New results on the effective string corrections to the interquark potential.

M. Caselle, M. Zago

Villasimius; June 2010

Plan of the Talk

- Universality of the effective corrections to the interquark potential
- Flux density in presence of two Polyakov loops
- Effective string corrections for the mean flux density.
- Comparison with high precision simulations in the 3d Ising model and estimate of the sixth order correction.

Universality of the effective string corrections

One of the most interesting recent results in effective string theory are the universality theorems for the quartic (Lüscher and Weisz 2004) and sextic (Aharony and Karzbrun 2009) corrections to the interquark potential. A numerical test of universality would be of great importance, but it turns out to be very difficult for two reasons:

- In the standard "zero temperature" interquark potential, higher order corrections are proportional to higher powers in $1/R$ and are thus visible only at very short distance where the effective string picture breaks down and perturbative contributions become important.

- The dominant string correction (the Lüscher term) may shadow the subleading ones

To solve these two problems we propose the following strategy:

1) We study the interquark potential at finite temperature (just below the deconfinement transition). In this regime the string corrections are proportional to R and act as a temperature dependent renormalization of the string tension.

Higher order corrections correspond to higher powers of T and can be observed much better than in the zero temperature limit.

2) In order to eliminate the dominant Lüscher term we shall not measure directly the interquark potential, but shall instead study the changes induced in the flux configuration by the presence of the Polyakov loops.

Flux density in presence of two Polyakov loops.

The flux through a plaquette p in presence of two Polyakov loops P, P' is:

$$\langle \phi(p; P, P') \rangle = \frac{\langle P P'^{\dagger} \, U_p \rangle}{\langle P P'^{\dagger} \rangle} - \langle U_p \rangle$$

The mean flux density is

$$\langle \Phi(R, L) \rangle = \frac{1}{N_p} \sum_p \frac{\langle P P'^{\dagger} \, U_p \rangle}{\langle P P'^{\dagger} \rangle}$$

where we have neglected the disconnected component $\langle U_p \rangle$ since we shall be interested in the following only to terms proportional to R.

If we define the partiton function of the system in presence of the two Polyakov loops as

$$Z(R, L) = \langle P^\dagger(R) P(0) \rangle$$

then the flux $\langle \Phi(R, L) \rangle$ can be written as:

$$\langle \Phi(R, L) \rangle = \frac{1}{N_p} \frac{d}{d\beta} \log Z(R, L) \quad .$$

If we neglect for the moment effective string corrections and keep only the area term in Z, i.e. $Z(L, R) \sim e^{-\sigma RL}$ we find a linearly rising behaviour for $\langle \Phi(R, L) \rangle$:

$$\langle \Phi(R, L) \rangle = \alpha R$$

with an angular coefficient

$$\alpha = \frac{L}{N_p} \frac{d\sigma}{d\beta}$$

Effective string corrections

$$\langle P^\dagger(R)P(0)\rangle \ = \ Z(L,R) \ \sim \ \int \mathcal{D}X e^{-S[X]} \ .$$

If we choose a Nambu-Goto action for the effective string (and set $d = 2{+}1$) then:

$$S[X] = \sigma \int_0^L d\tau \int_0^R d\varsigma \sqrt{1 + (\partial_\tau X)^2 + (\partial_\varsigma X)^2} \ .$$

Perturbative expansion in powers of $1/(\sigma RL)$

$$Z(L,R) = e^{-\sigma RL} \cdot Z_1 \cdot \left(1 + \frac{F_4}{\sigma R^2} + \frac{F_6}{(\sigma R^2)^2} + \cdots\right)$$

The leading order of this expansion: Z_1 corresponds to the partition function of a free boson in two dimensions and thus is scale invariant.

Hence, since $Z(R,L)$ may depend on β only through the string tension σ the leading correction to $\langle \Phi(R,L) \rangle$ (the Lüscher term) vanishes.

A straightforward calculation (Dietz-Filk 1983) gives:

$$F_4 = \frac{\pi^2 L}{1152\sigma R^3}\left[2E_4\left(i\frac{L}{2R}\right) - E_2^2\left(i\frac{L}{2R}\right)\right] \quad ,$$

where E_k are Eisentein functions of order k

We obtain in the large R limit:

$$\langle\Phi(R,L)\rangle \;=\; \alpha\left(\frac{\pi^2}{72\sigma^2 L^4}R + \frac{\pi}{12\sigma^2 L^3} + \frac{1}{8\sigma^2 L^2}\frac{1}{R}\right)$$

which must be added to the "classical" contribution:

$$\langle\Phi(R,L)\rangle = \alpha R$$

All orders calculations in the Nambu-Goto case

In $d = 2+1$ dimensions one finds that $Z(R,L)$ is given by a tower of K_0 Bessel functions (Lüscher-Weisz 2004):

$$Z(R,L) = \langle P(0,0)P(0,R)\rangle = \sum_{n=0}^{\infty} c_n K_0(E_n R).$$

where E_n are the closed string energy levels:

$$E_n = \sigma L \left\{ 1 + \frac{8\pi}{\sigma L^2} \left[-\frac{1}{24}(d-2) + n \right] \right\}^{1/2}.$$

In the large R limit only the lowest state $(n = 0)$ survives

$$\lim_{R\to\infty} \langle P(0,0)P(0,R)\rangle = c_0 K_0(E_0 R).$$

with

$$E_0 = \sigma L \left(1 - \frac{\pi}{3\sigma L^2}\right)^{1/2}.$$

and

$$c_0 = \frac{L}{2}\sqrt{\frac{\sigma}{\pi}}$$

where σ denotes the zero temperature string tension.

From which we find:

$$\langle \Phi(R,L) \rangle = \frac{1}{N_p} \frac{d \log Z(R,L)}{d\beta} = \frac{1}{N_p} \left(\frac{1}{2\sigma} + R \frac{K_0'}{K_0} \frac{dE_0}{d\sigma} \right) \frac{d\sigma}{d\beta}$$

where we used again the fact that $Z(R,L)$ is a function of β only through the string tension σ

Setting

$$x \equiv \frac{\pi}{3\sigma L^2}$$

we find

$$\frac{dE_0}{d\sigma} = \frac{L(1 - x/2)}{\sqrt{1-x}}$$

Recalling that

$$K_0' = -K_1$$

and using the asymptotic expansion of the modified Bessel functions:

$$\frac{K_1(E_0R)}{K_0(E_0R)} = 1 + \frac{1}{2E_0R} - \frac{1}{8(E_0R)^2} + ...$$

Collecting all the terms together we find

$$\langle \Phi(R,L) \rangle \ = \ \alpha \left(RA(x) + B(x) + \frac{C(x)}{R} + ... \right)$$

where we defined:

$$A(x) = \frac{(1-x/2)}{\sqrt{1-x}}$$
$$B(x) = -\frac{1}{\sigma L}\frac{x}{4(1-x)}$$
$$C(x) = \frac{1-x/2}{8(L\sigma)^2(1-x)^{3/2}}$$

in (2+1) dimensions $N_p = 3N_s^2 L$ and hence:

$$\alpha = -\frac{1}{3N_s^2}\frac{d\sigma}{d\beta} \quad .$$

Expanding in $x \equiv \frac{\pi}{3\sigma L^2}$ we find

$$A(x) = \left(1 + \frac{x^2}{8} + \frac{x^3}{8} + \ldots\right)$$

$$B(x) = \frac{1}{\sigma L}\left(\frac{x}{4} + \frac{x^2}{4} + \ldots\right)$$

$$C(x) = -\frac{1}{8(\sigma L)^2}\left(1 + \frac{3}{2}x + \ldots\right)$$

which at the first order coincide with the corrections obtained with the Dietz-Filk approach. In addition we find the next to leading corrections of order $1/L^6$.

Comparison with the numerical data

To test these results we performed a set of Montecarlo simulations in the 3d gauge Ising model for various values of R and L.

We used duality to map the Polyakov loops correlator into the partition function of a 3d Ising spin model in which we changed the sign of the coupling of all the links dual to the surface bordered by the two Polyakov loops.

We then estimated $\langle \Phi(R, L) \rangle$ by simply evaluating the mean energy in presence of these frustrated links.

The results are in remarkable agreement with the string calculation at the quartic order but disagree with the sextic order correction.

For this model the scaling function $\sigma(\beta)$ is known with high precision

$$\sigma(\beta) = \sigma_c t^{2\nu} \times (1 + a t^{\theta} + b t) \; .$$

with: $a = -0.479(26)$ $b = -2.12(19)$ $\theta = 0.5241(33)$ $\sigma_c = 10.083(8)$ $\nu = 0.63002(10)$

t is the reduced temperature of the dual spin model: $t = \beta_s - \beta_{c,s}$ where the duality relation is

$$\beta_s = -\frac{1}{2} \log th\beta$$

and the critical coupling for the spin model is: $\beta_{c,s} = 0.22165455(5)$

Simulation setting

We simulated the model at $\beta = 0.75180$ for which $L_c = 1/T_c = 8$, choosing $N_s = 128$ which implies $\alpha = 2.792 \ \ 10^{-5}$.

We chose two sets of values for L

- Low T set ($\frac{T_c}{3} < T < \frac{T_c}{2}$): $L = 16, 20, 24$. For these values the sextic and higher order terms are negligible within the errors
- High T set ($\frac{2T_c}{3} < T < T_c$): $L = 10, 11, 12$. For these values the sextic term is larger than the statistical uncertainties.

Results:

- Low T set:

  for each L we fit the R dependence of the data with

  $$\Phi(R, L) = a(L)R + b(L)$$

  (the $c(L)/R$ was always negligible within the errors) and always found very good χ^2 Then we fitted the values of $a(L)$ with:

  $$a(L) = \alpha(1 + \gamma x^2)$$

  we found $\alpha = 2.7918(17)\ 10^{-5}$ and $\gamma = 0.132(7)$ with a very good χ^2. Both these values nicely agree with the predictions $\alpha = 2.792\ \ 10^{-5}$ and $\gamma = 1/8$.

- High T set: In this case we must include also the $1/R$ term in the fit:

$$\Phi(R, L) = a(L)R + b(L) + c(L)/R$$

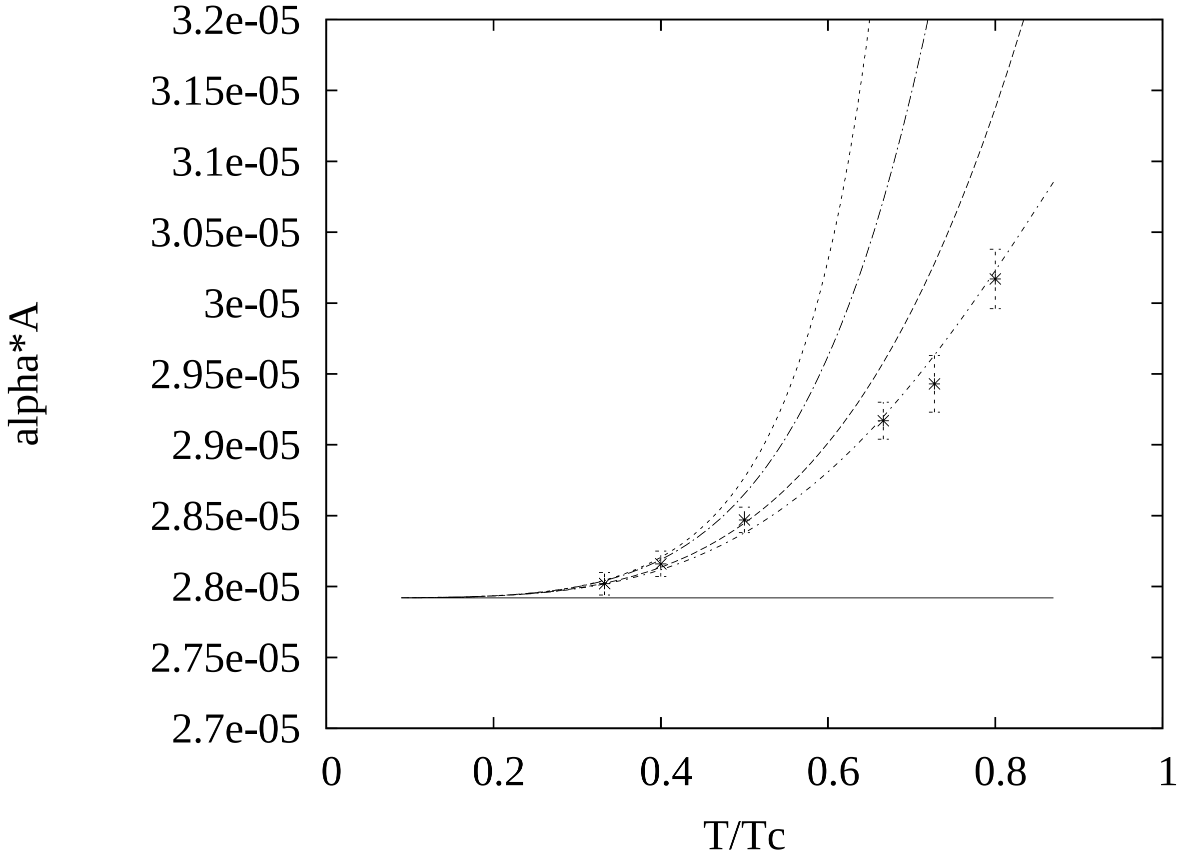
Then we fitted the values of $a(L)$ (including also those at low T) with:

$$a(L) = \alpha(1 + \gamma x^2 + \delta x^3)$$

and find

$$\alpha = 2.796(5)\ 10^{-5}, \quad \gamma = 0.127(25), \quad \delta = -0.051(27)$$

The first two values agree again very well with the predictions but the coefficient of the sextic correction, which should be $\delta = 1/8$ completely disagrees.



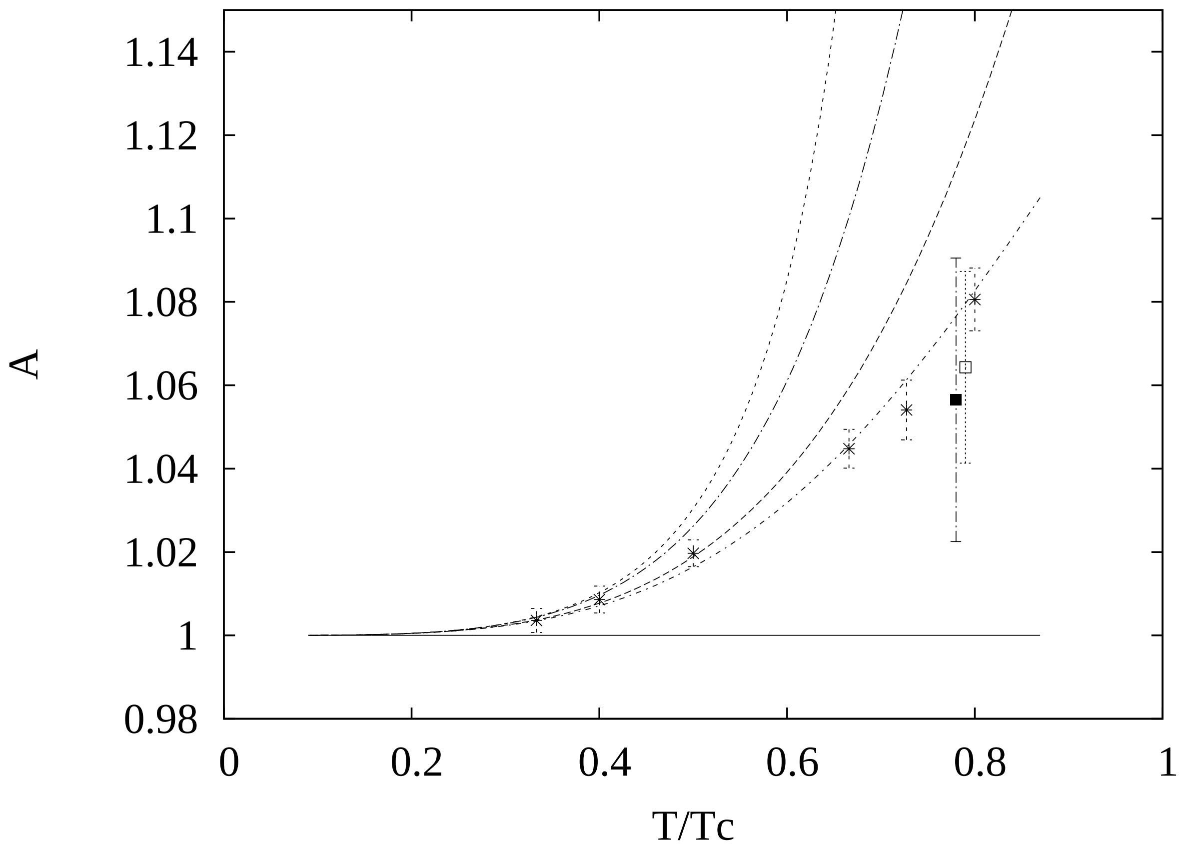
3.2e-05
3.15e-05
3.1e-05
3.05e-05
3e-05
2.95e-05
2.9e-05
2.85e-05
2.8e-05
2.75e-05
2.7e-05
alpha*A
0
0.2
0.4
0.6
0.8
1
T/Tc

Conclusions

- We confirm the universality of the subleading effective string corrections at the quartic order

- We found deviations at the sextic order with respect to the Nambu-Goto predictions thus suggesting a failure of the universality proof by Aharony and Karzbrun at this order. We see three possible reasons for this failure
  - It could be due to the presence of an irrelevant operator in the Ising gauge model with a very large overlap with our observable
  - It could be due to the failure of the weak coupling assumption for the effective string model
  - It could be due to the anomaly which affects the physical gauge in $d \neq 26$

- Duality plays a crucial role in the simulation and for this reason our approach is particularly suited for abelian gauge theories, but in principle, given enough computational power, there is no obstruction to apply it also to non-abelian models.



1.14
1.12
1.1
1.08
1.06
1.04
1.02
1
0.98
A
0
0.2
0.4
0.6
0.8
1
T/Tc