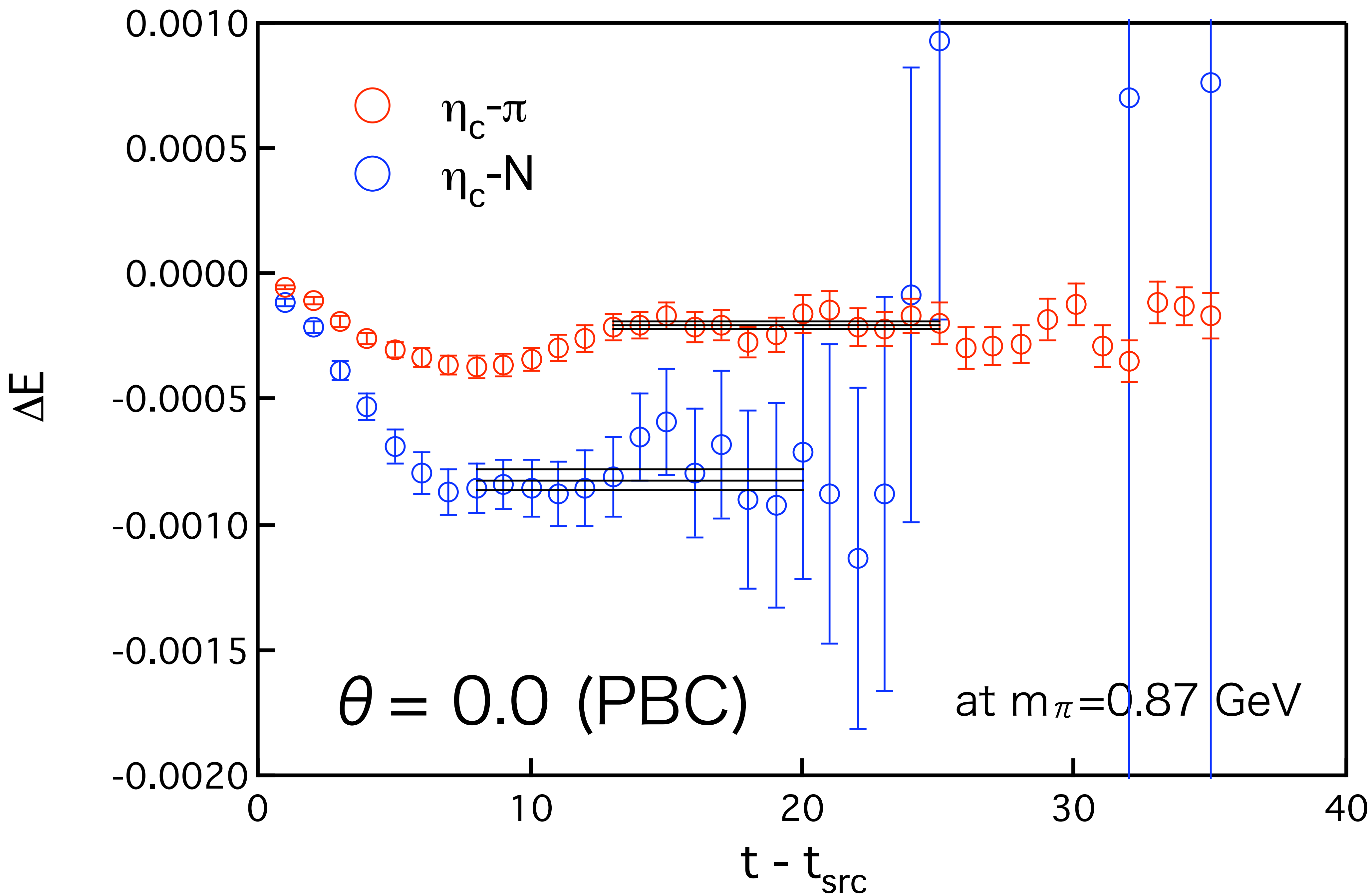
2-point correlator

$$G_h(t, t_{\rm src}) = \langle \mathcal{O}^h(t)\mathcal{O}^h(t_{\rm src})\rangle$$



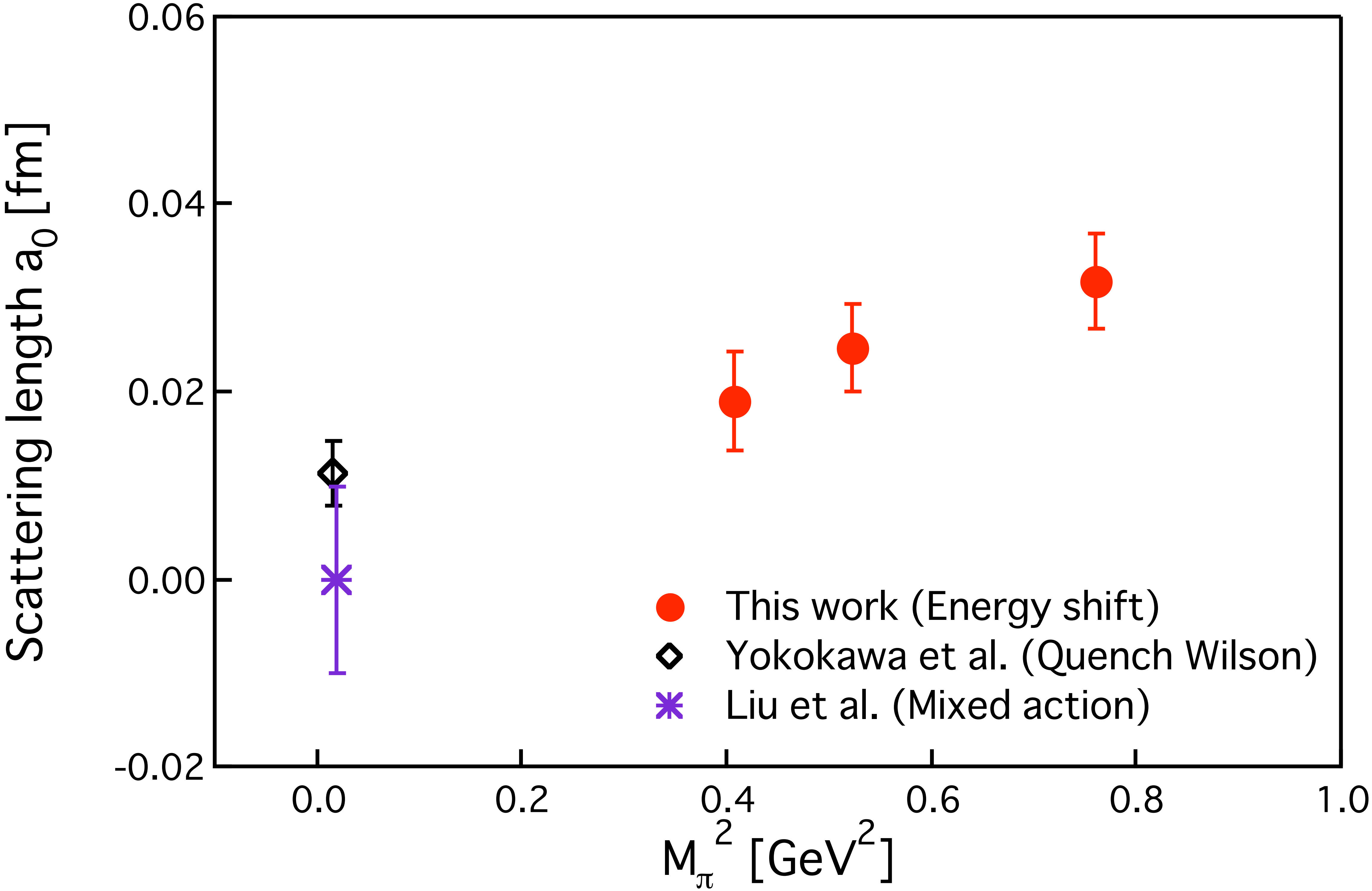
$$R_{h_1\text{-}h_2}(t) = \frac{G_{h_1\text{-}h_2}(t, t_{\rm src})}{G_{h_1}(t, t_{\rm src})G_{h_2}(t, t_{\rm src}+1)} \to \exp(-\Delta E \cdot t)$$

Measurement of Energy Shift ΔE



$$R_{\eta_c\text{-}N}(t) \to \exp(-\Delta E \cdot t) \text{ for } t \gg t_{\rm src}$$

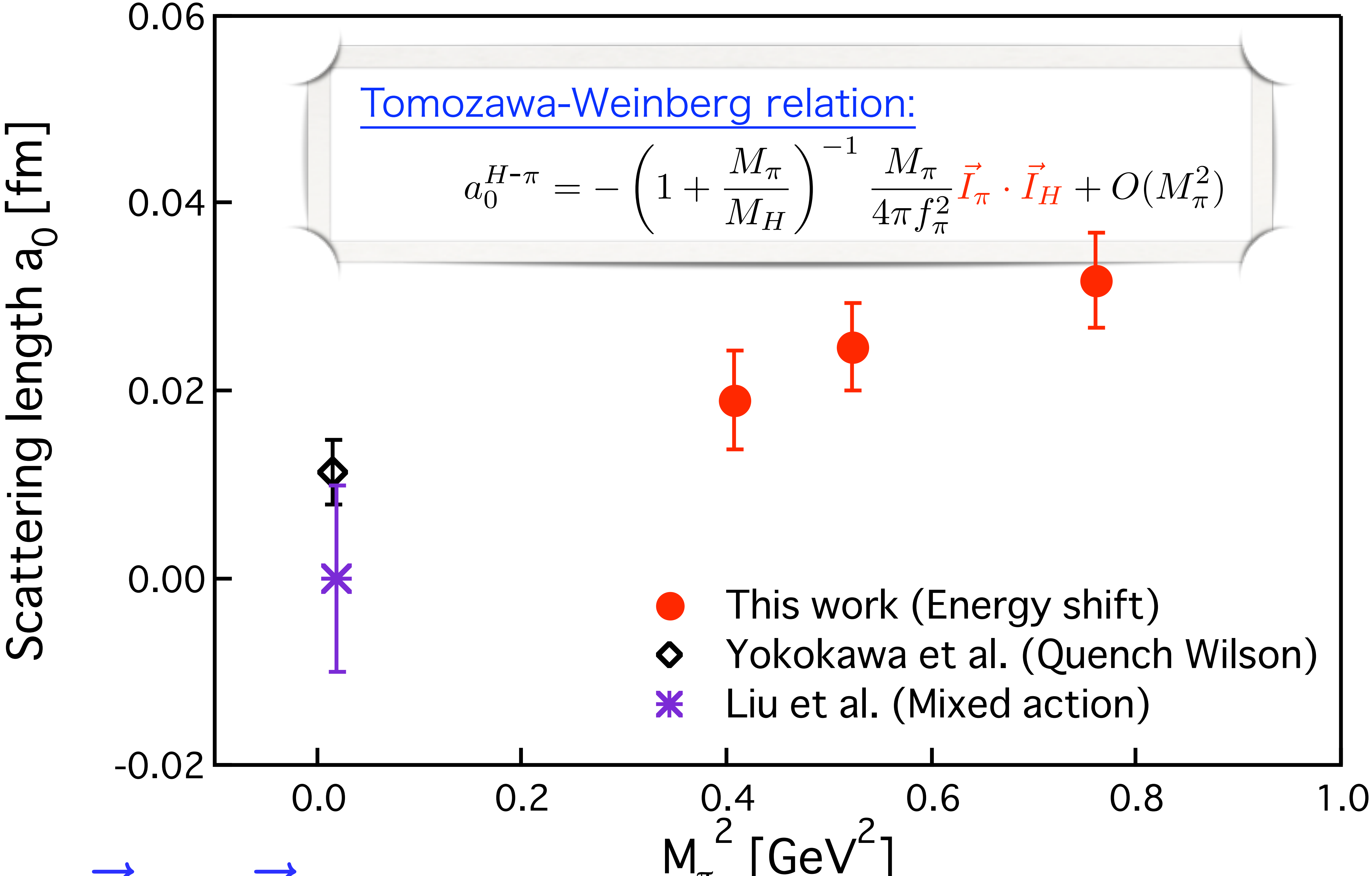
Summary of η_c-π scattering length



Yokokawa et al., Phys.Rev.D74:034504,2006.

Liu et al., arXiv:0810.5412.

Summary of η_c-π scattering length

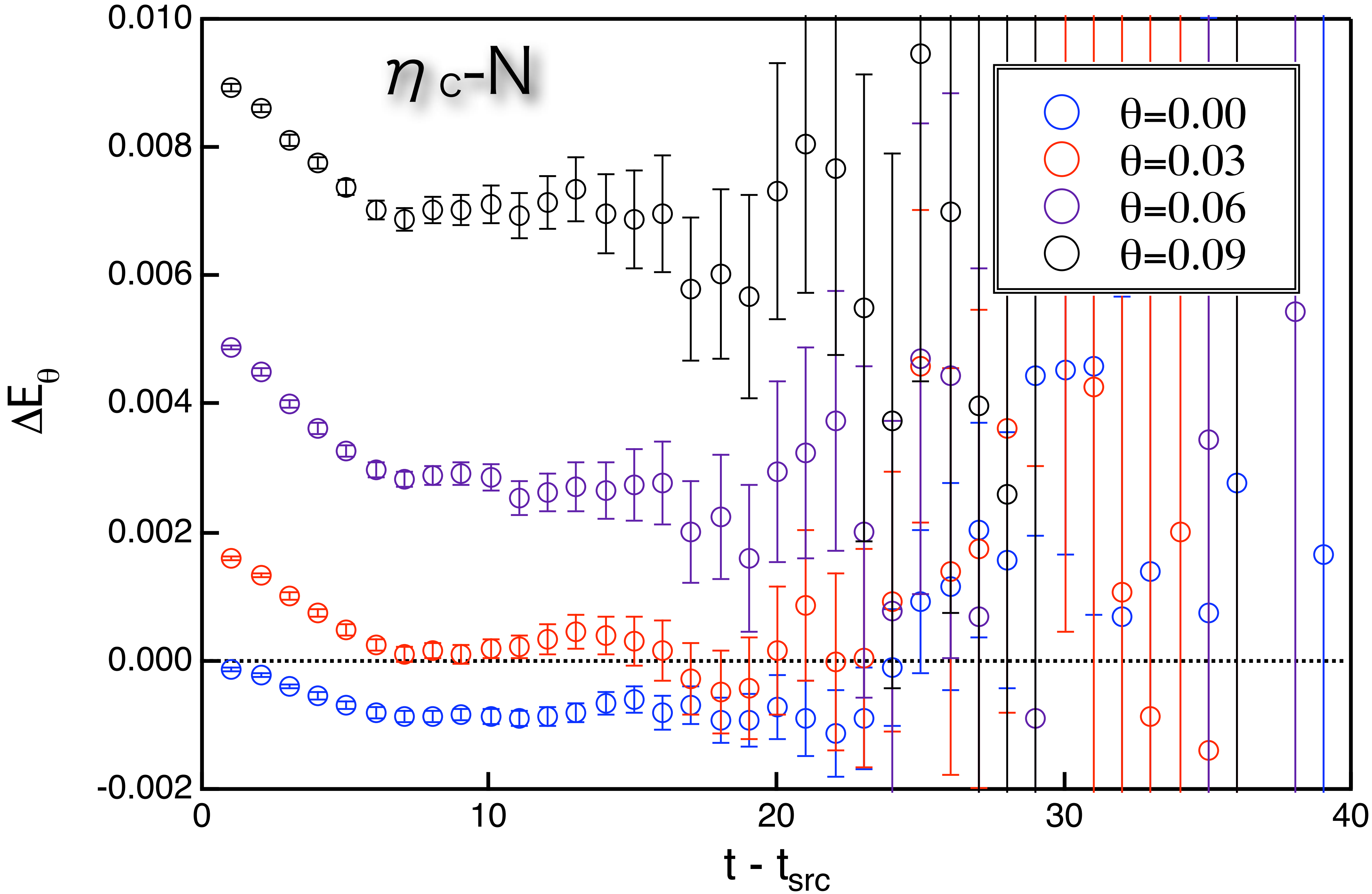


$$\vec{I}_\pi\cdot\vec{I}_{\eta_c} = 0$$

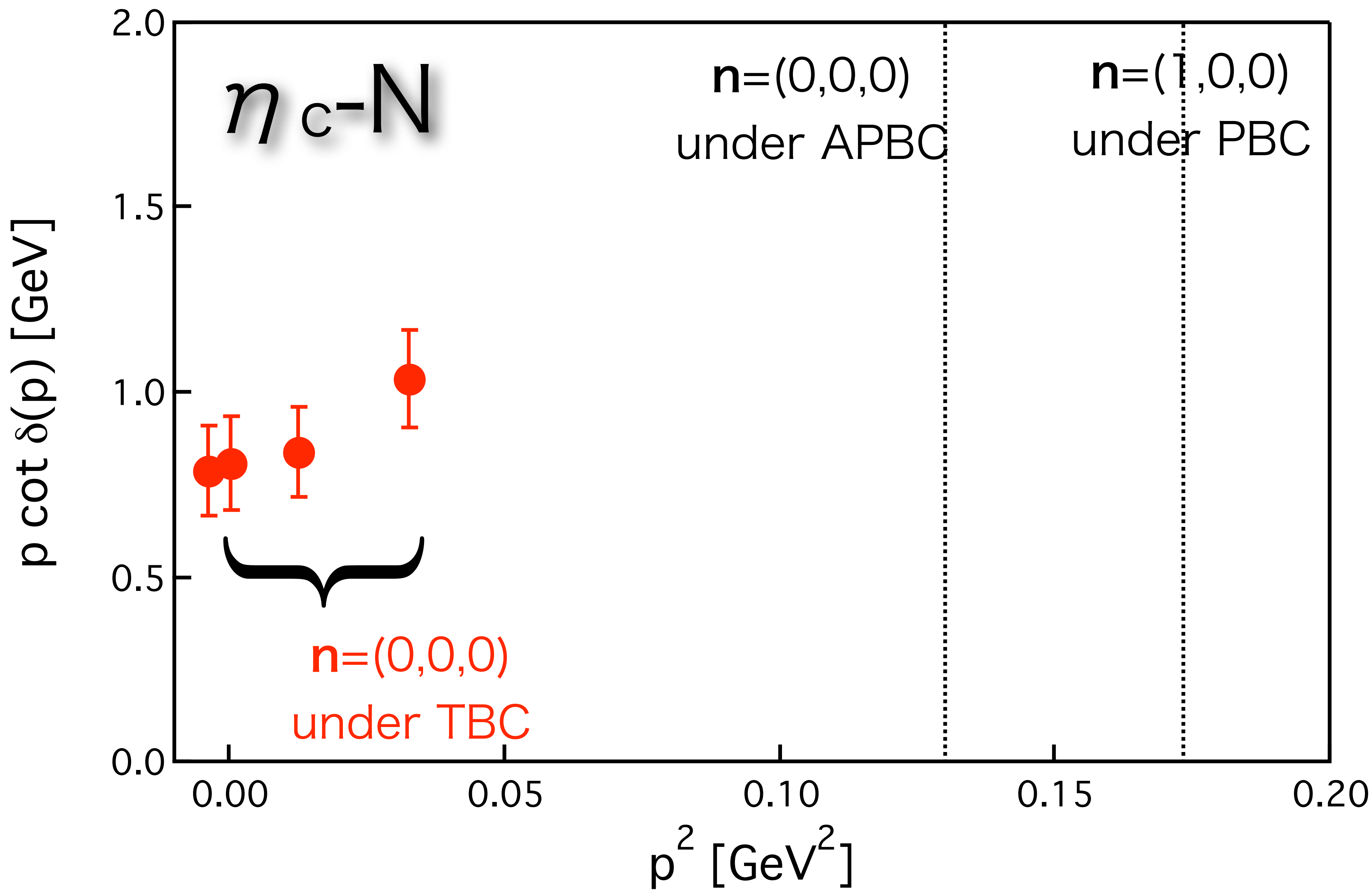
Yokokawa et al., Phys.Rev.D74:034504,2006.

Liu et al., PoS Lattice 2008 112 (arXiv:0810.5412).

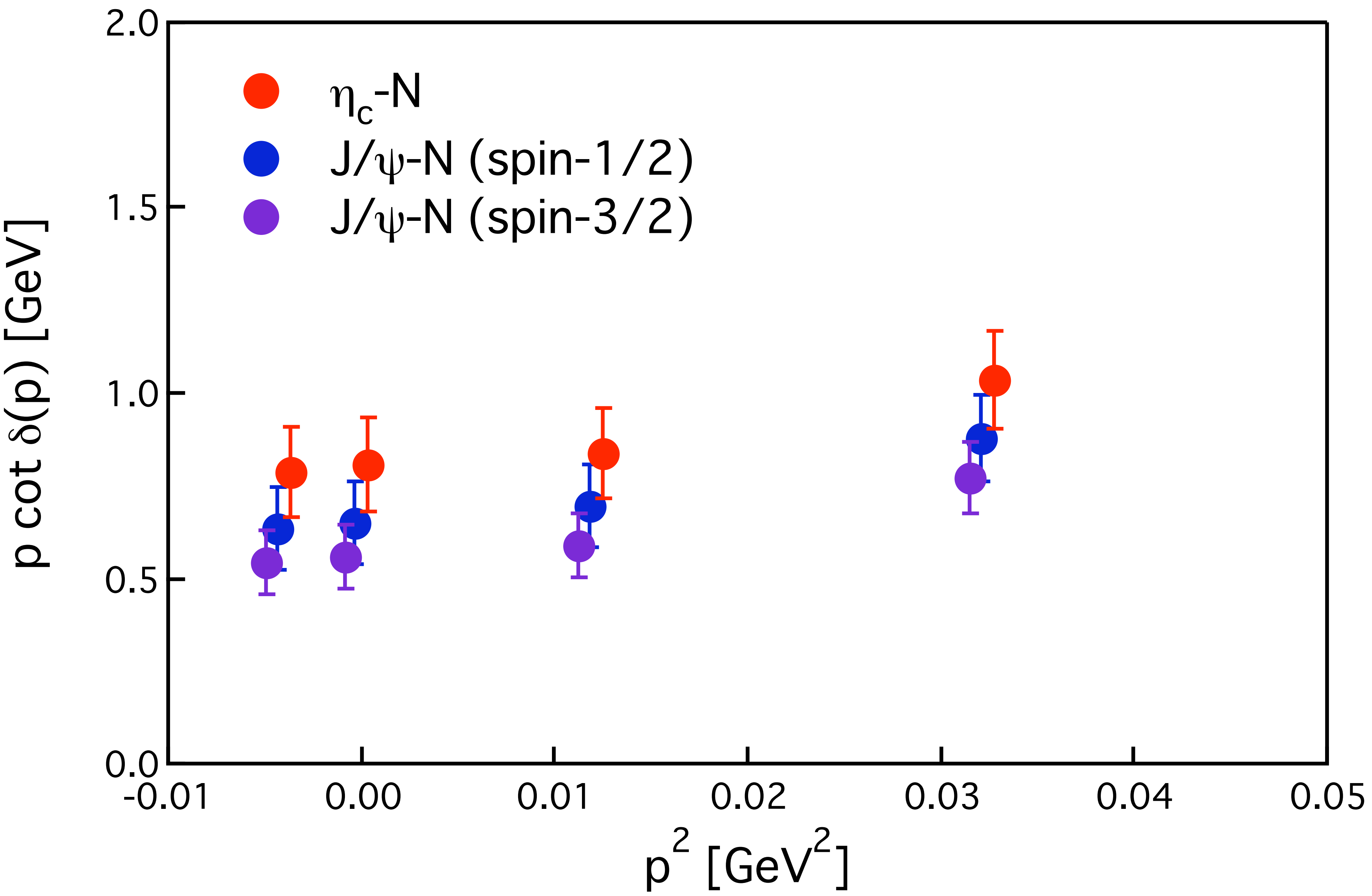
Energy Shift ΔE_θ under twisted BC



$$R^{\theta}_{\eta_c\text{-}N}(t) = \frac{G^{\theta}_{\eta_c\text{-}N}(t, t_{\rm src})}{G_{\eta_c}(t, t_{\rm src})\, G_N(t, t_{\rm src}+1)} \rightarrow \exp(-\Delta E_\theta \cdot t)$$

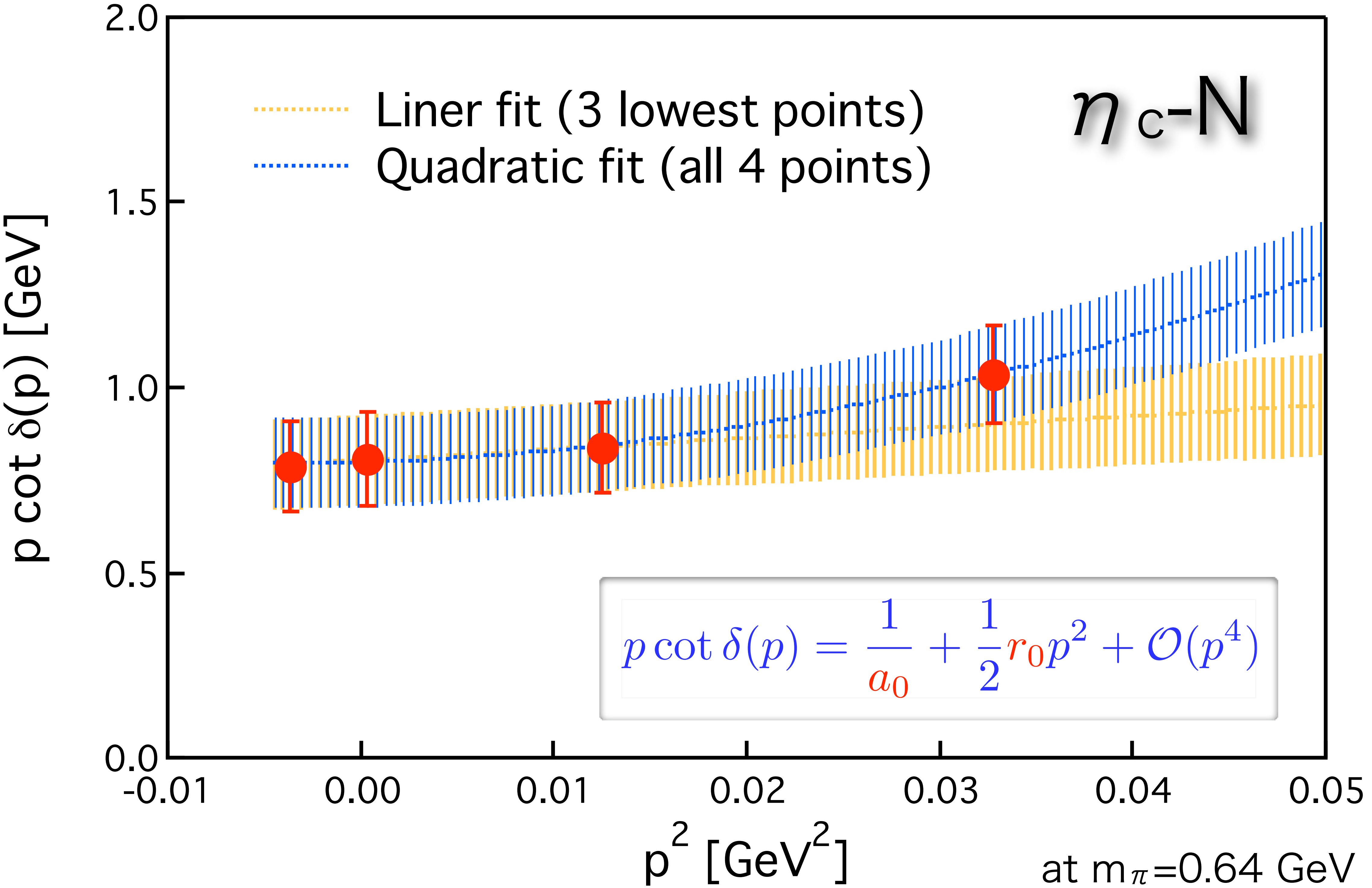


at m_π=0.64 GeV



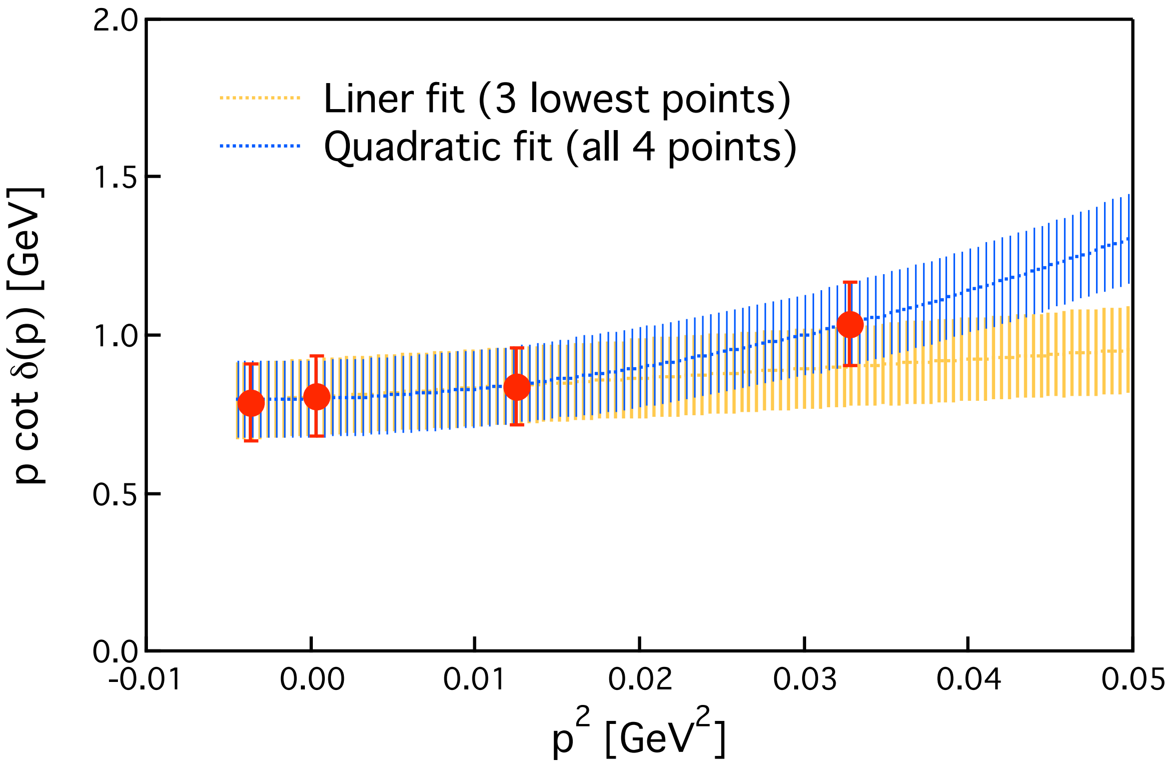
at m_{π}=0.64 GeV

Effective range expansion



at m_π=0.64 GeV

Effective-range expansion

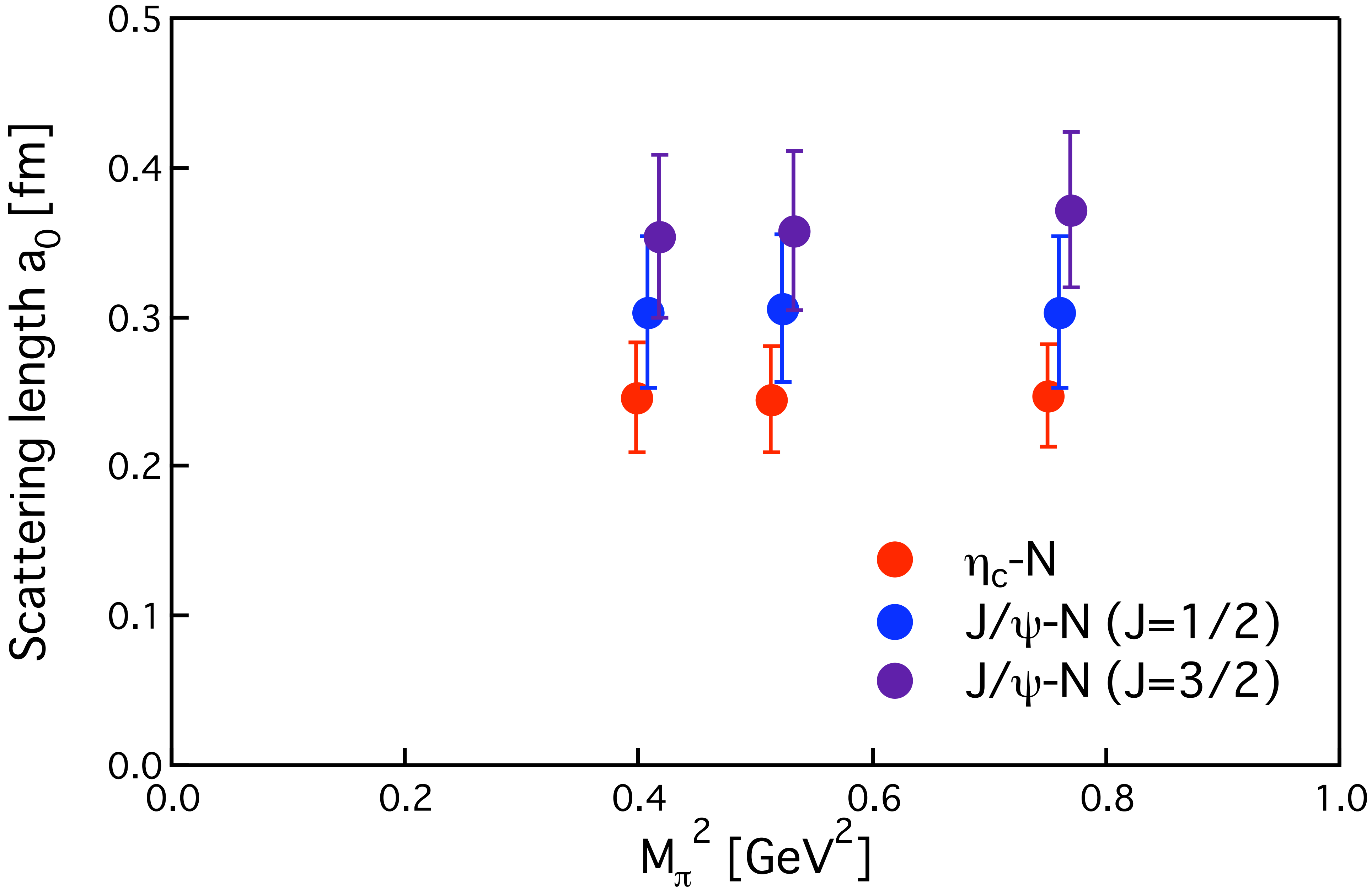


$$p \cot \delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2 + \mathcal{O}(p^4)$$

fit	a_0 [fm]	r_0 [fm]	χ^2/ndf
Linear	0.245(37)	1.18(53)	0.003
Quadratic	0.247(37)	0.56(64)	0.012

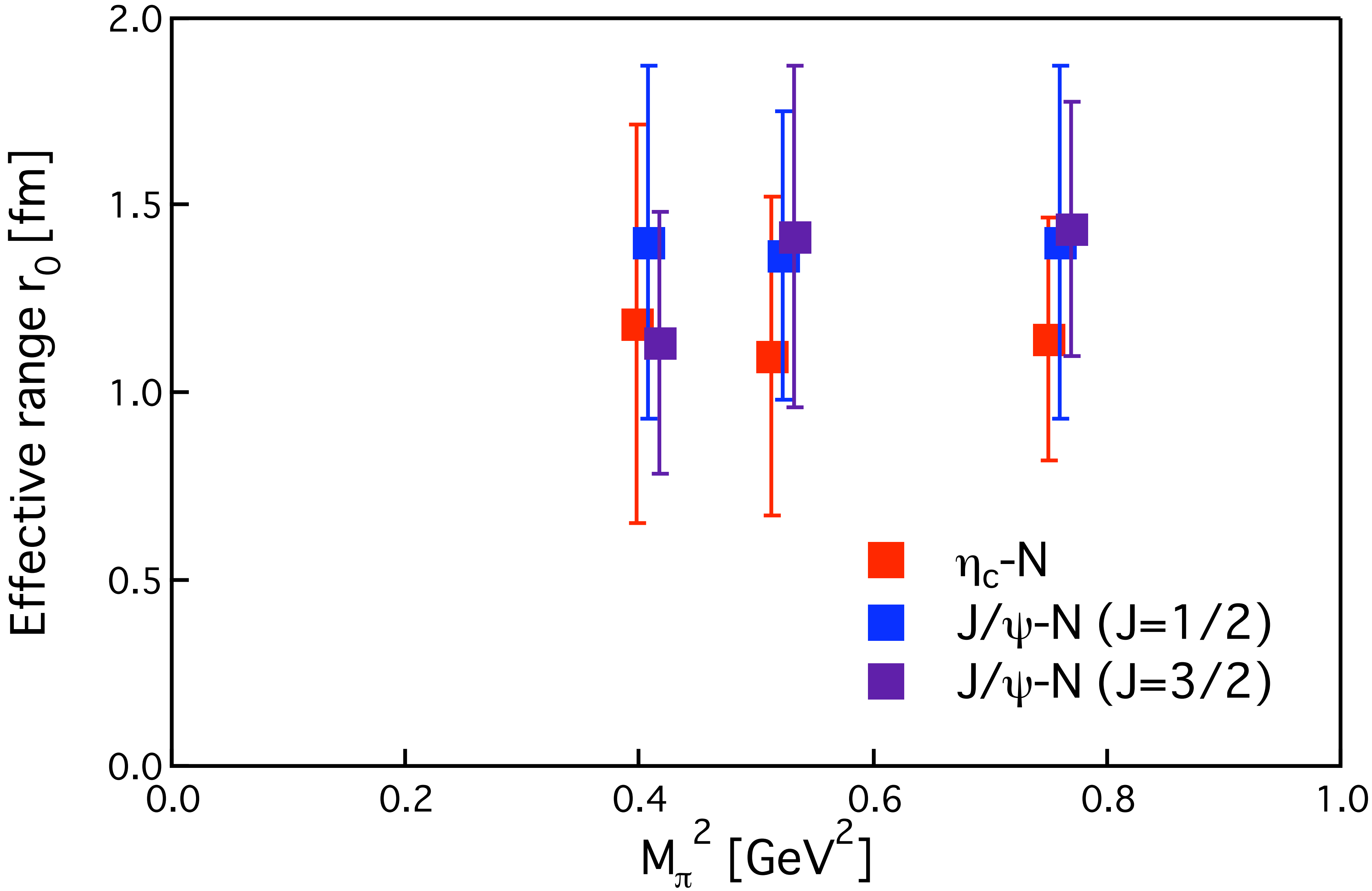
at m_π=0.64 GeV

scattering length a_0



$$a_0^{\eta_c\text{-}N} \approx 0.25 \text{ fm} < a_0^{J/\psi\text{-}N} \approx 0.34 \text{ fm}$$

effective range r_0



$$r_0^{\eta_c\text{-}N} \approx r_0^{J/\psi\text{-}N} \sim 1.2 - 1.4 \text{ fm}$$

Summary

- We have studied the charmonium-nucleon interactions at low energies by extended Lüscher formula with twisted boundary conditions

✓ demonstrate the feasibility of this new approach

✓ successfully determine both scattering lengths and effective ranges of the η_c-N and J/ψ-N scatterings

$$a_0^{\eta_c\text{-}N} \approx 0.25 \text{ fm} < a_0^{J/\psi\text{-}N} \approx 0.34 \text{ fm}$$

$$r_0^{\eta_c\text{-}N} \approx r_0^{J/\psi\text{-}N} \sim 1.2 - 1.4 \text{ fm}$$

✓ dynamical simulations ($m_\pi \leq 0.41$ GeV) on PACS-CS 2+1 flavor gauge configurations are now under way