An extention to the Luescher's finite volume method above inelastic threshold (formalism)

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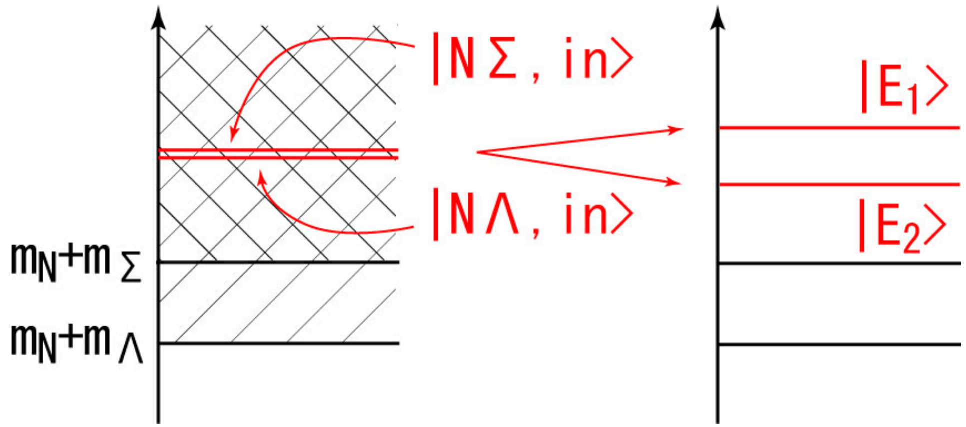
Introduction

- Scattering phases are calculable in lattice QCD by Luscher's finite volume method. However, its standard use is restricted to the elastic region. Several works exist to go above the inelastic threshold.

  S. He, et al., JHEP0507, 011 (2005).
  M.Lage, et al., PLB681, 439 (2009).

- To go above inelastic thresholds, we have to struggle with a problem, i.e.,
  - Degeneracy with respect to different incomming states in the infinite volume disappears in a finite volume with cubic boundary condition.
    Ex) NΛ(I=1/2) above the NΣ threshold



- ☐ There are no obvious relations between
  |NΛ,in>, |NΣ,in> in infinite volume and |E_1>, |E_2> in finite volume
- ☐ If we stick to a single energy in a finite volume,
  number of equations are less than needed in order to obtain the S-matrix.

- We propose to avoid this problem by extending HAL-QCD method to construct the interaction potential (single channel) to the coupled channel version.

NΛ−NΣ coupled system

To be specific, we consider NΛ−NΣ coupled system (I=1/2)

$m_N \sim 940$ MeV (I=1/2)

$m_\Lambda \sim 1115$ MeV (I=0)

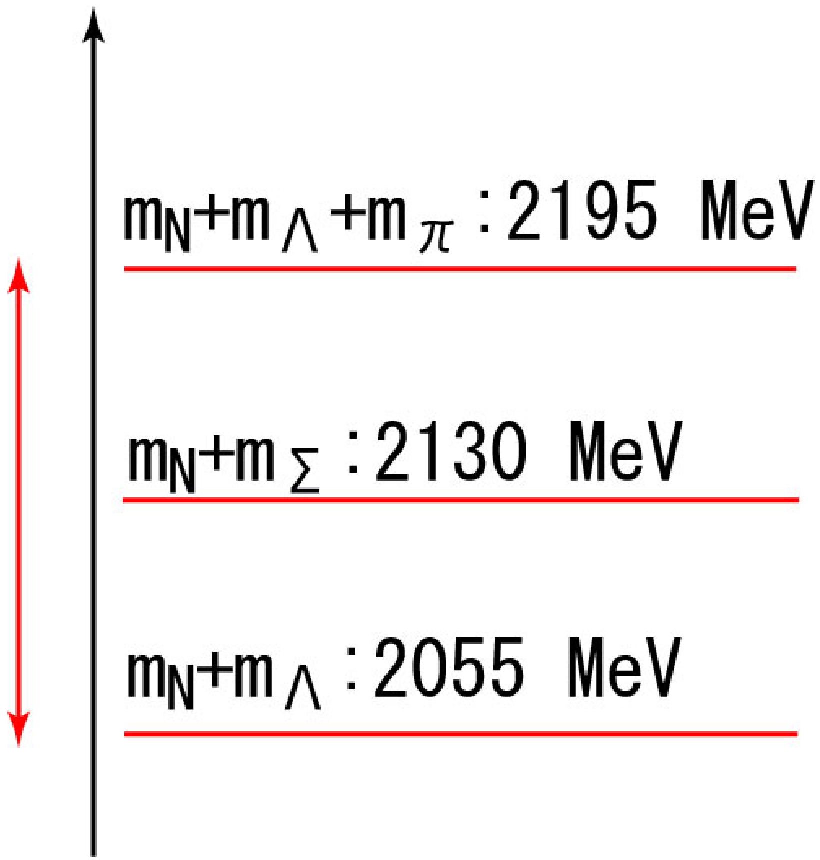
$m_\Sigma \sim 1190$ MeV (I=1)

$$m_N < m_\Lambda < m_\Sigma$$

To simplify, we treat them as bosons.

We first consider it in **infinite** volume.

We then proceed to **finite** volume.



Bethe-Salpeter (BS) wave functions

[infinite volume]

- (equal-time) BS wave functions associated with |NΛ,in> and |NΣ,in> incomming states

$$\begin{cases}\langle 0|N(\vec{x})\Lambda(0)|N(\vec{p})\Lambda(-\vec{p}),in\rangle \\ \langle 0|N(\vec{x})\Sigma(0)|N(\vec{p})\Lambda(-\vec{p}),in\rangle\end{cases} \qquad \begin{cases}\langle 0|N(\vec{x})\Lambda(0)|N(\vec{q})\Sigma(-\vec{q}),in\rangle \\ \langle 0|N(\vec{x})\Sigma(0)|N(\vec{q})\Sigma(-\vec{q}),in\rangle\end{cases}$$

N(x), Λ(x), Σ(x): local composite interpolating fields for N, Λ, Σ

- Their long distance behavior are derived similarly as single channel case:

C.-J.D.Lin et al.,NPB619, 467 (2001).
CP-PACS Coll., PRD71, 094504 (2005).
S.Aoki et al., PTP123, 89 (2010).

For instance,

◆ NΛ-NΛ BS wave function

This is related to T-matrix through reduction formula

$$\langle 0|N(\vec{x})\Lambda(0)|N(\vec{p})\Lambda(-\vec{p}),in\rangle$$

$$= \int \frac{d^3k}{(2\pi)^3 2E_N(\vec{k})}\langle 0|N(\vec{x})|N(\vec{k})\rangle\, \underline{\langle N(\vec{k})|\Lambda(0)|N(\vec{p})\Lambda(-\vec{p}),in\rangle} + \cancel{I(\vec{x})}$$

$$\simeq Z_N^{1/2} Z_\Lambda^{1/2}\left(e^{i\vec{p}\cdot\vec{x}} + \int \frac{d^3k}{(2\pi)^3 2E_N(\vec{k})} \times \frac{1}{E_\Lambda(\vec{k}) - E_N(\vec{k}) + E_N(\vec{p}) + E_\Lambda(\vec{p})} \times \frac{\mathcal{T}(N(\vec{k})\Lambda(-\vec{k}); N(\vec{p})\Lambda(-\vec{p}))\, e^{i\vec{k}\cdot\vec{x}}}{E_N(\vec{k}) + E_\Lambda(\vec{k}) - E_N(\vec{p}) - E_\Lambda(\vec{p}) - i\epsilon}\right)$$

$$\simeq Z_N^{1/2} Z_\Lambda^{1/2}\left(e^{i\vec{p}\cdot\vec{x}} + \frac{\lambda^{1/2}(s, m_N^2, m_\Lambda^2)}{s}\mathcal{T}_{N\Lambda,N\Lambda}(s)\frac{e^{ipr}}{pr}\right)$$

the Kallen function

$$\boxed{\lambda(x,y,z) \equiv x^2 + y^2 + z^2 - 2xy - 2yz - 2zx}$$

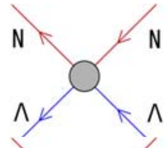
Helmholtz equation at long distance

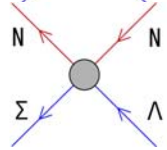
[infinite volume] (5)

➢ BS wave functions at long distance

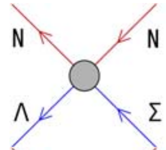
$$E = \sqrt{m_N^2 + \vec{p}^{\,2}} + \sqrt{m_\Lambda^2 + \vec{p}^{\,2}} = \sqrt{m_N^2 + \vec{q}^{\,2}} + \sqrt{m_\Sigma^2 + \vec{q}^{\,2}}$$

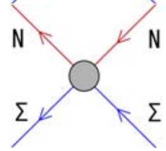
$$\begin{cases} \psi_{N\Lambda,N\Lambda}(\vec{x};E) \equiv Z_N^{-1/2} Z_\Lambda^{-1/2} \langle 0 | N(\vec{x})\Lambda(0) | N(\vec{p})\Lambda(-\vec{p}), in\rangle \sim e^{i\vec{p}\cdot\vec{r}} + \dfrac{\lambda^{1/2}(s,m_N^2,m_\Lambda^2)}{s} \mathcal{T}_{N\Lambda,N\Lambda}(s) \dfrac{e^{ipr}}{pr} + \cdots \\ \psi_{N\Sigma,N\Lambda}(\vec{x};E) \equiv Z_N^{-1/2} Z_\Sigma^{-1/2} \langle 0 | N(\vec{x})\Sigma(0) | N(\vec{p})\Lambda(-\vec{p}), in\rangle \sim \dfrac{\lambda^{1/2}(s,m_N^2,m_\Sigma^2)}{s} \mathcal{T}_{N\Sigma,N\Lambda}(s) \dfrac{e^{iqr}}{qr} + \cdots \end{cases}$$





$$\begin{cases} \psi_{N\Lambda,N\Sigma}(\vec{x};E) \equiv Z_N^{-1/2} Z_\Lambda^{-1/2} \langle 0 | N(\vec{x})\Lambda(0) | N(\vec{q})\Sigma(-\vec{q}), in\rangle \sim \dfrac{\lambda^{1/2}(s,m_N^2,m_\Lambda^2)}{s} \mathcal{T}_{N\Lambda,N\Sigma}(s) \dfrac{e^{ipr}}{pr} + \cdots \\ \psi_{N\Sigma,N\Sigma}(\vec{x};E) \equiv Z_N^{-1/2} Z_\Sigma^{-1/2} \langle 0 | N(\vec{x})\Sigma(0) | N(\vec{q})\Sigma(-\vec{q}), in\rangle \sim e^{i\vec{q}\cdot\vec{r}} + \dfrac{\lambda^{1/2}(s,m_N^2,m_\Sigma^2)}{s} \mathcal{T}_{N\Sigma,N\Sigma}(s) \dfrac{e^{iqr}}{qr} + \cdots \end{cases}$$





➢ Helmholtz eq. is satisfied by BS wave functions at long distance (|x| >> R).

$$(\vec{\nabla}^2 + p^2)\psi_{N\Lambda,N\Lambda}(\vec{x};E) \equiv K_{N\Lambda,N\Lambda}(\vec{x};E)$$

$$(\vec{\nabla}^2 + q^2)\psi_{N\Sigma,N\Lambda}(\vec{x};E) \equiv K_{N\Sigma,N\Lambda}(\vec{x};E)$$

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$$(\vec{\nabla}^2 + q^2)\psi_{N\Sigma,N\Sigma}(\vec{x};E) \equiv K_{N\Sigma,N\Sigma}(\vec{x};E)$$

|x|>>R

$$K_{N\Lambda,N\Lambda}(\vec{x};E) \sim 0$$

$$K_{N\Sigma,N\Lambda}(\vec{x};E) \sim 0$$

$$K_{N\Lambda,N\Sigma}(\vec{x};E) \sim 0$$

$$K_{N\Sigma,N\Sigma}(\vec{x};E) \sim 0$$

- Propagating degrees of freedoms are filtered out.
  ➔ K(x,E) is a localized object.
- Helmholtz eq. is satisfied |x|>>R.

Energy-independent, non-local interaction kernel

[infinite volume] (6)

- For $|x| \leqq R$, K does not vanish. We wish to factorize K such that

$$K_{N\Lambda,N\Lambda}(\vec{x};E) = \int d^3y U_{N\Lambda,N\Lambda}(\vec{x},\vec{y}) \psi_{N\Lambda,N\Lambda}(\vec{y};E) + \int d^3y U_{N\Lambda,N\Sigma}(\vec{x},\vec{y}) \psi_{N\Sigma,N\Lambda}(\vec{y};E)$$

$$K_{N\Sigma,N\Lambda}(\vec{x};E) = \int d^3y U_{N\Sigma,N\Lambda}(\vec{x},\vec{y}) \psi_{N\Lambda,N\Lambda}(\vec{y};E) + \int d^3y U_{N\Sigma,N\Sigma}(\vec{x},\vec{y}) \psi_{N\Sigma,N\Lambda}(\vec{y};E)$$

$$K_{N\Lambda,N\Sigma}(\vec{x};E) = \int d^3y U_{N\Lambda,N\Lambda}(\vec{x},\vec{y}) \psi_{N\Lambda,N\Sigma}(\vec{y};E) + \int d^3y U_{N\Lambda,N\Sigma}(\vec{x},\vec{y}) \psi_{N\Sigma,N\Sigma}(\vec{y};E)$$

$$K_{N\Sigma,N\Sigma}(\vec{x};E) = \int d^3y U_{N\Sigma,N\Lambda}(\vec{x},\vec{y}) \psi_{N\Lambda,N\Sigma}(\vec{y};E) + \int d^3y U_{N\Sigma,N\Sigma}(\vec{x},\vec{y}) \psi_{N\Sigma,N\Sigma}(\vec{y};E)$$

U(x,y) denotes E-independent and non-local interaction kernel.

- These relations are compactly written as

$$\begin{bmatrix} K_{N\Lambda,N\Lambda}(\vec{x};E) & K_{N\Lambda,N\Sigma}(\vec{x};E) \\ K_{N\Sigma,N\Lambda}(\vec{x};E) & K_{N\Sigma,N\Sigma}(\vec{x};E) \end{bmatrix} = \int d^3y \begin{bmatrix} U_{N\Lambda,N\Lambda}(\vec{x},\vec{y}) & U_{N\Lambda,N\Sigma}(\vec{x},\vec{y}) \\ U_{N\Sigma,N\Lambda}(\vec{x},\vec{y}) & U_{N\Sigma,N\Sigma}(\vec{x},\vec{y}) \end{bmatrix} \cdot \begin{bmatrix} \psi_{N\Lambda,N\Lambda}(\vec{x};E) & \psi_{N\Lambda,N\Sigma}(\vec{x};E) \\ \psi_{N\Sigma,N\Lambda}(\vec{x};E) & \psi_{N\Sigma,N\Sigma}(\vec{x};E) \end{bmatrix}$$

Energy-independent, non-local interaction kernel (2) [infinite volume]

➢ The factorization is possible.

◆ Assume that BS wave functions are linearly independent, .i.e.,

$$\left\{ \begin{bmatrix} \psi_{N\Lambda,N\Lambda}(\vec{x};E,\alpha) \\ \psi_{N\Sigma,N\Lambda}(\vec{x};E,\alpha) \end{bmatrix}, \begin{bmatrix} \psi_{N\Lambda,N\Sigma}(\vec{x};E,\alpha) \\ \psi_{N\Sigma,N\Sigma}(\vec{x};E,\alpha) \end{bmatrix} \right\}_{E,\alpha}$$

(α is to distinguish states with same E).

BS wave functions have a **"left inverse"** as an intergration operator as

$$\int d^3y \begin{bmatrix} \tilde{\psi}_{N\Lambda,N\Lambda}(\vec{x};E',\alpha') & \tilde{\psi}_{N\Lambda,N\Sigma}(\vec{x};E',\alpha') \\ \tilde{\psi}_{N\Sigma,N\Lambda}(\vec{x};E',\alpha') & \tilde{\psi}_{N\Sigma,N\Sigma}(\vec{x};E',\alpha') \end{bmatrix} \cdot \begin{bmatrix} \psi_{N\Lambda,N\Lambda}(\vec{x};E,\alpha) & \psi_{N\Lambda,N\Sigma}(\vec{x};E,\alpha) \\ \psi_{N\Sigma,N\Lambda}(\vec{x};E,\alpha) & \psi_{N\Sigma,N\Sigma}(\vec{x};E,\alpha) \end{bmatrix} = (2\pi)\delta(E-E')\delta_{\alpha,\alpha'}$$

◆ **Factorization** is possible

$$\begin{bmatrix} K_{N\Lambda,N\Lambda}(\vec{x};E,\alpha) & K_{N\Lambda,N\Sigma}(\vec{x};E,\alpha) \\ K_{N\Sigma,N\Lambda}(\vec{x};E,\alpha) & K_{N\Sigma,N\Sigma}(\vec{x};E,\alpha) \end{bmatrix}$$

$$= \sum_{\alpha'} \int \frac{dE'}{2\pi} \begin{bmatrix} K_{N\Lambda,N\Lambda}(\vec{x};E',\alpha') & * \\ * & * \end{bmatrix} \cdot \int d^3y \begin{bmatrix} \tilde{\psi}_{N\Lambda,N\Lambda}(\vec{y};E',\alpha') & * \\ * & * \end{bmatrix} \cdot \begin{bmatrix} \psi_{N\Lambda,N\Lambda}(\vec{y};E,\alpha) & * \\ * & * \end{bmatrix}$$

$$= \int d^3y \begin{bmatrix} U_{N\Lambda,N\Lambda}(\vec{x},\vec{y}) & U_{N\Lambda,N\Sigma}(\vec{x},\vec{y}) \\ U_{N\Sigma,N\Lambda}(\vec{x},\vec{y}) & U_{N\Sigma,N\Sigma}(\vec{x},\vec{y}) \end{bmatrix} \cdot \begin{bmatrix} \psi_{N\Lambda,N\Lambda}(\vec{y};E,\alpha) & \psi_{N\Lambda,N\Sigma}(\vec{y};E,\alpha) \\ \psi_{N\Sigma,N\Lambda}(\vec{y};E,\alpha) & \psi_{N\Sigma,N\Sigma}(\vec{y};E,\alpha) \end{bmatrix}$$

◆ Here, we defined the **E-independent and non-local interaction kernel** U

$$\begin{bmatrix} U_{N\Lambda,N\Lambda}(\vec{x},\vec{y}) & * \\ * & * \end{bmatrix} \equiv \sum_{\alpha'} \int \frac{dE'}{2\pi} \begin{bmatrix} K_{N\Lambda,N\Lambda}(\vec{x};E',\alpha') & * \\ * & * \end{bmatrix} \cdot \begin{bmatrix} \tilde{\psi}_{N\Lambda,N\Lambda}(\vec{y};E',\alpha') & * \\ * & * \end{bmatrix}$$

Energy-independent, non-local interaction kernel (2)

[infinite volume]

➢ The factorization is possible.

◆ Assume that BS wa

$$\left\{ \begin{bmatrix} \psi_{N\Lambda,N\Lambda}(\vec{x};E,\alpha) \\ \psi_{N\Sigma,N\Lambda}(\vec{x};E,\alpha) \end{bmatrix} \right.$$

BS wave functions l

$$\int d^3y \begin{bmatrix} \tilde{\psi}_{N\Lambda,N\Lambda}(\vec{x};E', \\ \tilde{\psi}_{N\Sigma,N\Lambda}(\vec{x};E', \end{bmatrix}$$

Comments:

◆ U does not depend on a particular value of E.
(U is constructed by integrating over E. U is an averaged object)

◆ U is most generally a non-local integration operator.

◆ The factorization is possible
with E-independent and non-local interaction kenel U.
(But U may not be unique.)

◆ **Factorization** is possible

$$\begin{bmatrix} K_{N\Lambda,N\Lambda}(\vec{x};E,\alpha) & K_{N\Lambda,N\Sigma}(\vec{x};E,\alpha) \\ K_{N\Sigma,N\Lambda}(\vec{x};E,\alpha) & K_{N\Sigma,N\Sigma}(\vec{x};E,\alpha) \end{bmatrix}$$

$$= \sum_{\alpha'} \int \frac{dE'}{2\pi} \begin{bmatrix} K_{N\Lambda,N\Lambda}(\vec{x};E',\alpha') & * \\ * & * \end{bmatrix} \cdot \int d^3y \begin{bmatrix} \tilde{\psi}_{N\Lambda,N\Lambda}(\vec{y};E',\alpha') & * \\ * & * \end{bmatrix} \cdot \begin{bmatrix} \psi_{N\Lambda,N\Lambda}(\vec{y};E,\alpha) & * \\ * & * \end{bmatrix}$$

$$= \int d^3y \begin{bmatrix} U_{N\Lambda,N\Lambda}(\vec{x},\vec{y}) & U_{N\Lambda,N\Sigma}(\vec{x},\vec{y}) \\ U_{N\Sigma,N\Lambda}(\vec{x},\vec{y}) & U_{N\Sigma,N\Sigma}(\vec{x},\vec{y}) \end{bmatrix} \cdot \begin{bmatrix} \psi_{N\Lambda,N\Lambda}(\vec{y};E,\alpha) & \psi_{N\Lambda,N\Sigma}(\vec{y};E,\alpha) \\ \psi_{N\Sigma,N\Lambda}(\vec{y};E,\alpha) & \psi_{N\Sigma,N\Sigma}(\vec{y};E,\alpha) \end{bmatrix}$$

◆ Here, we defined the E-independent and non-local interaction kernel U

$$\begin{bmatrix} U_{N\Lambda,N\Lambda}(\vec{x},\vec{y}) & * \\ * & * \end{bmatrix} \equiv \sum_{\alpha'} \int \frac{dE'}{2\pi} \begin{bmatrix} K_{N\Lambda,N\Lambda}(\vec{x};E',\alpha') & * \\ * & * \end{bmatrix} \cdot \begin{bmatrix} \tilde{\psi}_{N\Lambda,N\Lambda}(\vec{y};E',\alpha') & * \\ * & * \end{bmatrix}$$

An effective Schrodinger equation (coupled channel version) [infinite volume]

- Combining the results so far, we arrive at

An effective Schrodinger eq. (coupled channel version)

$$(\vec{\nabla}^2 + p_E^2)\psi_{N\Lambda}(\vec{x};E) = \int d^3y U_{N\Lambda,N\Lambda}(\vec{x},\vec{y})\psi_{N\Lambda}(\vec{y};E) + \int d^3y U_{N\Lambda,N\Sigma}(\vec{x},\vec{y})\psi_{N\Sigma}(\vec{y};E)$$

$$(\vec{\nabla}^2 + q_E^2)\psi_{N\Sigma}(\vec{x};E) = \int d^3y U_{N\Sigma,N\Lambda}(\vec{x},\vec{y})\psi_{N\Lambda}(\vec{y};E) + \int d^3y U_{N\Sigma,N\Sigma}(\vec{x},\vec{y})\psi_{N\Sigma}(\vec{y};E)$$

$$E = \sqrt{m_N^2 + \vec{p}_E^{\,2}} + \sqrt{m_\Lambda^2 + \vec{p}_E^{\,2}} = \sqrt{m_N^2 + \vec{q}_E^{\,2}} + \sqrt{m_\Sigma^2 + \vec{q}_E^{\,2}}$$

- At each E, this coupled equation generates the following BS wave function as solutions, which contain T-matrix of QCD in their long distance parts.

$$\begin{cases} \psi_{N\Lambda,N\Lambda}(\vec{x};E) \equiv Z_N^{-1/2} Z_\Lambda^{-1/2} \langle 0 | N(\vec{x})\Lambda(0) | N(\vec{p})\Lambda(-\vec{p}), in\rangle \sim e^{i\vec{p}\cdot\vec{r}} + \dfrac{\lambda^{1/2}(s,m_N^2,m_\Lambda^2)}{s} \mathcal{T}_{N\Lambda,N\Lambda}(s) \dfrac{e^{ipr}}{pr} + \cdots \\ \psi_{N\Sigma,N\Lambda}(\vec{x};E) \equiv Z_N^{-1/2} Z_\Sigma^{-1/2} \langle 0 | N(\vec{x})\Sigma(0) | N(\vec{p})\Lambda(-\vec{p}), in\rangle \sim \dfrac{\lambda^{1/2}(s,m_N^2,m_\Sigma^2)}{s} \mathcal{T}_{N\Sigma,N\Lambda}(s) \dfrac{e^{iqr}}{qr} + \cdots \end{cases}$$

$$\begin{cases} \psi_{N\Lambda,N\Sigma}(\vec{x};E) \equiv Z_N^{-1/2} Z_\Lambda^{-1/2} \langle 0 | N(\vec{x})\Lambda(0) | N(\vec{q})\Sigma(-\vec{q}), in\rangle \sim \dfrac{\lambda^{1/2}(s,m_N^2,m_\Lambda^2)}{s} \mathcal{T}_{N\Lambda,N\Sigma}(s) \dfrac{e^{ipr}}{pr} + \cdots \\ \psi_{N\Sigma,N\Sigma}(\vec{x};E) \equiv Z_N^{-1/2} Z_\Sigma^{-1/2} \langle 0 | N(\vec{x})\Sigma(0) | N(\vec{q})\Sigma(-\vec{q}), in\rangle \sim e^{i\vec{q}\cdot\vec{r}} + \dfrac{\lambda^{1/2}(s,m_N^2,m_\Sigma^2)}{s} \mathcal{T}_{N\Sigma,N\Sigma}(s) \dfrac{e^{iqr}}{qr} + \cdots \end{cases}$$

- T-matrix of QCD is obtained by solving this coupled effective Schrodinger equation, once the E-independent and non-local interaction kernel U has been constructed.

Construction of the interaction kernel U in the finite volume [finite volume] (10)

- Choose a sufficiently large L (>> R/2) so as not to modify the internal region.
- **Derivative expansion** (an approximate construction of the E-independent and non-local interaction kernel.) For simplicity, keep only the local contribution.

$$U_{N\Lambda,N\Lambda}(\vec{x},\vec{y}) = \left\{V_{N\Lambda,N\Lambda}(\vec{x}) + \cdots\right\}\delta^3(\vec{x}-\vec{y}), \quad \text{etc.}$$

- BS wave functions for two energy eigenstate $E=E_0$ and E_1. (**variational method**)

lowest-lying state

$$\psi_{N\Lambda}(\vec{x};E_0) \equiv Z_N^{-1/2} Z_\Lambda^{-1/2}\langle 0 | N(\vec{x})\Lambda(0) | E_0\rangle$$

$$\psi_{N\Sigma}(\vec{x};E_0) \equiv Z_N^{-1/2} Z_\Sigma^{-1/2}\langle 0 | N(\vec{x})\Sigma(0) | E_0\rangle$$

1st excited state

$$\psi_{N\Lambda}(\vec{x};E_1) \equiv Z_N^{-1/2} Z_\Lambda^{-1/2}\langle 0 | N(\vec{x})\Lambda(0) | E_1\rangle$$

$$\psi_{N\Sigma}(\vec{x};E_1) \equiv Z_N^{-1/2} Z_\Sigma^{-1/2}\langle 0 | N(\vec{x})\Sigma(0) | E_1\rangle$$

- These BS wave functions should satisfy **the coupled effective Schrodinger eq.**

$$\begin{cases} (\vec{\nabla}^2 + p_i^2)\psi_{N\Lambda}(\vec{x};E_i) = V_{N\Lambda,N\Lambda}(\vec{x})\psi_{N\Lambda}(\vec{x};E_i) + V_{N\Lambda,N\Sigma}(\vec{x})\psi_{N\Sigma}(\vec{x};E_i) \\ (\vec{\nabla}^2 + q_i^2)\psi_{N\Sigma}(\vec{x};E_i) = V_{N\Sigma,N\Lambda}(\vec{x})\psi_{N\Lambda}(\vec{x};E_i) + V_{N\Sigma,N\Sigma}(\vec{x})\psi_{N\Sigma}(\vec{x};E_i) \end{cases} \quad \text{(i=0,1)}$$

$$E_i = \sqrt{m_N^2 + \vec{p}_i^{\,2}} + \sqrt{m_\Lambda^2 + \vec{p}_i^{\,2}} = \sqrt{m_N^2 + \vec{q}_i^{\,2}} + \sqrt{m_\Sigma^2 + \vec{q}_i^{\,2}}$$

- **Solve this coupled equations (i=0,1) back for the interaction kernels** by inserting the BS wave functions. (4 unknown from 4 equations)
- It is important to examine the **convergence of derivative expansion**.

Construction of the interaction kernel U in the finite volume [finite volume]

- Choose a sufficient
- **Derivative expansio**
  interaction kernel.)

$$U_{N\Lambda,N\Lambda}(\vec{x},\vec{y}) = \{V$$

- BS wave functions

lowest-lying state

$$\psi_{N\Lambda}(\vec{x};E_0) \equiv Z_N^{-1}$$

$$\psi_{N\Sigma}(\vec{x};E_0) \equiv Z_N^{-1}$$

Comments:

Once it is constructed on the lattice,

- ◆ take the infinite volume limit
- ◆ solve the coupled effective Schrodinger equation
  in the infinite volume to reconstruct T-matrix.
- ◆ The resulting T-matrix interpolates the QCD T-matrix
  which was originally contained in the BS wave functions.

- These BS wave functions should satisfy **the coupled effective Schrodinger eq.**

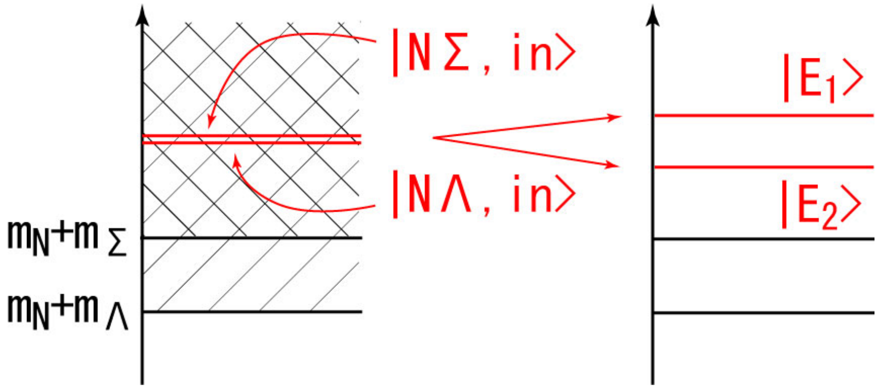
$$\begin{cases} (\vec{\nabla}^2 + p_i^2)\psi_{N\Lambda}(\vec{x};E_i) = V_{N\Lambda,N\Lambda}(\vec{x})\psi_{N\Lambda}(\vec{x};E_i) + V_{N\Lambda,N\Sigma}(\vec{x})\psi_{N\Sigma}(\vec{x};E_i) \\ (\vec{\nabla}^2 + q_i^2)\psi_{N\Sigma}(\vec{x};E_i) = V_{N\Sigma,N\Lambda}(\vec{x})\psi_{N\Lambda}(\vec{x};E_i) + V_{N\Sigma,N\Sigma}(\vec{x})\psi_{N\Sigma}(\vec{x};E_i) \end{cases} \quad \text{(i=0,1)}$$

$$E_i = \sqrt{m_N^2 + \vec{p}_i^2} + \sqrt{m_\Lambda^2 + \vec{p}_i^2} = \sqrt{m_N^2 + \vec{q}_i^2} + \sqrt{m_\Sigma^2 + \vec{q}_i^2}$$

- **Solve this coupled equations (i=0,1) back for the interaction kernels**
  by inserting the BS wave functions. (4 unknown from 4 equations)
- It is important to examine the **convergence of derivative expansion**.

The point

- How our method avoids the problem, which was mentioned in the introduction



- If we stick to a single energy eigen state in the finite volume, all we can do is to impose a constraint on the S-matrix. (It is not easy to obtain each S-matrix element separately.)
  S.He et al., JHEP0507, 011 (2004).

- In our method, the key role is played by the **E-independent, non-local interaction kernel**.

$$\begin{cases} (\vec{\nabla}^2 + p_i^2)\psi_{N\Lambda}(\vec{x};E_i) = V_{N\Lambda,N\Lambda}(\vec{x})\psi_{N\Lambda}(\vec{x};E_i) + V_{N\Lambda,N\Sigma}(\vec{x})\psi_{N\Sigma}(\vec{x};E_i) \\ (\vec{\nabla}^2 + q_i^2)\psi_{N\Sigma}(\vec{x};E_i) = V_{N\Sigma,N\Lambda}(\vec{x})\psi_{N\Lambda}(\vec{x};E_i) + V_{N\Sigma,N\Sigma}(\vec{x})\psi_{N\Sigma}(\vec{x};E_i) \end{cases}$$

[LO derivative expansion]

- It gathers the information scattered around different energies in finite volume.
- Reconstruction of T-matrix is performed by using the coupled effective Schrodinger equation in infinite volume.
- The reconstructed T-matrix is an interpolation of the T-matrix contained in the BS wave functions generated by lattice QCD. (Interpolation is performed by E-independent, non-local interaction kernel.)

Summary

- We have proposed a new method to calculate T-matrix of QCD above inelastic thresholds.
- In our method,
  - E-independent and non-local interaction kernel plays a key role.
    It gathers information scattered around different energies to calculate S-matrix.
  - Numerically, the most challenging part is the variational method for BS wave functions of excited states.
    This turns out to be feasible. [K.Sasaki at previous session]
- Outlook
  - The method can be applied to more complicated systems such as $\Lambda\Lambda$-$N\Xi$-$\Sigma\Sigma$ coupled system. [K.Sasaki at previous session].
  - The method may be also applied to much more complicated systems such as NN-NNπ coupled system and NK-NKπ coupled system. (← future plan)
    However, once three hadron BS wave function is involved, numerical cost becomes huge. [T.Doi at previous session]
  - The method can be, in principle, applied to NK^{bar}-$\Sigma\pi$ coupled system for $\Lambda(1405)$.
    However, it is so far challenging to obtain BS wave function with annihilation diagram (sink). [Efficient algorithm has to be developed.]