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Lattice Index Theorem and Fractional Topological Charge

in coop. with Manfried Faber and Urs M. Heller

June 15, Lattice 2010, Villasimius

Center Vortices

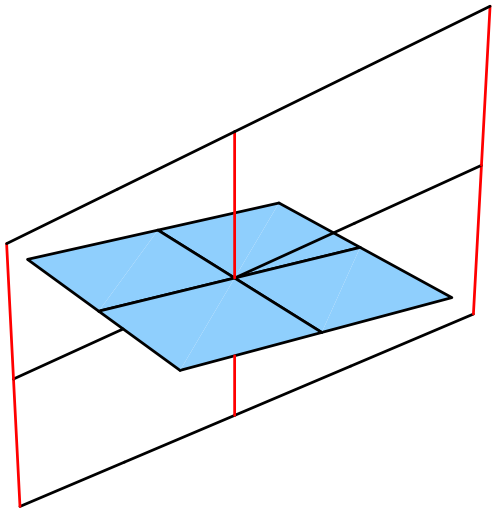
➜ *'t Hooft 1979, Nielsen, Ambjorn, Olesen, Cornwall, 1979 Mack, 1980; Feynman, 1981*

- QCD vacuum is a *condensate of closed magnetic flux-lines*, they have topology of tubes (3D) or surfaces (4D),
- magnetic flux corresponds to the *center of the group*,
- Vortex model may explain ...
  - **Confinement** $\rightarrow$ *piercing of Wilson loop* $\equiv$ crossing of static electric flux tube and moving closed magnetic flux
  - **Topological charge**: vortices carry topological charge at intersection points and writhing points
  - **Spontaneous chiral symmetry breaking**: also center-projected configurations show SCSB

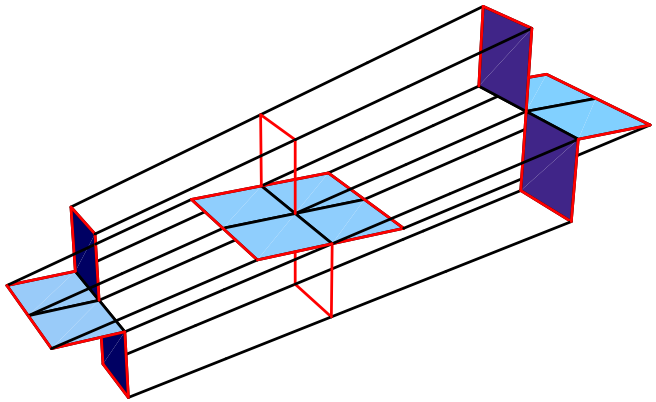
➜ *R.H., Faber, Greensite, Heller, Olejnik 2008*

Contributions to topological charge Q

Vortices:



intersections



writhing points

➜ *Engelhardt, Reinhardt (2000)*

Axial anomaly $\Rightarrow$ Topological charge:

$$\partial_\mu j^5_\mu = -\frac{N_f}{16\pi^2}\,\mathrm{tr}(\mathcal{F}_{\mu\nu}\tilde{\mathcal{F}}_{\mu\nu}) \quad \Rightarrow \quad Q := \int \mathrm{d}^4x \partial_\mu j^5_\mu$$

Atiyah-Singer index theorem

- Index theorem (wilson, overlap fermions):

  n_-, n_+: number of left-/right-handed zeromodes

  $$\text{ind}\, D[A] = n_- - n_+ = Q[A]$$

- (Asqtad) staggered fermions:

  $$\text{ind}\, D[A] = 2Q[A] \text{ (SU(2), double degeneracy)}$$

- Adjoint fermions:

  $$\text{ind}\, D[A] = 2NQ[A] = 4Q[A] \text{ (real representation)}$$

- Lüscher-condition:

  $$\text{tr}(\mathbb{1} - U_{\mu\nu}) < 0.03.$$

  ➜ *Lüscher (1998), Neuberger, Fukaya (1999)*

Thick Spherical SU(2)-vortices

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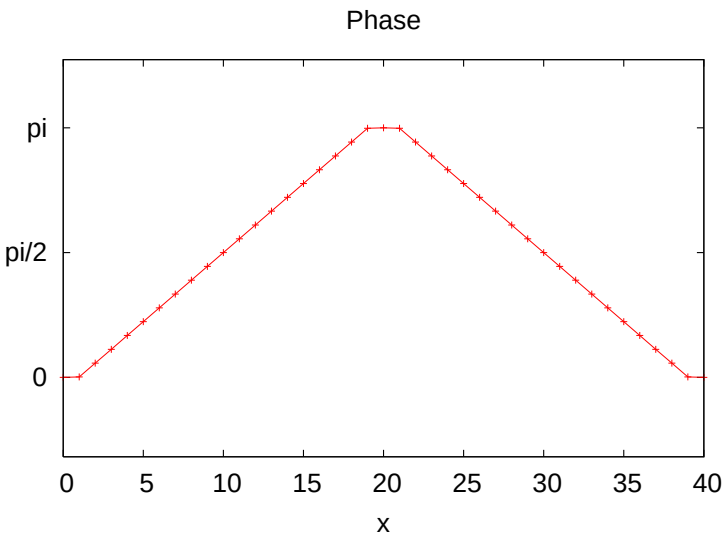
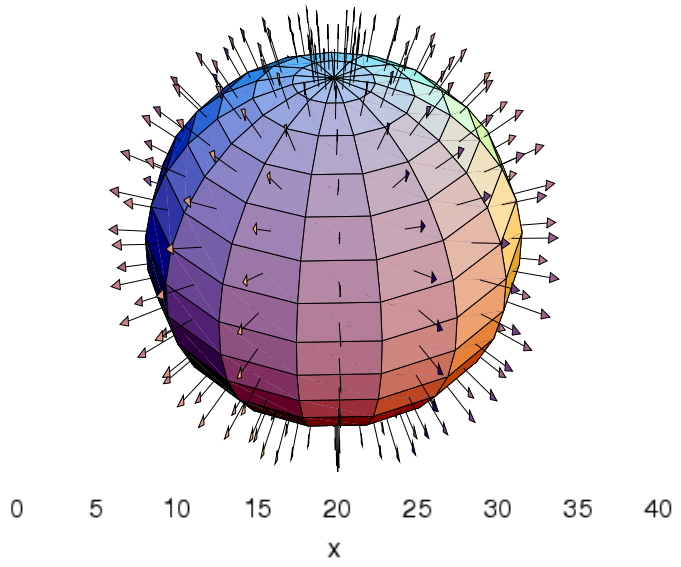


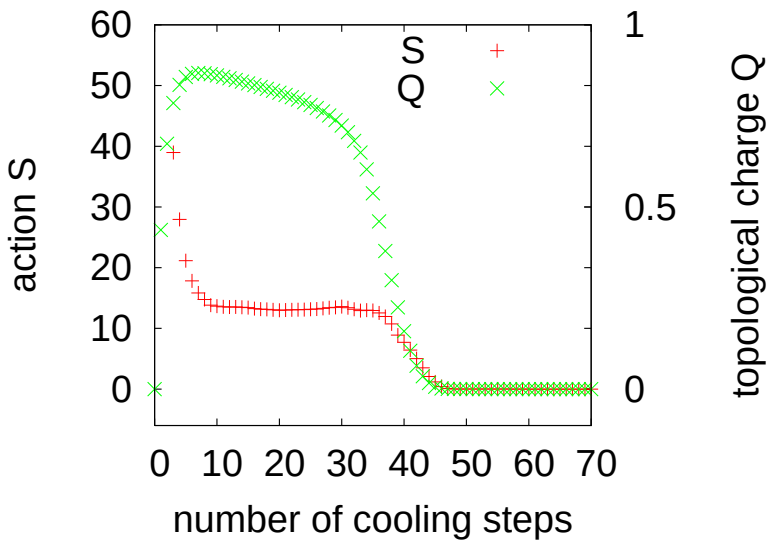


$$U_\mu(x^\nu) = \begin{cases} \exp\{i\alpha(r)\frac{\vec{r}}{r}\vec{\sigma}\}, & t = 1, \mu = 4 \\ 1 & \text{else} \end{cases}$$

$$\alpha_\pm(r) = \begin{cases} \pi/0 & r < R - \frac{\Delta}{2} \\ \frac{\pi}{2}\left(1 \mp \frac{r-R}{\frac{\Delta}{2}}\right) & R - \frac{\Delta}{2} < r < R + \frac{\Delta}{2} \\ 0/\pi & R + \frac{\Delta}{2} < r \end{cases}$$

Discrepancy for Overlap Dirac Operator

- Index of the overlap operator:
$$\text{ind } D = 1$$
- Topological charge before cooling:
$\vec{E} \neq 0, \vec{B} = 0$
$$\Longrightarrow \quad Q = 0$$
- Topological charge after cooling:
$$Q \approx 1$$



$40^3 \times N_t$-lattice fulfills Lüscher-condition!

Conclusion: lattice index theorem inapplicable?

Overlap, Asqtad & Adjoint Results

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negative spherical vortex

	fundamental:				adjoint:			
lattice:	ovl:	apbc:	stag:	apbc:	ovl:	apbc:	stag:	apbc:
8^4:	3+ 4-	0+ 1-	6+ 8-	0+ 2-	4+ **6-**	0+ **2-**	8+ **12-**	0+ **4-**
12^4:	3+ 4-	0+ 1-	6+ 8-	0+ 2-	4+ **6-**	0+ **2-**	8+ **12-**	0+ **4-**
16^4:	3+ 4-	0+ 1-	6+ 8-	0+ 2-	4+ 8-	0+ 4-	8+ 16-	0+ 8-
20^4:	3+ 4-	0+ 1-	6+ 8-	0+ 2-	4+ 8-	0+ 4-	8+ 16-	0+ 8-
$40^3 \times 2$:	3+ 4-	0+ 1-	6+ 8-	0+ 2-	4+ 8-	0+ 4-	8+ 16-	0+ 8-
cooled:	3+ 4-	0+ 1-	6+ 8-	0+ 2-	4+ **6-**	0+ **2-**	8+ **12-**	0+ **4-**
$40^3 \times 4$:	3+ 4-	0+ 1-	6+ 8-	0+ 2-	4+ 8-	0+ 4-	8+ 16-	0+ 8-

positive spherical vortex

	fundamental:				adjoint:			
lattice:	ovl:	apbc:	stag:	apbc:	ovl:	apbc:	stag:	apbc:
8^4:	1+ 0-	4+ 3-	2+ 0-	8+ 6-	**6+** 4-	**2+** 0-	**12+** 8-	**4+** 0-
12^4:	1+ 0-	4+ 3-	2+ 0-	8+ 6-	**6+** 4-	**2+** 0-	**12+** 8-	**4+** 0-
16^4:	1+ 0-	4+ 3-	2+ 0-	8+ 6-	8+ 4-	4+ 0-	16+ 8-	8+ 0-
20^4:	1+ 0-	4+ 3-	2+ 0-	8+ 6-	8+ 4-	4+ 0-	16+ 8-	8+ 0-
$40^3 \times 2$:	1+ 0-	4+ 3-	2+ 0-	8+ 6-	8+ 4-	4+ 0-	16+ 8-	8+ 0-
cooled:	1+ 0-	4+ 3-	2+ 0-	8+ 6-	**6+** 4-	**2+** 0-	**12+** 8-	**4+** 0-
$40^3 \times 4$:	1+ 0-	4+ 3-	2+ 0-	8+ 6-	8+ 4-	4+ 0-	16+ 8-	8+ 0-

Alternative index theorem for topological excitations on $R^3 \times S^1$

An L2-Index Theorem for Dirac Operators on $R^3 \times S^1$

➜ *Nye, Singer: arXiv:math/0009144*

Index theorem for topological excitations on $R^3 \times S^1$ and Chern-Simons theory

➜ *Poppitz, Ünsal: arXiv:hep-th/0812.2085*

Index $(n_- - n_+)$ includes:
- topological charge contribution
- surface term contribution:
takes into account the asymtotic holonomies of Wilson loops

$\Rightarrow$ all results are in exact agreement with this index theorem

Cooling a spherical vortex on a $40^3 \times 2$ lattice

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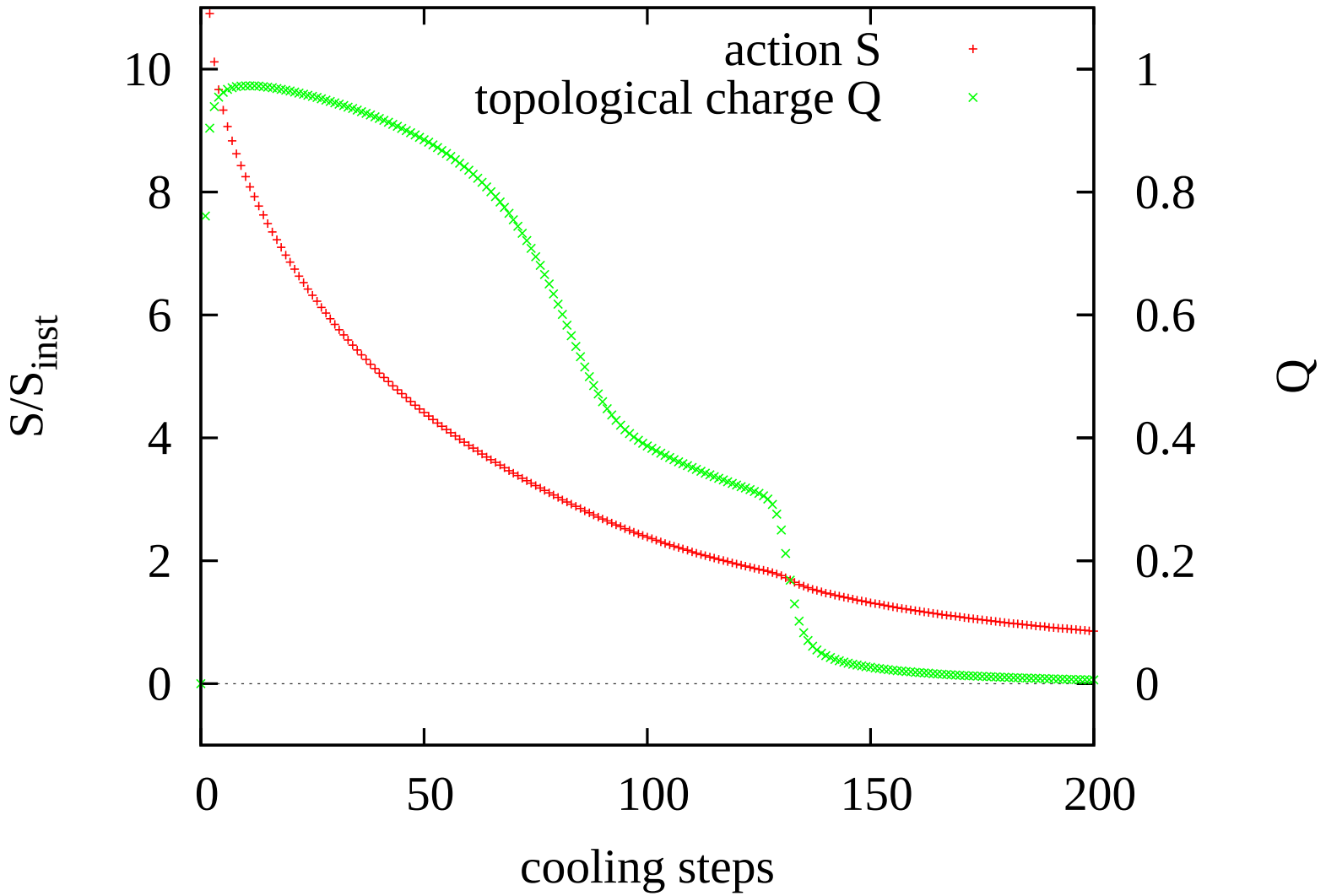
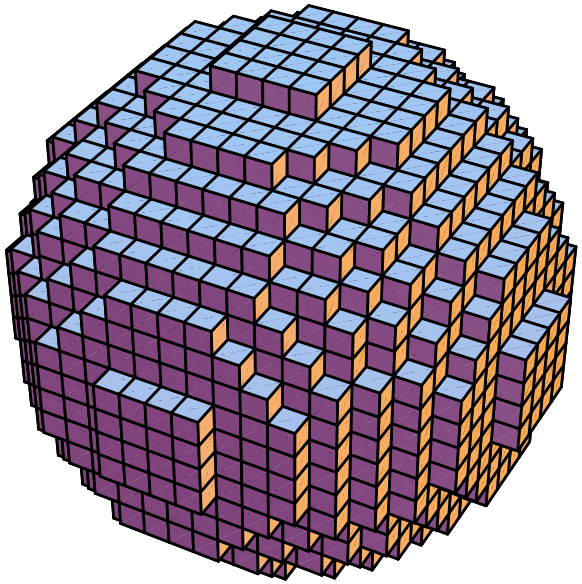
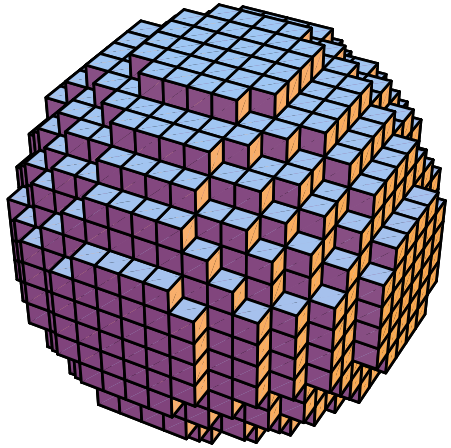


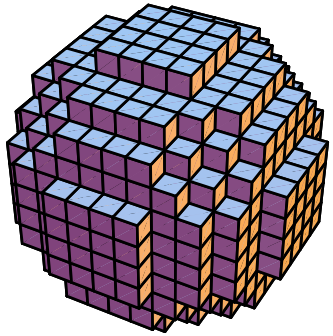
Cooling a spherical vortex on a $40^3 \times 2$ lattice



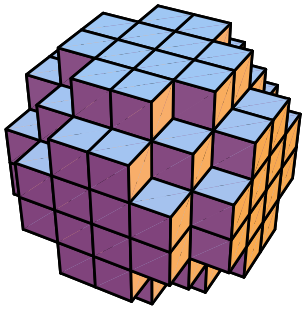
0 cooling steps



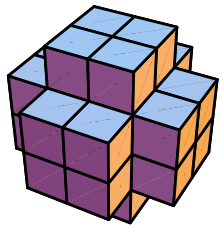
20 cooling steps



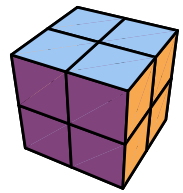
40 cooling steps



60 cooling steps

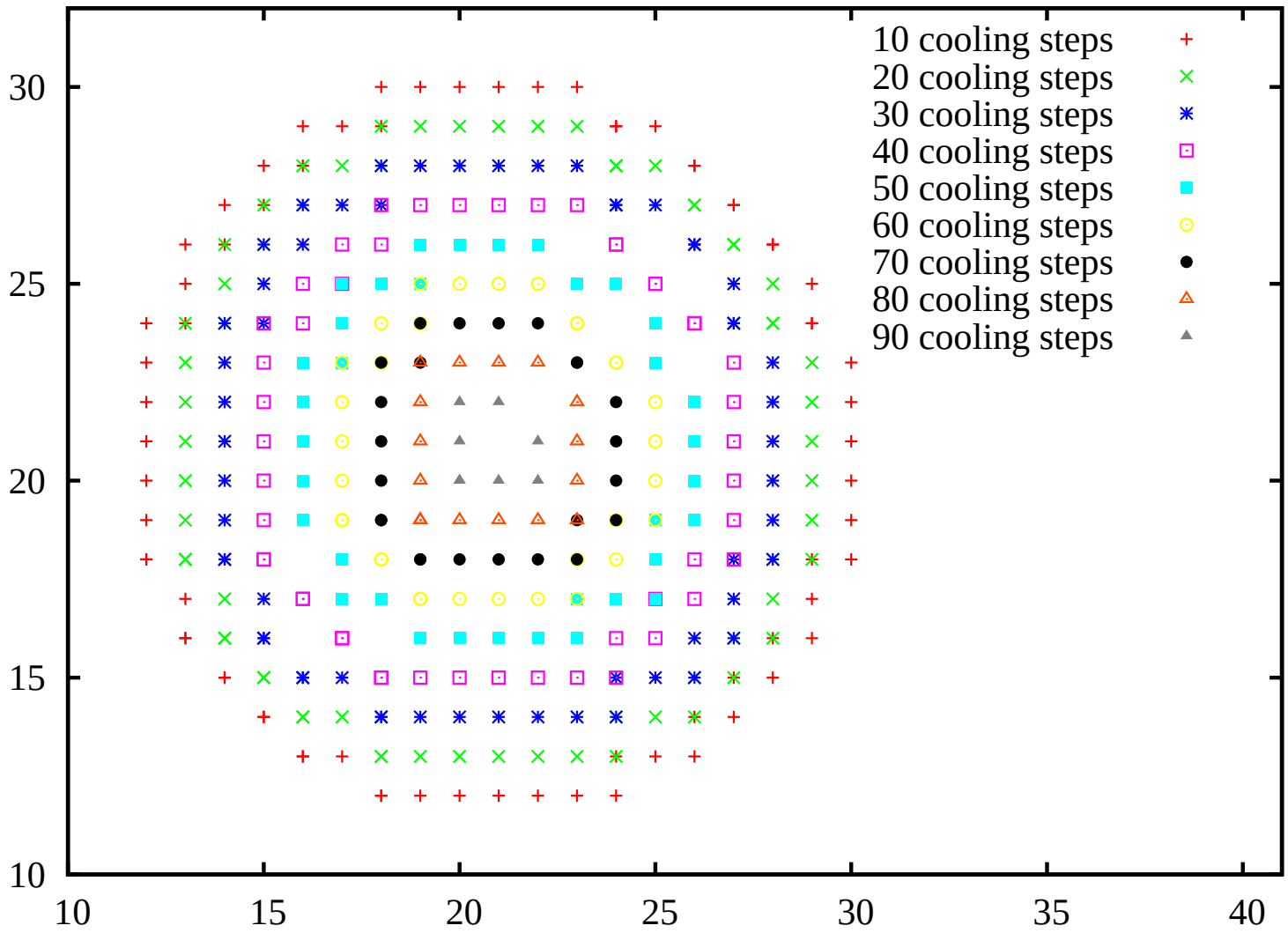


70 cooling steps



78 cooling steps

Cooling process under Abelian projection



Profile functions of the Polyakov loop

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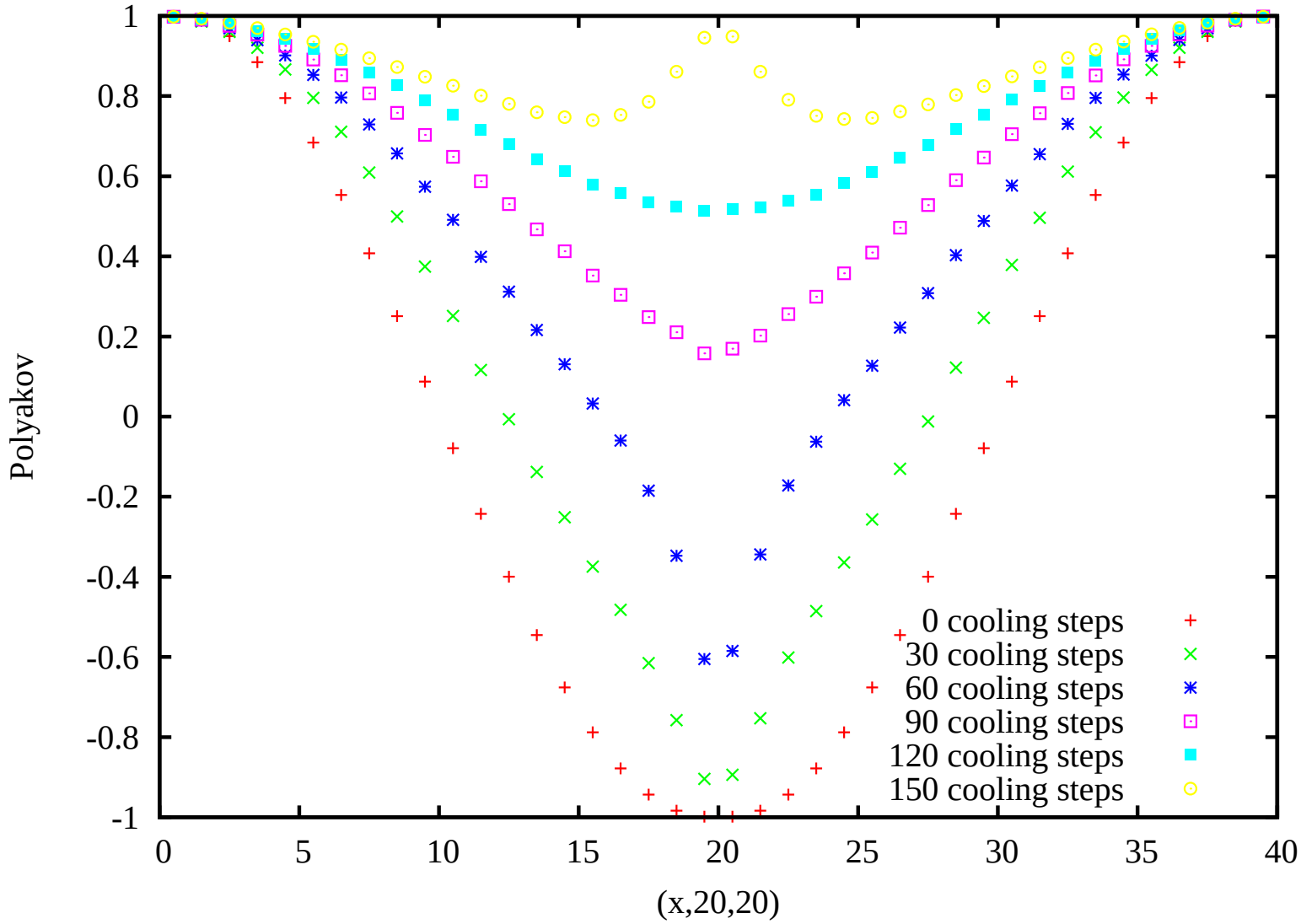

Central cube after 120 cooling steps

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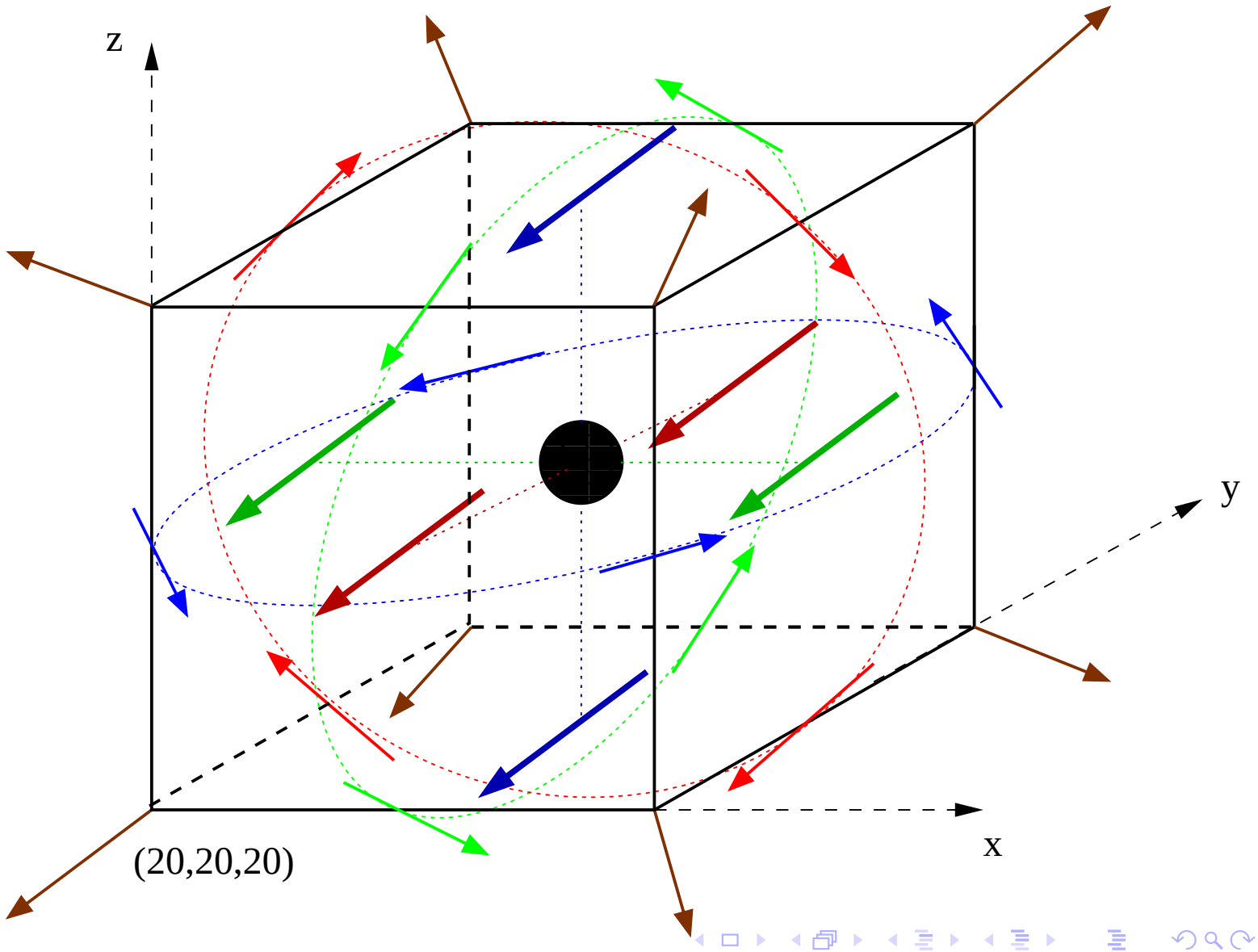

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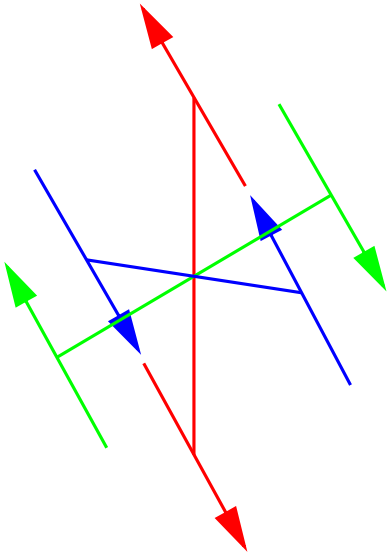
arXiv: hep-lat/1005.1015

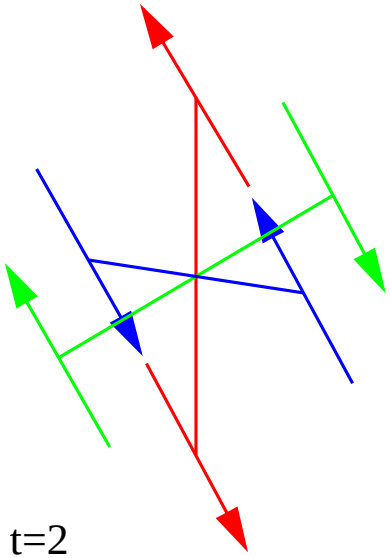
Thank you for your attention!

Questions?

Color flux through central plaquettes



t=1



t=2

xy-plaquette
xz-plaquette
yz-plaquette

Polyakov loop components during cooling

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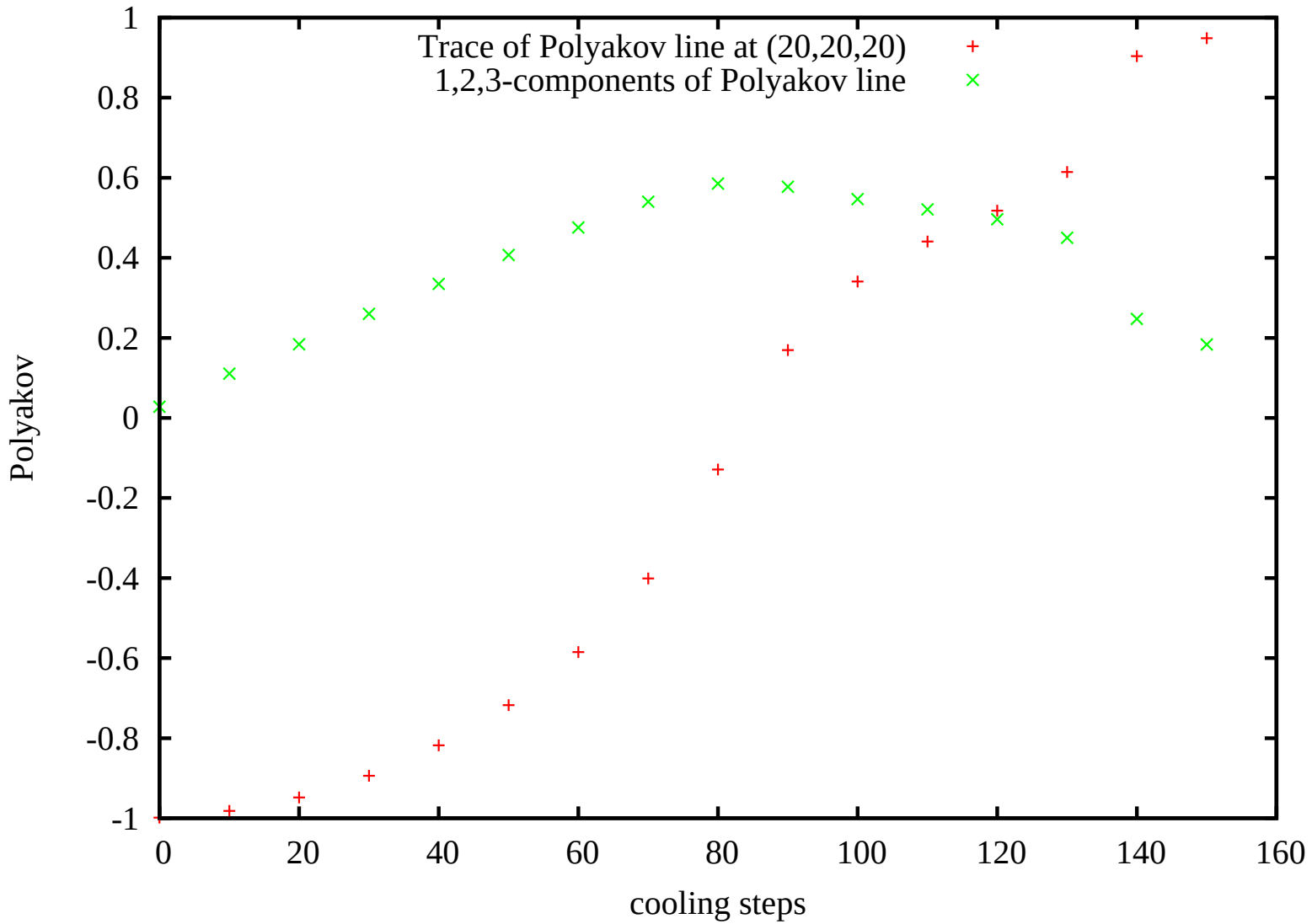

x-links at z=20 in Landau gauge

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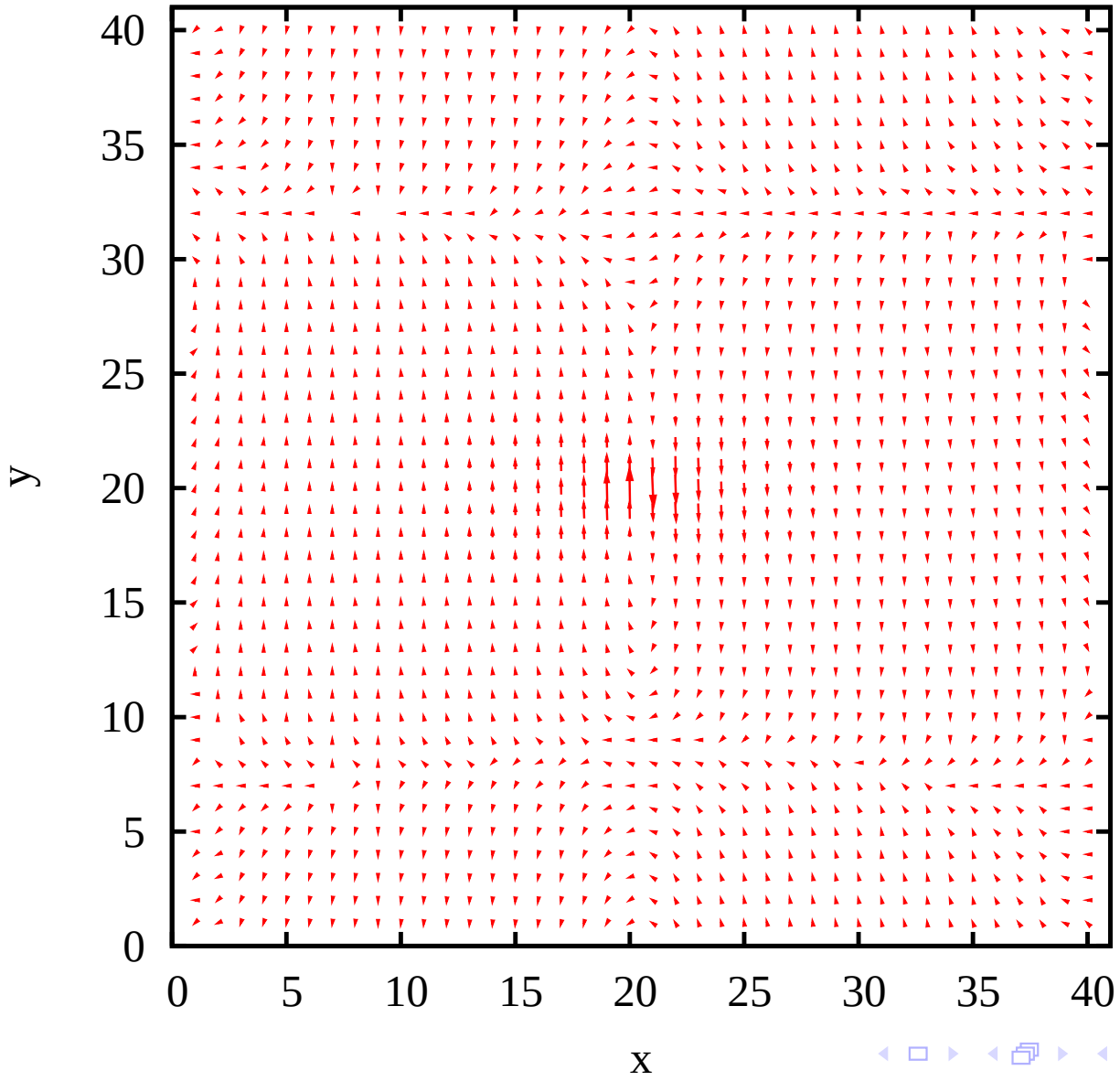

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