Masses and decay constants from relativistic highly improved staggered quarks in full lattice QCD

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Lattice 2010

HPQCD collaboration

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Thanks: The MILC collaboration for making their configurations publicly available.

Outline

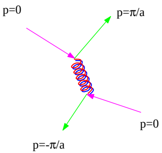
- Motivation.
- Quark discretizations. Staggered quarks.
- Heavy quarks.
- Results for masses and decay constants.

Motivation

- To test lattice field theory as a tool for studying strongly coupled field theories. Compare precision calculations of masses and decay constants with experimental results.
- To calculate theoretical quantities needed in the analysis of experimental data, for example, in the determination of elements of the CKM matrix.

Improved Staggered Quarks

- The staggered action describes 4 tastes (in 4D). The spectrum on the lattice has a multiplicity of states corresponding to the same continuum state. There are unphysical taste-changing interactions that lift the degeneracy between such states.
- These effects are lattice artifacts, of order a^2, and vanish in the continuum limit $a \to 0$. They involve at leading order the exchange of a gluon of momentum $q \approx \pi/a$.
- Such interactions are perturbative for typical values of the lattice spacing, and can be corrected systematically a la Symanzik.



Smear the gauge field to remove the coupling between quarks and gluons with momentum π/a.

- In an unquenched simulation, $\sqrt[4]{\text{det}}$. → "Rooting trick".

Improved Staggered Actions

- ASQ(TAD) (S. Naik, the MILC collaboration, G.P. Lepage.)
  - Discretization errors $\approx \mathcal{O}(\alpha_s \mathrm{a}^2, \mathrm{a}^4)$.
- HYP (A. Hasenfratz, F. Knechtli.)
  - Discretization errors $\approx \mathcal{O}(\mathrm{a}^2)$.
  - Substantially reduced (by a factor $\sim$ 3) taste-changing.
- HISQ (E.F., Q. Mason, C. Davies, K. Hornbostel, G.P. Lepage, H. Trottier.)
  - Discretization errors $\approx \mathcal{O}(\alpha_s \mathrm{a}^2, \mathrm{a}^4)$.
  - Substantially reduced (by a factor of $\sim$ 3) taste-changing.
  - Can be used to study heavy quarks.

Heavy Quarks

- The discretization errors grow with the quark mass as powers of am.
- For a direct simulation, we need:

  $am_h \ll 1$ (heavy quarks)
  $La \gg m_\pi^{-1}$ (light quarks)

- Two scales. Difficult to do directly.
- If m_h is large enough, ⇒ effective field theory (NRQCD, HQET). Very successful for b physics.

Relativistic Heavy Quarks

A relativistic formulation has many advantages:

- The parameters of an effective theory have to be matched to QCD, which is both difficult and introduces another sistematic error.
- Some symmetries of the relativistic theory can be lost in the effective theory. For example, for staggered quarks, meson decay constants do not renormalize because of PCAC.
- Using the same formulation for the heavy and the light quarks is simpler, and allows, for example, to calculate accurate ratios of quark masses.
- Using the same formulation for the heavy and the light quarks also provides very stringent tests of the lattice methods, because there are very few free parameters: all the calculations should give the right answers once those are fixed.

Relativistic Heavy Quarks

- m_c, m_b quite large for relativistic quark actions, $am_h \lesssim 1$.
- However, if we use a very accurate action (HISQ) and fine enough lattices (MILC), it is possible to get accurate results.
- Errors for HISQ: $\mathcal{O}((am)^4, \alpha_s (am)^2)$.
- Non-relativistic system: further suppression by factors of (v/c), by a simple adjustment of the coefficient of the Naik term which makes $c^2 = 1$.
- Can reduce the errors to the few percent level.
- We use the same action for heavy-heavy and heavy-light systems → extensive consistency checks.

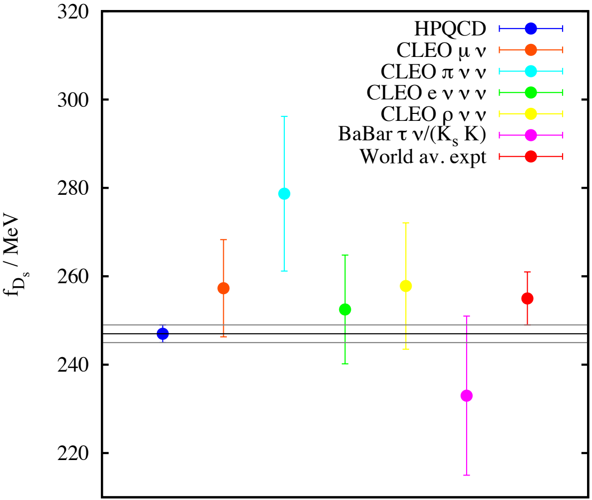
Fixing the parameters

- Scale: lattice spacing a:
  - We use r_1, related to the heavy quark potential. The value of r_1/a is calculated with high precision on the lattice.
  - We use several quantities to then calculate r_1 in physical units: $m_{\Upsilon'} - m_{\Upsilon}$, $m_{D_s} - m_{\eta_c}/2$, f_{η_s}.
- Quark masses: $m_{u,d}, m_s, m_c, m_b$.
  Fixed by $m_\pi, m_K, m_{\eta_c}, m_{\eta_b}$.
- In the HISQ heavy quark formulation: improvement parameter ϵ. Fixed by requiring relativistic dispersion relation, $c^2 = 1$.

Ensembles

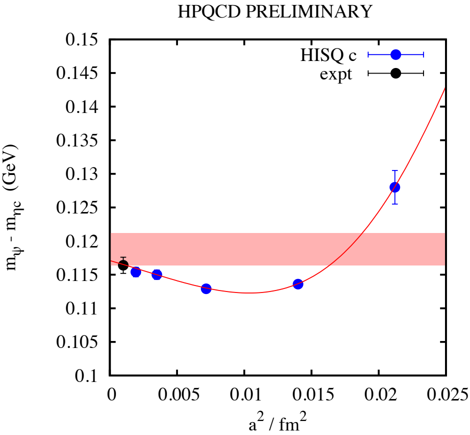
- MILC ensembles: Improved gluon action and $2+1$ ASQTAD sea quarks: $(m_l,\ m_l,\ m_s)$.
- Five values of the lattice spacing a ranging from 0.044 to 0.15 fm.
- Physical m_s and m_c.
- For now an extrapolation in $m_h \Rightarrow m_b$ is needed. Finer lattices would allow us to simulate at the physical m_b mass.
- $m_l = m_s/10$ to $m_s/2.5$. Chiral extrapolation.

f_{D_s} update



Preliminary: $f_{D_s} = 247(2)$.

Hyperfine splitting



Preliminary: $m_\psi - m_{\eta_c} = 118.8(2.4)$ MeV.

Quark masses

- We can calculate very accurate quark mass ratios in the lattice.
- Because we use the same formalism for different quarks:

$$\frac{\overline{m_b}(\mu)}{\overline{m_c}(\mu)} = \left(\frac{m_{0b}}{m_{0c}}\right)_{a\to 0}$$

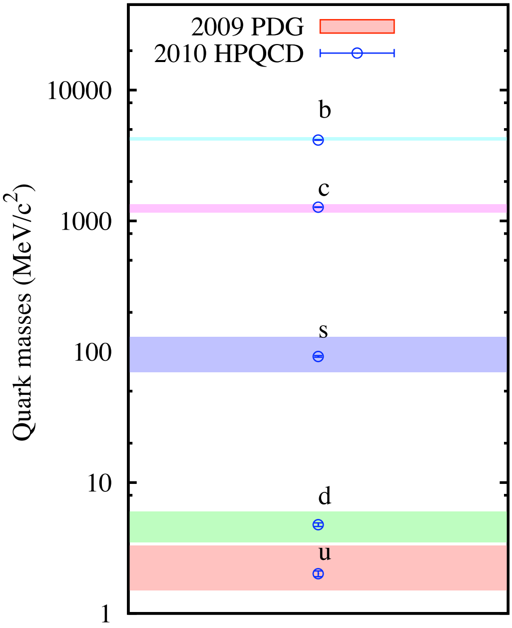
- Leverage the precision in m_c determination from current-current correlators + continuum PT (collaboration with K.G. Chetyrkin et al), $m_c(m_c) = 1.273(6)$, to get m_s:

$$\frac{m_c}{m_s} = 11.85(16), \quad m_s(2GeV) = 92.2(1.3)MeV$$

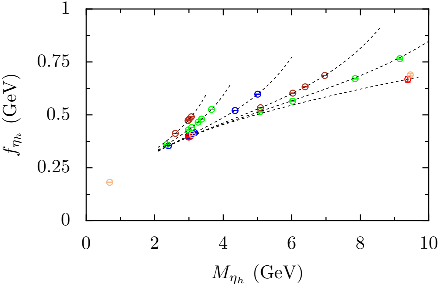
- Completely non-perturbative crosscheck of current-current correlators + continuum PT m_b, m_c calculation:

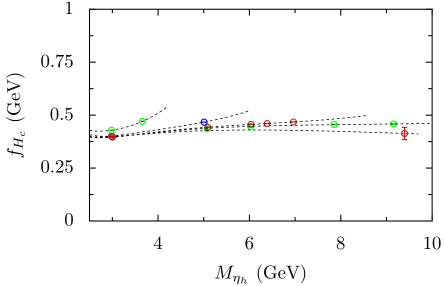
$$\left(\frac{m_b}{m_c}\right)_{lattice} = 4.49(4), \quad \left(\frac{m_b}{m_c}\right)_{correlators} = 4.53(4)$$

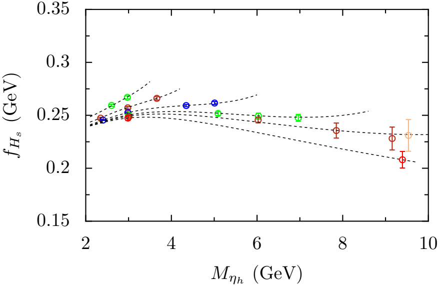
Quark masses

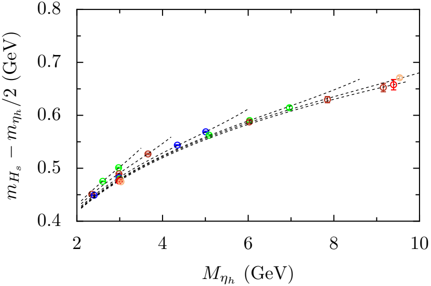


Heavy meson masses and decay constants (preliminary)









Conclusions and outlook

- The use of a highly improved quark action and fine enough lattices provides a very good way of doing precision calculations in heavy-heavy and heavy-light systems.
- Using the same, relativistic action for heavy and light quarks allows us to leverage the precision results for the heavy quark masses to obtain precision results for the light quark masses.
- It provides a stringent test of the lattice methods, because by fixing a small number of parameters in the lattice formulation we should reproduce a large number of experimental measurements.
- This should increase our confidence in the lattice as a precision tool.
- If we had 0.03 fm ensembles we should be able to simulate at the b mass without extrapolation. This is at the moment the most promising way of achieving precision b physics calculations from the lattice.