Study of finite Temperature *QCD* with $(2+1)$ flavors via Taylor expansion and imaginary chemical potential

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Outline

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   - Taylor expansion
   - Technical Information
2. **Observables**
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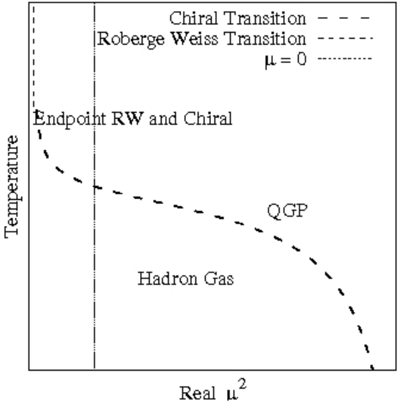
   - Critical Line
   - Quark Density
3. **Outlook**

QCD phase diagram

Sketchy view of the QCD phase diagram in the $T - \mu^2$ plane



[M. D'Elia, F. Di Renzo and M.P. Lombardo,Phys.Rev.D76(2007)114509]

The QCD partition function

$$\mathcal{Z} = \int \mathcal{D}U(\text{det}M(m_u,\mu_u))^{N_f/4}(\text{det}M(m_d,\mu_d))^{N_f/4}(\text{det}M(m_s,\mu_s))^{N_f/4}e^{-S_g}$$

$$\overset{(2+1)}{=} \int \mathcal{D}U(\text{det}M(m_q,\mu_q))^{1/2}(\text{det}M(m_s,\mu_s))^{1/4}e^{-S_g}$$

$$\mu_q = \mu_u = \mu_d$$

On the lattice

$$\begin{array}{lcll} U_t & \rightarrow & e^{a\mu}U_t & \textit{forward temporal link} \\ U_t^\dagger & \rightarrow & e^{-a\mu}U_t^\dagger & \textit{backward temporal link} \end{array}$$

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$$\mu_q = \mu_u = \mu_d$$

On the lattice at imaginary chemical potential

$$\begin{array}{lcll} U_t & \rightarrow & e^{ia\mu_I} U_t & \textit{forward temporal link} \\ U_t^\dagger & \rightarrow & e^{-ia\mu_I} U_t^\dagger & \textit{backward temporal link} \end{array}$$

$\Longrightarrow$ *detM* *real and positive* !

Taylor expansion

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln\mathcal{Z} \quad \text{pressure}$$

$$\frac{n_i}{T^3} = \frac{1}{VT^2}\frac{\partial \ln\mathcal{Z}}{\partial\mu_i} \quad \text{quark density}$$

$$\frac{p}{T^4}(\hat{\mu}) = \sum_{k,l,n} c_{kln} \quad (\hat{\mu}_u - \hat{\mu}_0)^k(\hat{\mu}_d - \hat{\mu}_0)^l(\hat{\mu}_s - \hat{\mu}_0)^n$$

where $\hat{\mu} = \frac{\mu}{T}$

$$c_{kln} = \frac{1}{k!l!n!}\frac{\partial^k}{\partial\hat{\mu_u}^k}\frac{\partial^l}{\partial\hat{\mu_d}^l}\frac{\partial^n}{\partial\hat{\mu_s}^n}\left(\frac{p}{T^4}\right)$$

$$\xrightarrow{\text{on the lattice}} c_{kln} = \frac{1}{k!l!n!}\frac{N_\tau^{3-k-l-n}}{N_\sigma^3}\frac{\partial^k}{\partial\mu_u^k}\frac{\partial^l}{\partial\mu_d^l}\frac{\partial^n}{\partial\mu_s^n}(\ln Z)$$

[S. Gottlieb,W. Liu,D. Toussaint,R.L. Renken,R.L. Sugar,Phys.Rev.D38(1988)2888]
[R.V. Gavai and S. Gupta,Phys.Rev.D68(2003)034506]
[C.R. Allton *et al.*,Phys.Rev.D68(2003)014507]

Lattice setup

- tree level Symanzik improved gauge action
- improved staggered fermions action (p4fat3)
- number of dynamical fermions: $2 + 1$
- lattice temporal extent $N_t = 4$

Configuration ensemble

- range of temperatures: $0.937 < T/T_c < 1.072$
- LCP : $m_{\bar{s}s} r_0 = 1.59$, $\hat{m}_s/\hat{m}_l = 10$

  $m_\pi \simeq 220(4)\ MeV \quad m_{\bar{s}s} \simeq 669(10) MeV \quad m_K \simeq 503(6) MeV$

  [M. Cheng *et al.*,Phys.RevD77(2008)014511]

Roberge Weiss transition [A. Roberge and N. Weiss, Nucl.Phys.B275(1986)734]

$$\theta = \frac{2\pi}{N_c}(k + \frac{1}{2}) \qquad k = 0, \pm 1, \pm 2 \ldots$$

$$\theta = \frac{\mu}{T} \rightarrow a\mu_I = \frac{2\pi}{3N_\tau}\left(k + \frac{1}{2}\right)$$

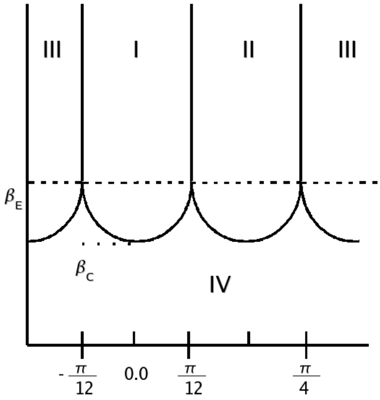
$$P(\vec{x}) \equiv |P(\vec{x})| e^{i\phi}$$

When $\mu_I \neq 0 \rightarrow P(\vec{x}) e^{i\theta}$

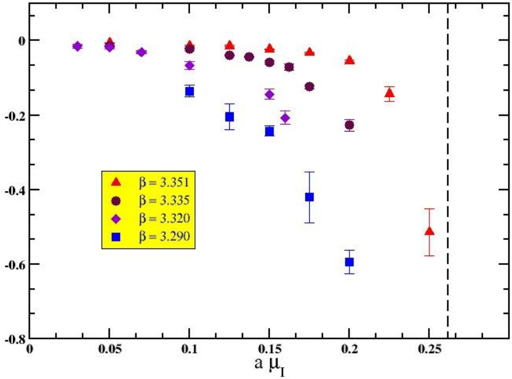
low T : $\langle \phi \rangle = -\theta = -4a\mu_I$

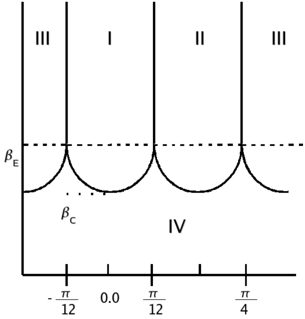
high T : $\langle \phi \rangle \simeq \frac{2k\pi}{3}$

for $\frac{\pi}{6}\left(k - \frac{1}{2}\right) < a\mu_I < \frac{\pi}{6}\left(k + \frac{1}{2}\right)$



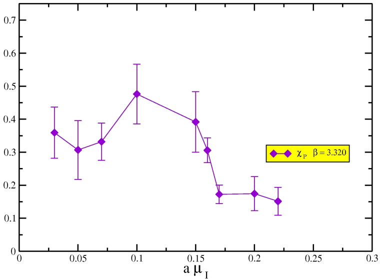
Polyakov Loop Phase



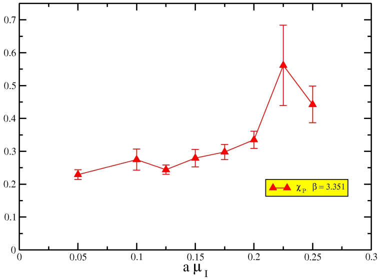


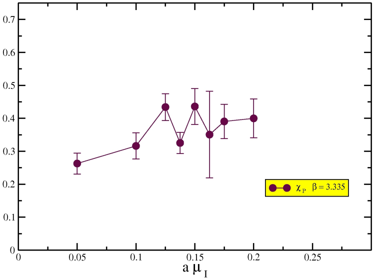
$$\beta_c(\mu_I = 0) = 3.3145(7)$$

Susceptibility χ_P



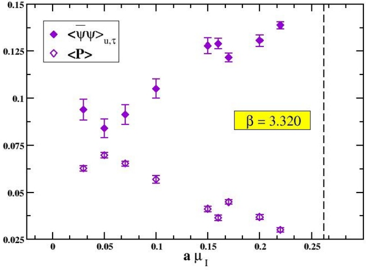
$$\chi_P = N_s^3 \left(\langle P^2 \rangle - \langle P \rangle^2 \right)$$

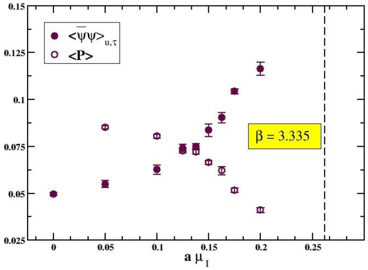


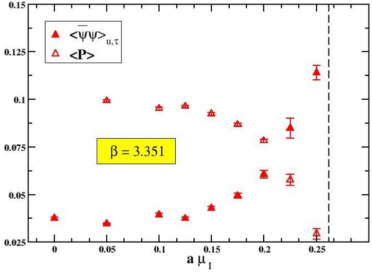


Chiral Condensate

$$\langle \hat{\bar\psi}\psi \rangle_{q,\tau} \equiv \frac{1}{4}\frac{1}{N_\sigma^3 N_t}\langle TrM^{-1}\rangle_\tau , \quad q = u, l, s$$





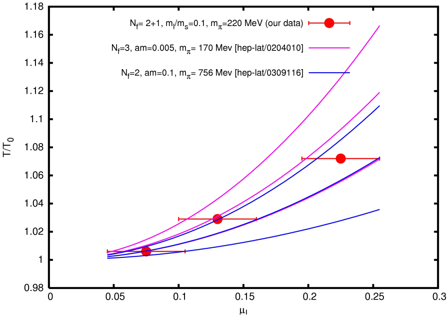


Critical Line

[O. Philpsen,Prog.Theor.Phys.Suppl.174(2008)206]

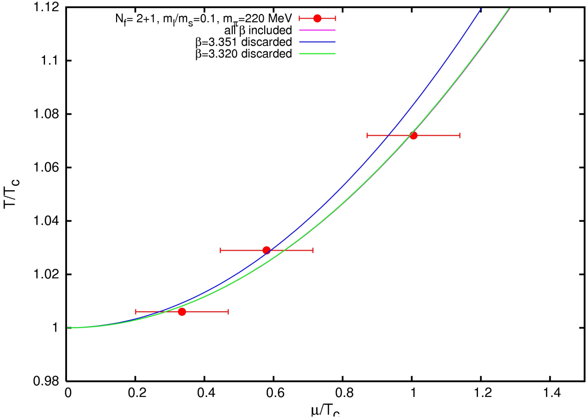
$$\frac{T_c(\mu)}{T_c(0)} = 1 - t_2(N_f, m_f)\left(\frac{\mu}{\pi T}\right)^2 + \mathcal{O}\left(\left(\frac{\mu}{\pi T}\right)^4\right)$$

$$T_c(0) \simeq 204 MeV$$



[hep-lat/0204010]: C.R Allton *et all*, Phys.RevD66(2002)074507 $t_2 = 0.69(35)$
[hep-lat/0309116]: C.R Allton *et all*, Nucl.Phys.Proc.Suppl.129(2004)614 $t_2 = 1.13(45)$

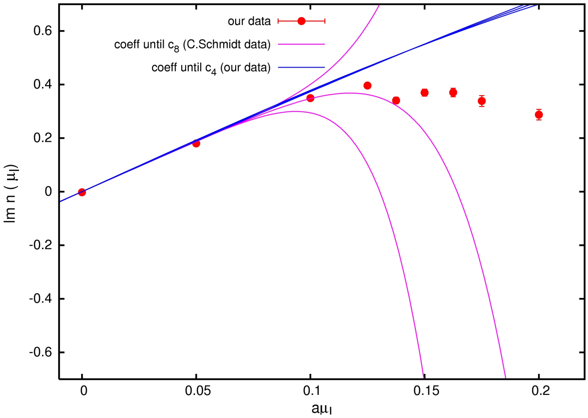
Critical Line



all β included	$\beta = 3.320$ discarded	$\beta = 3.351$ discarded
$t_2 = 0.89(4)$	$t_2 = 0.89(5)$	$t_2 = 1.02(12)$

Taylor Expansion

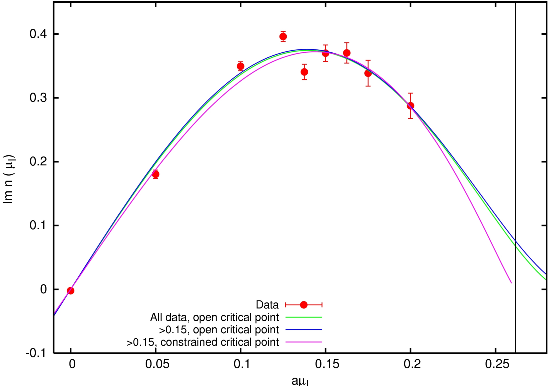
$$Im(n(\mu_I)) = 2N_\tau c_{200}\mu_I - 4N_\tau^3 c_{400}\mu_I^3 + 6N_\tau^5 c_{600}\mu_I^5 - 8N_\tau^7 c_{800}\mu_I^7 + O(\mu_I^9)$$



Critical Fit

$$n(\mu_I) = A\mu_I(\mu_I^{c2} - \mu_I^2)^{\alpha}$$

[M. D'Elia, F. Di Renzo and M.P. Lombardo,Phys.RevD76(2007)114509]



$$\mu_I^c = \frac{\pi T}{3}$$

On the lattice

$$a\mu_I^c = \frac{\pi}{12}$$

$a\mu_I < 0.15$ constrained critical point

$$A = 85(19)$$
$$\alpha = 1.15(7)$$
$$\chi^2/dof < 1$$

Outlook

- improve statistics and data
- improve determination of the critical line
- crosschecking the radius of convergence
- study convergence of the Taylor expansion close to the endpoint and in the hadronic phase
- study the endpoint at the transition in the complex plane

Thank you!