Nucleon strange quark content in 2+1 flavor QCD

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for JLQCD Collaboration

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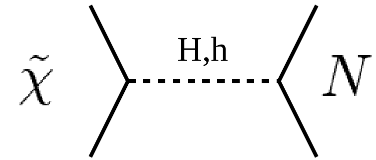
1. Introduction

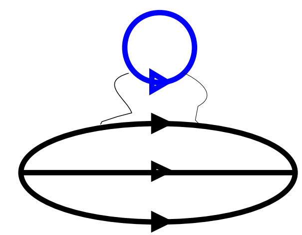
- *Nucleon strange quark content* $\langle N|\bar{s}s|N\rangle$ *is scalar form factor of nucleon at zero recoil.*

- *Important parameter for..*
  - *Nucleon structure* $$f_{T_s} = \frac{m_s\langle N|\bar{s}s|N\rangle}{m_N}$$
  - *Cross section between dark matter and nucleon*

e.g.) Dark matter = neutralino



- *We calculate nucleon strange quark content from lattice QCD.*
  - *Calculation involve " disconnected " diagram*
    - *Quark loop at arbitrary lattice sites*
    - *Computational demanding, statistical noisy*
  - *Scalar need VEV subtraction*



1. Introduction (Cont'd)

- *This Work*
  - ✔ *We calculate the nucleon strange quark content from the disconnected three-point function.*
  - ✔ *Disconnected diagram* ⇐ *all-to-all propagator*
  - ✔ *Signal improvement* ⇐ *low-mode averaging technique*

2. Measurement

- *overlap fermion + extra-Wilson fermion*

H.Fukaya et al. JLQCD
Phys. Rev. D 74, 094505 (2006)

- *Iwasaki gauge*
- $\beta=2.30$, $a \sim 0.112$ fm ⇐ $m_\Omega = 1672$ [MeV]

➢ *Two heavier ud quark masses*
  $am_{ud} = 0.035, 0.050$

➢ *Two strange quark masses*
  $am_s = 0.080, 0.100$

➢ $16^3 \times 48$, $L \sim 1.8$ fm

➢ *Two lighter ud quark masses*
  $am_{ud} = 0.015, 0.025$

➢ *One strange quark mass*
  $am_s = 0.080 \sim am_{s,phys} = 0.081$

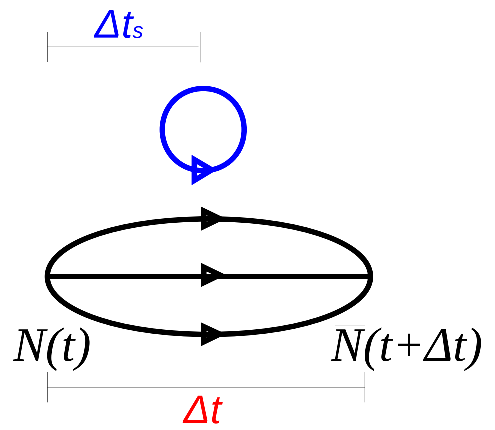
➢ $24^3 \times 48$, $L \sim 2.6$ fm

- $m_\pi L \gtrsim 4$ *Finite size effect might be small.*
- ~50 *independent confs./each quark mass*

2.1 Ratio Method

- *Ratio (3pt. function / 2pt. function) is calculated.*

$$R(\Delta t, \Delta t_s) = \frac{C_{3pt}(t, \Delta t, \Delta t_s)}{C_{2pt}(t, \Delta t)} \to \langle N|\bar{s}s|N\rangle \quad (\Delta t, \Delta t_s \to \infty)$$



- $C_{2pt} = \mathrm{Tr}[P_\pm \langle N(t+\Delta t)\bar{N}(t)\rangle]$

$$P_\pm = \frac{1 \pm \gamma_4}{2}$$

⇐ *Low-mode averaging*

L.Giusti et.al. (2004)
T.A.DeGrand and S.Schaefer (2004)

$$D_L^{-1} = \sum_k^{N_{\rm ev}} \frac{1}{\lambda_k} v^{(k)} v^{(k)\dagger} \qquad D v^{(k)} = \lambda_k v^{(k)}$$

Averaged over 16 spacial point at every time-slices

- $C_{3pt} = \mathrm{Tr}[P_\pm \langle N(t+\Delta t)\bar{N}(t) S(t+\Delta t_s)\rangle] - \langle S(t+\Delta t_s)\rangle \mathrm{Tr}[P_\pm \langle N(t+\Delta t)\bar{N}(t)\rangle]$

$$S(t) = \bar{s}s(t)$$

⇐ *Low-mode averaging + all-to-all propagator*

2.2 all-to-all propagator

TrinLat Collaboration
Comput.Phys.Commun,172(2005)

Calculate quark loops at arbitrary lattice sites.

- Low-mode is treated exactly.

$$D_L^{-1} = \sum_k^{N_{\rm ev}} \frac{1}{\lambda_k} v^{(k)} v^{(k)\dagger} \qquad D v^{(k)} = \lambda_k v^{(k)} \qquad (k = 1, \cdots, N_{\rm ev})$$

- High-mode is constructed stochastically.
  - Produce complex Z_2 noise $\eta_{[r]}$ $\quad \sum_{r=1}^{\infty} \eta_{[r]}(x) \eta_{[r]}^{\dagger}(y) = \delta_{x,y}$

    (Diluted in spin, color and time)
  - Solve linear equation $\quad D\psi_{[r]} = (1 - \mathcal{P}_{low}) \eta_{[r]} \Rightarrow D_H^{-1} = \sum_{r=1}^{N_r} \psi_{[r]} \eta_{[r]}^{\dagger}$

- We use N_{ev}=160 or 240 and N_r=1 / configuration
  - High-mode is probably small since low-mode is well dominated.

2.3 Smearing function

- *To enhance ground state contribution, we apply the smearing technique.*
- *In previous study, we find sink smearing is useful.*
- *Gaussian smearing is employed for source and sink.*

$$q_{smr}(x) = \sum_{\vec{y}} \left(1 + \frac{\omega}{4N} H\right)^N_{\vec{x},\vec{y}} q(\vec{y}, t)$$

$$N_\alpha = \epsilon_{abc} u^a (C\gamma_5) d^b u^c_\alpha$$

$$H_{\vec{x},\vec{y}} = \sum_{i=1}^{3} (\delta_{\vec{x},\vec{y}-\hat{i}} + \delta_{\vec{x},\vec{y}+\hat{i}})$$

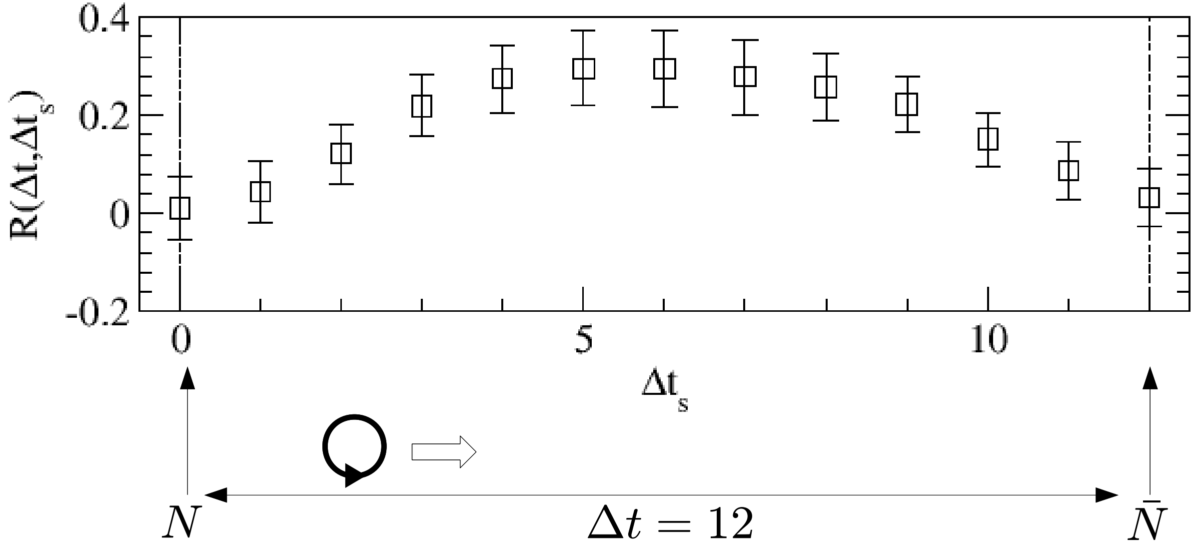
- *ω=20 , N=400*
- *Gauge links in the smearing function is set to unity.*

  (Configuration is fixed to Coulomb gauge)

3. Results at simulation points

- $R(\Delta t, \Delta t_s) = \dfrac{C_{3pt}(t, \Delta t, \Delta t_s)}{C_{2pt}(t, \Delta t)} \to \langle N|\bar{s}s|N\rangle \qquad (\Delta t, \Delta t_s \to \infty)$
- *We omit the noisy high-mode contribution to the quark loop.*

am_{ud} = 0.050, am_s = 0.080



- *Non-zero signal is shown.*

3. Results at simulation points

- $R(\Delta t, \Delta t_s) = \dfrac{C_{3pt}(t, \Delta t, \Delta t_s)}{C_{2pt}(t, \Delta t)} \to \langle N|\bar{s}s|N\rangle \qquad (\Delta t, \Delta t_s \to \infty)$
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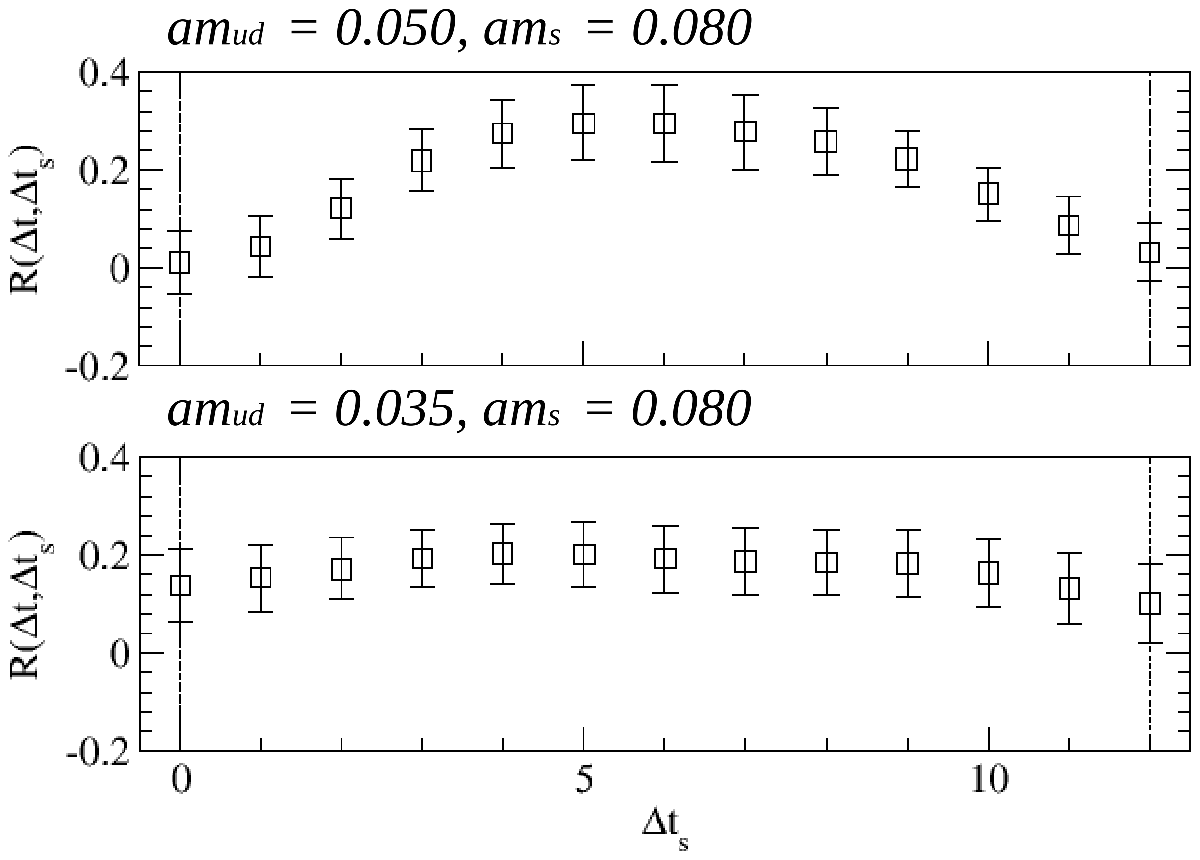
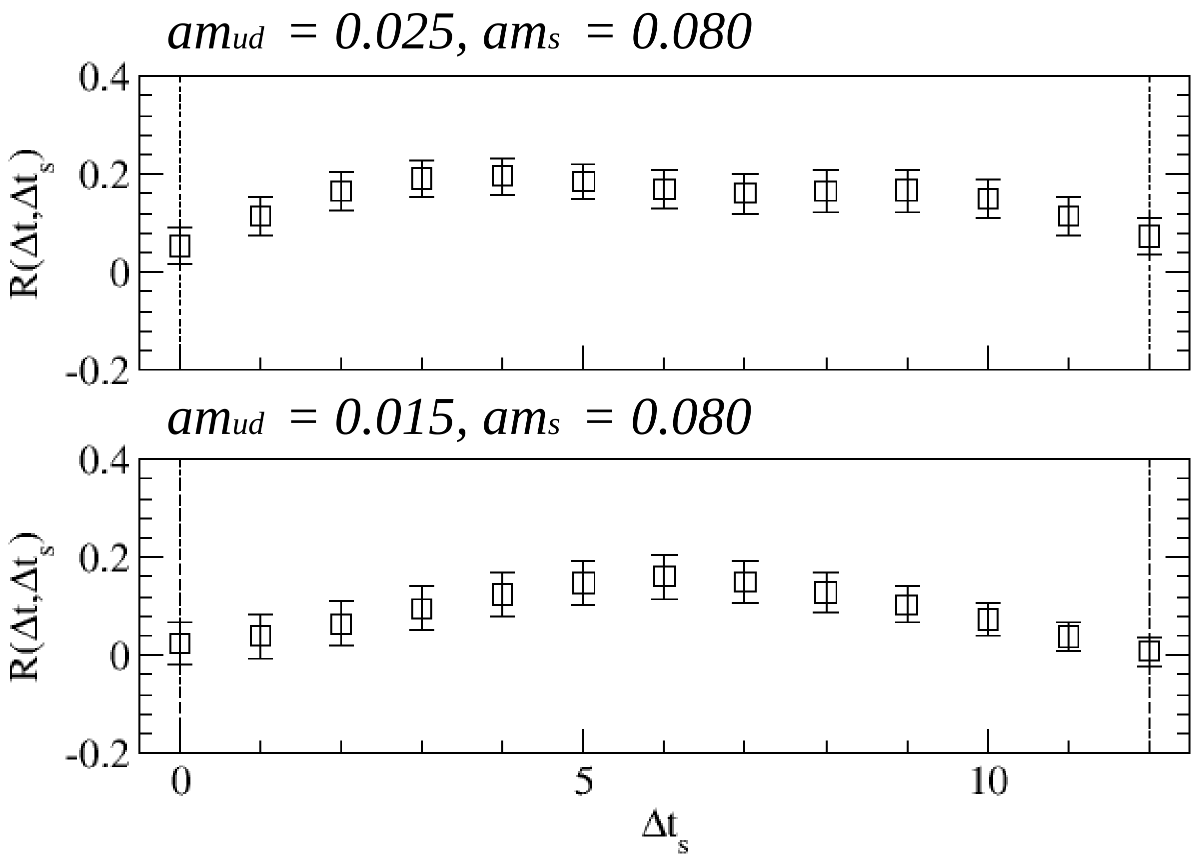


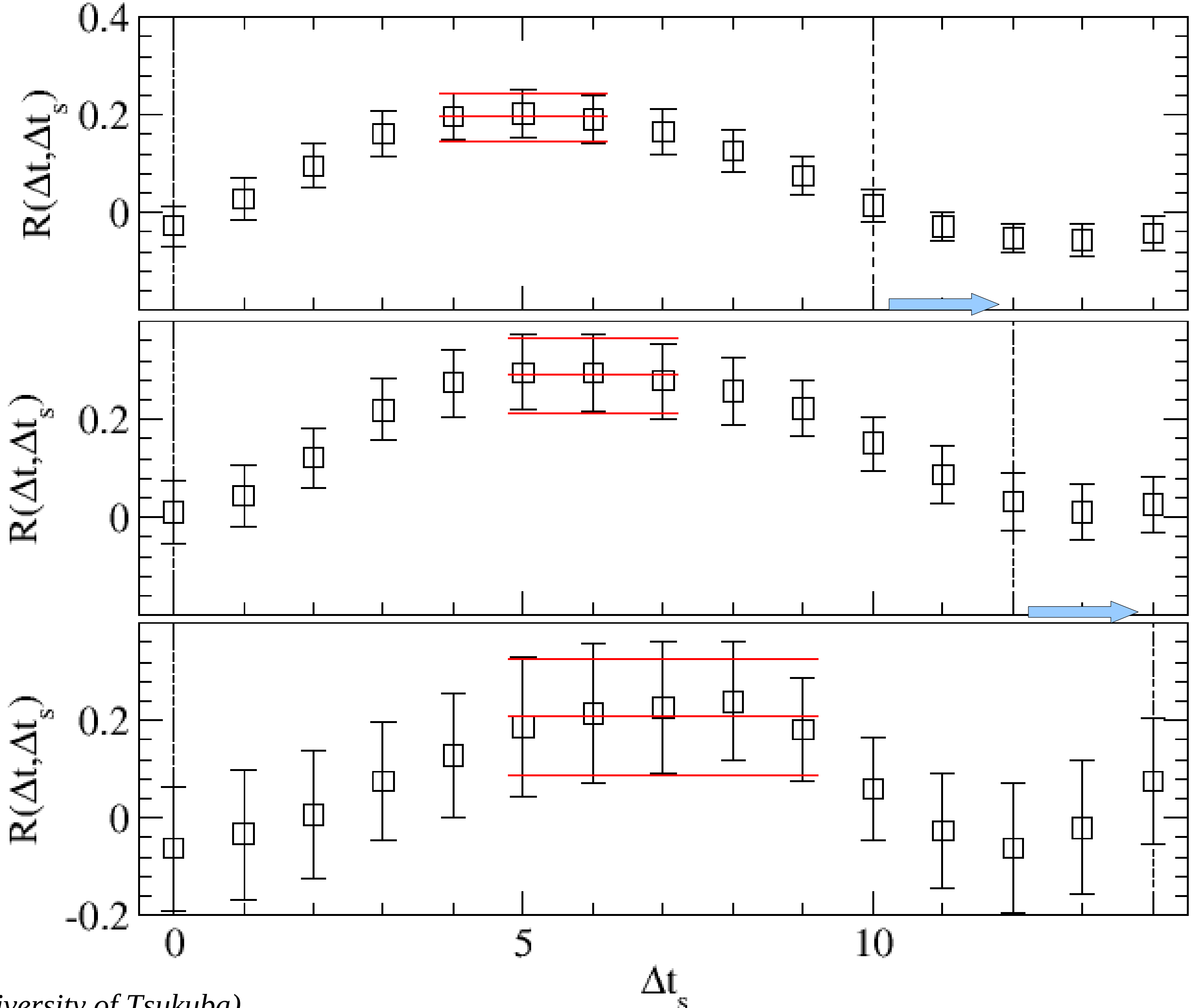
3. Results at simulation points

- *Lighter ud quark masses on large lattice ($24^3 \times 48$, $L \sim 2.6$ fm)*



3. Results at simulation points $R(\Delta t, \Delta t_s) \to R(\Delta t)$

$am_{ud} = 0.050$, $am_s = 0.080$



$\Delta t = 10$

$\Delta t = 12$

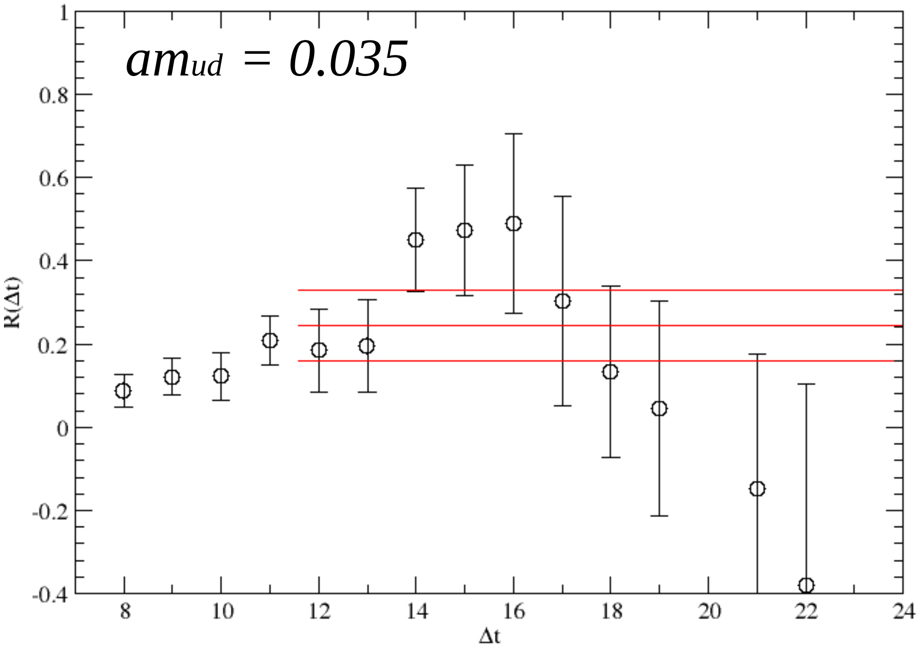
$\Delta t = 14$

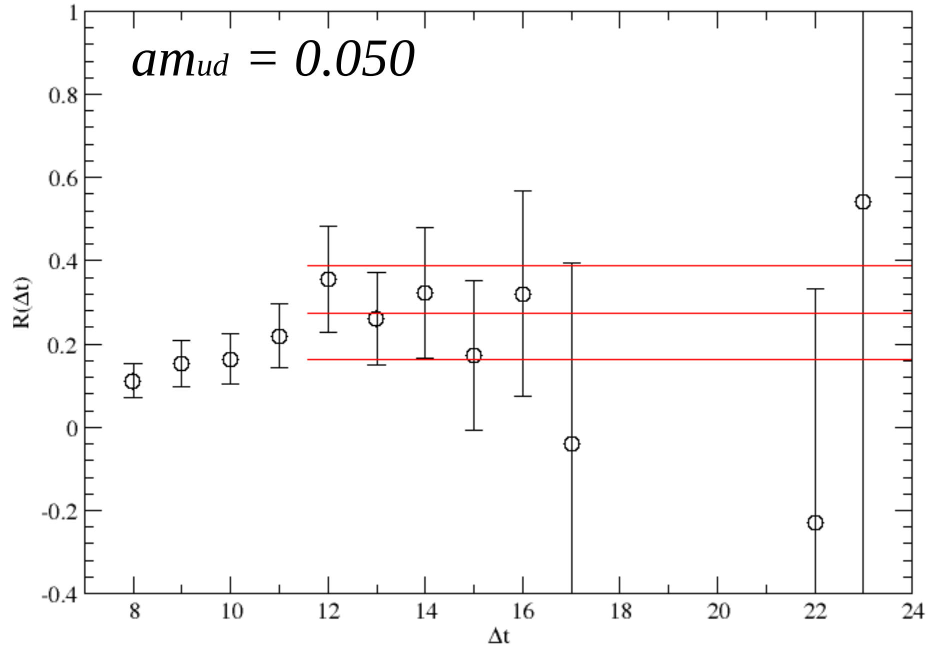
$R(\Delta t)$

3. *Results at simulation points*

- *For two heavier quark masses, we include high-mode contribution of quark loop.*
- *Fit range ; Δt=[12,23]*

am_s = 0.080





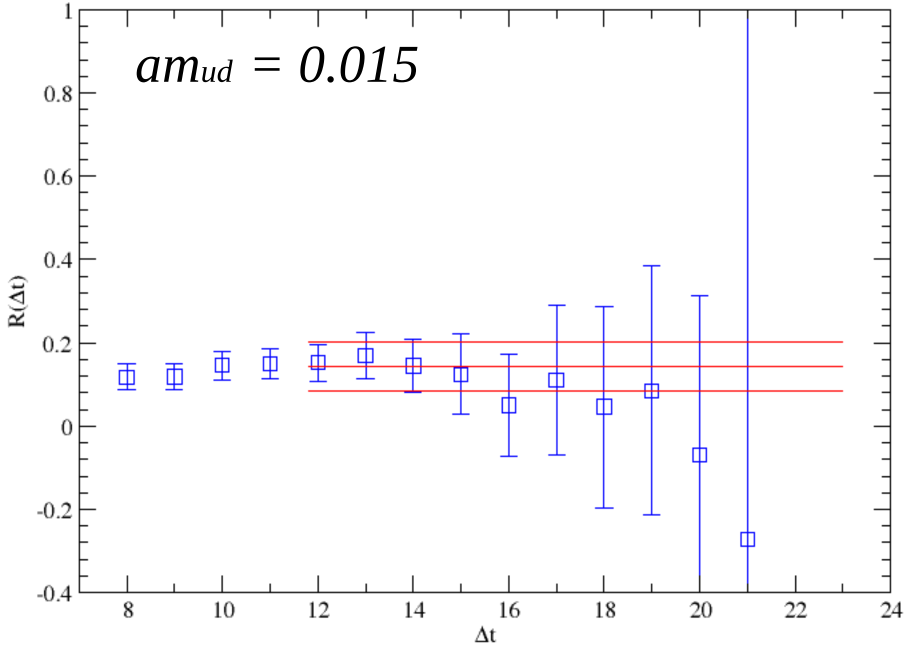
3. Results at simulation points

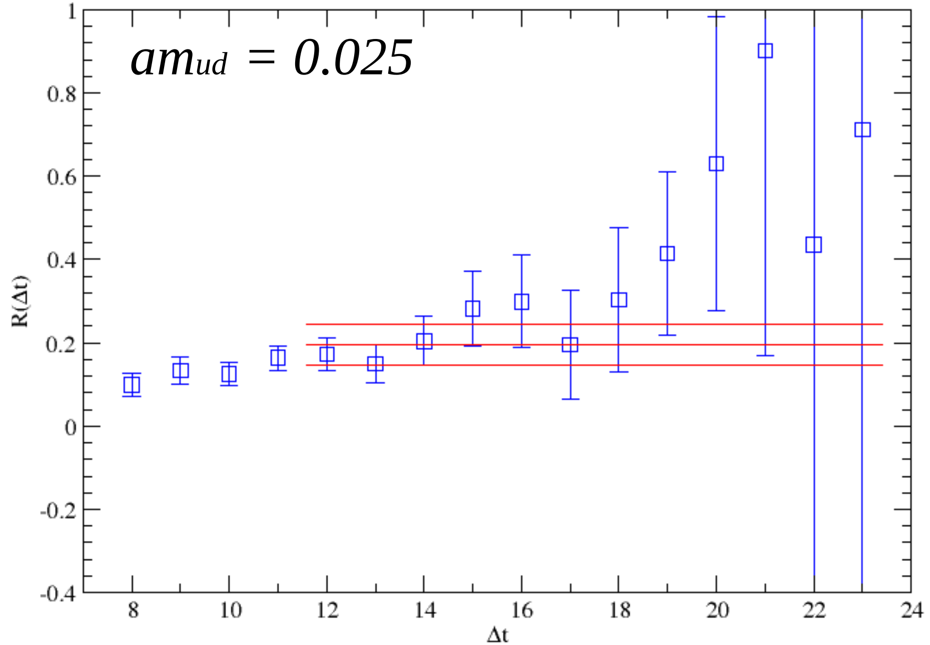
- *For two lighter quark masses,*

  calculation of the high-mode contribution is on-going.

- *Use only low-mode contribution*

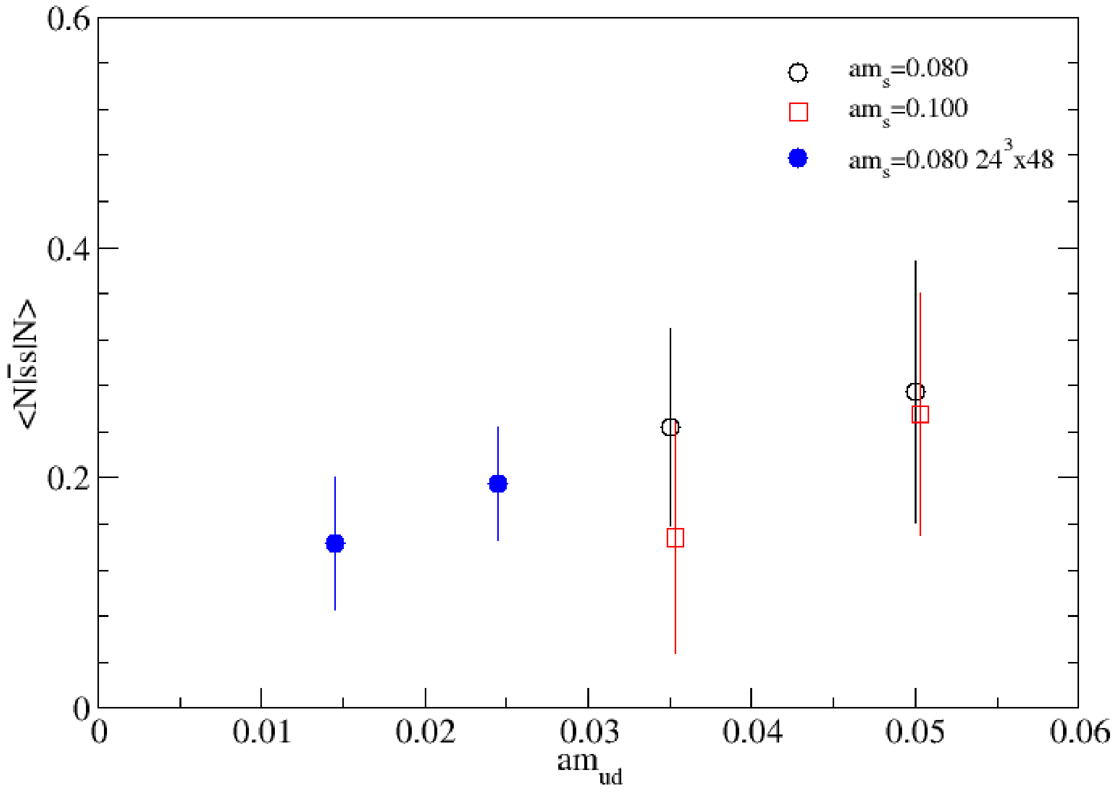
$am_s = 0.080$





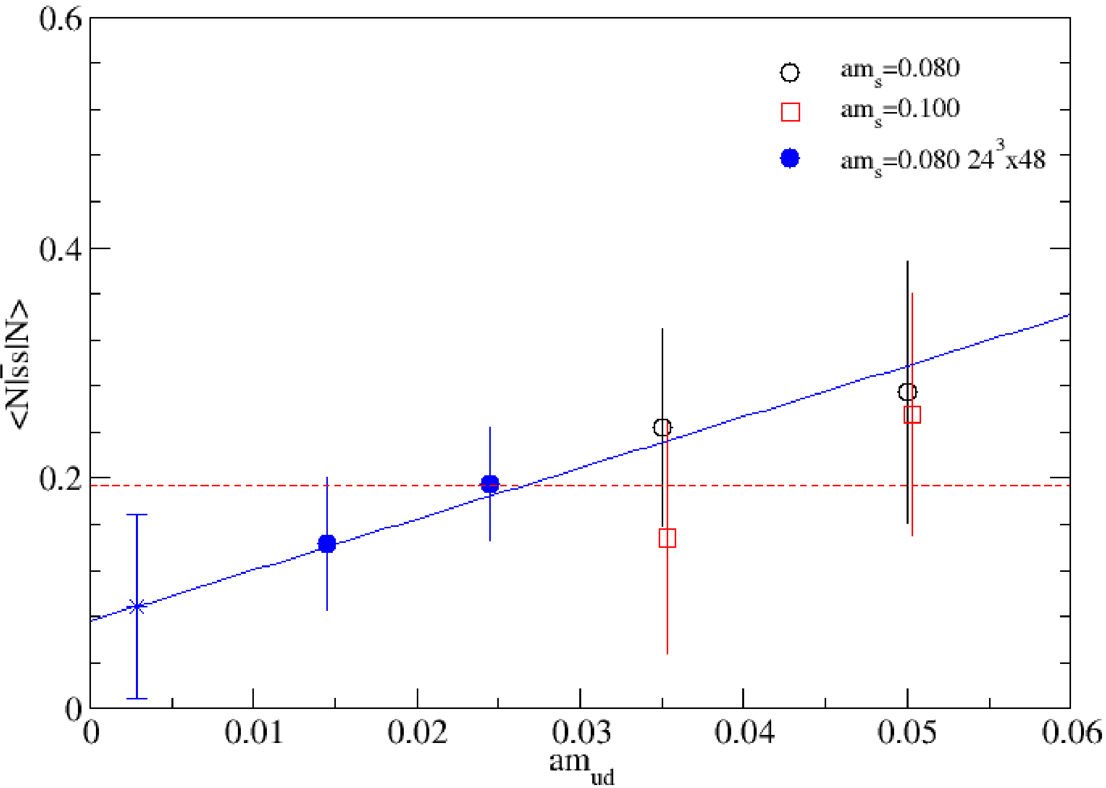
4. Chiral extrapolation

- *We observe weak ud quark mass dependence.*



4. Chiral extrapolation (Cont'd)

- *Linear extrapolation to the physical point.*



- *Result is;*

$\langle N|\bar{s}s|N\rangle^{lat.}$ = *0.085(80)(104)*

difference with constant fit

- *Convert to phenomenological parameter*

$$f_{T_s} = \frac{m_s \langle N|\bar{s}s|N\rangle}{m_N}$$

= *0.013(12)(15)*

5. Renormalization

- *We consider the renormalization of* $\bar{s}s$.

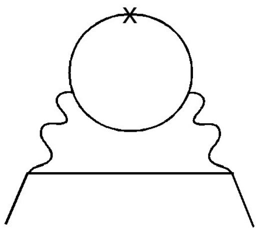
$$(\bar{s}s)^{phys} = \frac{1}{3}\Big[\underbrace{(\bar{\psi}\psi)^{phys}}_{\text{flavor singlet}} - \sqrt{3}\,\underbrace{(\bar{\psi}\lambda^8\psi)^{phys}}_{\text{octet}}\Big]$$

$$= \frac{1}{3}\Big[(Z_0 + 2Z_8)\,(\bar{s}s)^{lat.} + (Z_0 - Z_8)\,(\bar{u}u + \bar{d}d)^{lat.}\Big]$$

$$(\bar{\psi}\psi)^{phys} = Z_0(\bar{\psi}\psi)^{lat.}$$
$$(\bar{\psi}\lambda^8\psi)^{phys} = Z_8(\bar{\psi}\lambda^8\psi)^{lat.}$$

- *Mixing of strange and ud quark operator.*
- $(Z_0 - Z_8)$ *comes from disconnected diagram*
  - *In the chiral limit, these diagrams vanish* *only if there are no additive mass shift.*
  - *Axial Ward identity also states* $(Z_0 - Z_8) = 0$

ex.)



5. Renormalization

- *In addition,*
  - *Additive renormalization appears*

$$(\bar{\psi}\psi)^{phys} = Z_0(\bar{\psi}\psi)^{lat.} + \frac{c_0}{a^3} + ac_1 \mathrm{Tr}(G_{\mu\nu}G_{\mu\nu})^{lat.}$$

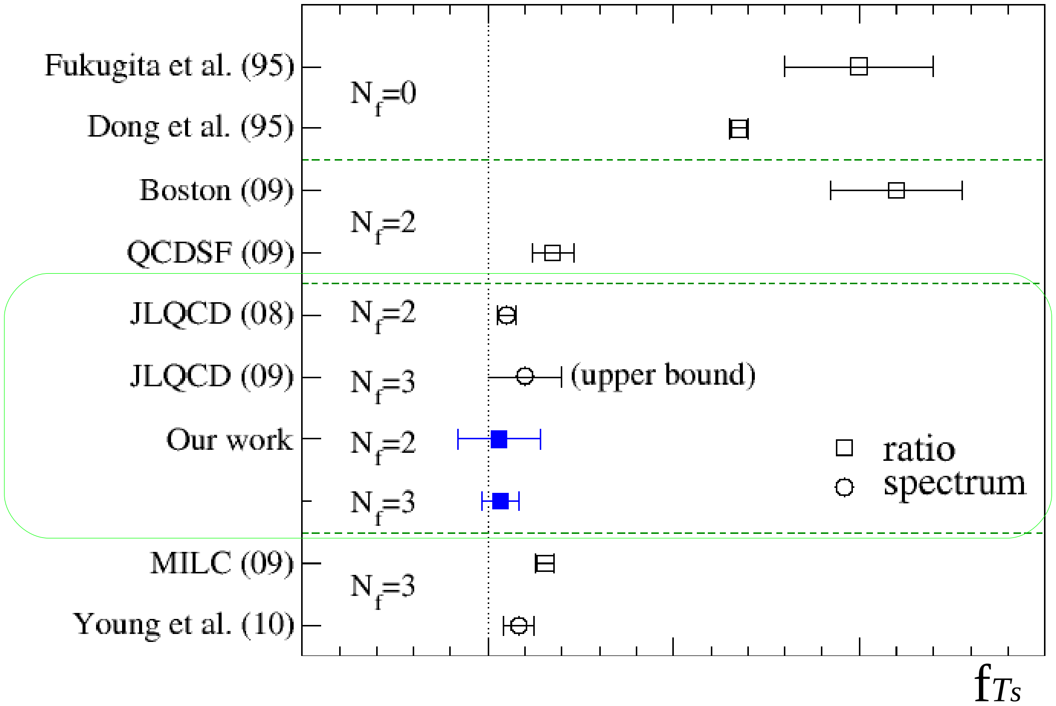
  - *Finally,*

$$(\bar{s}s)^{phys} = \frac{1}{3}\left[(Z_0 + 2Z_8)(\bar{s}s)^{lat.} + \frac{c_0}{a^3} + ac_1 \mathrm{Tr}(G_{\mu\nu}G_{\mu\nu})^{lat.} \right. \\ \left. + (Z_0 - Z_8)(\bar{u}u + \bar{d}d)^{lat.}\right]$$

- *If chiral symmetry is not exact, one must subtract extra terms.*
- *Chiral symmetry simplifies the renormalization.*

5. Renormalization

- *Compare* $f_{T_s} = \frac{m_s\langle N|\bar{s}s|N\rangle}{m_N}$



cf. spectrum method

$$\langle N|\bar{s}s|N\rangle = \frac{\partial m_N}{\partial m_s}$$

- *Consistent with result from spectrum method.*
- *Smaller than the results of previous works.*

6. Summary

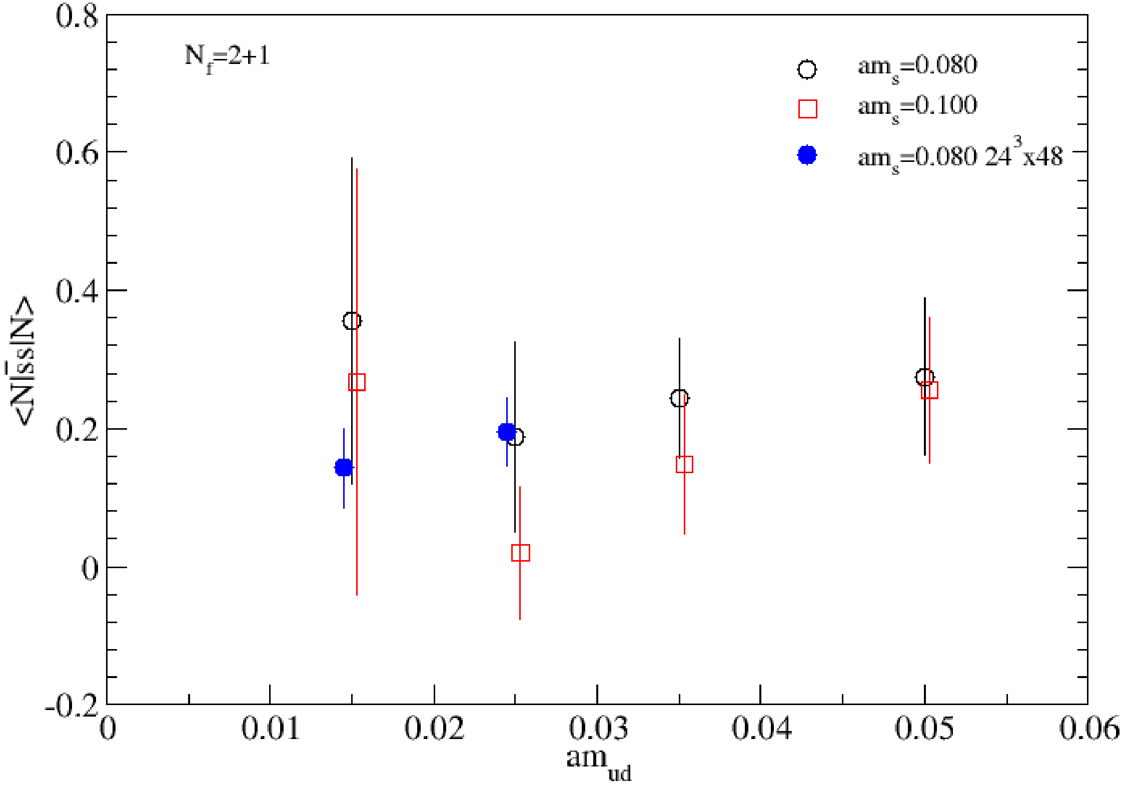
- *We calculate nucleon strange quark content with exact chiral symmetry.*
    - *LMA for nucleon propagator*
- f_{Ts}*=0.013(12)(15)*
    - *Consistent with the spectrum method.*
- *Exact chiral symmetry is crucial to simplify the renormalization procedure.*

- *Future direction ;*

  Other matrix element *e.g) strange spin fraction of the nucleon*

$$\langle N|\bar{s}\gamma_i\gamma_5 s|N\rangle$$

- *Finite size effect is not obvious.*

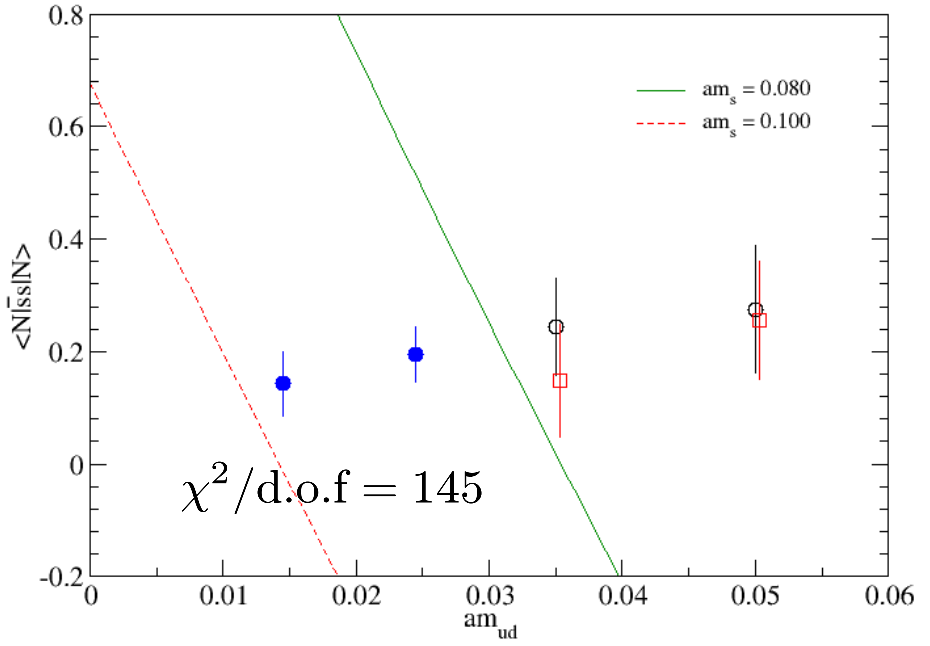


- *NLO HBChPT fit* $D = 0.81, F = 0.47$

$$\langle N|\bar{s}s|N\rangle = -c_s - B\left\{\frac{3}{2}C_{NNK}\,M_K + 2C_{NN\eta}\,M_\eta\right\}$$

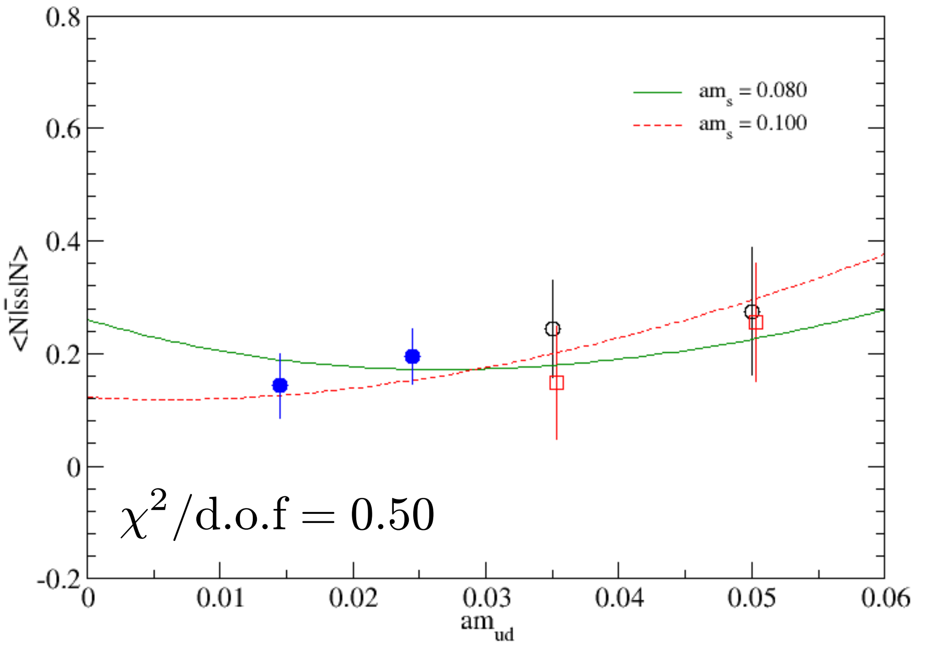
$$C_{NNK} = \frac{1}{8\pi f^2}\frac{(5D^2 - 6DF + 9F^2)}{3},$$

$$C_{NN\eta} = \frac{1}{8\pi f^2}\frac{(D-3F)^2}{6}$$

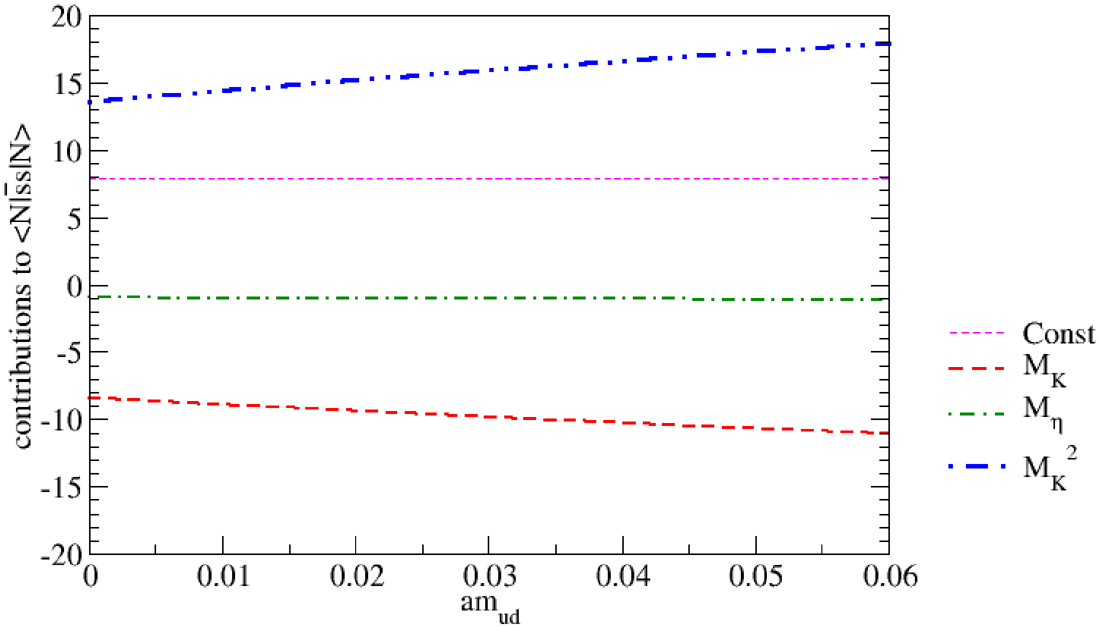


- *NLO+higher term*

$$\langle N|\bar{s}s|N\rangle = -c_s - B\left\{\frac{3}{2}C_{NNK}\,M_K + 2C_{NN\eta}\,M_\eta\right\} + c_2 M_K^2$$



- *Chiral expansion shows a poor convergence*



$$\langle N|\bar{s}s|N\rangle = -c_s - B\left\{\frac{3}{2}C_{NNK}\,M_K + 2C_{NN\eta}\,M_\eta\right\} + c_2 M_K^2$$