Lattice QCD analysis for the instantaneous interquark potential in the generalized Landau gauge

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Quantum Chromodynamics and Gauge

Quantum Chromodynamics — $\mathrm{SU}(N_c)$ gauge theory

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2}\mathrm{Tr}\ G_{\mu\nu}G^{\mu\nu} + \bar{q}\left(i\gamma_\mu D^\mu - m\right)q$$

For actual calculations ⇒ fix the gauge

For example

- Landau gauge
- Coulomb gauge
- Maximally Abelian gauge
- ...

physical phenomena ⇒ gauge invariant

- confinement
- chiral symmetry breaking
- hadron mass
- ...

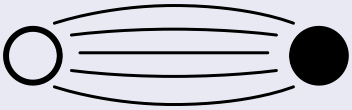
But physical pictures depend on gauges.

Confinement in the Various Gauge

Confinement in the Maximally Abelian gauge

Dual-superconductor picture — Nambu, 't Hooft, Mandelstam

- QCD vacuum ⇒ Dual superconductor ⇒ Dual Meissner effect



squeezing of the color flux tube

⇓

linear potential between quarks

Confinement in the Landau gauge

Kugo-Ojima criterion — T.Kugo, and I.Ojima, Suppl.Prog.Theor. Phys. **66**, 1-130(1979).

If ghost propagator $D_G(p^2)$ becomes more singular than free propagator

$$\lim_{p^2 \to 0} p^2 D_G(p^2) = \infty$$

then

- BRS singlet ⇒ color singlet

Confinement in the Coulomb Gauge — Gribov-Zwanziger's Scenario

— V.Gribov, Nucl, Phys. **B139**, 1(1978); D.Zwanzgier, Nucl.Phys. **B518**, 237(1998).

QCD Hamiltonian in the Coulomb gauge

$$H = \frac{1}{2}\int d^3x \left(\vec{E}^{a,tr}\cdot\vec{E}^{a,tr} + \vec{B}^a\cdot\vec{B}^a\right) + \underline{\frac{1}{2}\int d^3x d^3y \rho^a(x) K^{ab}(x,y)\rho^b(y)}$$

$K \equiv [M^{-1}(-\nabla^2)M^{-1}]^{ab}_{x,y}$: instantaneous Coulomb propagator
M : Faddeev-Popov operator
ρ : color charge density

Coulomb energy part — $\rho K \rho$

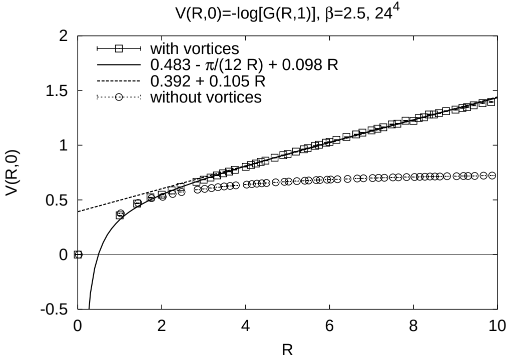
- Gribov horizon $\Leftarrow$ zero eigenvalues of M
- near Gribov horizon $\Rightarrow$ strongly enhancement of the Coulomb energy
- $\Rightarrow$ Confinement force

Instantaneous Potential in the Coulomb Gauge

numerical result of the Coulomb energy

Instantaneous potential

$$V(R,0) = -\log[\mathrm{Tr}\langle U_4(0,0)U_4^\dagger(R,0)\rangle]$$



J.Greensite, S.Olejnik, PRD.67, 094503(2003).

- linearly rising potential
- 2-3 times larger than physical string tension
- Zwanziger's inequality

  D.Zwanziger, PRL.90, 102001(2003).

  $$V_{\mathrm{phys}}(R) \le V_{\mathrm{Coul}}(R)$$

Gauge and Properties of Gluons

The role of gluons is changed according to gauges

- Landau gauge
  - $A_\mu^a \Rightarrow$ equal roles
- Coulomb gauge
  - spatial component $\mathbf{A} = (A_1, A_2, A_3) \Rightarrow$ dynamical variable
  - temporal component $A_0 \Rightarrow$ potential
- Maximally Abelian gauge
  - diagonal part $A_\mu^3, A_\mu^8 \Rightarrow$ dominant
  - off-diagonal part $\Rightarrow$ inactive — K.Amemiya, and H.Suganuma, PRD.60, 114509 (1999)

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In this talk, we discuss the gluon properties
from Landau gauge toward Coulomb gauge.

Landau Gauge and Coulomb Gauge

Landau gauge

$$\partial_\mu A_\mu = 0$$

global form (in the Euclidean metric)

$$\text{minimize} \quad R \equiv \int d^4x \mathrm{Tr}\left\{A_\mu(x) A_\mu(x)\right\}$$

- Lorentz covariance is kept
- gauge field fluctuation $\Rightarrow$ suppressed

Coulomb gauge

$$\nabla \cdot \mathbf{A} = 0$$

- Lorentz covariance is broken

  $A_\mu \to A_0 \quad$ and $\quad \mathbf{A}$
- compatible with canonical quantization

Landau Gauge and Coulomb Gauge

Similarity of gauge fixing condition

- Landau gauge

$$\partial_1 A_1 + \partial_2 A_2 + \partial_3 A_3 + \partial_4 A_4 = 0$$

- Coulomb gauge

$$\partial_1 A_1 + \partial_2 A_2 + \partial_3 A_3 = 0$$

Generalization of the Landau Gauge

generalized Landau gauge (λ-gauge)

— cf. C.Bernard, *et al.*, Nucl.Phys.B (Proc.Suppl.) 17 (1990) 593; 20 (1991) 410.

$$\partial_i A_i + \lambda \partial_4 A_4 = 0$$

- $\lambda = 0 \Rightarrow$ Coulomb gauge
- $\lambda = 1 \Rightarrow$ Landau gauge

- by varying λ-parameter : $\lambda = 1 \longrightarrow 0$
- we can analyze continuous change of gluon properties
- from Landau gauge $\Rightarrow$ Coulomb gauge

on lattice QCD

maximize

$$R_\lambda[U] \equiv \sum_x \sum_{i=1}^{3} \mathrm{Re}\ \mathrm{Tr}\ U_i(x) + \lambda \sum_x \mathrm{Re}\ \mathrm{Tr}\ U_4(x)$$

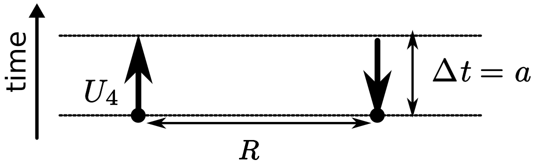
using a gauge transformation of link-variables

“Instantaneous” Interquark Potential

Definition of the “instantaneous” potential

$$V(R) \equiv -\frac{1}{a} \log \mathrm{Tr}\langle U_4(\vec{x},a) U_4^\dagger(\vec{y},a)\rangle, \qquad R = |\vec{x}-\vec{y}|$$

a:lattice spacing



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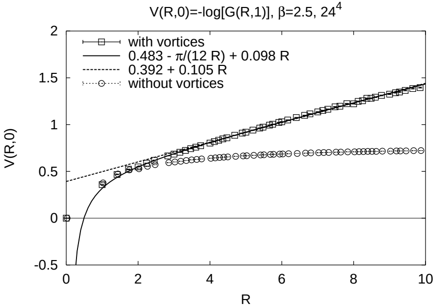

$\Leftarrow$ Coulomb gauge result

J.Greensite, S.Olejnik, PRD.67, 094503(2003).

Coulomb + linear potential
(2-3 times larger than physical string tension)

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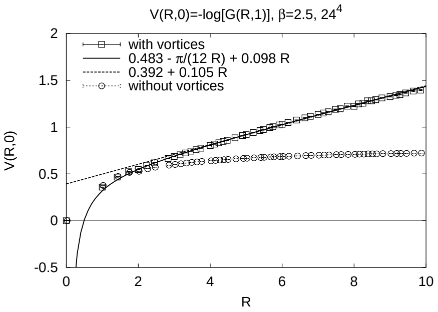

⇐ Coulomb gauge result

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⇒ Analyze gauge dependence of $V(R)$ in the generalized Landau gauge.

Lattice QCD Calculation

lattice QCD result ($\beta = 5.8$, $a = 0.152$ fm, 16^4 lattice)

"instantaneous" potential

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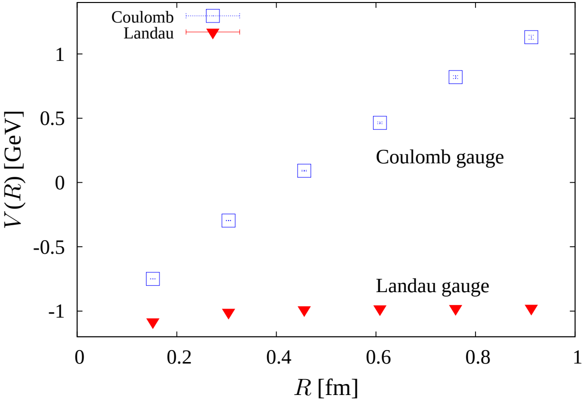

- Coulomb gauge ($\lambda = 0$)
  Coulomb + Linear pot.
  $\sigma \sim 2.3$ [GeV/fm]
- Landau gauge ($\lambda = 1$)
  No linear part
  $\sigma \sim 0$ [GeV/fm]

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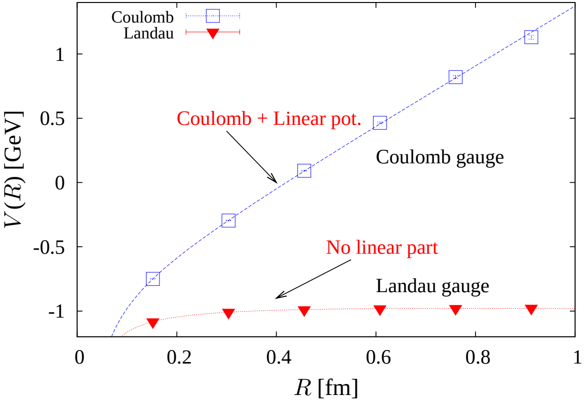

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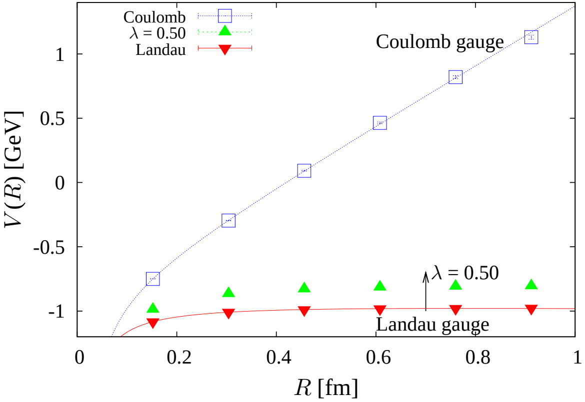

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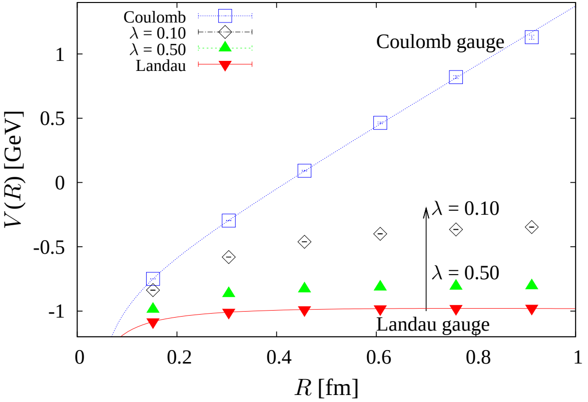

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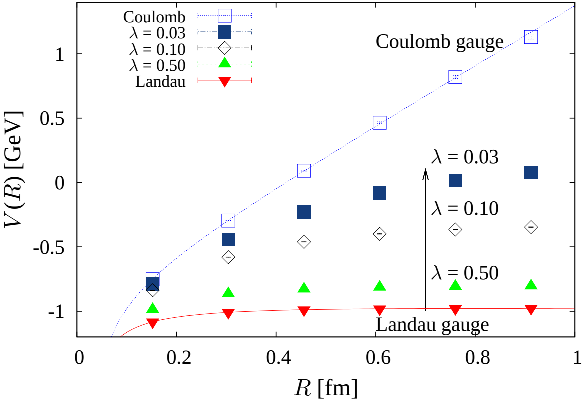

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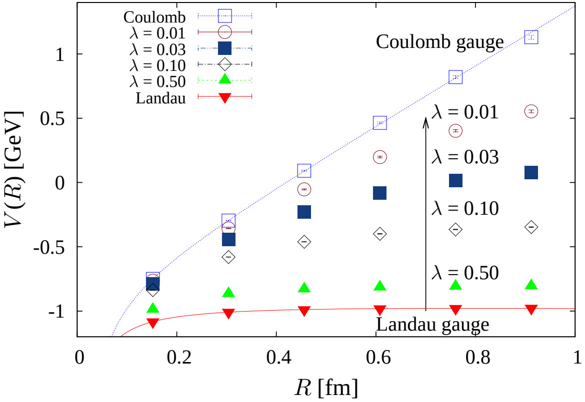

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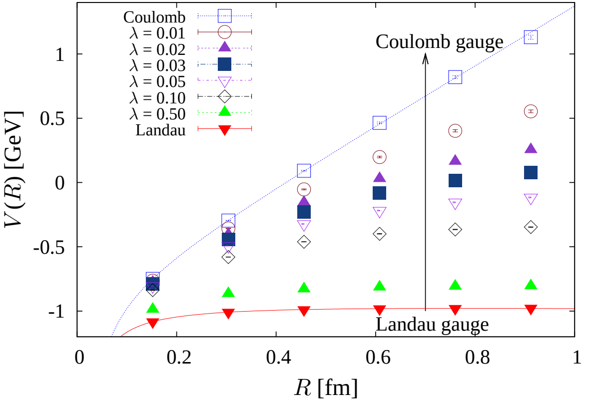

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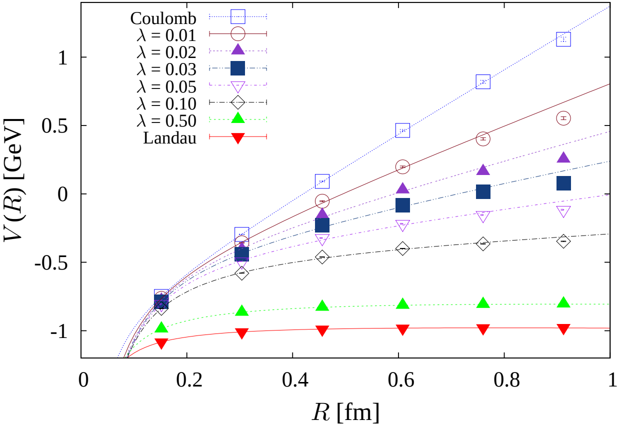
- Landau gauge ($\lambda = 1$)
  No linear part
  $\sigma \sim 0$ [GeV/fm]
- large gap between Landau and Coulomb
- $\lambda = 1 \to 0$
  potential is changing continuously

Instantaneous Potential Analysis

Coulomb plus linear functional form

$$V(R) = -\frac{A}{R} + \sigma R + \text{Const.}$$

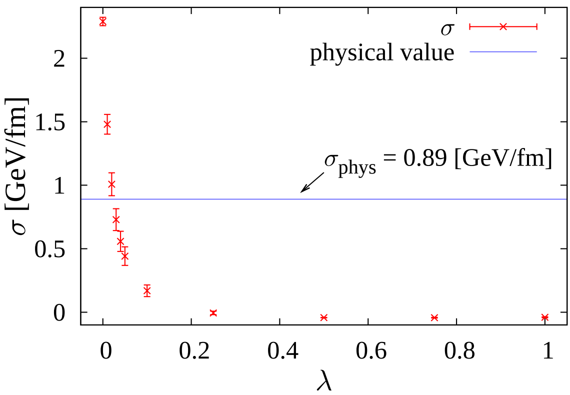


- Landau and Coulomb gauge is connected continuously
- Coulomb + linear pot. works well for $R < 0.8$ fm

String Tension in the Generalized Landau Gauge

String tension of the instantaneous potential

- Coulomb gauge ($\lambda = 0$) $\Rightarrow \sigma \sim 2.5\sigma_{\text{phys}}$ over confinement
- Landau gauge ($\lambda = 1$) $\Rightarrow \sigma \sim 0$ no linear part



- drastically changing near the Coulomb gauge ($\lambda \sim 0.1 \to 0$)

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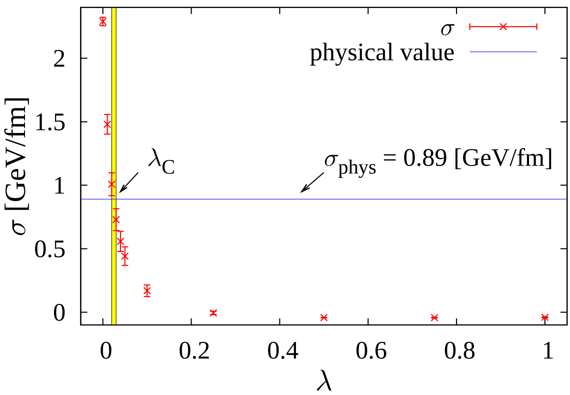

- drastically changing near the Coulomb gauge ($\lambda \sim 0.1 \to 0$)
- $\lambda_C \sim 0.02$
  $\sigma(\lambda) \simeq \sigma_{\rm phys}$

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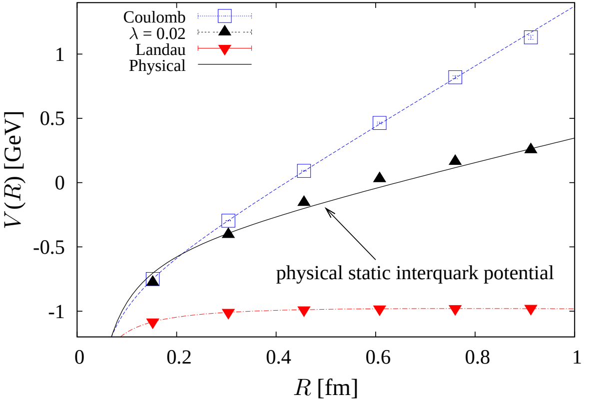
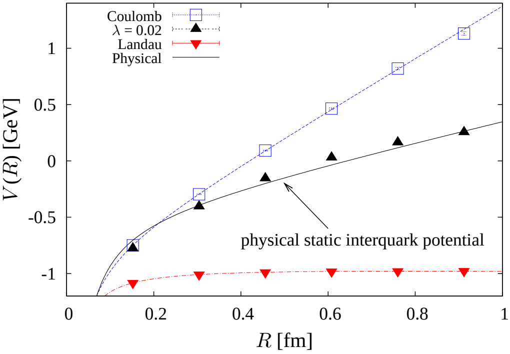


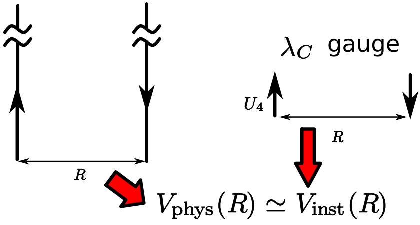
- drastically changing near the Coulomb gauge ($\lambda \sim 0.1 \to 0$)
- $\lambda_C \sim 0.02$
  $\sigma(\lambda) \simeq \sigma_{\text{phys}}$
- in λ_C gauge instantaneous potential reproduces the physical interquark potential

λ_C gauge

in λ_C gauge

instantaneous potential reproduces the physical interquark potential



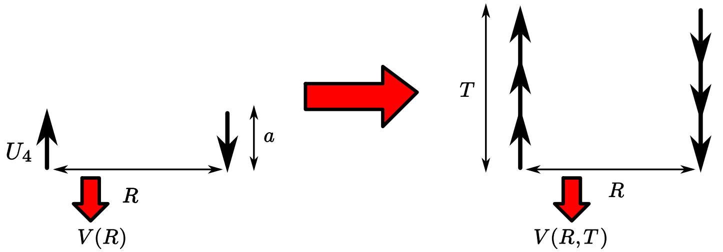


useful for modeling QCD ?

T-length Potential

Instantaneous potential

$$V(R) \equiv -\frac{1}{a} \log \mathrm{Tr} \langle U_4(\vec{x}, a) U_4^\dagger(\vec{y}, a) \rangle, \quad R = |\vec{x} - \vec{y}|$$



instantaneous potential $\qquad$ T-length potential

T-length potential

$$V(R,T) \equiv -\frac{1}{T} \log \mathrm{Tr} \langle L(\vec{x}, T) L^\dagger(\vec{y}, T) \rangle, \quad R = |\vec{x} - \vec{y}|$$

Polyakov line: $L(\vec{x}, T) \equiv U_4(\vec{x}, a) U_4(\vec{x}, 2a) \cdots U_4(\vec{x}, T)$

T-length Potential in the Coulomb Gauge

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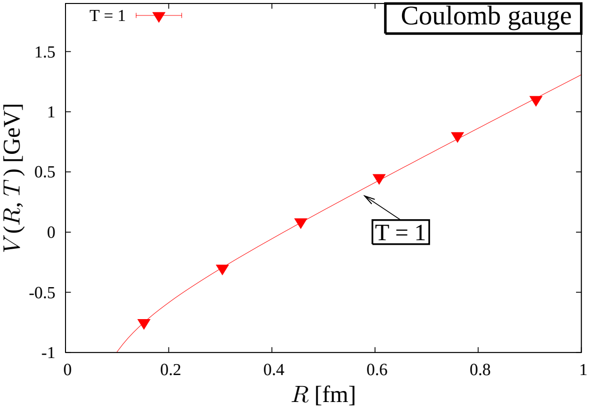

- Coulomb + linear

for SU(2) Coulomb gauge J.Greensite, S.Olejnik, PRD.67, 094503(2003).

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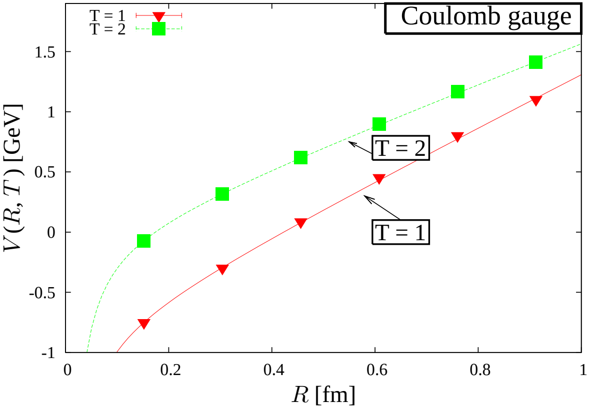

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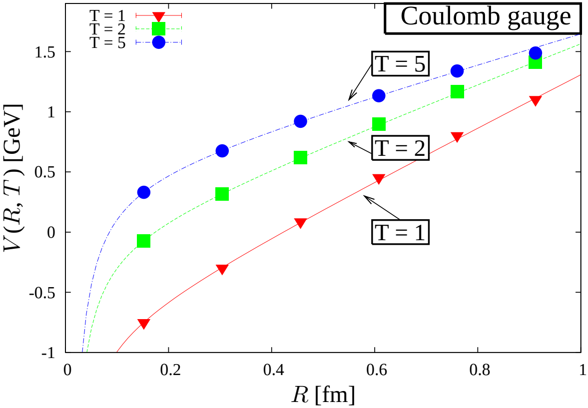

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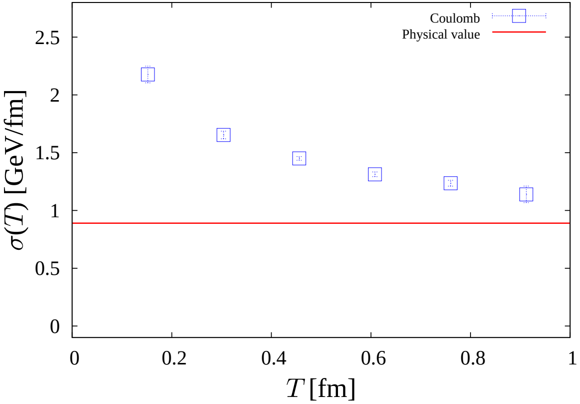


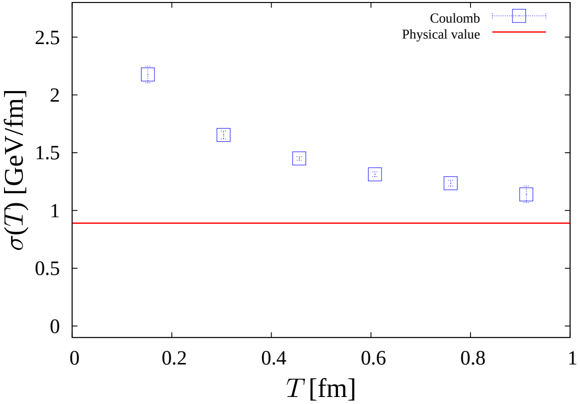
- Coulomb + linear
- string tension is changed

for SU(2) Coulomb gauge J.Greensite, S.Olejnik, PRD.67, 094503(2003).

T-length String Tension in the Generalized Landau Gauge

T-length potential analysis

$$V_\lambda(R,T) = -\frac{A_\lambda(T)}{R} + \sigma_\lambda(T)R + \text{Const.}$$



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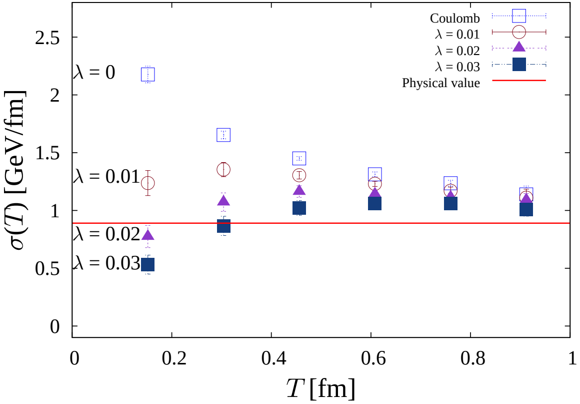

- Near the Coulomb gauge $(\lambda \sim 0)$ $\sigma_\lambda(T) \to \sigma_{\text{phys}}$ $T \sim 1$ [fm]

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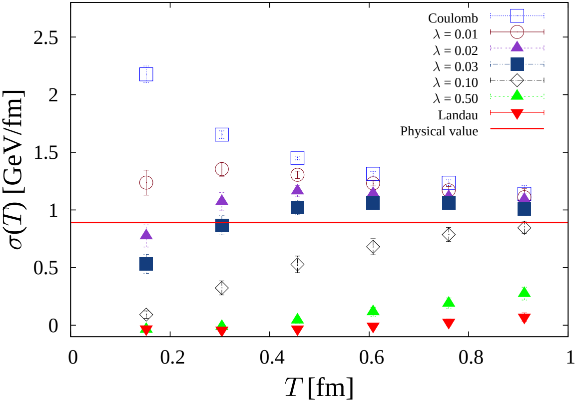

- Near the Coulomb gauge $(\lambda \sim 0)$ $\sigma_\lambda(T) \to \sigma_{\text{phys}}$ $T \sim 1$ [fm]
- $T \to \infty$ $\sigma_\lambda(T) \to \sigma_{\text{phys}}$ for all λ ?

Summary

We have analyzed the instantaneous potential in the generalized Landau gauge (λ-gauge)

- The potential changes continuously from Landau toward Coulomb gauge.
- Coulomb + linear ($V(R) = -A/R + \sigma R + \mathrm{Const.}$)
- String tension is drastically changed near the Coulomb gauge.
- In specified λ-parameter, physical interquark potential is reproduced

We also have analyzed T-length potential.

- Coulomb + linear potential also works well.
- The asymptotic value seems to coincide with the physical value near the Coulomb gauge.