The study of the Three Nucleon Force in full QCD Lattice calculations

Takumi Doi

(Univ. of Tsukuba)

for HAL QCD Collaboration

S. Aoki, K. Sasaki (Univ. of Tsukuba)
T. Hatsuda, N. Ishii (Univ. of Tokyo)
Y. Ikeda (RIKEN)
T. Inoue (Nihon Univ.)
K. Murano (KEK)
H. Nemura (Tohoku Univ.)

HAL QCD strategy

- Calculate BS wave func from 4pt
  - ➔ $\psi(\vec{r}) = \langle 0|N(\vec{x}+\vec{r})N(\vec{x})|2N\rangle$
  - The choice of nucleon op. corresponds to the choice of “scheme” (N.B. Observables do not depend on “scheme”)

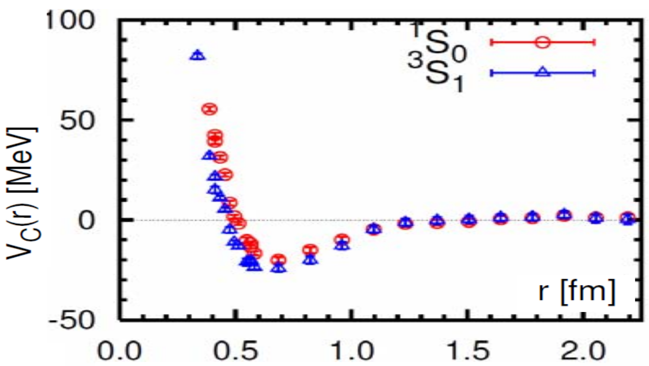
C.-J.D.Lin et al., NPB619(2001)467
CP-PACS Coll. PRD71(2005)094504

- Define the non-local pot. $(E-H_0)\psi(\vec{r}) = \int d\vec{r'}\, U(\vec{r},\vec{r'})\psi(\vec{r'})$
- Velocity expansion (Okubo-Marshak(1958))

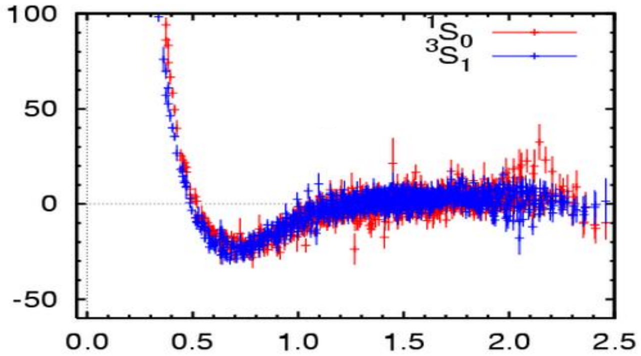
$$U(\vec{r},\vec{r'}) = V_c(r) + S_{12}V_T(r) + \vec{L}\cdot\vec{S}V_{LS}(r) + \mathcal{O}(\nabla^2)$$

LO LO NLO

Formulation:
Aoki-Hatsuda-Ishii,
PTP123(2010)89



Quenched QCD
m_π = 530MeV, L=4.4fm



Nf=2+1 Full QCD
m_π = 570MeV, L=2.9fm

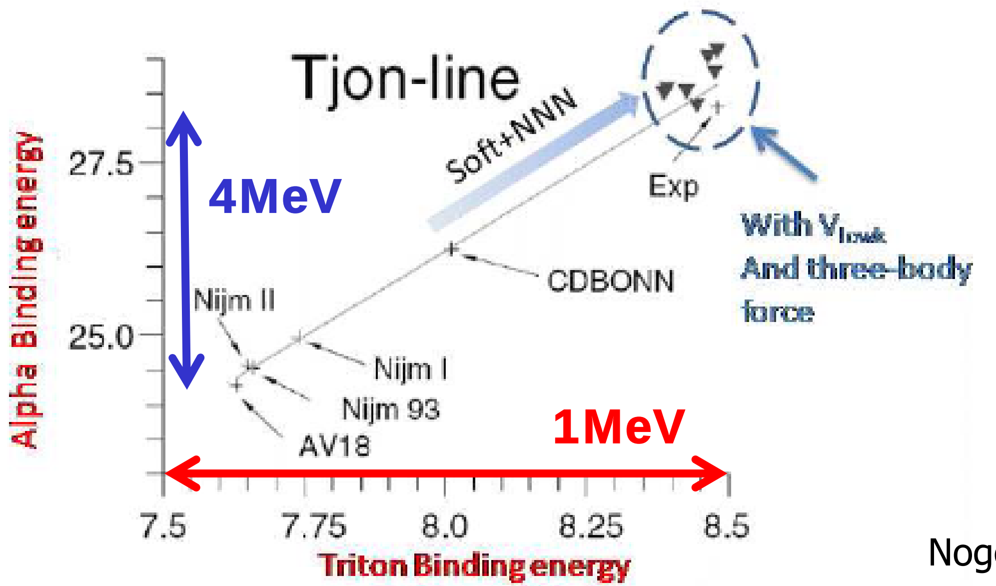
Ishii-Aoki-Hatsuda,
PRL99(2007)022001
PoS LAT2008(2008)155

Importance of Three Nucleon Force (TNF)

- Precise few-body calc:
  NN force cannot reproduce B.E.

δB.E.= 0.5-1MeV for ^{3}H
δB.E.= 2-4 MeV for ^{4}He

Attractive TNF necessary



Nogga et al., PRL85(2000)944

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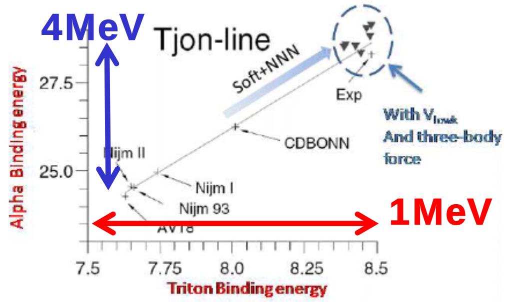


Nogga et al., PRL85(2000)944

- Saturation density/energy of nuclear matter also requires TNF
  - EOS of neutron star
    - Flavor universal TNF (repulsive) ?

  Repulsive TNF also necessary

  A.Akmal et al., PRC58(1998)1804

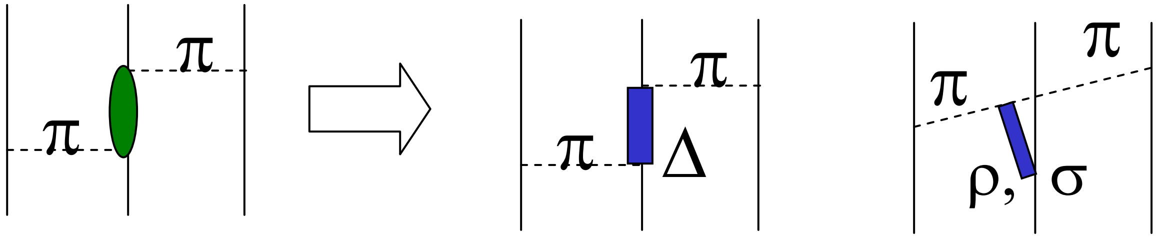
  Takatsuka et al., Prog.Theor.Phys.Suppl.174(2008)80

- Ay puzzle in N-d scatt. N-A scatt. etc.
- The effect on the nuclear chart
  - Oxygen anomaly (drip line ^{28}O → ^{24}O) by TNF

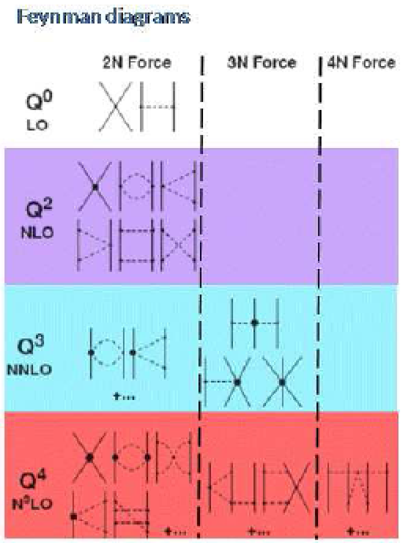
  T.Otsuka et al., arXiv:0908.2607

Three Nucleon Force (TNF)

- It is natural to expect the existence of TNF
- It is very nontrivial to determine TNF from QCD
- 2πE-TNF Fujita-Miyazawa, Prog.Theor.Phys.17(1957)360
  - Off-energy-shell πN scatt



- EFT expansion
  - ➔ TNF appears at NNLO order



U.v.Kolck, PRC49(1994)2932

How can we tackle TNF in Lattice QCD ?

- **In the case of 2N system…**
  - Calc 4pt func ➔BS amp.

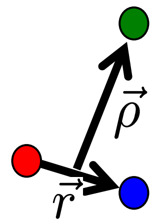
$$\psi(\vec{r}) = \langle 0|N(\vec{x}+\vec{r};t)N(\vec{x};t)|2N\rangle$$

$$|2N\rangle = \bar{N}_{src}(t=0)\bar{N}_{src}(t=0)|0\rangle$$

➔ $$(E - H_0)\psi(\vec{r}) = [V_c(r) + S_{12}V_T(r) + \cdots]\psi(r)$$

- **Extention to 3N system**
  - Calc 6pt func ➔BS amp. of NNN

$$\psi(\vec{r},\vec{\rho}) = \langle 0|N(\vec{x}+\vec{r})N(\vec{x})N(\vec{x}+\vec{r}/2+\vec{\rho})|3N\rangle$$



  - Obtain TNF through

$$(E-H_0^r-H_0^\rho)\psi(\vec{r},\vec{\rho}) = \left[\sum_{i<j} V_{ij}(\vec{r}_{ij}) + V_{TNF}(\vec{r},\vec{\rho})\right]\psi(\vec{r},\vec{\rho})$$

⇧ by 2N calc

  - Difficulty(1): volume factor
    - 2N: naïve $O(L^6)$ calc ➔ $O(L^3 \log L^3)$
    - 3N: naïve $O(L^9)$ calc ➔ $O(L^6 \log L^6)$

    $O(10^4$-$10^5)$ factor

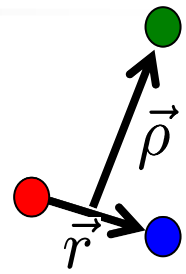
  - Difficulty(2): naïve calc of quark dof grows in factorial ($\sim N_u!\ N_d!$)
    - 2N: $O(L^3)$ X N_{wick} X color/spinor loops
    - 3N: $O(L^6)$ X N_{wick} X color/spinor loops

    $O(L^3)$ X O(4000) factor

TNF is exceptionally challenging problem !

How can we tackle TNF in Lattice QCD ?

- Consider the effective 2N-potential in 3N system



$$\phi(\vec{r}) \equiv \int d\vec{\rho}\,\psi(\vec{x},\vec{\rho})$$

Integrate out 3rd spectator DoF

$$\begin{aligned}(E - H_0^r)\phi(\vec{r}) &= \left[V_{12}(\vec{r}) + \delta V_{eff}(\vec{r})\right]\phi(\vec{r}) \\ &= \left[V_{12}(\vec{r})\phi(\vec{r}) + \int d\vec{\rho}\,(V_{13}(\vec{r},\vec{\rho}) + V_{23}(\vec{r},\vec{\rho}) + V_{TNF}(\vec{r},\vec{\rho}))\psi(\vec{r},\vec{\rho})\right]\end{aligned}$$

  - **"finite density effect"** for 2N-potential

As 3N system, we consider ^{3}H (Triton)

  - ➔ Difficulty(1) (Volume factor) is resolved

$$\mathcal{O}(L^6 \log L^6)\ \text{calc} \longrightarrow \mathcal{O}(L^3 \log L^3)\ \text{calc}$$

  - How about Difficulty(2) (contraction/loops) ?
    - Use symmetry as much as possible & skip the calc of redundant components ➔ **factor of 16 improvement**
    - We propose to use non-rela limit op for nucleon source op. ➔ **factor of 8**

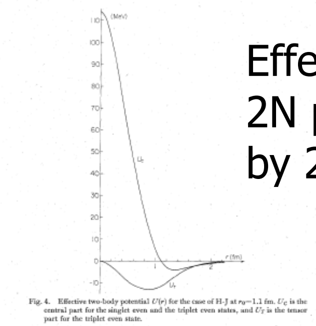
$$N^{src} = \epsilon_{abc}(u_a^T C\gamma_5 \tfrac{1+\gamma_4}{2} d_b)\tfrac{1+\gamma_4}{2} u_c$$

Effective 2N potential

- For the nuclear matter, 2πE-TNF can produce effective repulsion (as well as attraction)

$$\mathcal{O}(10-100)\ \text{MeV potential ?}$$

T.Kasahara, Y.Akaishi, H.Tanaka,
Prog.Theor.Phys.Suppl.56(1974)96



Effective
2N potential
by 2πE-TNF

- Additionally universal repulsive TNF ? Tamagaki, Takatsuka
  - SJM(string-junction model) ➔ $\mathcal{O}(10)\text{MeV at } \rho_0$
  - N.B. SJM may contradicts lattice results for 2N repusive core
- Nijmegen ESC
  - TNF as finite density effect to vector mesons

$$M_V(\rho) = M_V \exp(-\alpha_V \rho/\rho_0), \alpha_V = 0.11 - 0.18$$

Lattice QCD Calculations

Numerical
Setup & Results

Configurations

- Nf=2 dynamical clover fermion + RG improved gauge configs (CP-PACS)
  - 599 configs X 16 measurements
  - beta=1.95, (a^{-1}=1.27GeV, a=0.156fm)
  - 16^3 X 32 lattice, L=2.5fm
  - Kappa(ud)=0.13750
    - $M(\pi)$ = 1.13GeV
    - M(N) = 2.15GeV
    - $M(\Delta)$ = 2.31GeV
  - Techniques
    - **Automatic Wick contraction code** to handle 4 up- and 5 down-quarks
    - Non-rela limit op is used to create 3N state at source

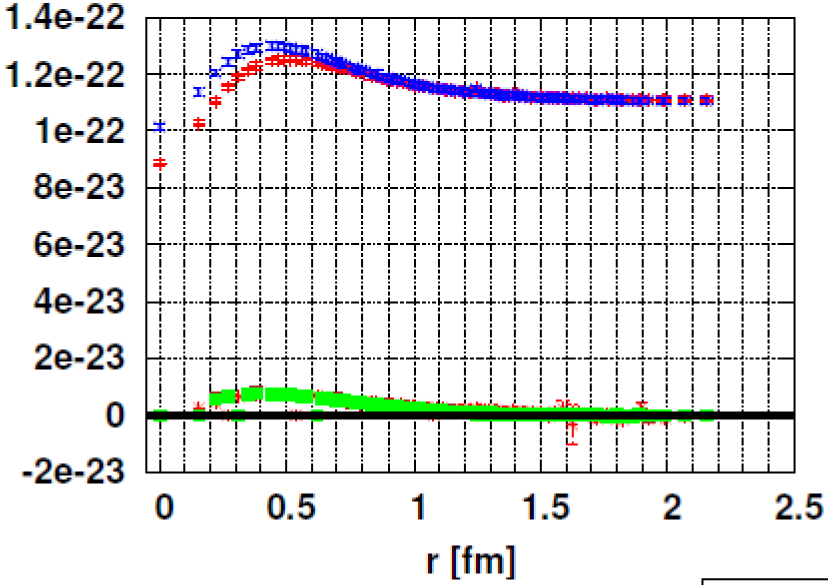
$$N^{src} = \epsilon_{abc}(u_a^T C\gamma_5 \frac{1+\gamma_4}{2} d_b)\frac{1+\gamma_4}{2} u_c$$

➔ Factor of 2^3=8 faster

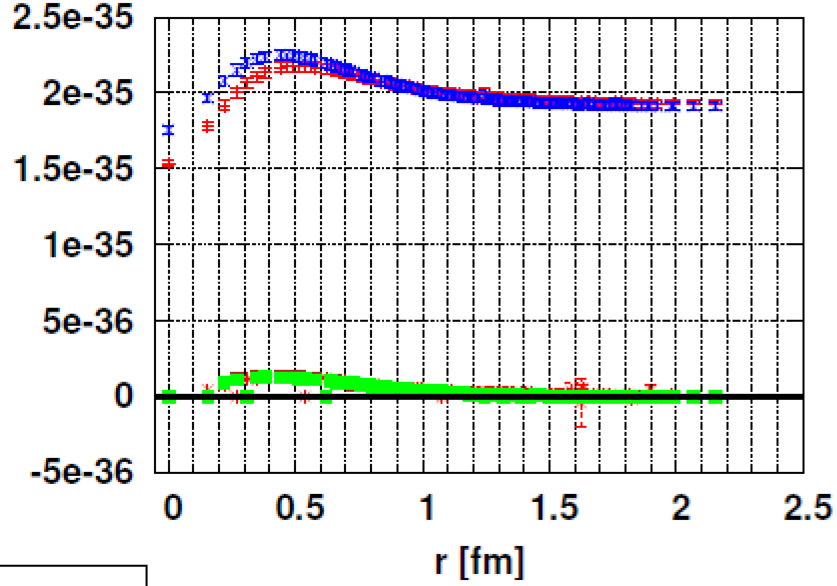
CP-PACS Coll. S. Aoki et al.,
Phys. Rev. D65 (2002) 054505
[E: D67 (2003) 059901]

Results for wave functions

Genuine 2N



Effective 2N in 3N

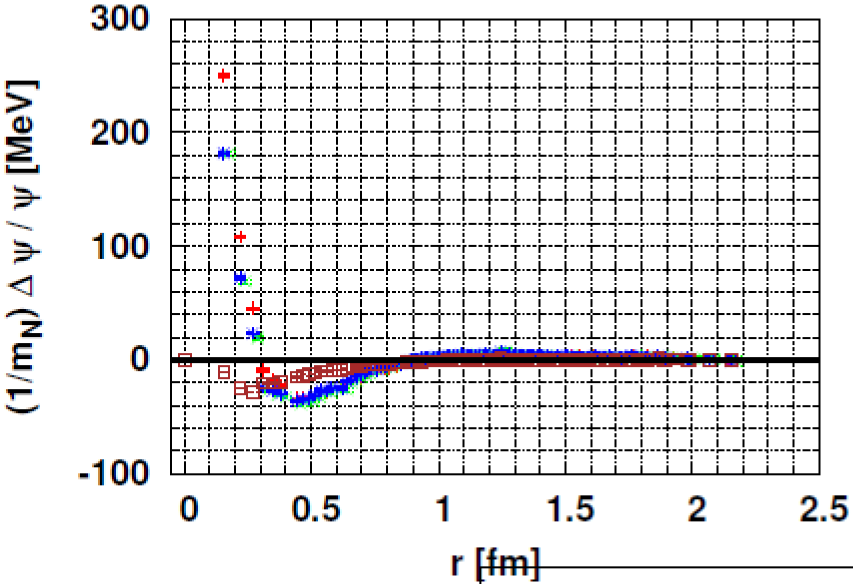


Red: 1S0
Blue: 3S1
Green: 3D1

M(π)=1.13GeV

Results for Potentials

Genuine 2N



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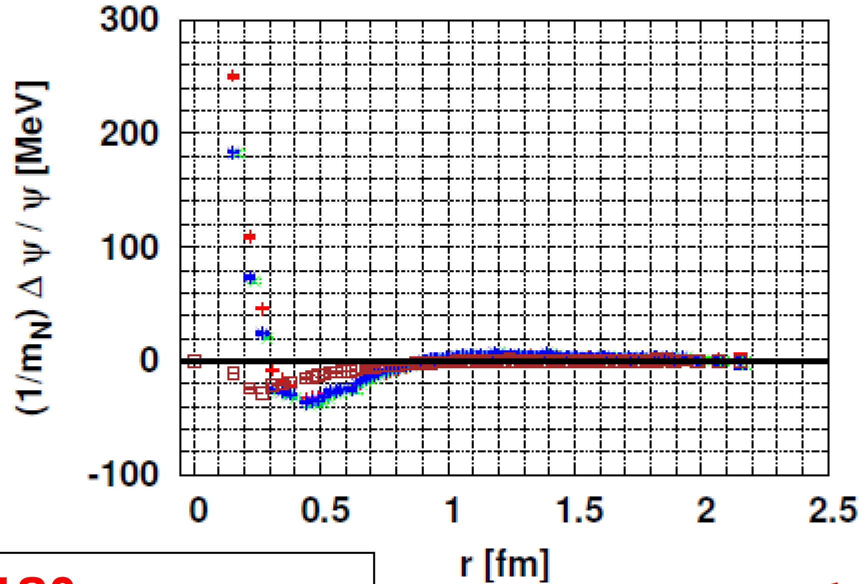


M(π)=1.13GeV

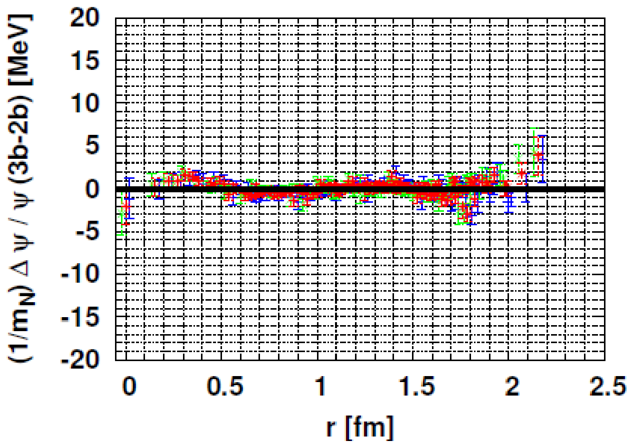
Red: central 1S0
Green: eff. central in 3S1-3D1
Blue: central in 3S1-3D1
Brown: tensor in 3S1-3D1

Good S/N even for 3N !

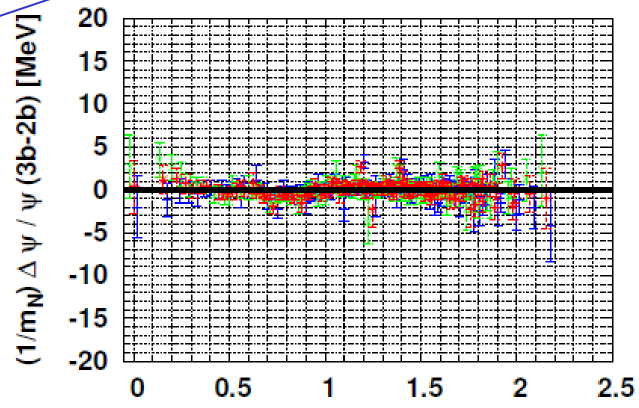
Potential difference (3N-2N)

Preliminary

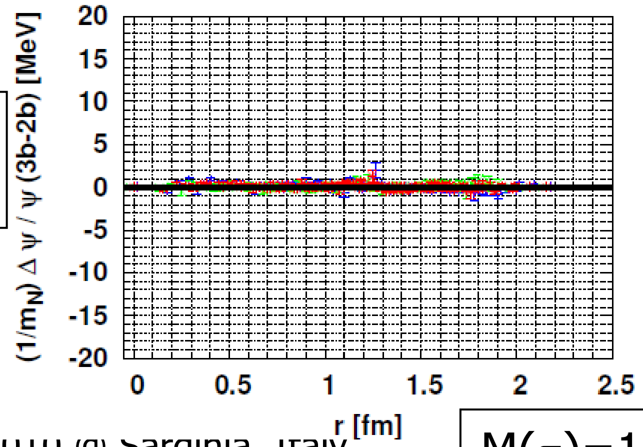
Central in 1S0



3S1-3D1



Central



Tensor

- All results are consistent with zero.
- Tensor is best constrained.

Preliminary:

GS saturation may not be enough

Constant energy shift not included

M(π)=1.13GeV

Additional calc w/ smaller mass

- Small TNF because of heavy quark mass ?

$$\left[e^{-m_\pi^{lat} r}/r\right] / \left[e^{-m_\pi^{phys} r}/r\right] = 0.08 \quad \text{at } r = 0.5\text{fm w/ } m_\pi^{lat} = 1.13\text{GeV}$$

2πE–TNF ➔ $(0.08)^2$ suppression ? *In practice, effective "r" also modified*

- Nf=2+1 dynamical clover fermion + RG improved gauge configs (PACS-CS)
  - ~400 configs X 4 measurements
  - beta=1.90, (a^{-1}=2.18GeV, a=0.0907fm)
  - 32^3 X 64 lattice, L=2.9fm
  - Kappa(ud)=0.13700, 0.13727
    - **M(π) = 0.70GeV, 0.57GeV**
    - M(N) = 1.58GeV, 1.41 GeV
    - M(Δ) = 1.79GeV, 1.67 GeV

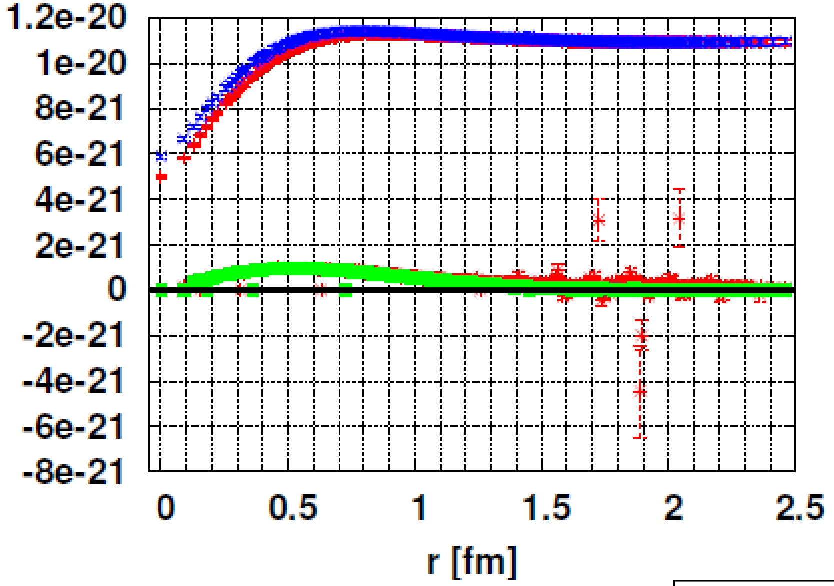
PACS-CS Coll. S. Aoki et al., Phys. Rev. D79 (2009) 034503

$$\left[e^{-m_\pi^{lat} r}/r\right] / \left[e^{-m_\pi^{phys} r}/r\right]$$
$$= 0.24 \quad (m_\pi^{lat} = 0.70\text{GeV})$$
$$= 0.34 \quad (m_\pi^{lat} = 0.57\text{GeV})$$

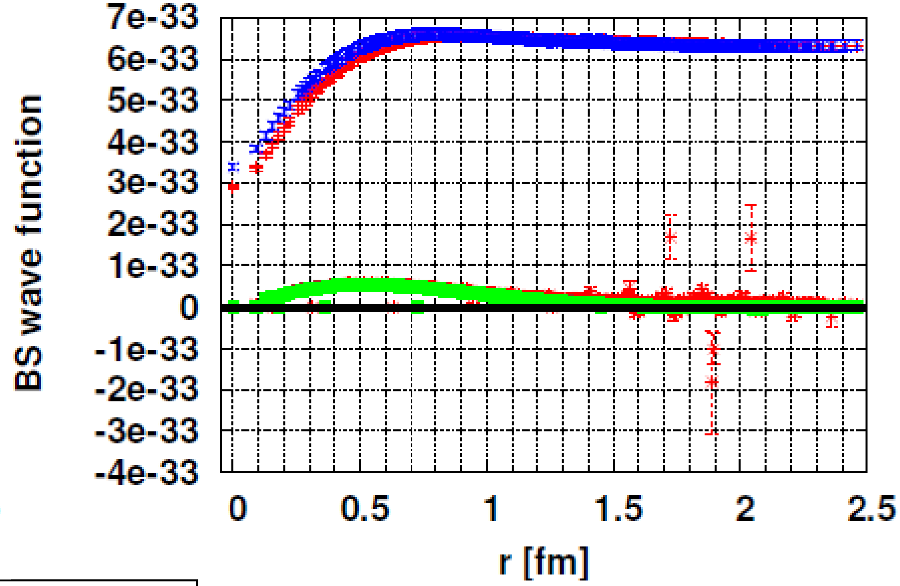
at $r = 0.5$fm

Results for wave functions

Genuine 2N



Effective 2N in 3N



Red: 1S0
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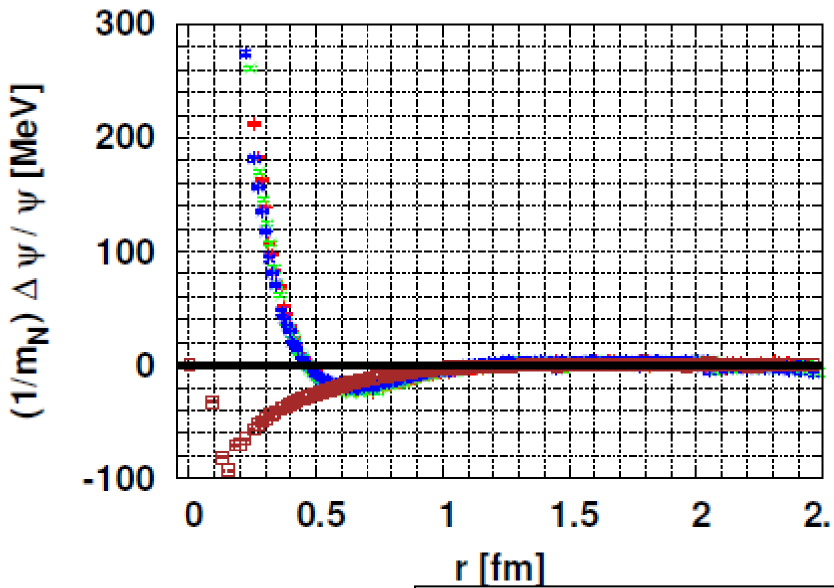
Mπ=0.7GeV

Results for Potentials

Genuine 2N



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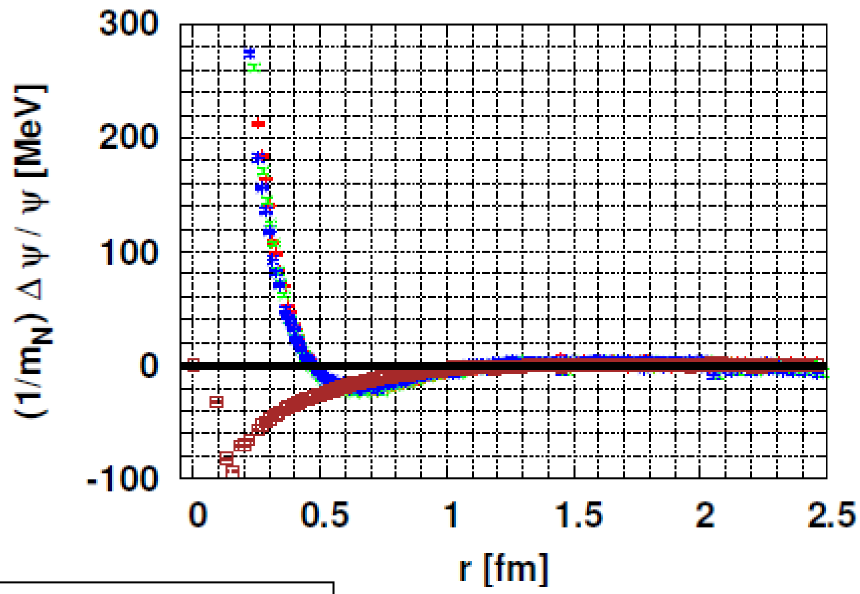

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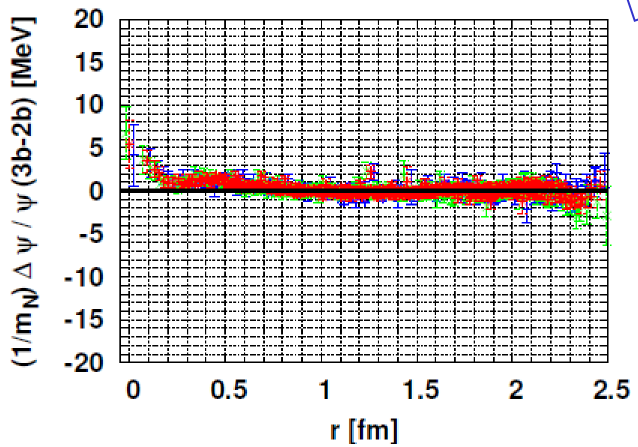
M_π=0.7GeV

Potential difference (3N-2N)

Preliminary

Central in 1S0



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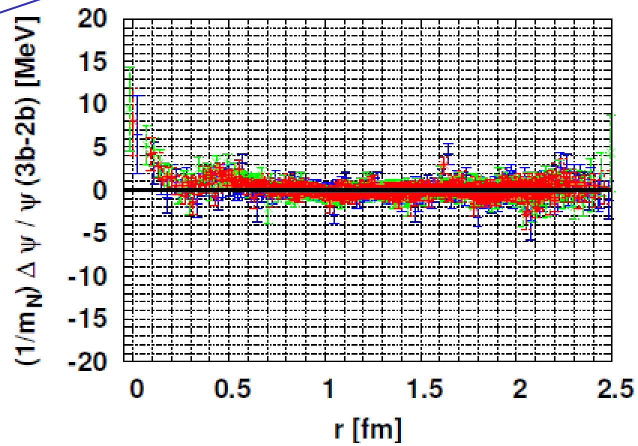
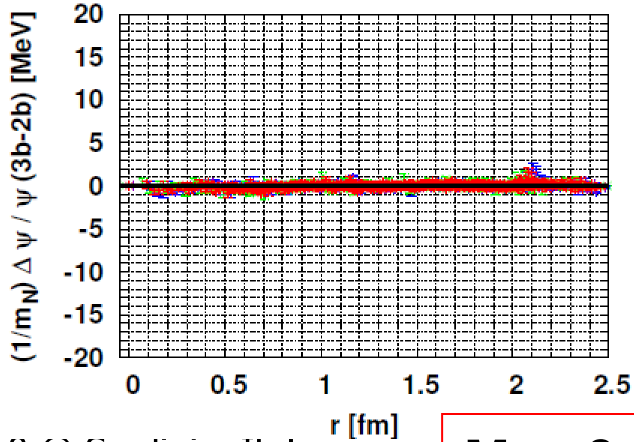


Central



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Constant energy shift not included

M_π=0.7GeV

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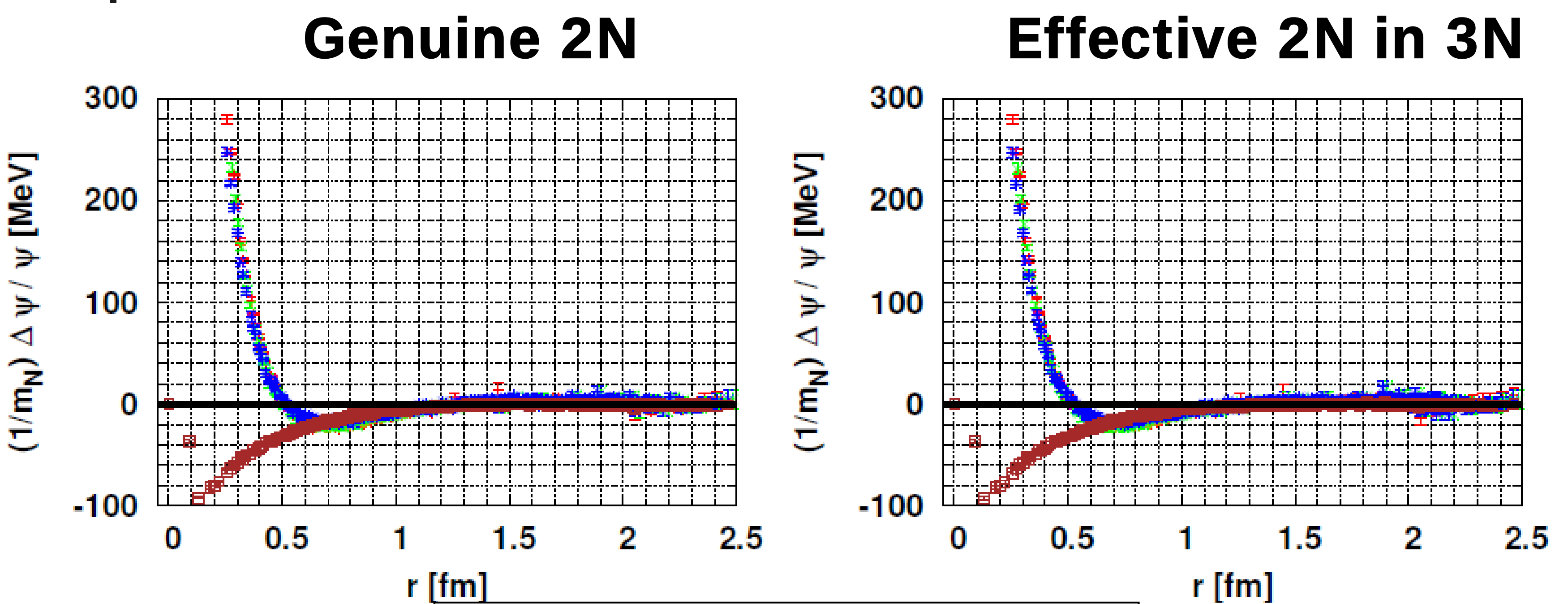

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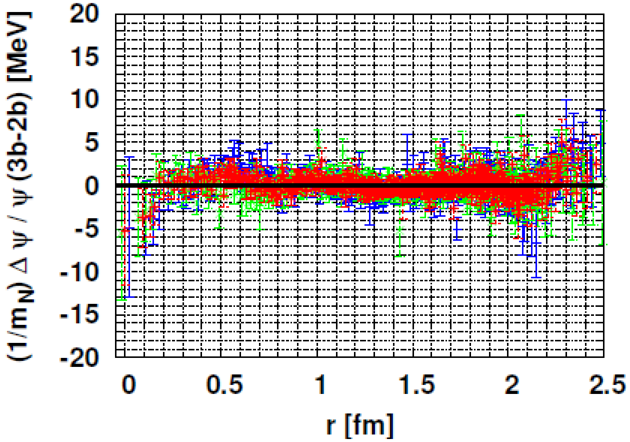

3S1-3D1

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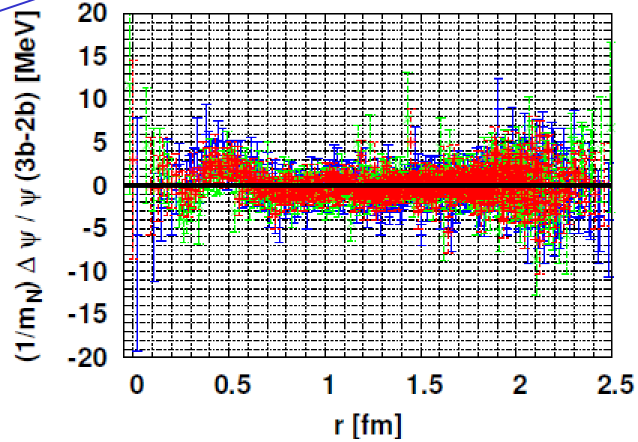

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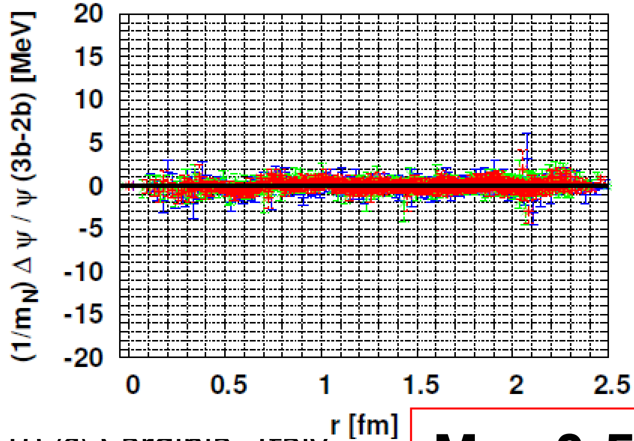

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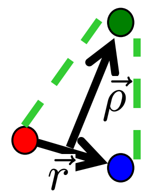
Mπ=0.57GeV

Summary/Outlook

- We have performed the lattice QCD study of the Three Nucleon Force (TNF)
  - We have calculated **"effective" 2N-potential** in 3N (^{3}H) system
  - Nf=2, 2+1 dynamical clover fermion with m_π= 1.13, 0.70, 0.57GeV
    - The effective 2N-potential have been obtained with good precision
      - Errors for the differences of potentials are several MeV order
    - So far, no indication of TNF has been observed
- Outlook
  - The evaluation of Jacobi-"off-diagonal" 2N potential contribution



    - ➔ genuine TNF can be extracted
  - Calc w/ lighter quark masses, different volume lat box
  - Different 3D-configurations of 3N in progress (linear, triangle, etc.)

Backup Slides

Nuclear matter/EoS of Neutron Star

- Repulsion from 2πE-TNF is not sufficient to sustain neutron star
  - Softening by Y-mixing ➔ M(max)~1.2M⊙ where Λ mixing causes a problem (No ΛΛπ coupling)
  - ➔ "Universal extra repulsion" is necessary
  - Example: string junction model (SJM)

T.Takatsuka, S.Nishizaki, R.Tamagaki
Prog.Theor.Phys.Suppl.174(2008)80