Scaling study of quenched quark mass using 2 HEX smeared fermions

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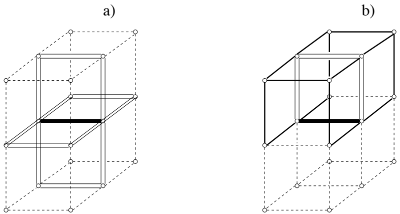
Outline

Why Smear?

- improves chirality of wilson fermions: eigenvalue spectrum closer to a chiral one
  $\longrightarrow$ improved stability of dynamical simulations
  $\longrightarrow$ suppressing exceptionals in quenched simulations
- simulations at smaller pion masses possible
- better agreement with perturbation theory (c_{sw} closer to 1)
  Hoffmann, Hasenfratz, Schaefer [PoSLAT 2007]

HEX smearing

- HYP smearing Hasenfratz, Knechtli [Phys.Rev.D 2001]



- HMC requires differentiable smearing: replace APE-links with EXP(stout)-links Morningstar and Peardon [Phys.Rev.D 2004]
- We choose 2 HEX smearing steps with moderate smearing parameters

Locality

- sufficient for Symanzik scaling: doubler free and local action
- two notions of locality
  1. local in coordinate space, i.e.

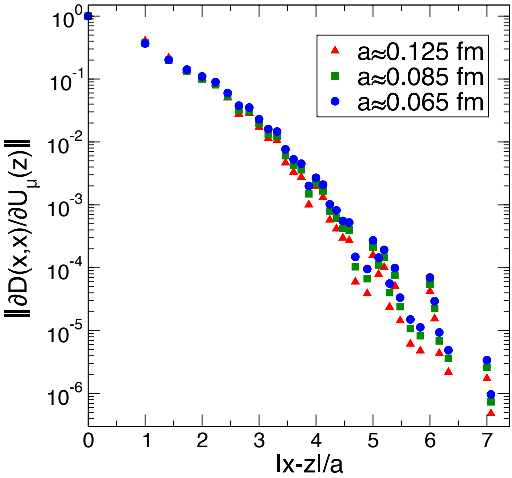
     $$\|D(x,y)\| < \mathrm{const.}\, e^{-\lambda|x-y|}$$

     with $\lambda = \mathcal{O}(a^{-1})$: trivially fulfilled, only nearest neighbour coupling in our case
  2. locality with respect to gauge fields, i.e.

     $$\left\| \frac{\delta D(x,x)}{\delta U(z)} \right\| < \mathrm{const.}\, e^{-\lambda|x-z|}$$

     also with $\lambda = \mathcal{O}(a^{-1})$
- our action is local (and in fact even ultralocal)

Gauge field locality



- we find $\lambda \simeq 2.2a^{-1}$ for $3 \leq |x-z| \leq 6$ in case of 6 EXP

  BMW [Science 2008]
- 2 HEX by construction even “more ultralocal“in gauge space

Setup for hadron masses scaling study

- $N_f = 3$ hadron mass scaling study at 4 betas (from $a \approx 0.06\,\mathrm{fm}$ to $0.2\,\mathrm{fm}$) and at least 4 masses per beta
- tree level improved Symanzik gauge action Lüscher, Weisz [Phys.Lett.B 1985] with smeared clover improved wilson operator
- RHMC with different optimizations → cf. BMW [Phys.Rev.D 2009] for details
- concerning stability (mass gap), topology → cf. also BMW [Phys.Rev.D 2009]
- valence sector: use same action and quark masses as in sea (unitary setup)
- compare to previously obtained 6 EXP results

Determination of hadron masses

- apply correlated cosh/sinh fits to correlators
- calculate PCAC-mass from plateau of $\langle \partial_0 A_0(t)\, P(0)\rangle / \langle P(t)\, P(0)\rangle$
- interpolate aM_N, aM_Δ in m_{PCAC} to obtain quantities at physically motivated ratio $M_\pi / M_\rho \doteq \sqrt{2(M_K^{\mathrm{phys}})^2 - (M_\pi^{\mathrm{phys}})^2} / M_\phi^{\mathrm{phys}} \approx 0.67$
- extrapolate resulting M_N, M_Δ to the continuum assuming $\mathcal{O}(\alpha a)$ or $\mathcal{O}(a^2)$ scaling

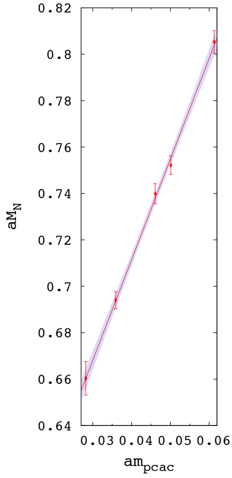
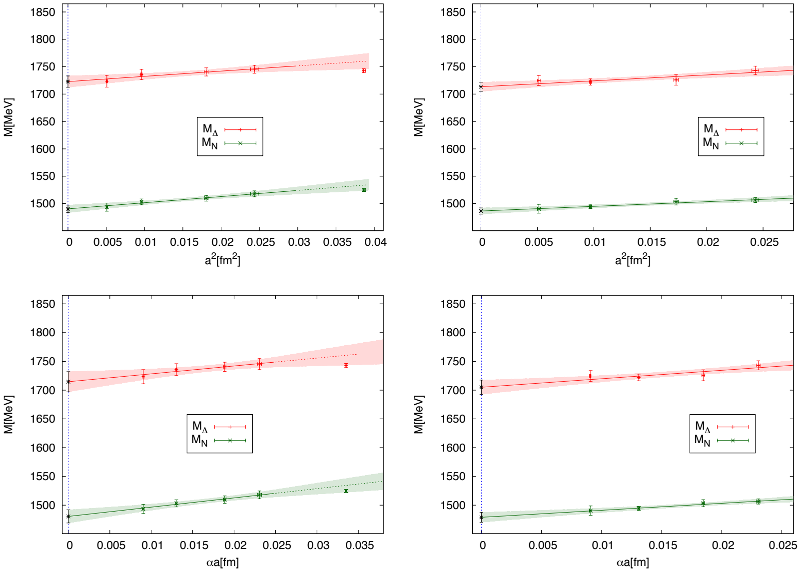


Figure: from BMW [Phys.Rev.D 2009]

Scaling Plots (6 EXP vs. 2 HEX and $\mathcal{O}(a^2)$ vs. $\mathcal{O}(\alpha a)$)



Renormalization

- quark masses Lagrangian parameters → renormalization needed
- using non-perturbative RI-MOM scheme Martinelli et al. [Nucl.Phys.B 1995]: renormalization constant for lattice operator $O(a)$ (gauge fixed to Landau gauge)

$$O(\mu) = Z_O(\mu a, g(a))\, O(a)$$

impose renormalization condition

$$Z_O(\mu a, g(a))\, Z_q^{-1}(\mu a, g(a))\, \Gamma_O(pa)|_{p^2=\mu^2} = 1$$

using

$$\Gamma_O(pa) = \frac{1}{12}\mathrm{Tr}(\Lambda_O(pa), P_O)$$

where

$$\Lambda_O(pa) = S^{-1}(pa)\, G_O(pa) S^{-1}(pa)$$

Renormalization II

- improve signal using trace subtraction Martinelli et al. [Phys.Rev.D 2000], Schierholz et al. [Nucl.Phys.B 2001], Martinelli et al. [Nucl.Phys.B 2001], Maillart, Niedermayer [hep-lat/0807.0030v1]: $S \to \bar{S} \doteq S - \mathrm{Tr}_\mathrm{D} S/4$
- calculate vector current renormalization Z_V via the 3-point/2-point function ratio Göckeler et al. [Phys.Lett.B 2004]:

$$\zeta(t) \doteq \frac{\sum_x \langle \bar{P}(T/2)\, V_4(x,t)\, P(0)\rangle}{\langle \bar{P}(T/2)\, P(0)\rangle}$$

  and using
  $(Z_V)_{3pt}(1 + am^W) = |\zeta(t_0 > T/2) - \zeta(t_0 - T/2)|^{-1}$
- obtaining $(Z_q)_{RI}$ by calculating $(Z_q/Z_V)_{RI} \cdot (Z_V)_{3pt}$
- using Z_V^{cons} or Z_q' in RI-MOM instead yields same results but are more expensive

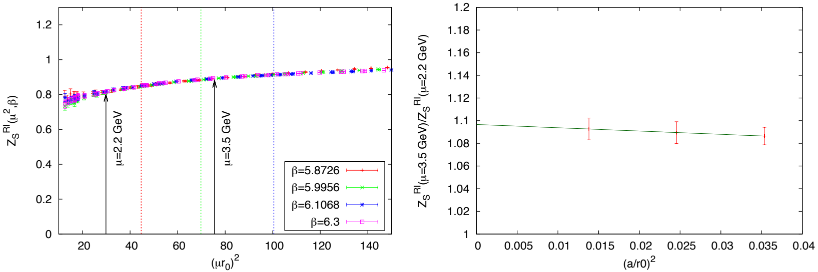
Renormalization III

- "Window condition" of RI-MOM: $\Lambda_{\mathrm{QCD}} \ll \mu \ll 2\pi/a \to$ safe using $\mu \leq \pi/(2a)$
- But: matching to continuum PT from $\mu \simeq 3\,\mathrm{GeV}$, not reachable on coarsest lattices.
- Idea: compute using only finest lattices $(\mu' > \mu'')$
  $R(\mu', \mu'') \doteq \lim_{a\to 0} Z_S(\mu', a)/Z_S(\mu'', a)$
- compute renormalization factor on all lattices by
  $Z_S(\mu', a) \doteq R(\mu', \mu'')\, Z_S(\mu'', a)$
- calculate renormalized quark mass via
  $m^{VWI}(\mu') = (1 - am^W/2)m^W/Z_S(\mu')$, where
  $m^W = m_{\mathrm{bare}} - m_{\mathrm{crit}}$

Setup for determination of the quenched quark mass

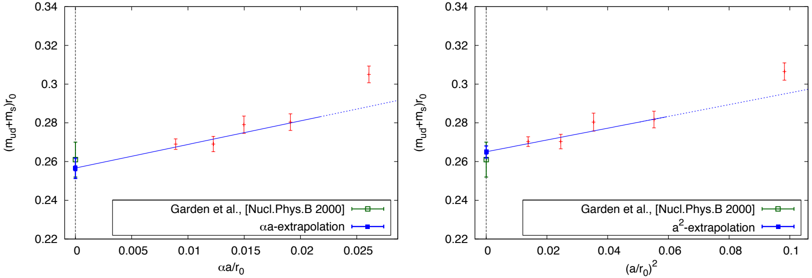
- generate quenched configs to compare against literature
- use the wilson plaquette action because very precise r_0-data available Necco, Sommer [Nucl.Phys.B 2002]
- use 5 betas (0.06 to $0.15\,\mathrm{fm}$) and at least 4 masses at each, furthermore $M_\pi L > 4$ for all masses and betas ($L \approx 1.84\,\mathrm{fm}$)
- extrapolate $Z_S^{RI}(M_\pi^2, \mu)$ linearly in M_π^2 to chiral limit $\forall\,\mu$
- extrapolate Z_S^{RI}-ratios vs αa and a^2 using $\mu' = 3.5\,\mathrm{GeV}$ and $\mu'' = 2.2\,\mathrm{GeV}$
- extrapolate $m^{RI}(3.5\,\mathrm{GeV}, a)$ linearly in αa and a^2
- convert $m^{RI}(3.5\,\mathrm{GeV})$ to $m^{\overline{MS}}(2\,\mathrm{GeV})$ perturbatively

Renormalization factors



- left panel: universality of scalar renormalization (colored vertical bars correspond to $\mu = \pi/(2a)$)
- right panel: continuum extrapolation of Z_S-ratios on 3 finest lattices

Scaling plot



- $(m_s + m_{ud})r_0 = 0.2608(42)(43)$ in perfect agreement with Garden et al. [Nucl.Phys.B 2000] (0.261(9)), good agreement with JLQCD [Phys.Rev.Lett. 1999] (0.274(18)) and Hölbling, Dürr [Phys.Rev.D 2005] (0.312(28))
- can hardly distinguish between $\mathcal{O}(\alpha a)$ or $\mathcal{O}(a^2)$
- continuum limit: $m_s^{\overline{MS}}(2\mathrm{GeV}) = 101.4(1.6)(1.7)$ ($r_0 = 0.49\,\mathrm{fm}$ used)

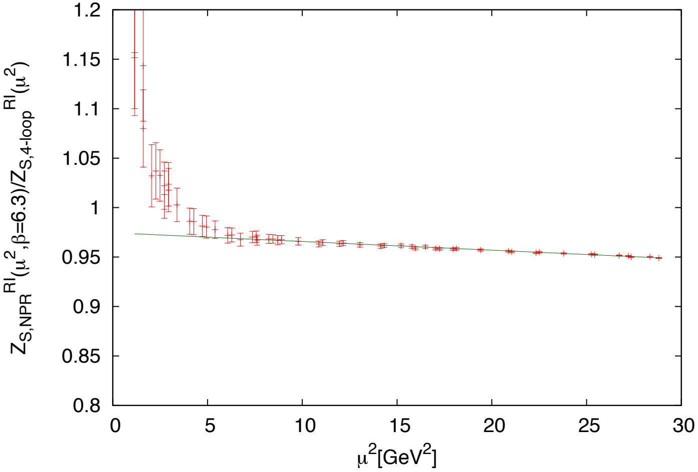
Error handling

- statistical errors: carry out analysis on 2000 bootstrap samples with blocksize 1
- systematical errors: carry out analysis using 3 different fitranges of correlators and assuming $\mathcal{O}(\alpha a)$ or $\mathcal{O}(a^2)$ scaling and accounting for non-vanishing slope in PT matched data $\rightarrow$ obtaining 18 different fits $\rightarrow$ calculate distribution from those, weighted by quality-of-fit Q BMW [Science 2008]
  - mean gives: best estimate of central value
  - variance: systematical error
  - one can hold one source of systematical error fixed and vary the other ones $\rightarrow$ disentangle systematic errors

Summary

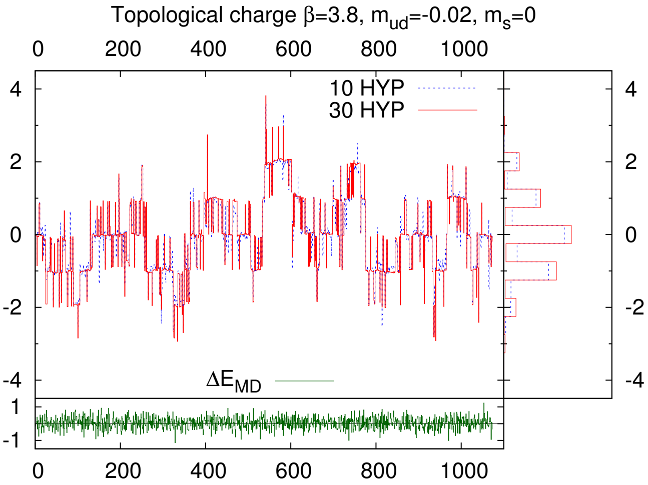
- 2 HEX action is ultralocal by construction
- scaling of hadron masses: very mild scaling and perfect agreement with previously used 6 EXP action Dürr et al. [Science 322,1224 (2008)]
- scaling of quark masses: fairly flat extrapolation, continuum limit in very good agreement with literature
- 2 HEX action has broad scaling region and small corrections
- for preliminary dynamical 2 HEX results, c.f. talks of Antonin Portelli, Alberto Ramos and Julien Frison

Perturbative matching



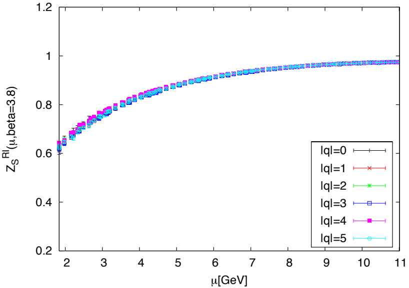
- Matching of $\beta = 6.3$ data to $N_f = 0$ continuum PT

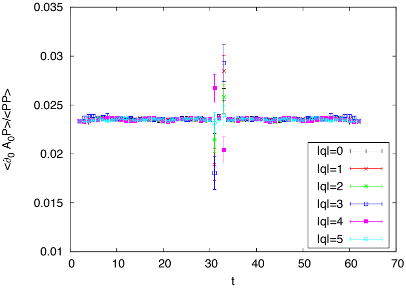
Topology



- Topological charge history for $N_f = 2 + 1$, $a \approx 0.05\,\mathrm{fm}$ and $M_\pi = 219(2)\mathrm{MeV}$

Topology II





- Left panel: topology dependence of quark mass renormalization factor Z_S^{RI}
- Left panel: topology dependence of m_{PCAC}