Properties of gauge orbits

Axel Maas

18th of June 2010

XXVIII International Symposium on Lattice Field Theory

Villasimius

Sardinia/Italy

Why gauge-fixing?

Why gauge-fixing?

- Questions requiring gauge-fixing
  - Properties of elementary particles – gluons, Higgs,...
  - Gauge-dependent mechanisms, e.g. confinement

Why gauge-fixing?

- Questions requiring gauge-fixing
  - Properties of elementary particles – gluons, Higgs,...
  - Gauge-dependent mechanisms, e.g. confinement
- Supporting methods beyond the lattice, e.g. DSEs,...
  - To access domains like
    - Cold, dense quark matter
    - Disparate scales, like in the standard model
    - Non-equilibrium, scattering cross-sections

Why gauge-fixing?

- Questions requiring gauge-fixing
  - Properties of elementary particles – gluons, Higgs,...
  - Gauge-dependent mechanisms, e.g. confinement
- Supporting methods beyond the lattice, e.g. DSEs,...
  - To access domains like
    - Cold, dense quark matter
    - Disparate scales, like in the standard model
    - Non-equilibrium, scattering cross-sections
- For simplification, like in hadron spectroscopy

Why gauge-fixing?

- Questions requiring gauge-fixing
  - Properties of elementary particles – gluons, Higgs,...
  - Gauge-dependent mechanisms, e.g. confinement
- Supporting methods beyond the lattice, e.g. DSEs,...
  - To access domains like
    - Cold, dense quark matter
    - Disparate scales, like in the standard model
    - Non-equilibrium, scattering cross-sections
- For simplification, like in hadron spectroscopy
- Requires (not yet achieved) non-perturbative control

Basics of gauge-fixing

- Important basic quantity: Correlation functions
  - Propagators and vertices
  - Gauge-dependent in general

Basics of gauge-fixing

- Important basic quantity: Correlation functions
  - Propagators and vertices
  - Gauge-dependent in general
- Requires gauge-fixing to determine

Basics of gauge-fixing

- Important basic quantity: Correlation functions
  - Propagators and vertices
  - Gauge-dependent in general
- Requires gauge-fixing to determine
- Local gauge conditions sufficient in perturbation theory
  - Landau gauge: $\partial^\mu A^a_\mu = 0$

Basics of gauge-fixing

- Important basic quantity: Correlation functions
  - Propagators and vertices
  - Gauge-dependent in general
- Requires gauge-fixing to determine
- Local gauge conditions sufficient in perturbation theory
  - Landau gauge: $\partial^\mu A^a_\mu = 0$
  - Equivalent: Condition on a correlation function, the gluon propagator: $p^\mu p^\nu D^{ab}_{\mu\nu} = 0$

Non-perturbative gauge-fixing [Gribov 1978, Singer 1978]

- Beyond perturbation theory local conditions insufficient
  - Gribov-Singer ambiguity due to Gribov copies
  - Requires a well-defined and method-independent resolution

Non-perturbative gauge-fixing [Gribov 1978, Singer 1978]

- Beyond perturbation theory local conditions insufficient
  - Gribov-Singer ambiguity due to Gribov copies
  - Requires a well-defined and method-independent resolution
- Correlation functions contain all information

Non-perturbative gauge-fixing [Gribov 1978, Singer 1978]

- Beyond perturbation theory local conditions insufficient
  - Gribov-Singer ambiguity due to Gribov copies
  - Requires a well-defined and method-independent resolution
- Correlation functions contain all information
- If two gauges are different they differ at least in one correlation function

Non-perturbative gauge-fixing [Gribov 1978, Singer 1978]

- Beyond perturbation theory local conditions insufficient
  - Gribov-Singer ambiguity due to Gribov copies
  - Requires a well-defined and method-independent resolution
- Correlation functions contain all information
- If two gauges are different they differ at least in one correlation function
- Gauges can be specified by imposing conditions on the correlation functions [Maas 2008, 2009]

Gauge (re)construction [Maas, 2009]

- Basic building blocks to (re)construct a gauge

Gauge (re)construction [Maas, 2009]

- Basic building blocks to (re)construct a gauge
- Gluon propagator $D^{ab}_{\mu\nu}$ [Zwanziger, 1990s+2000s, many others]
  - Total trace $(\ -\ ) \int d^d\, p\, D^{aa}_{\mu\mu}$
  - Connected to the fundamental modular domain

Gauge (re)construction [Maas, 2009]

- Basic building blocks to (re)construct a gauge
- Gluon propagator $D^{ab}_{\mu\nu}$ [Zwanziger, 1990s+2000s, many others]
  - Total trace $(\ -\ )\int d^d\, p\, D^{aa}_{\mu\mu}$
  - Connected to the fundamental modular domain
- Ghost propagator D^{ab}_{G} [Fischer, Maas, Pawlowski 2008, Maas, 2009]
  - B-parameter $B = \lim_{p\to 0} p^2 D^{aa}_G(p)/\mu^2 D^{aa}_G(\mu)$
  - Generates a one-parameter family of correlation functions in the continuum [Fischer, Maas, Pawlowski 2008]
  - Assume: Positive only in the 1st Gribov region

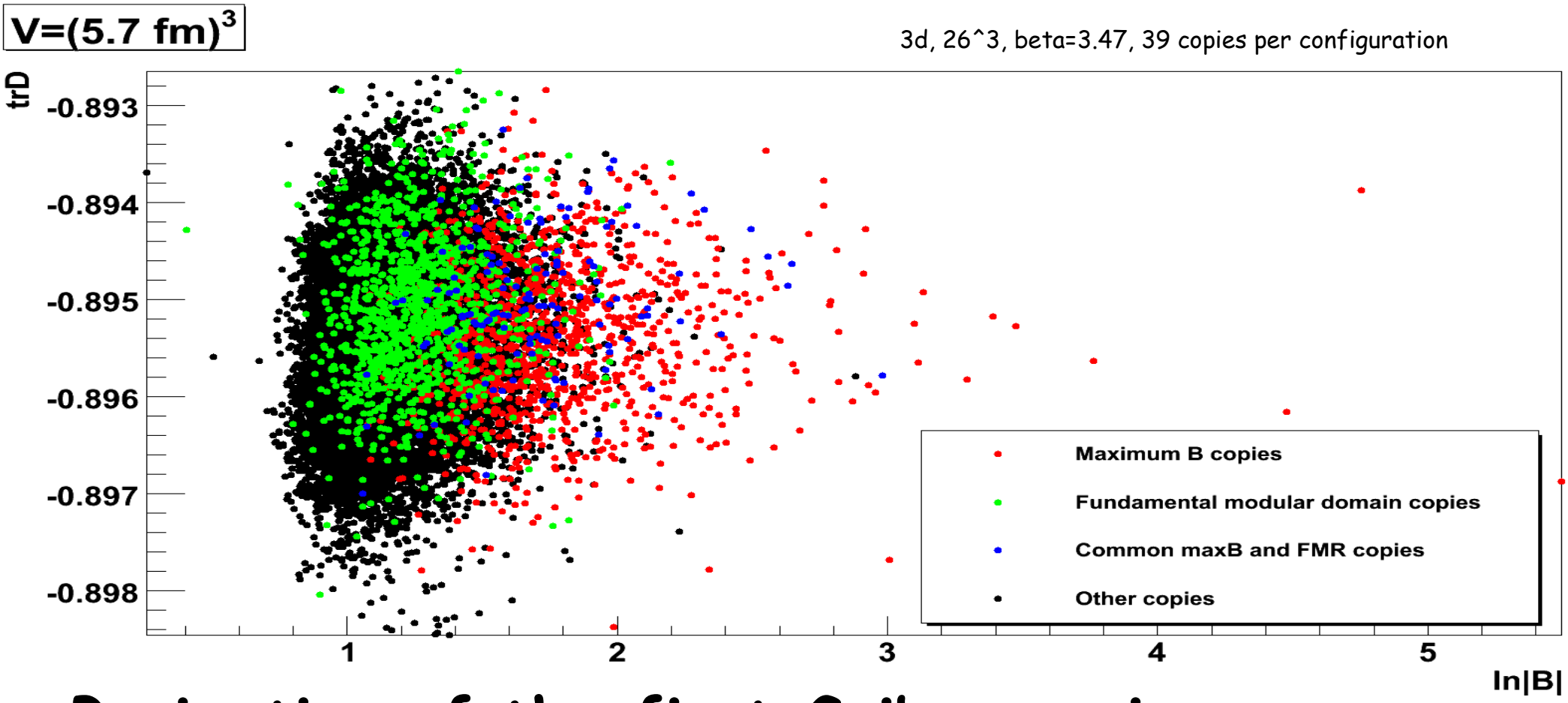
Distribution in trD-B-space [Maas 2009, unpublished]

3d, 26^3, beta=3.47, 39 copies per configuration

- Projection of the first Gribov region
  - Positive Faddeev-Popov operator

Distribution in trD-B-space

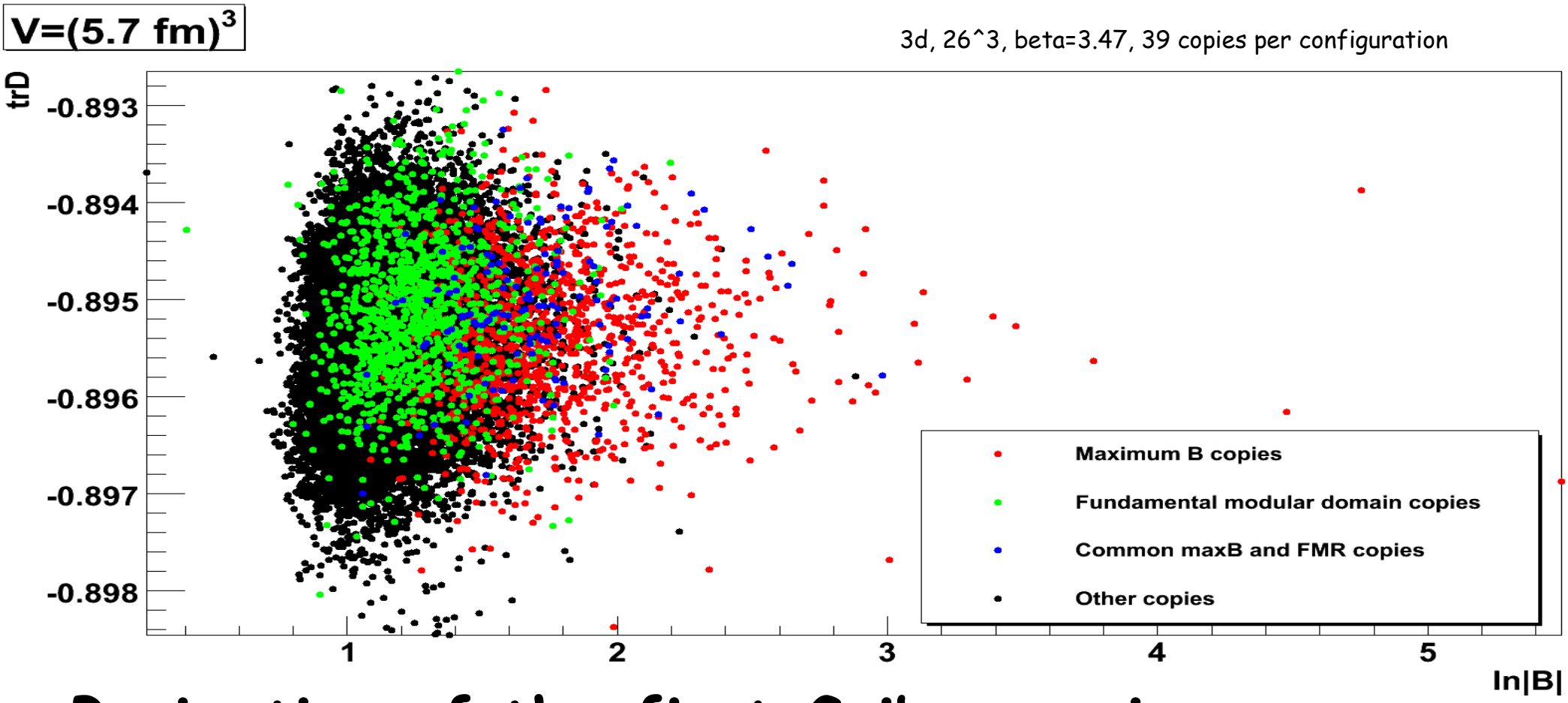
[Maas 2009, unpublished]



- Projection of the first Gribov region
  - Positive Faddeev-Popov operator

Distribution in trD-B-space

[Maas 2009, unpublished]



- Projection of the first Gribov region
  - Positive Faddeev-Popov operator
- Uncorrelated for different Gribov copies

Constructing gauges – independent of a method

[Maas 2009]

- Select a permitted (set of) constraint(s)

Constructing gauges – independent of a method

[Maas 2009]

- Select a permitted (set of) constraint(s)
  - If a complete specification: Done

Constructing gauges – independent of a method

[Maas 2009]

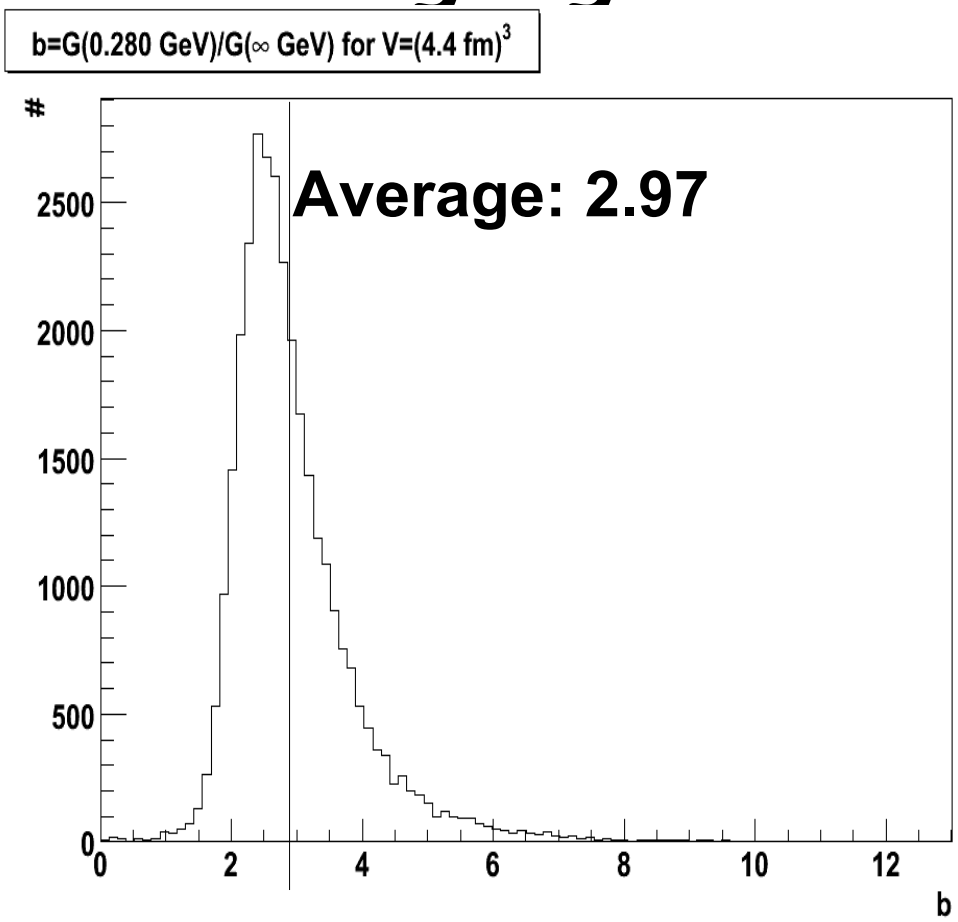
- Select a permitted (set of) constraint(s)
  - If a complete specification: Done
  - If not: There exist Gribov copies degenerate in the constraints
    - Select randomly among degenerate Gribov copies
    - Resulting correlation functions will be averages over all other possible constraints

Possible gauges [Maas 2009, unpublished]

- Minimal Landau gauge:
  No further constraint

Possible gauges

[Maas 2009, unpublished]

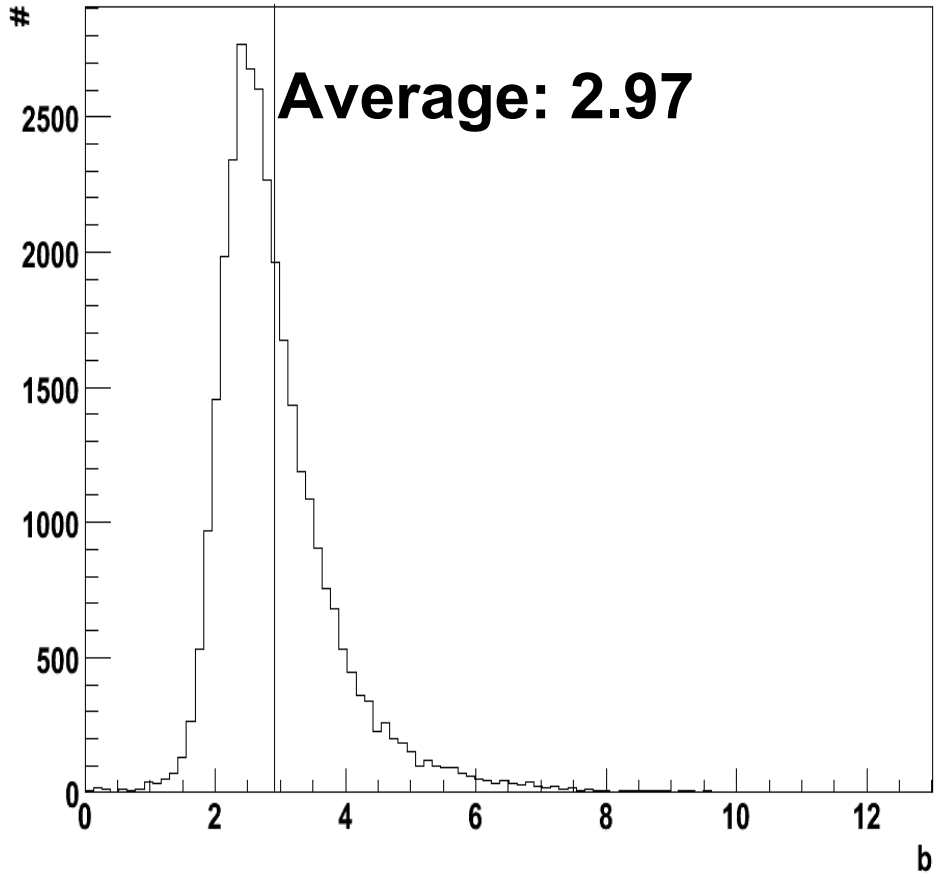


- Minimal Landau gauge:
  No further constraint

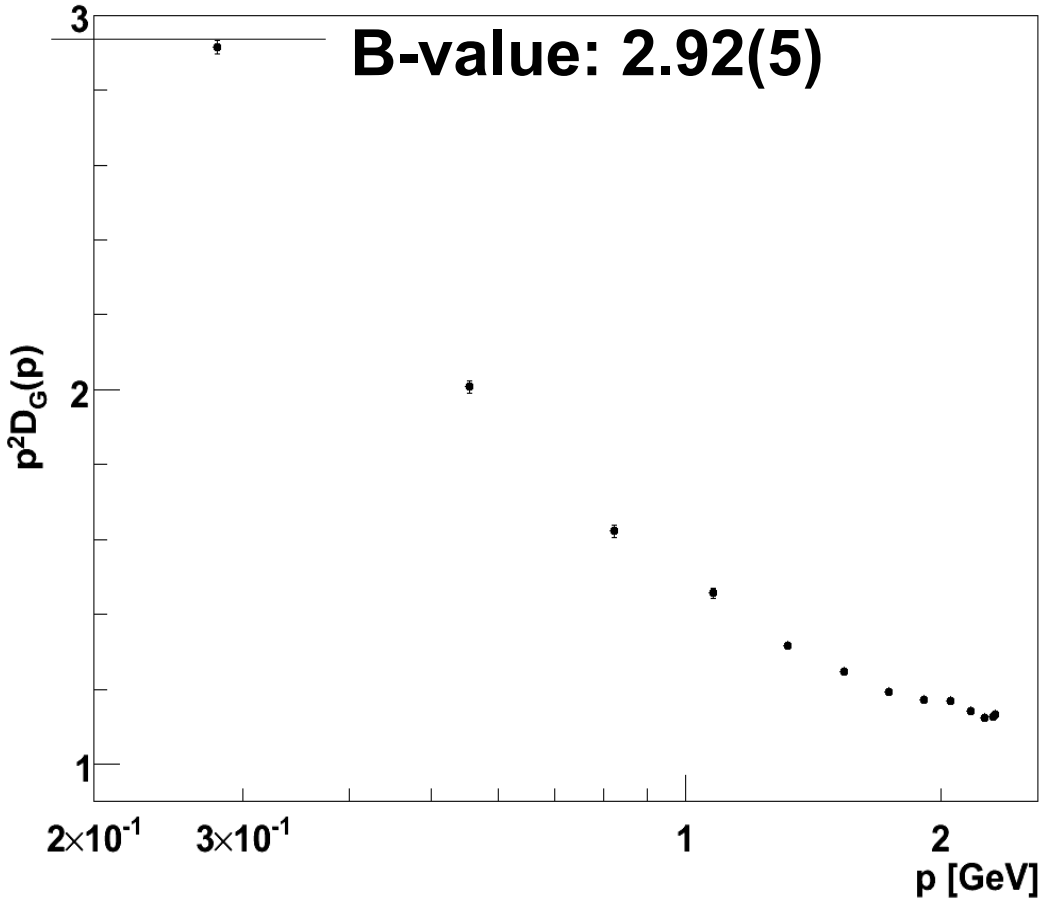
Possible gauges

[Maas 2009, unpublished]

b=G(0.280 GeV)/G(∞ GeV) for V=(4.4 fm)^3



Ghost dressing function



- Minimal Landau gauge:
  No further constraint

Constructing gauges – independent of a method

[Maas 2009]

- Select a permitted (set of) constraint(s)
  - If a complete specification: Done
  - If not: There exist Gribov copies degenerate in the constraints
    - Select randomly among degenerate Gribov copies
    - Resulting correlation functions will be averages over all other possible constraints
- Always: Inside first Gribov region
  - Can be implemented in all methods

Possible gauges

- Minimal Landau gauge:
  No further constraint

Possible gauges

- Minimal Landau gauge:
  No further constraint
- Absolute Landau gauge:
  Require minimal |trD|

Possible gauges

- Minimal Landau gauge:
  No further constraint
- Absolute Landau gauge:
  Require minimal |trD|
- MaxB gauge:
  Require maximum B

Possible gauges

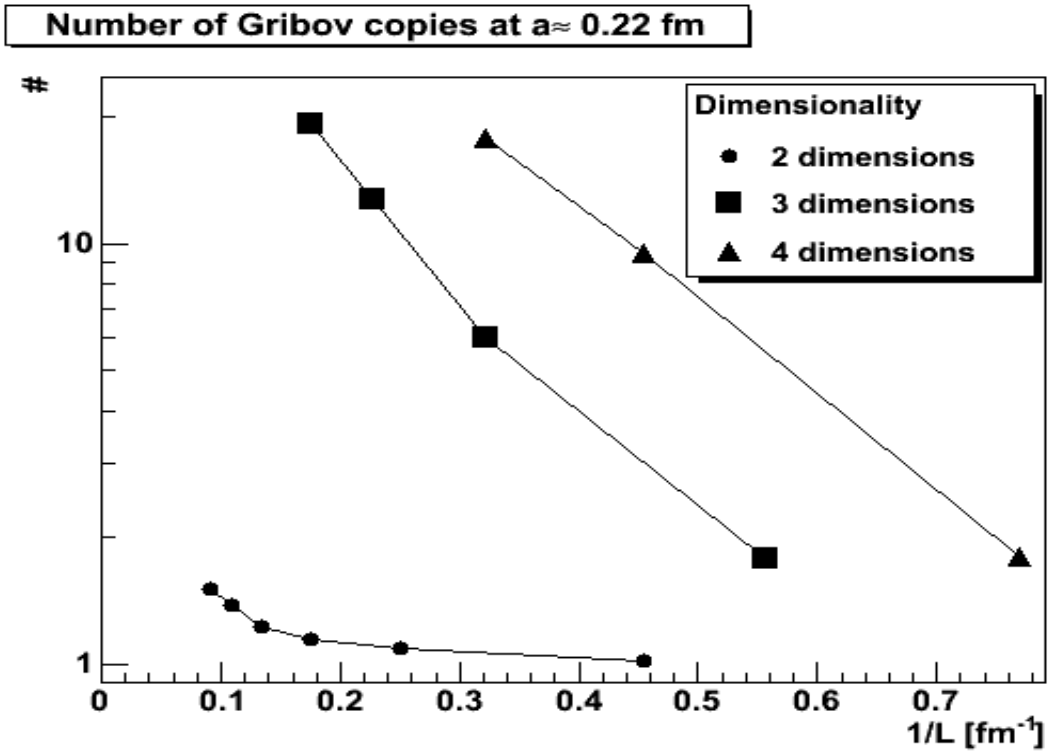
- Minimal Landau gauge:
  No further constraint
- Absolute Landau gauge:
  Require minimal |trD|
- MaxB gauge:
  Require maximum B
- Others possible
  - Minimize B or trD, combined constraints, averages,...

Obstacles...

- What are permitted constraints?
  - Requires knowledge of all Gribov copies: Gribov problem

Severity of the Gribov problem

[Maas 2009, Mehta et al. 2009]

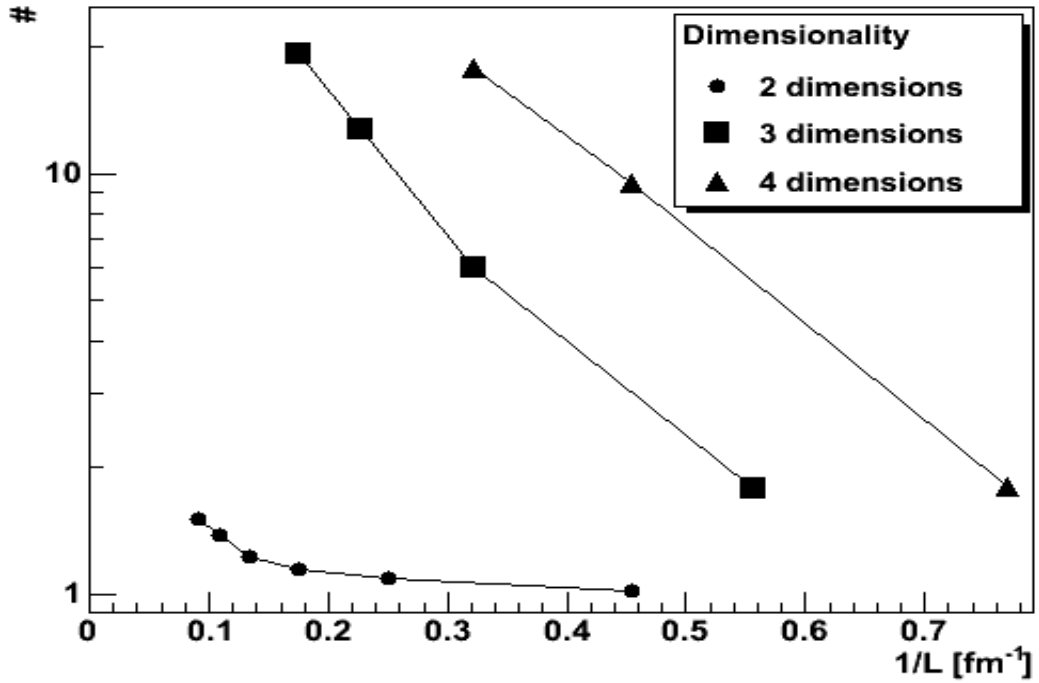


- Number of Gribov copies rises strongly with volume

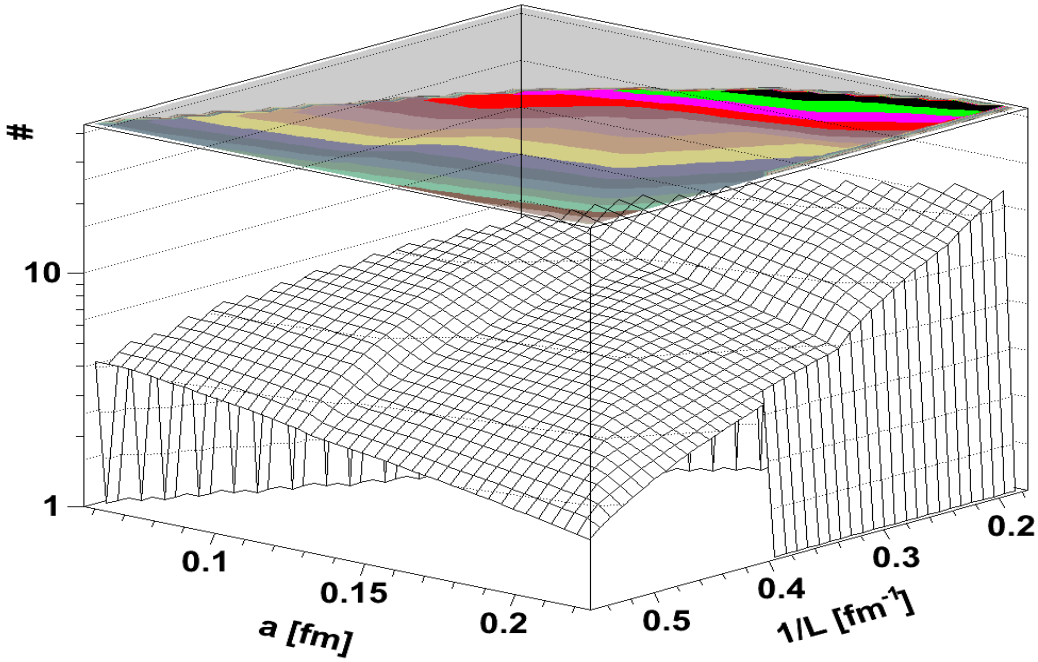
Severity of the Gribov problem

[Maas 2009, Mehta et al. 2009]

Number of Gribov copies at a≈ 0.22 fm



Number of Gribov copies



- Number of Gribov copies rises strongly with volume...
- ...but also with discretization!

UNI GRAZ

Obstacles...

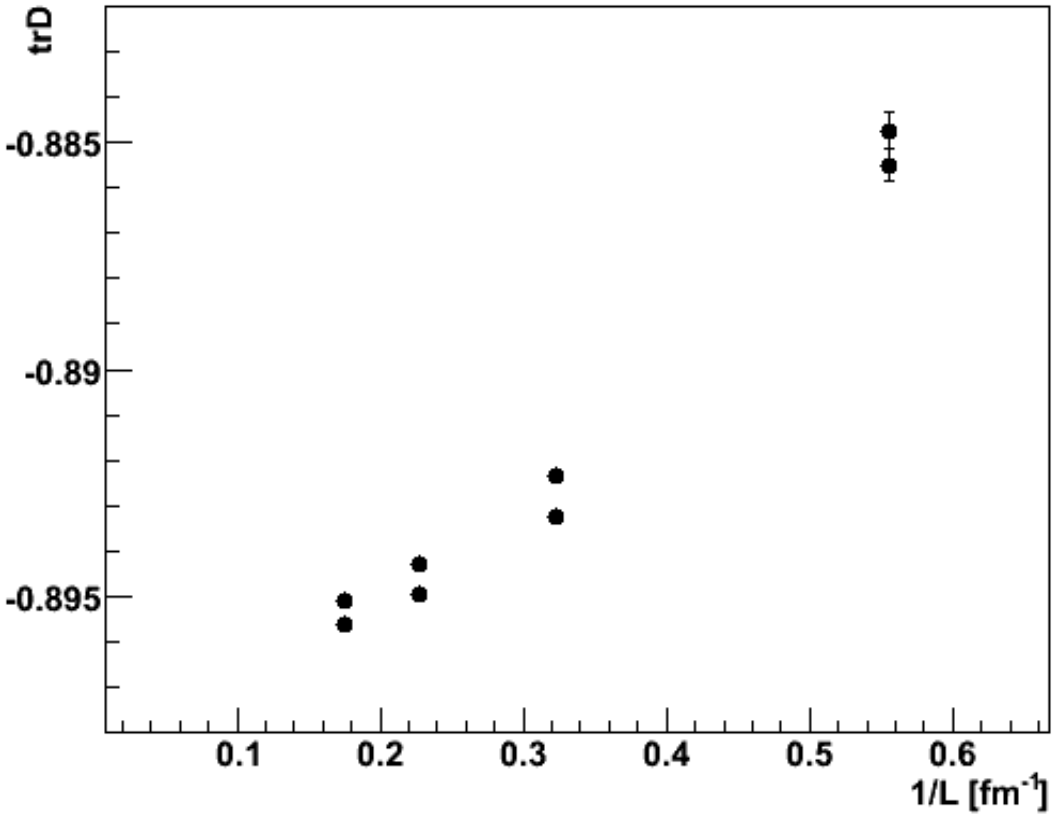
- What are permitted constraints?
  - Requires knowledge of all Gribov copies: Gribov problem
  - Requires eg to know permitted range of B or trD value

Permitted corridors

[Maas 2009, unpublished]

TrD corridor

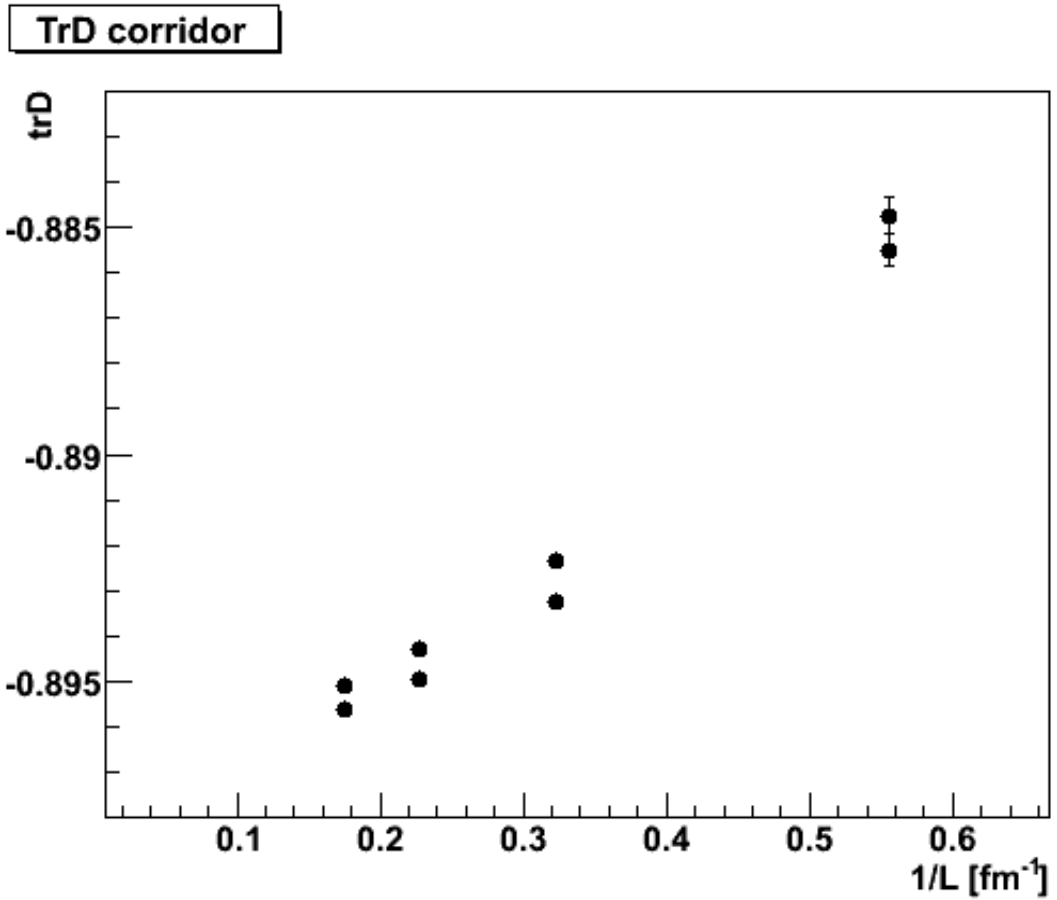
3d, a=0.22 fm

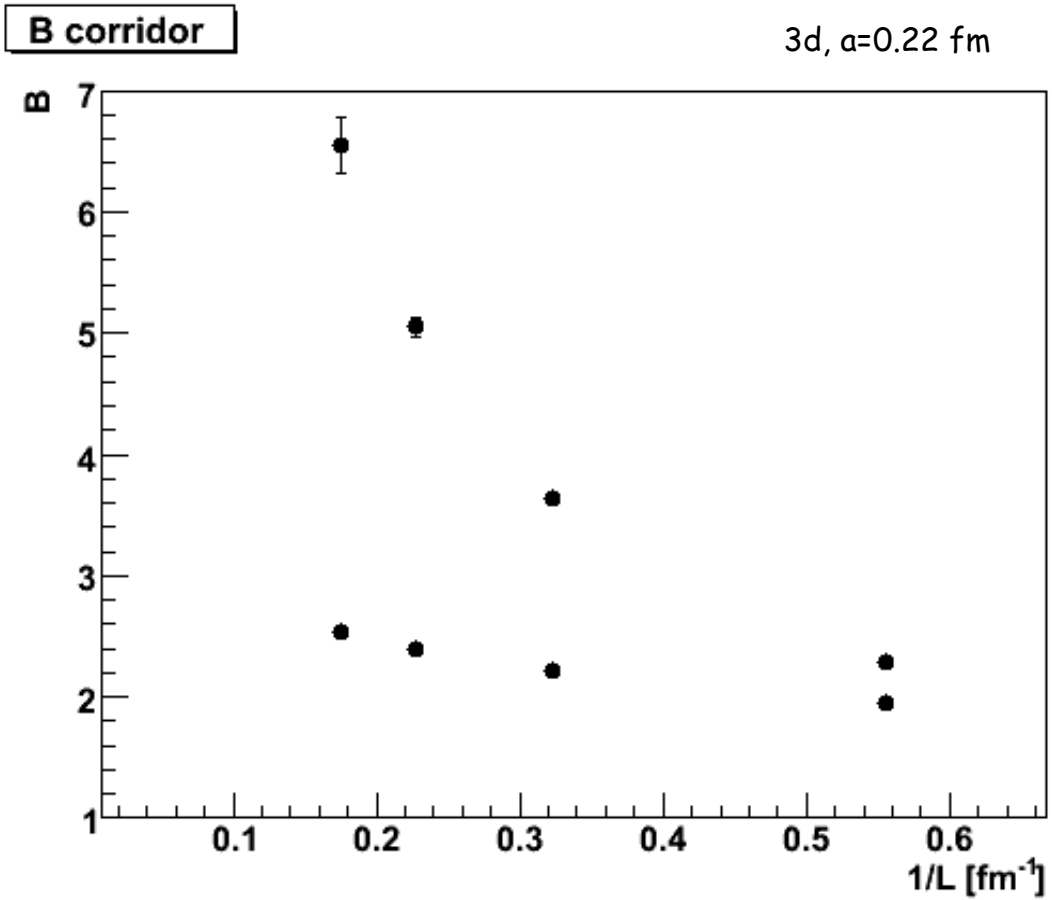


- trD: (small) range scales strongly with discretization

Permitted corridors

[Maas 2009, unpublished]





- trD: (small) range scales strongly with discretization
- B: Opens up with volume

UNI GRAZ

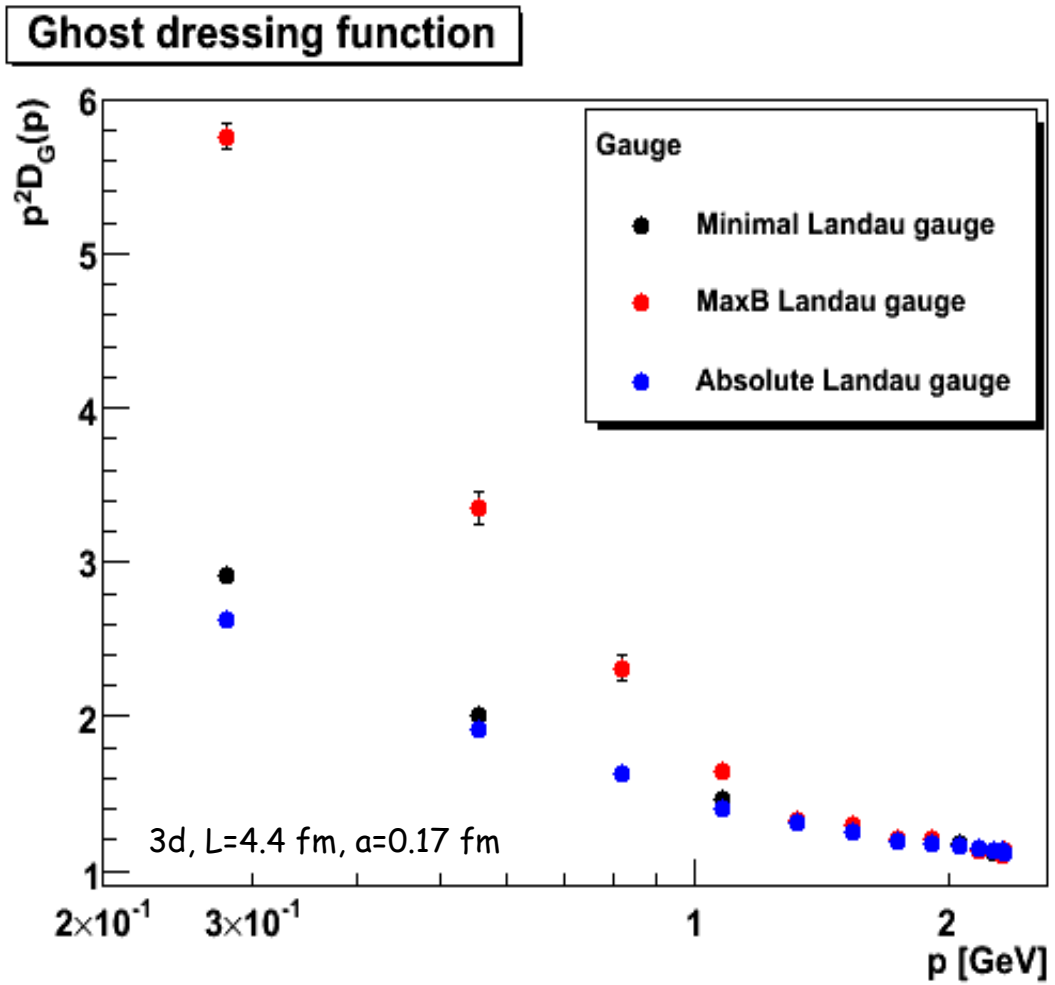
Obstacles...

- What are permitted constraints?
  - Requires knowledge of all Gribov copies: Gribov problem
  - Requires eg to know permitted range of B or trD value
- How many constraints are possible?
  - Unknown – but no hints for more than just one
  - Known constraints are related to free renormalization conditions – relevance?

Obstacles...

- What are permitted constraints?
  - Requires knowledge of all Gribov copies: Gribov problem
  - Requires eg to know permitted range of B or trD value
- How many constraints are possible?
  - Unknown – but no hints for more than just one
  - Known constraints are related to free renormalization conditions – relevance?
- Unspecified constraints: Outside lattice gauge theory knowledge of undetermined averages required

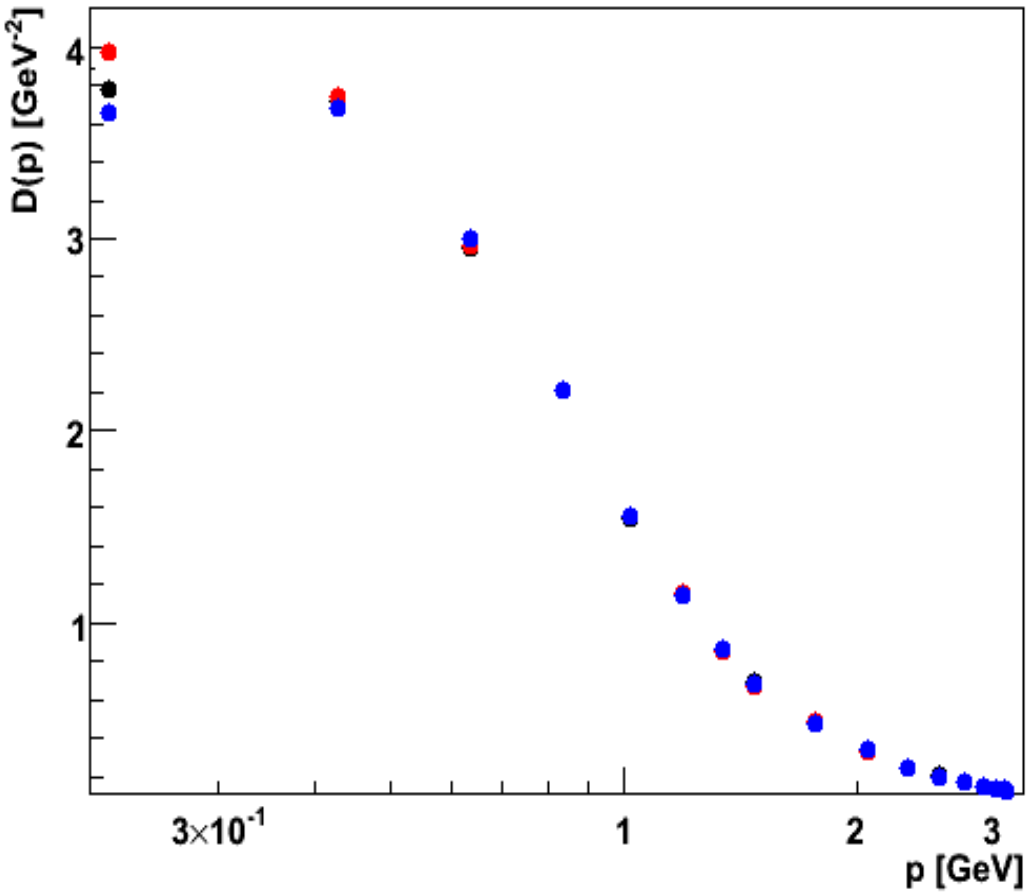
Ghosts and gluons (in 3d) [Maas 2009, unpublished]



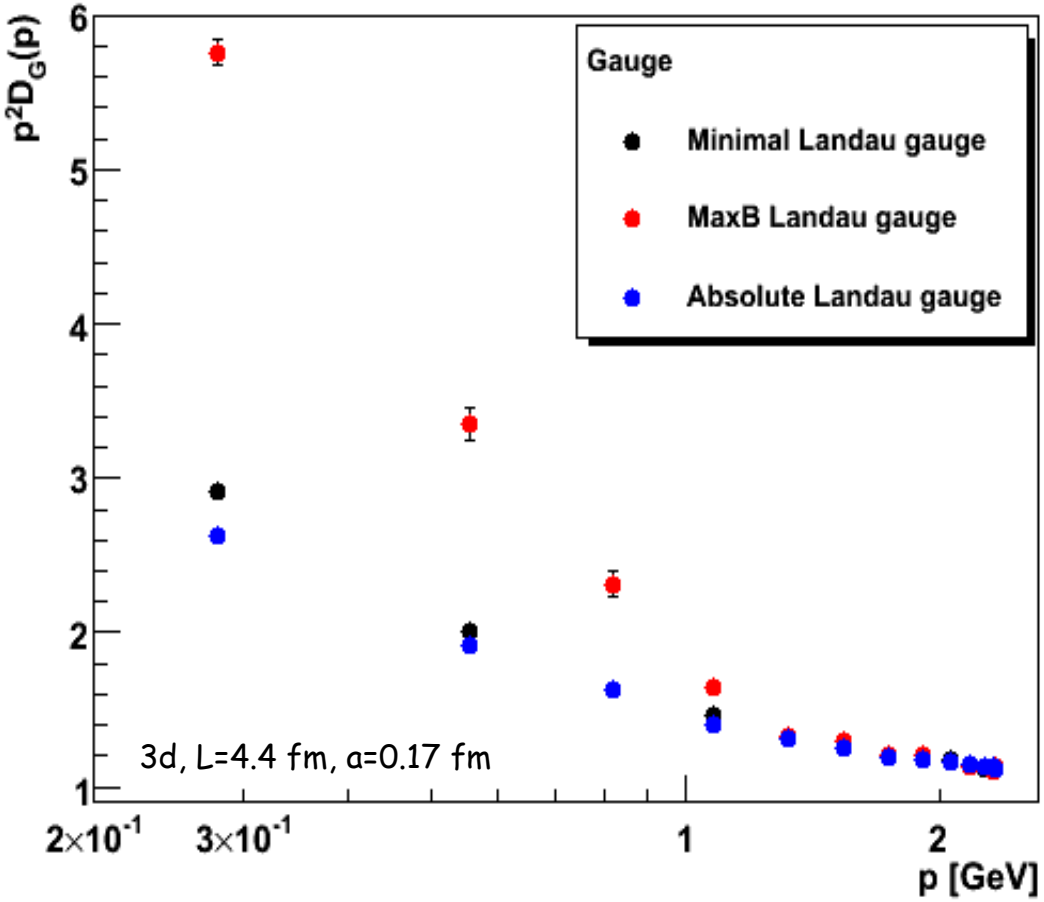
- Ghosts strongly dependent up to 1 GeV [Compare: e.g. Bornyakov et al. 2009]

Ghosts and gluons (in 3d) [Maas 2009, unpublished]

Gluon propagator



Ghost dressing function



- Ghosts strongly dependent up to 1 GeV
- Gluons not very sensitive beyond 100 MeV

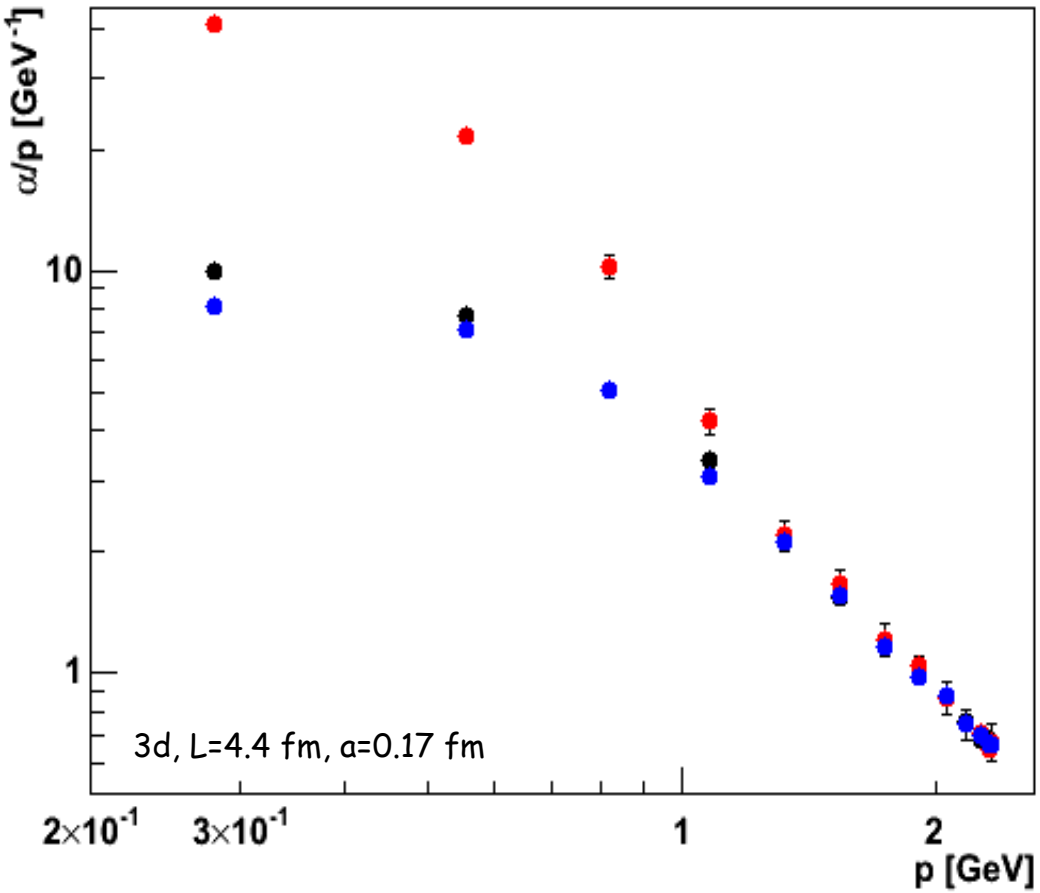
[Compare: e.g. Bornyakov et al. 2009]

UNI GRAZ

Coupling and matter

[Maas 2009, unpublished]

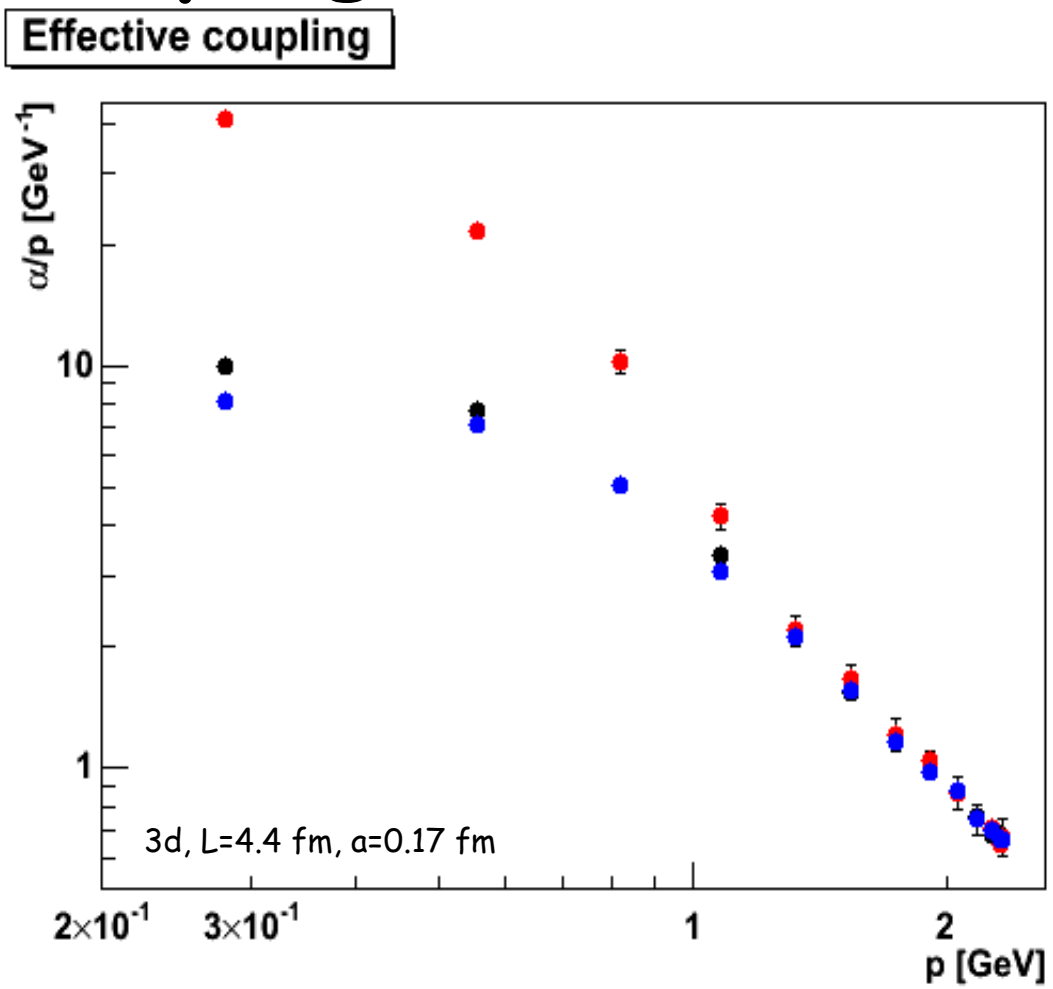
Effective coupling

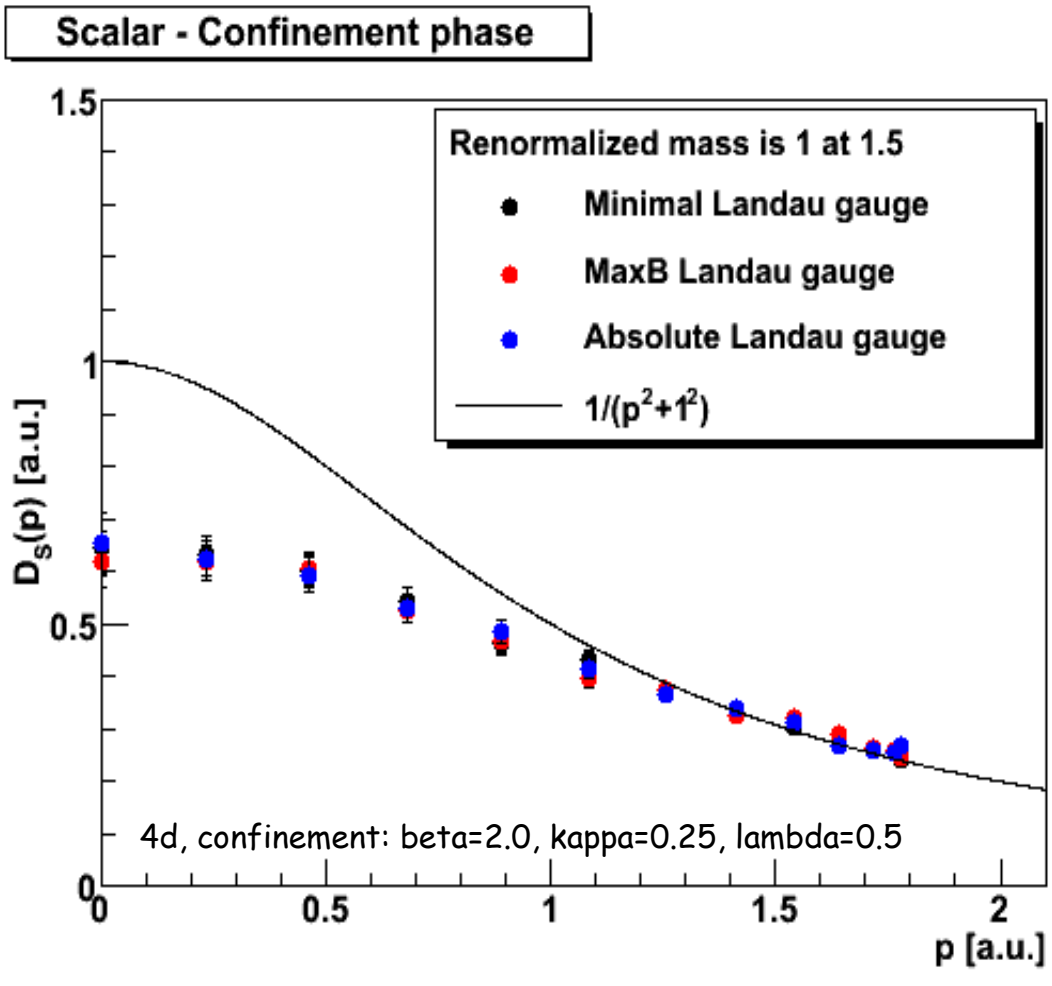


- Derived quantites inherit dependencies

Coupling and matter

[Maas 2009, unpublished]





- Derived quantites inherit dependencies
- Matter fields weakly influenced

Summary

Read more: arXiv: 0808.3047, 0810.1987, 0907.5185

Summary

- Gauge-fixing is necessary for many questions and practical applications

Read more: arXiv: 0808.3047, 0810.1987, 0907.5185

Summary

- Gauge-fixing is necessary for many questions and practical applications
- Understanding gauge-fixing is required to have control in these cases
- Beyond perturbation theory effects up-to the characteristic scale (1 GeV for QCD) in some quantities

Read more: arXiv: 0808.3047, 0810.1987, 0907.5185

Summary

- Gauge-fixing is necessary for many questions and practical applications
- Understanding gauge-fixing is required to have control in these cases
- Beyond perturbation theory effects up-to the characteristic scale (1 GeV for QCD) in some quantities
- Well-defined non-perturbative gauges possible
  - Construction based on correlation functions
  - Can provide a well-defined framework
  - Decoupling vs. scaling is possibly just a gauge choice?

Read more: arXiv: 0808.3047, 0810.1987, 0907.5185