Worm algorithm for the O(N) Gross-Neveu model

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Outline

Massive Thirring Model

Theory of a self-interacting fermionic field in 2 dimensions

$$\mathcal{L} = \overline{\psi}(\gamma_\mu \partial_\mu + m)\psi - \frac{\tilde{g}^2}{2}(\overline{\psi}\gamma_\mu\psi)(\overline{\psi}\gamma_\mu\psi)$$

γ and charge conjugation matrices

$$\gamma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \gamma_2 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \quad \mathcal{C} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

- spectrum: massive fermions, one bosonic bound state [Dashen et al '74]
- equivalent to sine-Gordon theory [Coleman '75]

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Gross Neveu Model

Model of N different interacting dirac fields

$$\begin{aligned} \mathcal{L} &= \overline{\psi}(\gamma_\mu \partial_\mu + m)\psi - \frac{g^2}{2}(\overline{\psi}\psi)^2 \\ \psi &= (\psi_1, \psi_2, ..., \psi_N)^T \end{aligned}$$

Equivalent to the **Thirring Model** for $N = 1$ and $\tilde{g} = 2g$.

Majorana fields

$$\begin{aligned} \psi &= \frac{1}{\sqrt{2}}(\xi_a + i\xi_b) \qquad \text{and} \qquad \overline{\psi} = \frac{1}{\sqrt{2}}(\xi_a^T - i\xi_b^T)\mathcal{C} \\ \mathcal{L} &= \frac{1}{2}\overline{\xi}_i(\gamma_\mu \partial_\mu + m)\xi_i - \frac{1}{8}g^2(\overline{\xi}_i\xi_i)^2 \end{aligned}$$

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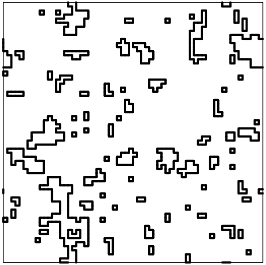
Loop gas representation

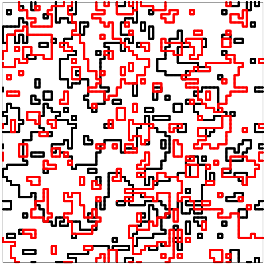
- Wilson fermions $\Rightarrow$ continuum limit, tune bare mass to critical mass
- Hubbard-Stratonovich transformation
- Expand the Boltzmann factor using nilpotency of Grassman elements
- Integrate out the auxillary field
- Loop gas representation [Gattringer '99]

Configurations

Configurations of non-oriented, self-avoiding loops



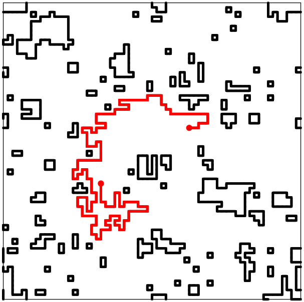
single majorana fermion



two majorana fermions

Worm Algorithm

[Prokof'ev, Svistunov '01; Wenger '08; Wolff '08]

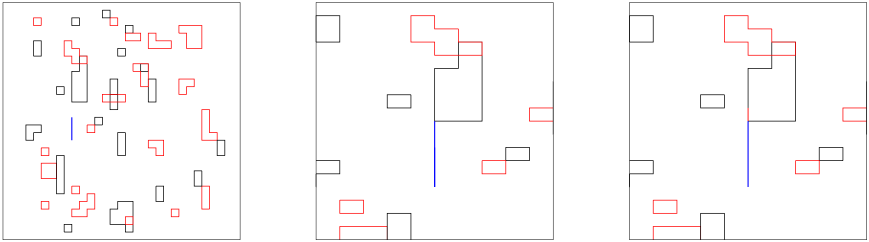


The open string corresponds to the insertion of a Majorana fermion pair $\{\xi^T(x)\mathcal{C}, \xi(y)\}$ at position x and y:
$\Rightarrow$ open string samples the correlation function

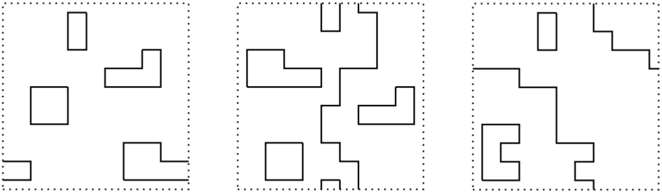
$$G(x,y) = \int \mathcal{D}\xi e^{-S} \xi(x)\xi(y)^T \mathcal{C}$$

Bound state

Measure correlation functions $\langle \mathcal{O}(x)\mathcal{O}(y)\rangle$,
$\mathcal{O}(x) = \xi_a^T(x)\,\mathcal{C}\Gamma\,\xi_b(x)$



Different Classes of Configurations



$\mathcal{L}_{00}$ $\mathcal{L}_{01}$ $\mathcal{L}_{11}$

Opportunity to study all possible boundry conditions:

$$
\begin{aligned}
Z_{\xi}^{p,p} &= Z_{\mathcal{L}_{00}} - Z_{\mathcal{L}_{10}} - Z_{\mathcal{L}_{01}} - Z_{\mathcal{L}_{11}} \\
Z_{\xi}^{p,ap} &= Z_{\mathcal{L}_{00}} - Z_{\mathcal{L}_{10}} + Z_{\mathcal{L}_{01}} + Z_{\mathcal{L}_{11}} \\
Z_{\xi_a,\xi_b}^{ap,p;p,p} &= Z_{\mathcal{L}_{00},\mathcal{L}_{00}} - Z_{\mathcal{L}_{00},\mathcal{L}_{10}} + Z_{\mathcal{L}_{10},\mathcal{L}_{00}} \cdots
\end{aligned}
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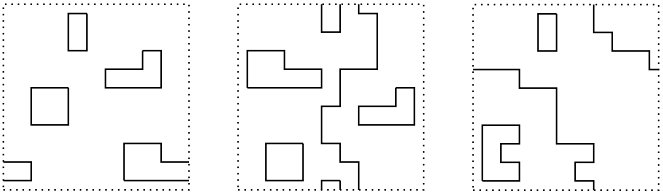

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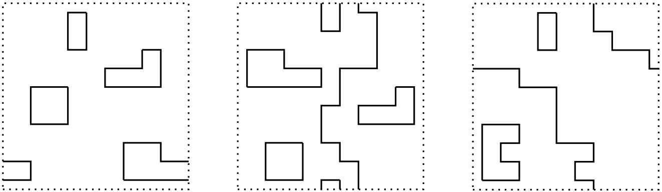

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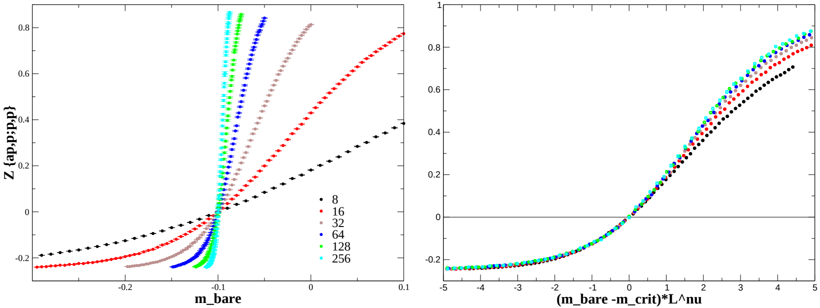
Results: Determination of Critical mass

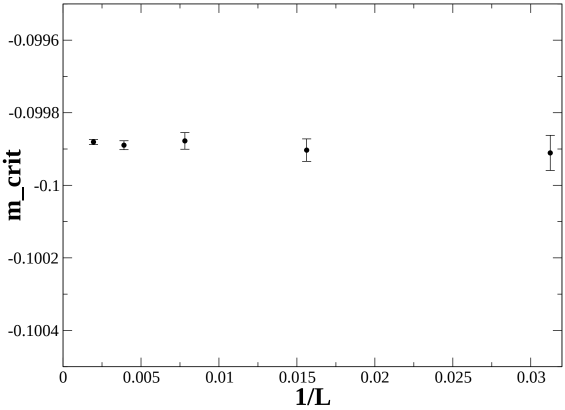
$$Z_{\xi}^{\{ap,p;p,p\}} = 0 \Rightarrow m_{bare} = m_{crit}$$

[Bär, Rath, Wolff '09]



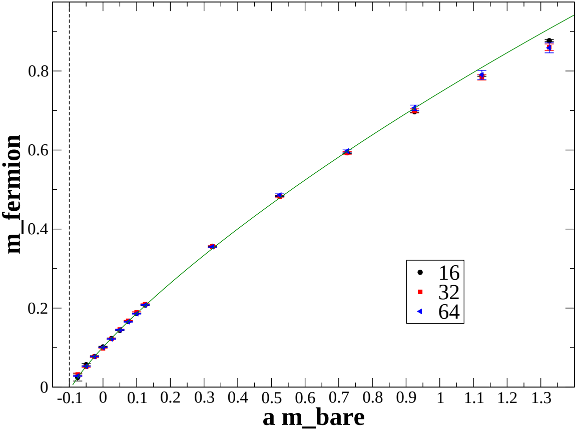
coupling $g = 0.5$

Critical mass



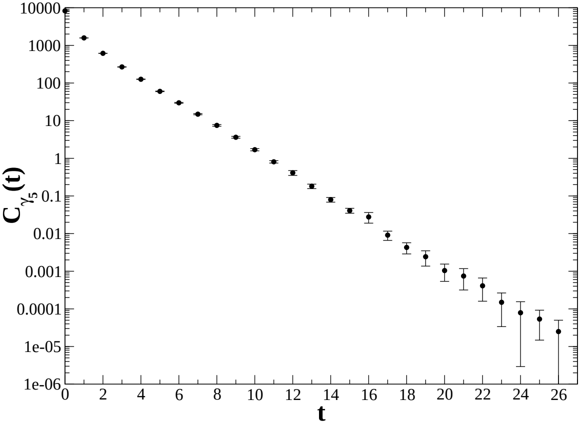
- no critical slowing down as $m_{bare} \Rightarrow m_{crit}$
- simulations at m_{crit} possible

Fermion mass vs. bare mass



Correlation Function of Bound State

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \langle \xi_a^T(x)\,\mathcal{C}\gamma_5\,\xi_b(x)\xi_a^T(x)\,\mathcal{C}\gamma_5\,\xi_b(x)\rangle$$



Bound state mass vs. fermion mass