Towards the $N_f = 2$ deconfinement transition temperature with $\mathcal{O}(a)$ improved Wilson fermions

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1. Setup

Almost all simulations so far were done with Staggered fermions
$\Rightarrow \quad T_C = 150 - 190$ MeV

But: Possible conceptual problems ($\Rightarrow$ rooting)

Tests with other discretizations are needed!

$\Rightarrow$ Use $\mathcal{O}(a)$ improved Wilson fermions (mtmF,clover)

First results are available from QCDSF-DIK and also from tmfT.

$\Rightarrow$ See previous talks! (Bornyakov, Zeidlewicz)

Action and algorithm

Here we use nonperturbatively $\mathcal{O}(a)$ improved Wilson fermions:

$$S_{SW} = S_W + c_{SW} \frac{a^5\, i\, \kappa}{4} \sum_x \bar{\psi}(x)\, \sigma_{\mu\nu}\, \hat{F}_{\mu\nu}(x)\, \psi(x) + \beta\, S_G$$

[Sheikoleslami, Wohlert (1985)]

Here $\hat{F}_{\mu\nu}(x)$ is some suitable representation of the field strength tensor.
S_W and S_G are Wilson's fermion and gauge field lattice actions.
c_{SW} is tuned such that spectral quantities are $\mathcal{O}(a)$ improved in the chiral limit.

We use the highly optimized deflation accelerated DD-HMC algorithm for the simulations.

[Lüscher, (2004-2005), e.g. hep-lat/0509125]

- Not optimal for small lattices, but very efficient for larger ones.
- Constrains the possible temporal extents to $12, 16, 20, 24, \ldots$

Strategy

Goal: Extract the transition temperature and the order of the $N_f = 2$ transition in the chiral limit.

- we scan in β with fixed 'mass' κ

  $\Rightarrow$ We might use modified Ferrenberg-Swendsen reweighting.
- $c_{SW}(\beta)$ is nonperturbatively tuned to ensure $\mathcal{O}(a)$ improvement of spectral quantities.

The scale is set via additional runs at $T = 0$.

2. Current status

Scans

- 12×24^3 $m_\pi \approx 600$ MeV (Test scan at $\kappa = 0.13595$)
  $\Rightarrow$ Equivalent to one of the finest QCDSF-DIK point
  ($\Rightarrow \beta_C = 5.29$).

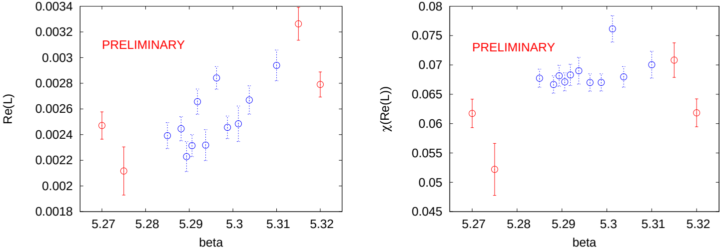
  Lower edge of the working region of the algorithm!

- 16×32^3 $m_\pi \approx 400$ MeV (First exploratory $N_t = 16$ scan.)
  Algorithm is stable and fast!

  The volume is still small ($m_\pi L \approx 3$; estimated using CLS-data).
  But: Reference point and benchmark for further $N_t = 16$ runs.

12×24^3 $m_\pi \approx 600$ MeV

Statistic: $\mathcal{O}(25000)$ $\mathcal{O}(7500)$ $\tau = 2.0$ (1 meas every tr.)

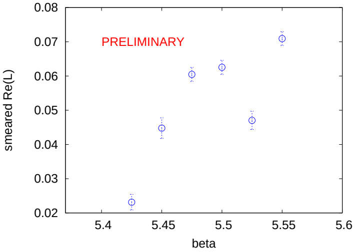


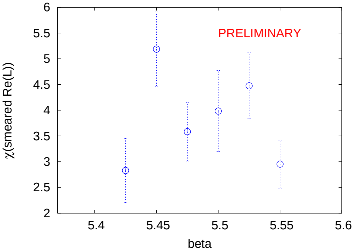
$$\beta_C \approx 5.301(3)$$

Error is taken naïvely from the nearest neighbors.
T_C is a little higher as in the analogous QCDSF-DIK scan.
(consistent with tmfT results)

16×32^3 $m_\pi \approx 400$ MeV

Statistic: $\mathcal{O}(1500)$ $\tau = 2.0$ (1 meas every 2 tr.)





$\beta_C \approx 5.45(3)$ very preliminary

3. Multi-Histogram methods

Reweighting

Reweighting is widely used in MC-Simulations.
(PHMC,RHMC,finite μ, ...)

Ferrenberg-Swendsen Reweighting:

Reweighting: We can use a Monte-Carlo simulation at β_0 to calculate the expectations value of some observable A at β

[Ferrenberg, Swendsen (1988)]

Problem: The reweighting factor for a configuration with averaged gauge action S_G is damped with $e^{-S_G\,(\beta-\beta_0)}$

$\Rightarrow$ for reweighting to work accurately: $\beta \approx \beta_0$

Usual multi-histogram methods

MH-method: Use a set of measurements with β_α close to β as a basis for the interpolation of expectation values.
[Huang, Moriarty, Myers, Potvin (1990)]

Limitation: The usual method only works if $\frac{d}{d\beta}\det(M) = 0$!

For $\mathcal{O}(a)$ improved Wilson fermions (and others) this is no longer true!

$\Rightarrow$ A modification is needed!

A naïve extention doesn't work, because one would need the full knowledge about the fermion determinant.
(Another problem: find a global density D)

Modified Multi-Histogram method

$\Rightarrow$ We have to use a trick and introduce a reference point $\beta = \beta_R$ for the fermionic part of the reweighting.

Appearing new factors:

$$R(\beta, \beta_0)[U] = \left[\frac{\det(M(\beta)[U])}{\det(M(\beta_0)[U])} \right]^{N_f}$$

(more precisely two factors $R(\beta, \beta_R)$ and $R(\beta_R, \beta_\alpha)$.)

Find a method to measure R without measuring the full determinant!

Approximation of R for clover fermions

For clover fermions the fermion matrix can be written as:

$$M(\beta)[U] = M_W[U] + c_{SW}(\beta)\, M_{SW}[U]$$

$$\Rightarrow \quad M(\beta) = M(\beta_0) + \Delta c_{SW}(\beta, \beta_0)\, M_{SW}$$

This means for the ratio:

$$R(\beta, \beta_0) = \left[\det\left(1 + \Delta c_{SW}(\beta, \beta_0)\, M^{-1}(\beta_0)\, M_{SW}\right)\right]^{N_f}$$

For typical scans $\Delta c_{SW} = \mathcal{O}(10^{-2})$ if β_R is set to β_C.

$\Rightarrow$ Expand $\det(\ldots)$ in Δc_{SW}!

$$\begin{aligned} R(\beta, \beta_0) = &\ 1 + N_f\, \Delta c_{SW}(\beta, \beta_0)\, \mathrm{Tr}\left(M^{-1}(\beta_0)\, M_{SW}\right) + \mathcal{O}\left((\Delta c_{SW})^2\right) \\ = &\ 1 - N_f\, \Delta c_{SW}(\beta_0, \beta)\, \mathrm{Tr}\left(M^{-1}(\beta)\, M_{SW}\right) + \mathcal{O}\left((\Delta c_{SW})^2\right) \end{aligned}$$

Approximation of R for clover fermions

$\Rightarrow$ It is sufficient to measure the operator

$$R_{op}(\beta_\alpha) \equiv \mathrm{Tr}\left(M^{-1}(\beta_\alpha)\, M_{SW}\right)$$

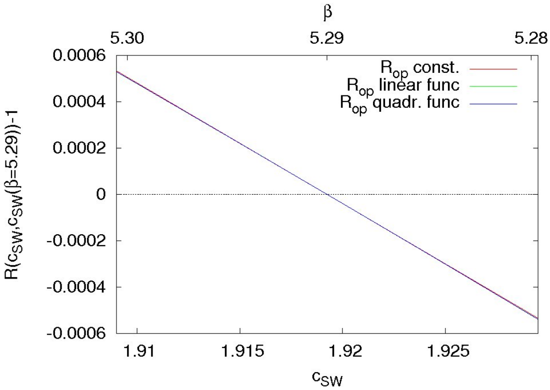
on each configuration to use the multi-histogram method.

Note:

- A similar expansion can be done for mtmF.
- $\dfrac{\partial R(\beta, \beta_0)}{\partial \Delta c_{SW}} = N_f\, R_{op}(\beta_0)$

$\Rightarrow$ Use R_{op} to model R around β_R and test effects of higher orders and the reliability.

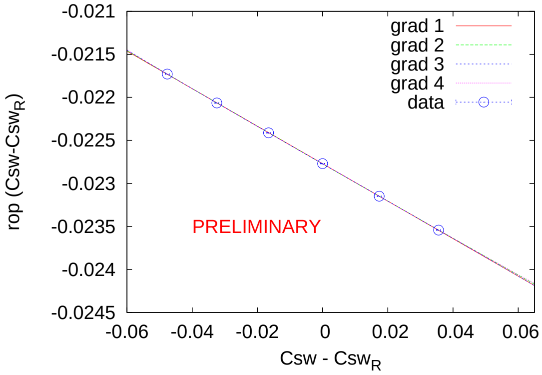
Behavior of R: 12×24^3 $m_\pi \approx 600$ MeV



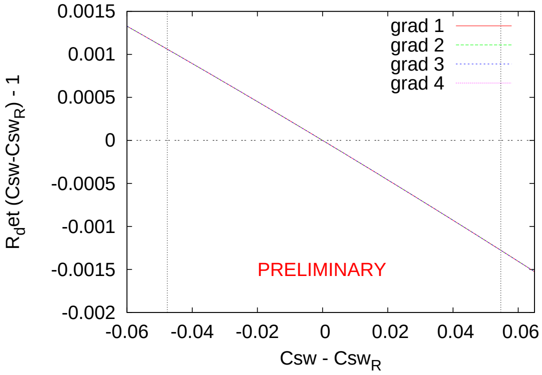
⇒ The variation is well below the statistical errors!

Naive reweighting should be sufficient!

Behavior of R_{op}: 16×32^3 $m_\pi \approx 400$ MeV



Behavior of R: 16×32^3 $m_\pi \approx 400$ MeV



4. Conclusions and outlook

Conclusions

- I showed first scans, where we approach the measurement of the transition temperature.
  (Including first $N_t = 16$ lattices for Wilson fermions!)
- The algorithm is optimal for large lattices and performs well for $N_t = 16$.
- We evolved and implemented a modified multi-histogram method for $\mathcal{O}(a)$ improved Wilson fermions.
  (Also possible to use it for twisted mass!)
- First results showed, that the method should work accurately in our current scans.

Outlook

- We are planing new runs with $m_\pi \approx 180 - 300$ MeV.
  - Including additional observables like screening masses ($\Rightarrow$ need larger volumes)
- We aim at future runs with lattices up to: $N_t = 20$ and $V = 64^3$.
- When the full set of measurements is available, we are going to extract the order of the transition via Binder cumulants and a finite size scaling analysis.