The chiral and angular momentum content of the ρ-meson

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[LYG,CBL,ML PRL103(2009)121601, FewBodySyst47(2010)91]

Introduction

- Composition of hadronic states in QFT is a subtle issue
- In non-relativistic approaches the concept of a wave function and a complete basis of states is well defined
- BUT: no suitable way of describing a single hadron beyond the ground state in QFT (scattering state)
- In LQCD studies of hadron limited to spectroscopy, axial couplings, form factors, etc.
- More demanding topic: composition of a hadron ⇒ understanding hadron structure in an *ab initio* QCD calculation
- Many Fock components present in a hadron, but only a few are dominant
- Best suited tool: variational method [Michael 1985; Lüscher,Wolff 1990]

Chiral classification and basis transformation

- Two possible local interpolators with the ρ quantum numbers $I, J^{PC} = 1, 1^{--}$ [Cohen,Ji 1997; LYG 2004, 2007]

$$\text{vector current:} \quad O_\rho^V = \overline{q}\,\gamma_i\,\boldsymbol{\tau}\,q \quad \rightsquigarrow \quad (0,1) \oplus (1,0)$$
$$\text{pseudo-tensor operator:} \quad O_\rho^T = \overline{q}\,\sigma^{0i}\,\boldsymbol{\tau}\,q \quad \rightsquigarrow \quad (1/2,1/2)_b$$

- Both are a complete set $\Rightarrow$ signal for chiral symmetry breaking if both couple
- Transformation to angular momentum basis [Nefediev,LYG 2007]

$$\left|(0,1) \oplus (1,0); 1\ 1^{--}\right\rangle = \sqrt{\frac{2}{3}}\left|1; {}^3S_1\right\rangle + \sqrt{\frac{1}{3}}\left|1; {}^3D_1\right\rangle$$
$$\left|(1/2,1/2)_b; 1\ 1^{--}\right\rangle = \sqrt{\frac{1}{3}}\left|1; {}^3S_1\right\rangle - \sqrt{\frac{2}{3}}\left|1; {}^3D_1\right\rangle$$

Variational method

- Representations $(0,1)\oplus(1,0)$ and $(1/2,1/2)_b$ form a complete (and orthogonal w.r.t. chiral group) basis $\Rightarrow$ usage of variational method
- Possibility to study mixing of the two representations
- Cross-correlation matrix

$$C_{ij}(t) = \langle O_i(t) O_j^\dagger(0)\rangle = \sum_{n=1}^{N} a_i^{(n)} a_j^{(n)*} \exp\left(-E^{(n)}t\right)$$

- Overlap of O_i with physical state $|n\rangle$ described by $a_i^{(n)} = \langle 0|O_i|n\rangle$
- Solve GEVP and get eigenvectors $u_j^{(n)}$
- Ratios of couplings of different lattice operators to physical state $|n\rangle$ via

$$\frac{a_i^{(n)}}{a_j^{(n)}} = \frac{C_{ik}(t) u_k^{(n)}}{C_{jk}(t) u_k^{(n)}}$$

Smearing and the resolution scale

- Hadron decomposition depends on resolution scale
- Point-like interpolators $\Rightarrow$ resolution scale given by lattice spacing a
- Interested in low (IR) resolution scales (i. e., large a) $\Rightarrow$ huge lattice artifacts
- Gauge-invariant smearing of quark fields to size R such that $R/a \gg 1$
- Substitute local interpolator by interpolator (with same quantum numbers) using smeared quark fields
- Want Gaussian profile of smeared fields $\Rightarrow$ Jacobi smearing [Güsken et al. 1989; Best et al. 1997]

$$S = M S_0, \quad M = \sum_{n=0}^{N} \kappa^n H^n$$

- Hopping term $H = \sum_{k=1}^{3} \left[U_k(x,t) \delta_{x+\hat{k},y} + U_k^\dagger(x-\hat{k},t) \delta_{x-\hat{k},y} \right]$

Simulation details

- Most details presented at talk of G. P. Engel (Monday, June 14) and in [Gattringer et al. 2009; Engel et al. 2010]
- Two mass-degenerate Chirally Improved fermions [Gattringer 2001; Gattringer,Hip,CBL 2001] and Lüscher-Weisz gauge action [Lüscher,Weisz 1985]
- Three different $16^3 \times 32$ lattices
- Jacobi smeared sources/sinks $\Rightarrow$ narrow (n) and wide (w) profile with widths of ≈ 0.34 fm and ≈ 0.67 fm

set	β_{LW}	m_0	#conf	a [fm]	m_π [MeV]	m_ρ [MeV]	$m_{\rho'}$ [MeV]
A	4.70	-0.050	200	0.150(1)	525(7)	914(10)	1881(63)
B	4.65	-0.060	200	0.150(1)	470(4)	872(11)	1905(71)
C	4.58	-0.077	200	0.144(1)	322(5)	791(15)	1799(72)

Interpolators

$$
\begin{aligned}
O_p^V &= \overline{u}_p \, \gamma_i \, d_p \\
O_n^V &= \overline{u}_n \, \gamma_i \, d_n \\
O_w^V &= \overline{u}_w \, \gamma_i \, d_w
\end{aligned}
$$

$$
\begin{aligned}
O_p^T &= \overline{u}_p \, \gamma_i \, \gamma_t \, d_p \\
O_n^T &= \overline{u}_n \, \gamma_i \, \gamma_t \, d_n \\
O_w^T &= \overline{u}_w \, \gamma_i \, \gamma_t \, d_w
\end{aligned}
$$

γ_i ... matrix in spatial direction

γ_t ... matrix in (Euclidean) time direction

Interpolators

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O_p^T &= \overline{u}_p \, \gamma_i \, \gamma_t \, d_p &\rightsquigarrow& \quad \left|(1/2,1/2)_b; 1 \; 1^{--}\right\rangle \\
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\end{aligned}
$$

γ_i ... matrix in spatial direction

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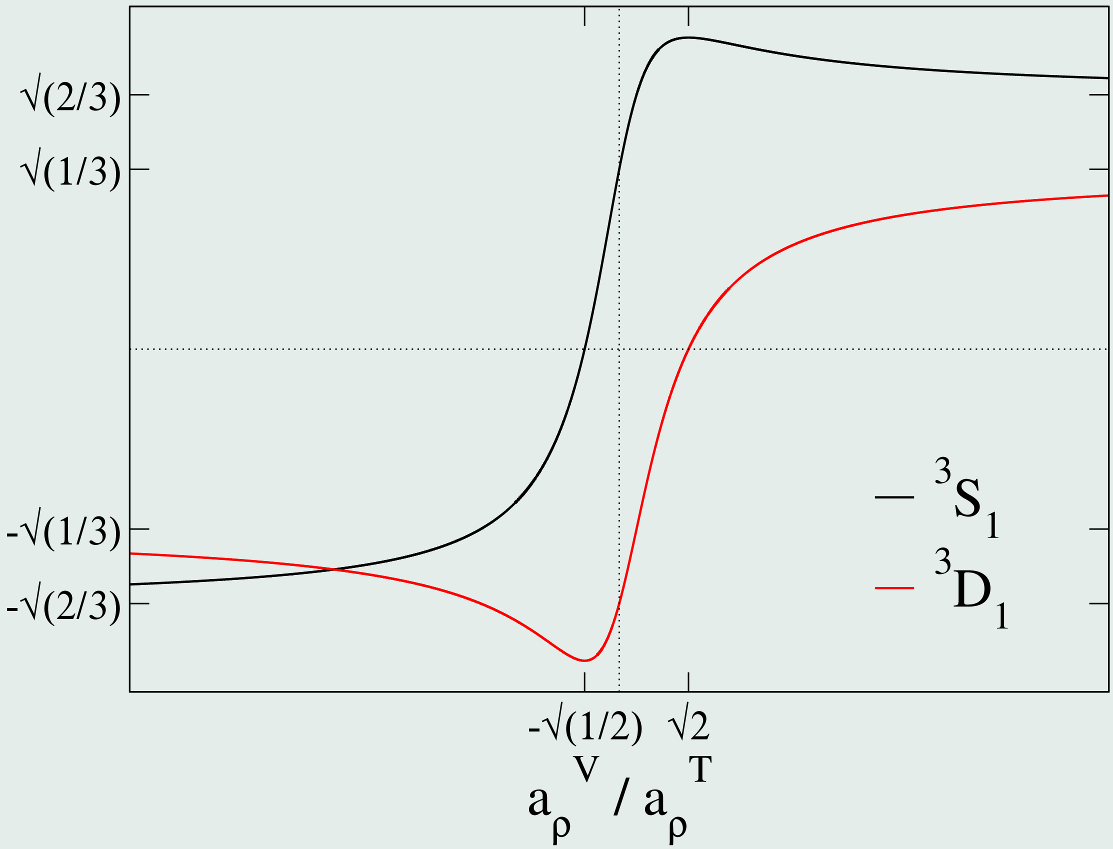
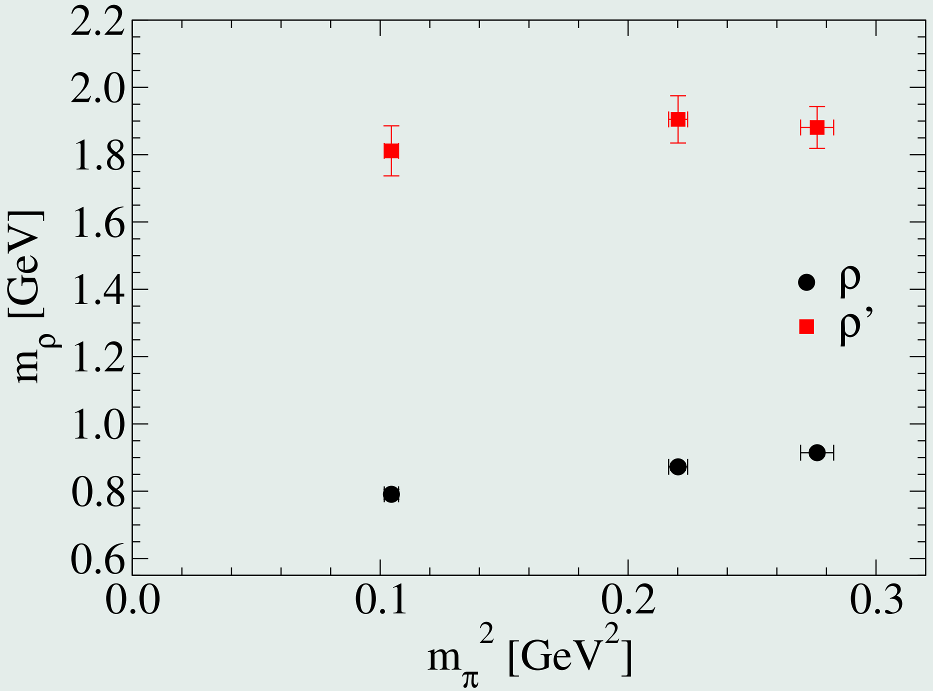
Coupling “strength”



Vector meson mass



- Masses extracted from 4×4 matrix, including $O^V_{n/w}, O^T_{n/w}$

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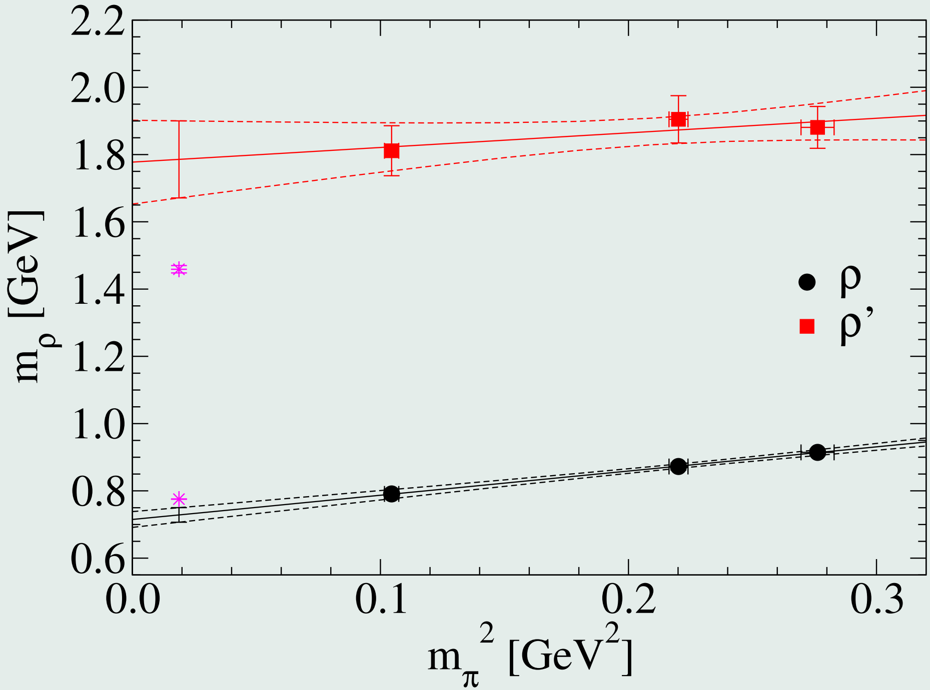
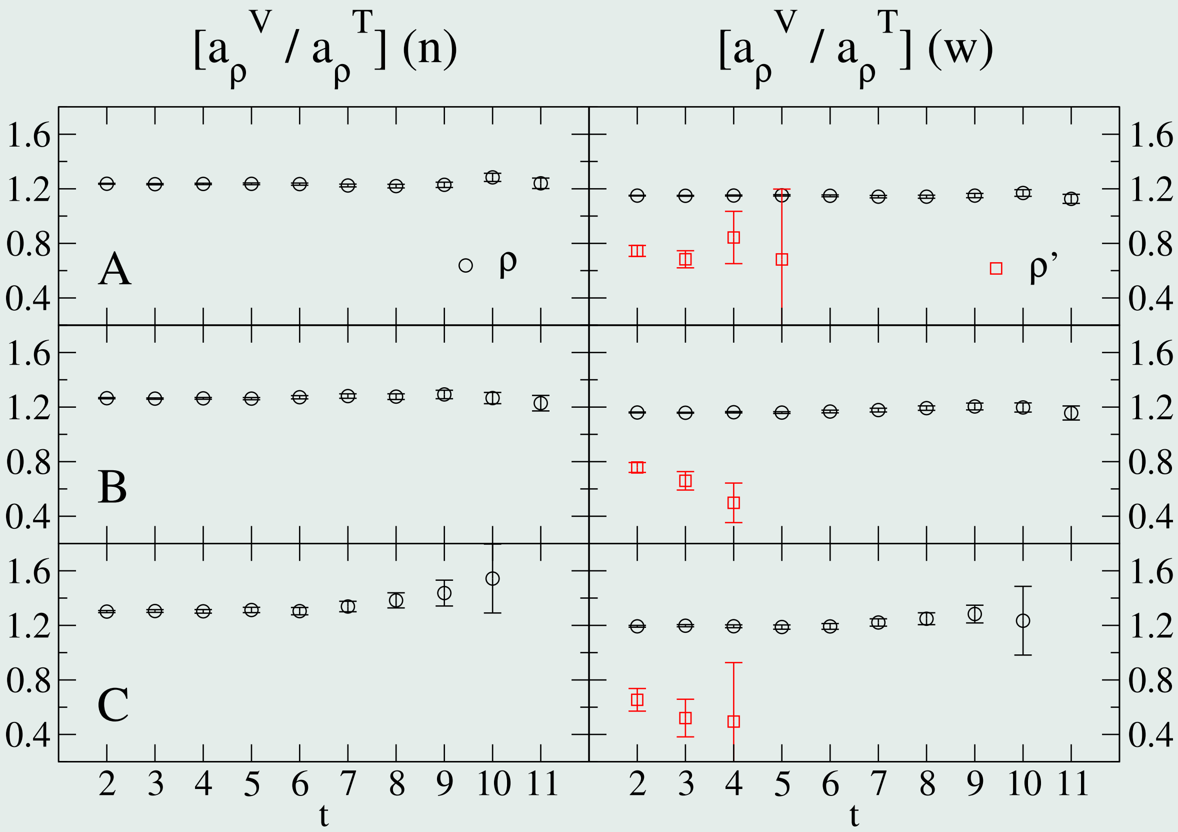
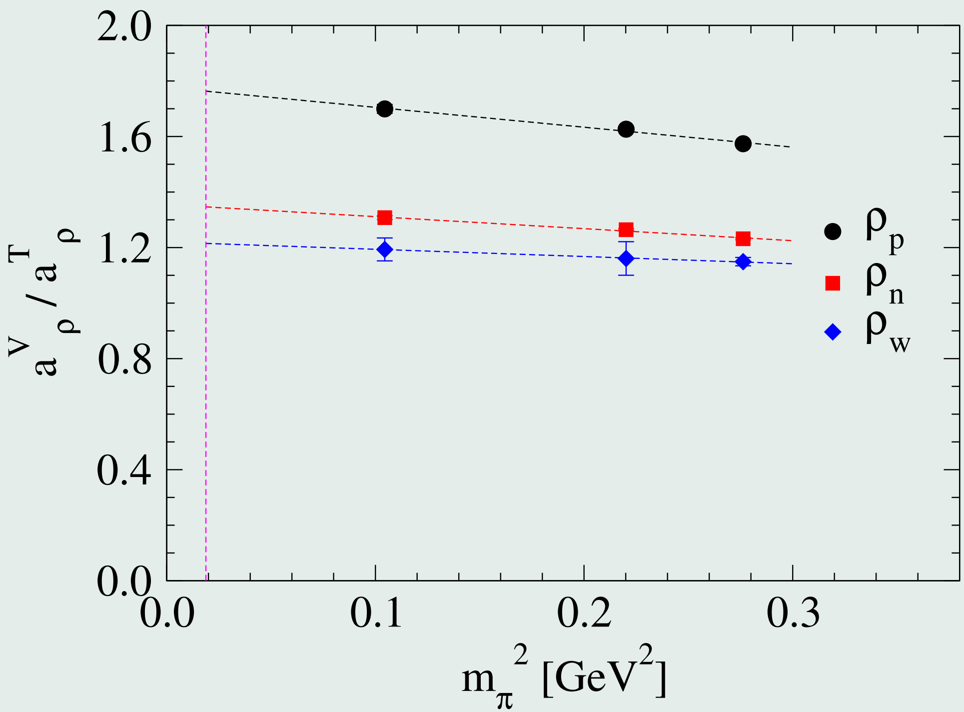


- Masses extracted from 4×4 matrix, including $O^V_{n/w}, O^T_{n/w}$
- "Chiral extrapolation" linear im m_π^2

Stability of the a-ratios



Vector meson decomposition



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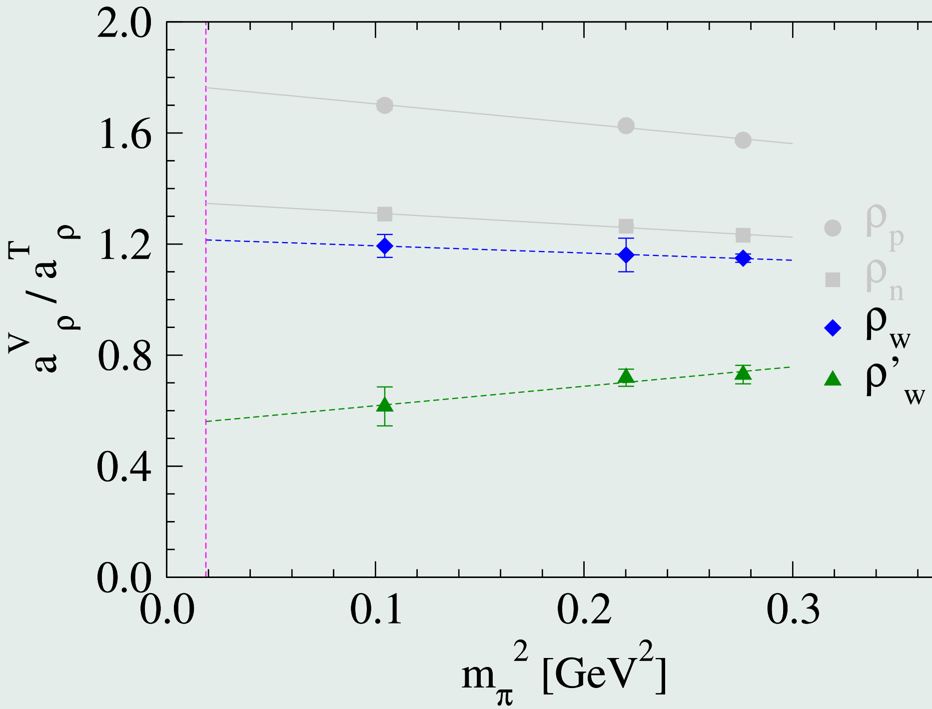

Vector meson decomposition

- Can read off fractions of $|1;{}^3S_1\rangle$ wave and $|1;{}^3D_1\rangle$ wave from a-ratios
- Our results:

$$\rho_p\colon \quad \frac{(0,1)\oplus(1,0)}{(1/2,1/2)_b} = 1.763 \quad \Rightarrow \quad 0.9950|1;{}^3S_1\rangle + 0.09943\,|1;{}^3D_1\rangle$$

$$\rho_n\colon \quad \frac{(0,1)\oplus(1,0)}{(1/2,1/2)_b} = 1.347 \quad \Rightarrow \quad 0.9997\,|1;{}^3S_1\rangle - 0.02328\,|1;{}^3D_1\rangle$$

$$\rho_w\colon \quad \frac{(0,1)\oplus(1,0)}{(1/2,1/2)_b} = 1.215 \quad \Rightarrow \quad 0.9973\,|1;{}^3S_1\rangle - 0.07311\,|1;{}^3D_1\rangle$$

$$\Rightarrow \rho \approx |1;{}^3S_1\rangle$$

Vector meson decomposition

- Can read off fractions of $\left|1;{}^3S_1\right\rangle$ wave and $\left|1;{}^3D_1\right\rangle$ wave from a-ratios
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$$\rho'_w: \quad \frac{(0,1)\oplus(1,0)}{(1/2,1/2)_b} = 0.5607 \quad \Rightarrow \quad 0.9029\left|1;{}^3S_1\right\rangle - 0.4298\left|1;{}^3D_1\right\rangle$$

$$\Rightarrow \rho \approx \left|1;{}^3S_1\right\rangle$$

$$\Rightarrow \rho' \not\approx \left|1;{}^3S_1\right\rangle$$

Summary

- Task: hadron composition from an *ab initio* QCD calculation
- Here: vector meson ρ $(I, J^{PC} = 1, 1^{--})$ was used
- Two different types of interpolators can be used: $O_\rho^V,\ O_\rho^T$
- Variational method applied
- Used smeared quark fields to vary resolution scale
- Simulation carried out on three $16^3 \times 32$ lattices with $a \approx 0.15$ fm and $m_\pi \approx 320 - 520$ MeV
- Results: ground state mostly $|1; {}^3S_1\rangle$, but first excitation has significant admixture of $|1; {}^3D_1\rangle$ (contradictory to quark model!)

Thank you!