

First Applications of the Stochastic LapH method in Lattice QCD Spectroscopy

Speaker: Chik Him Wong*

Authors:

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(Hadron Spectrum Collaboration)

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Lattice 2010

Outline

- Objectives
- Reviews
 - Stochastic LapH Method
 - Simulation Details
- First Applications
 - Applications on Connected n -pt Correlators
 - Nucleon N
 - Pion π
 - Applications on Disconnected Diagrams in Isoscalars
 - Isoscalar Pseudoscalar η
 - Isoscalar Scalar σ
 - Application on Multihadron Diagrams
 - $\pi\pi \rightarrow \pi\pi$ Correlator
 - $\rho \rightarrow \pi\pi$ Decay
- Current Work & Future Plans

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Objectives

- **Main Goal: Compute the Hadron Spectrum that includes Isoscalar and Multihadron states**
- Via Applications of the Stochastic Laplacian Heaviside(LapH) method on a variety of Diagrams, we can:
 - Verify the feasibility & capability of the innovative method
 - Determine the dilution schemes to be used in current & future runs
 - Study the improvement in efficiency compared with ordinary LapH method

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Stochastic LapH Method

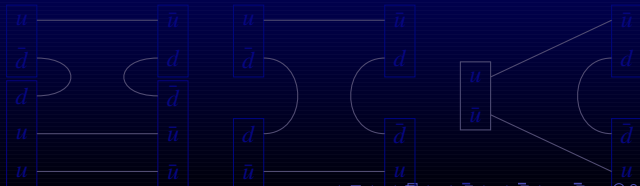
- Stochastic estimation of “Distillation” perambulator M^{-1} by introducing $Z_N = e^{i2\pi n/N}$ noises ρ in Time \times Spin \times LapH Subspace improved with “Dilution” technique:

$$M\varphi_D = P^{[D]}\rho, \sum_D E(P^{[D]}\rho(P^{[D]}\rho)^*) = 1 \rightarrow M^{-1} = \sum_D E(\varphi_D(P^{[D]}\rho)^*)$$

- Expected Advantages:

- Suppressed High-lying mode contamination due to the LapH projection
- Number of Inversions stays almost constant for any lattice size L for a given maximum cutoff eigenvalue due to Dilution technique
- Factorization into Single Hadron Operators provides a convenient way of constructing different diagrams without storage of huge perambulators:

Store:



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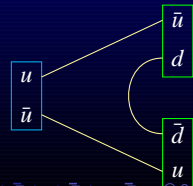
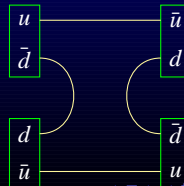
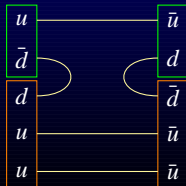
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Simulation Details

- Action:

- Anisotropic Clover Action
- Symanzik Improved Gauge Action
- $a_s/a_t = 3.5, a_s = 0.12$ fm

- Available and relevant gauge configurations:

m_π (MeV)	m_{q_l}	Lattice Size($N_x^3 \times N_t$)	N_f	
380	-0.0840	$16^3 \times 128$	2+1	First Application
360	-0.0840	$24^3 \times 128$	2+1	Current Work
220	-0.0860	$24^3 \times 128$	2+1	Current Work
220	-0.0860	$32^3 \times 128$	2+1	Current Work

- Selected Dilution Schemes:

- Z_4 Noise

Dilution Subspace	t_0 - t Propagator	t - t Propagator
Time	Full	Interlace 16
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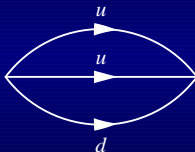
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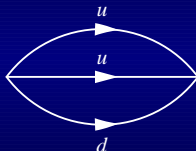
- Nucleon N :



- Connected Diagram
- 3 Noises, $1t_0$
- 52 configs
- $V = 16^3 \times 128$, 32 Laplacian Eigenvectors

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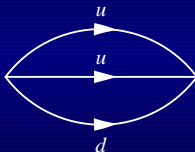
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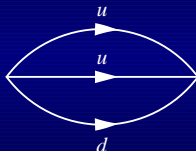
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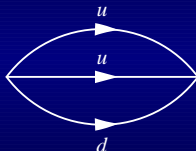
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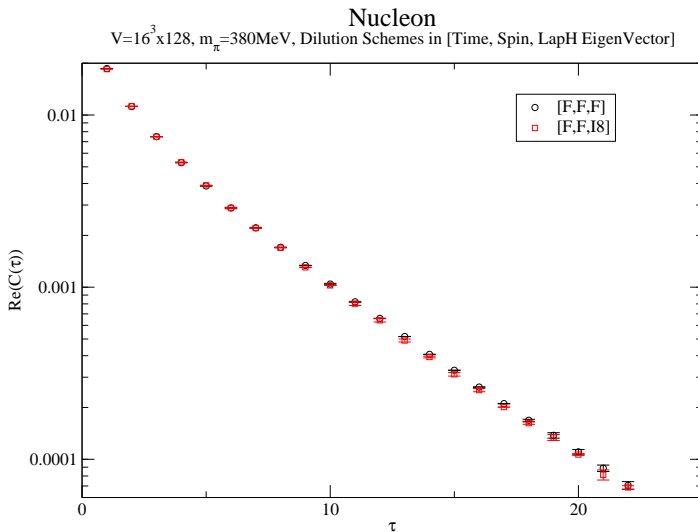
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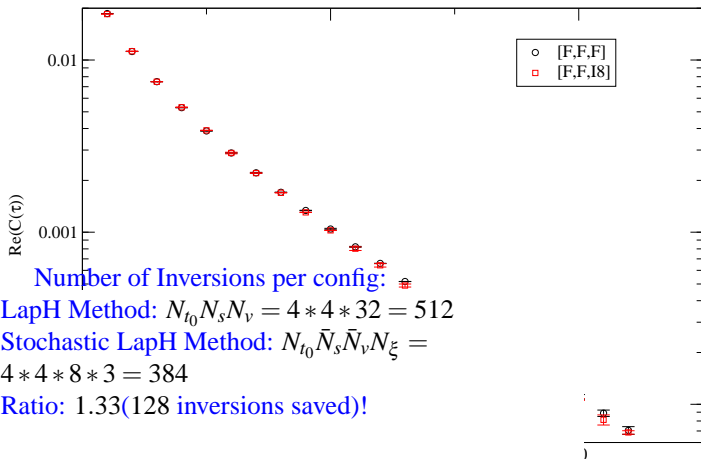
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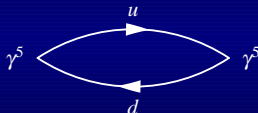
Nucleon

$V=16^3 \times 128$, $m_\pi=380\text{MeV}$, Dilution Schemes in [Time, Spin, LapH EigenVector]



Applications on Connected 2-pt Correlators

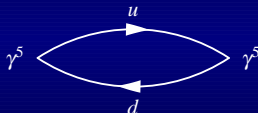
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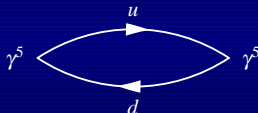
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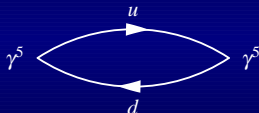
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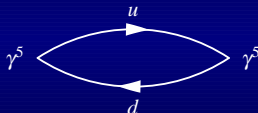
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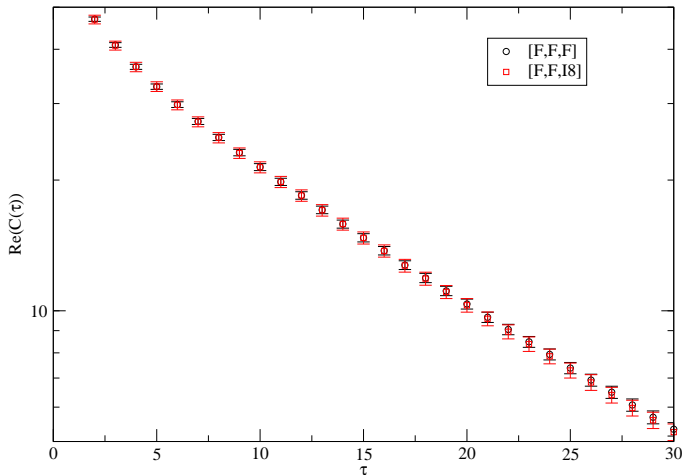


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Applications on Connected 2-pt Correlators: π

Pion π

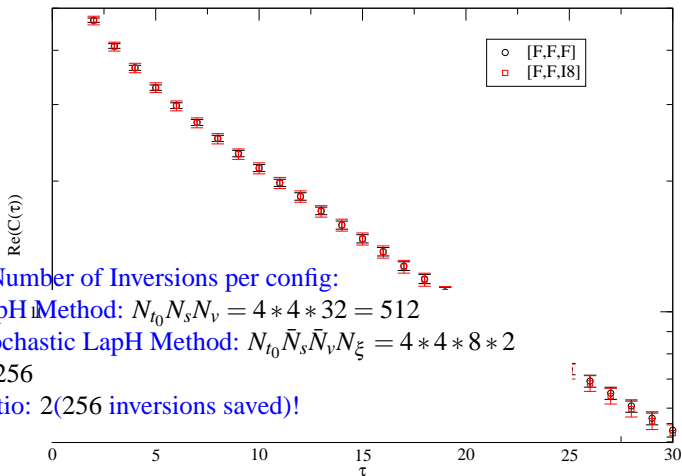
$V=16^3$, $m_\pi=380\text{MeV}$, Dilution Schemes in [Time, Spin, LapH EigenVector]



Applications on Connected 2-pt Correlators: π

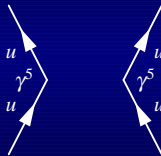
Pion π

$V=16^3 \times 128$, $m_\pi=380\text{MeV}$, Dilution Schemes in [Time, Spin, LapH EigenVector]



Applications on Disconnected Diagrams in Isoscalars

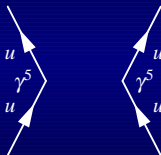
- Isoscalar Pseudoscalar η :



- Disconnected Diagram
- 1 Noise, all t_0 s
- 52 configs
- $V = 16^3 \times 128$, 32 Laplacian Eigenvectors

Applications on Disconnected Diagrams in Isoscalars

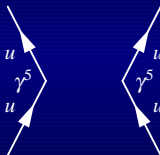
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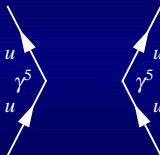
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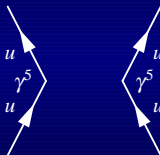
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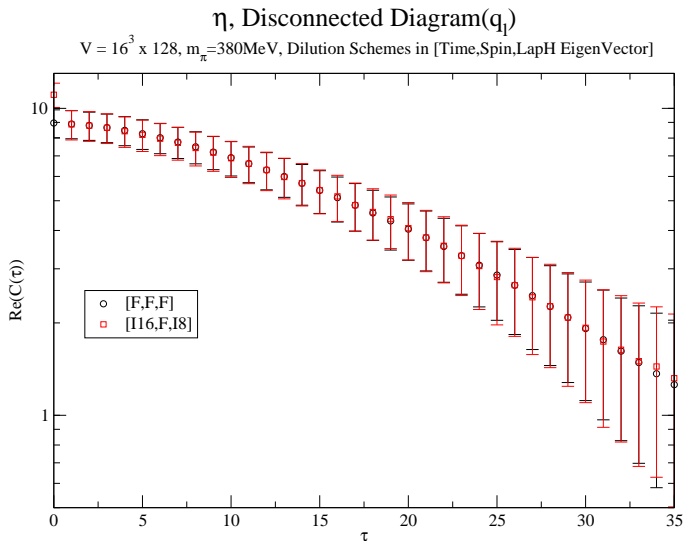
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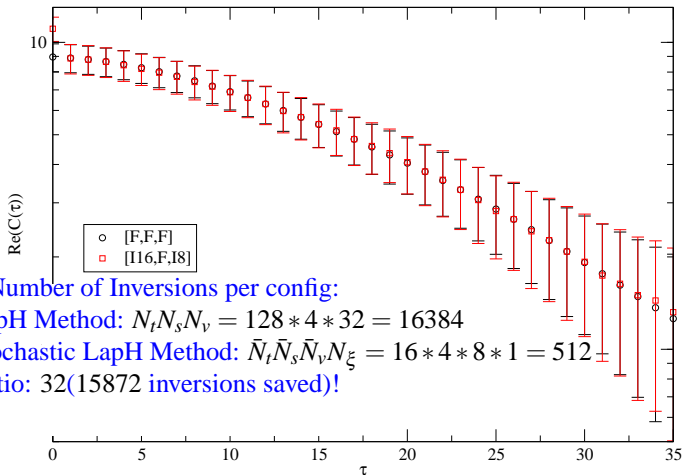
Applications on Disconnected Diagrams in Isoscalars: η



Applications on Disconnected Diagrams in Isoscalars: η

η , Disconnected Diagram(q_l)

$V = 16^3 \times 128$, $m_{\pi} = 380\text{MeV}$, Dilution Schemes in [Time, Spin, LapH EigenVector]



Number of Inversions per config:

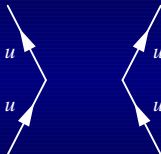
LapH Method: $N_t N_s N_v = 128 * 4 * 32 = 16384$

Stochastic LapH Method: $\bar{N}_t \bar{N}_s \bar{N}_v N_\xi = 16 * 4 * 8 * 1 = 512$

Ratio: 32(15872 inversions saved)!

Applications on Disconnected Diagrams in Isoscalars

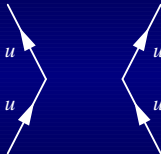
- Isoscalar Scalar σ :



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- Vacuum Expectation Value subtracted
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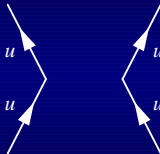
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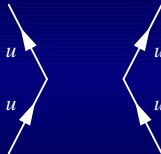
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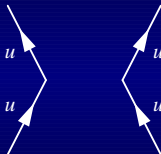
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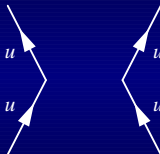
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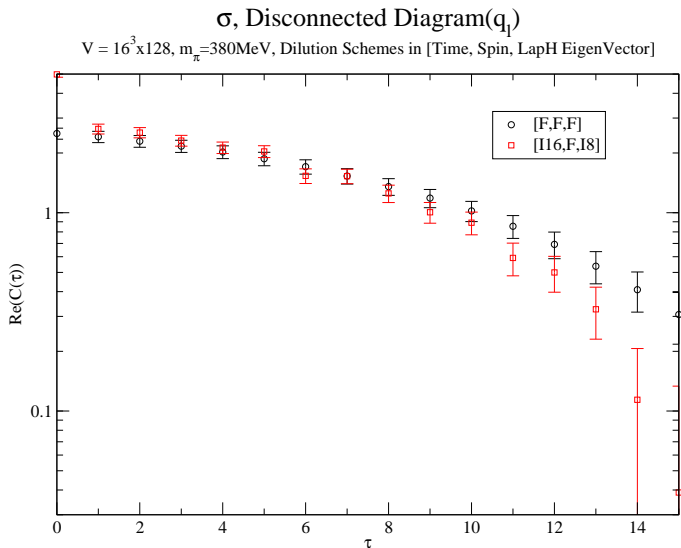
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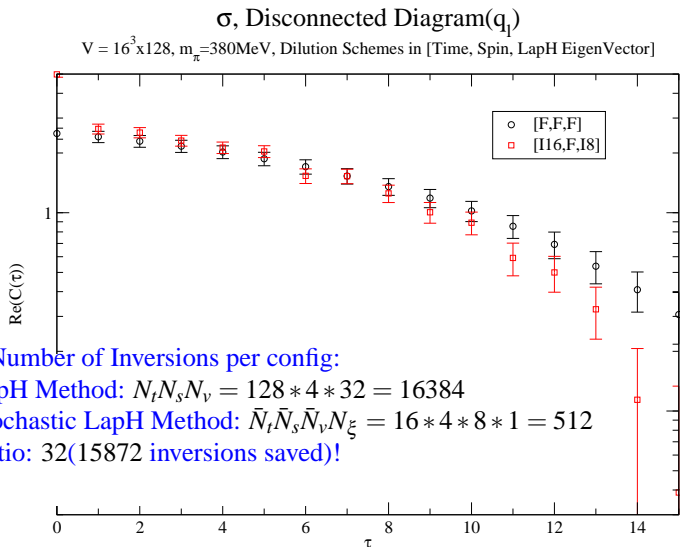


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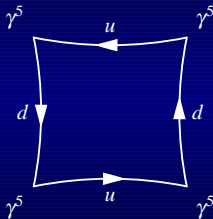


Applications on Disconnected Diagrams in Isoscalars: σ



Applications on Multihadron Diagrams

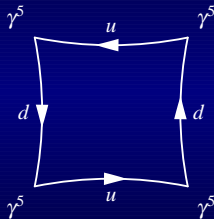
- $\pi\pi \rightarrow \pi\pi$ Correlator:



- Box Diagram
- 4 Noises, 4 t_0 s
- 52 configs
- $V = 16^3 \times 128$, 32 eigenvectors

Applications on Multihadron Diagrams

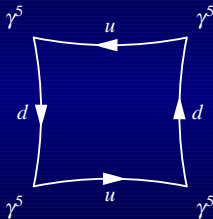
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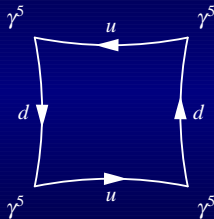
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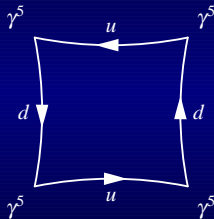
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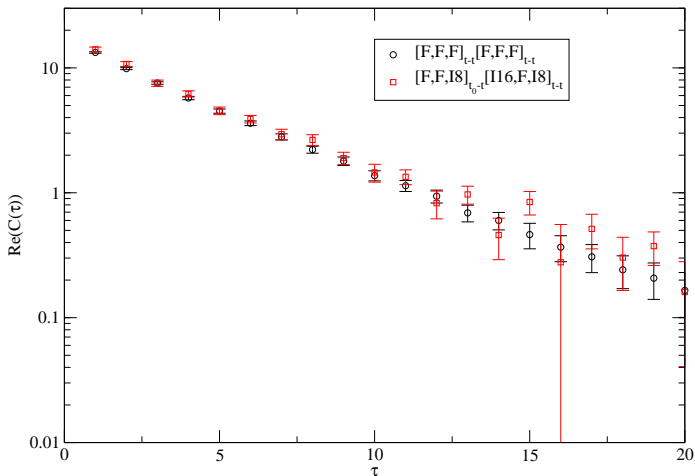


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$\pi\pi \rightarrow \pi\pi$, Box Diagram

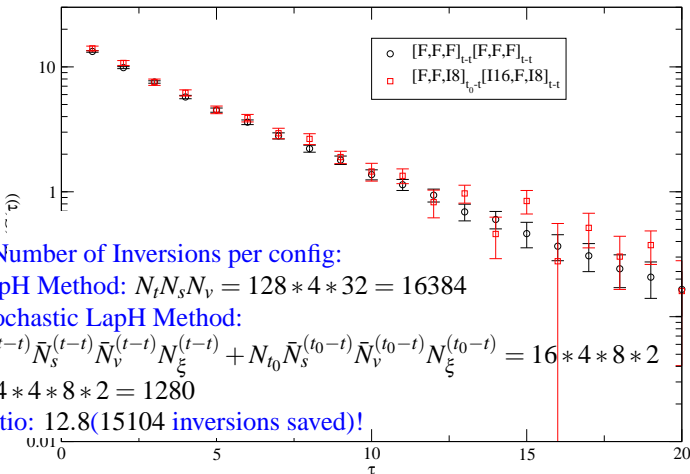
$V=16^3 \times 128$, $m_\pi=380\text{MeV}$, Dilution Schemes in [Time, Spin, LapH EigenVector]



Applications on Multihadron Diagrams: $\pi\pi \rightarrow \pi\pi$

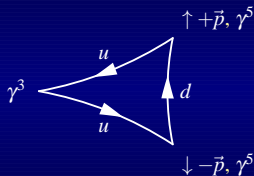
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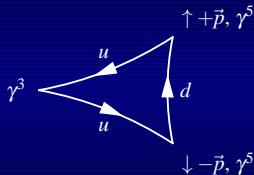
- $\rho \rightarrow \pi\pi$ Decay :



- Triangle Diagram
- $\vec{p} = (0, 0, 1)$
- 2 sets of 3 Noises, 4 t_0 s each set
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Applications on Multihadron Diagrams

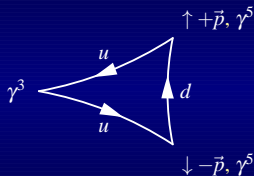
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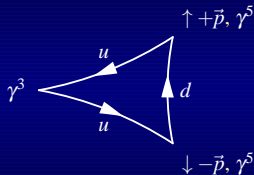
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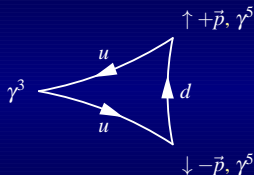
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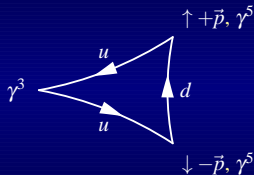
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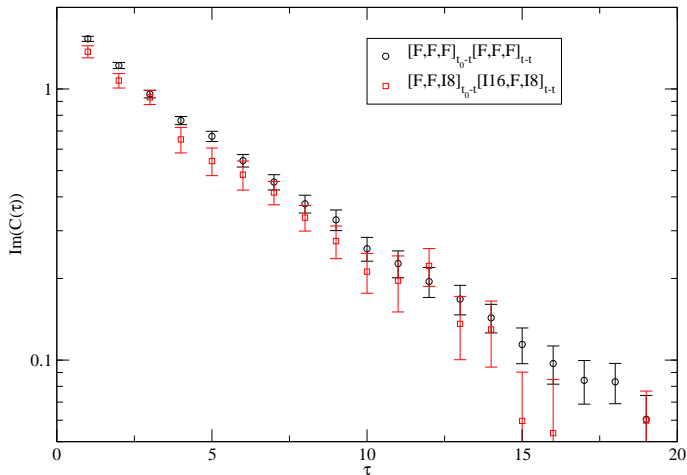


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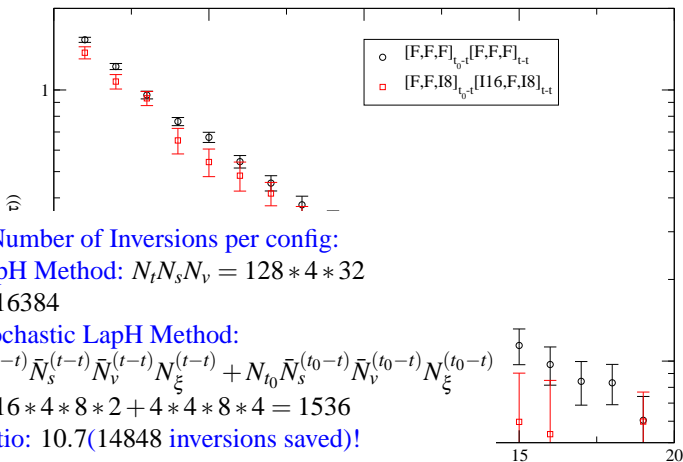
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Current Work & Future Plans

• Summary

- The Stochastic LapH method provides a promising approach for multihadron & isoscalar calculations
- The Stochastic LapH method saves a lot of work compared with the LapH method
- $V = 16^3$: It is an alternative; Larger volumes: Ordinary LapH is not possible!

• Current Work

- Generation of Quark Sources and Sinks
- Generation of Hadron “Source”s and “Sink”s
- Gauge Configurations used:

m_π (MeV)	m_{q_l}	Lattice Size($N_s^3 \times N_t$)	N_f
360	-0.0840	$24^3 \times 128$	2+1
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- Hadronic Spectrum with Multiparticle states
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Q & A