HQET parameters at the $1/m$ order with $n_f = 2$ flavors

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for the ALPHA Collaboration

HQET at zero velocity on the lattice

The static part is given by the Eichten-Hill action [Eichten & Hill 90]

$$S_{\rm stat} = a^4 \sum_x \overline{\psi}_{\rm h}(x) D_0 \psi_{\rm h}(x)$$

with $P_+ \psi_{\rm h} = \psi_{\rm h}\,, \quad \overline{\psi}_{\rm h} P_+ = \overline{\psi}_{\rm h}\,, \quad P_+ = \frac{1}{2}(1+\gamma_0)$

The static energy contains a linear divergence ($\propto 1/a$) which is absorbed by $m_{\rm bare}$

$$m_{\rm B} = E^{\rm stat} + m_{\rm bare}$$

The $1/m$ corrections are the kinetic and chromomagnetic terms

$$\mathcal{O}_{\rm kin} = -\overline{\psi}_{\rm h}(\mathbf{D}^2)\psi_{\rm h} \qquad \mathcal{O}_{\rm spin} = -\overline{\psi}_{\rm h}(\sigma\cdot\mathbf{B})\psi_{\rm h}$$

with coefficient $\omega_{\rm kin}, \omega_{\rm spin}$ $\Rightarrow$ Classically $\omega_{\rm kin} = \omega_{\rm spin} = 1/(2m)$

HQET coefficients $m_{\rm bare}, \omega_{\rm kin}, \omega_{\rm spin}$ are determined non-perturbatively $\Rightarrow$ renormalizability

HQET computation on the lattice

We want to compute hadronic quantities at the $1/m$ order of hqet, for example

$$
\begin{aligned}
m_{\rm B} &= m_{\rm bare} + E^{\rm stat} + \omega_{\rm kin} E^{\rm spin} + \omega_{\rm spin} E^{\rm spin} \\
\langle 0|A_0^{\rm HQET}|{\rm B}\rangle &= Z_{\rm A}^{\rm HQET}\Big(\langle 0|A_0^{\rm stat}|{\rm B}\rangle + \omega_{\rm kin}\langle 0|{\rm A}_0^{\rm kin}|{\rm B}\rangle + \omega_{\rm spin}\langle 0|{\rm A}_0^{\rm spin}|{\rm B}\rangle\Big)
\end{aligned}
$$

$\Rightarrow$ To achieve such a computation, one needs:

- large volume matrix element and energies $E^{\rm stat}, E^{\rm kin}, \langle 0|A_0^{\rm stat}|{\rm B}\rangle, \ldots$
  $\rightarrow$ see Benoît Blossier's talk
- HQET parameters $m_{\rm bare}, \omega_{\rm kin}, Z_{\rm A}^{\rm HQET}, \ldots$
  $\rightarrow$ this talk

Status of the project

- $n_f = 0$
  - ▷ b-quark mass (Alpha '06)
  - ▷ I. HQET parameters (Alpha '10)
  - ▷ II. Spectoscopy in the quenched approximation (Alpha '10)
  - ▷ III. Decay constants in the quenched approximation (in preparation)
- $n_f = 2$
  - ▷ HQET parameter : almost finished
  - ▷ Large volume part: preliminary results (1 lattice spacing)

Strategy [Heitger & Sommer 03]

QCD ($am_b \ll 1$) | HQET

$L_1 \sim 0.5$ fm: $\Phi^{\rm QCD}(L_1, am_{\rm b})$ ←Matching→ $\Phi^{\rm HQET}(L_1)$

↕

$L_2 = 2L_1$: $\left[\Phi^{\rm HQET}(L_2) - \Phi^{\rm HQET}(L_1)\right]$

↕

$L_{\rm inf}$: $\Phi(L_{\rm inf}, am_{\rm b})$ ↔ $\left[\Phi^{\rm HQET}(L_{\rm inf}) - \Phi^{\rm HQET}(L_2)\right]$

Experiment

Strategy [Heitger & Sommer 03]

- Simulate QCD in small volume $L_1 \sim 0.5\ \mathrm{fm}$ with $am_{\mathrm{b}} \ll 1$.
  Compute a set of observables and take the continuum limit $\Phi(L_1, m_{\mathrm{q}})$

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  Impose the matching $\Rightarrow$ HQET parameters for these values of the lattice spacings .

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- Perform another simulation of HQET, with the same a's but in a larger volume, for example $L_2 = 2L_1$.
  Use the HQET parameters computed in the previous step, to obtain the observables in the volume L_2, and take their continuum limit $\Phi(L_2, m_\mathrm{q})$ (cancelation of the divergences).

  Meson mass:

$$\begin{aligned}\Gamma(L_2, m_\mathrm{q}) &= \lim_{a\to 0}\left(\Gamma^\mathrm{stat}(L_2, a) - \Gamma^\mathrm{stat}(L_1, a)\right) + \Gamma^\mathrm{QCD}(L_1, m_\mathrm{q}) \\ &= L_1\,\sigma^\mathrm{m}(\bar{g}^2(L_1)) + \Gamma^\mathrm{QCD}(L_1, m_\mathrm{q})\end{aligned}$$

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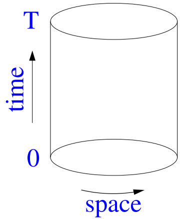
- Restart from step 1, with $\Phi^{\mathrm{QCD}}(L_1, m_{\mathrm{q}}) \to \Phi(L_2, m_{\mathrm{q}})$ until the volume is large enough to compute hadronic quantities

Implementation : Schrödinger functional [Lüscher '92, Sint '94]

Implementation: Schrödinger functional of size $T \times L^3$

- Dirichlet boundary conditions in time (at $x_0 = 0$ and $x_0 = T$)
- Periodic boundary conditions in space, up to a phase $\Psi(x + \hat{k}L) = e^{i\theta}\Psi(x)$.



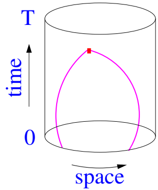
Transition amplitude for $C(x_0 = 0) \to C'(x_o = T)$

$$\begin{aligned} \mathcal{Z}[C', C] &= \langle C' | \mathrm{e}^{-\mathbb{H}\mathrm{T}} \mathbb{P} | \mathrm{C} \rangle \\ &= \sum_{n=0}^{\infty} \mathrm{e}^{-E_n T} \psi_n[C'] \psi_n[C]^* \end{aligned}$$

Implementation: 2-point functions in QCD

Boundary to current correlators

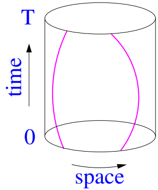
$$f_{\rm A}(x_0) \quad = \quad -\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}} \left\langle (A_{\rm I})_0(x)\,(\bar{\zeta}_{\rm b}(\mathbf{y})\gamma_5\zeta_{\rm l}(\mathbf{z}))\right\rangle$$



and boundary to boundary correlator

$$f_1 \quad = \quad -\frac{a^{12}}{2L^6}\sum_{\mathbf{y},\mathbf{z},\mathbf{y}',\mathbf{z}'} \left\langle (\bar{\zeta}'_{\rm b}(\mathbf{y}')\gamma_5\zeta'_{\rm l}(\mathbf{z}'))\,(\bar{\zeta}_{\rm b}(\mathbf{y})\gamma_5\zeta_{\rm l}(\mathbf{z}))\right\rangle$$

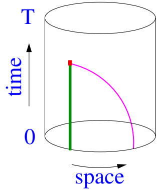
$$k_1 \quad = \quad -\frac{a^{12}}{2L^6}\sum_{\mathbf{y},\mathbf{z},\mathbf{y}',\mathbf{z}'} \left\langle (\bar{\zeta}'_{\rm b}(\mathbf{y}')\gamma_{\rm k}\zeta'_{\rm l}(\mathbf{z}'))\,(\bar{\zeta}_{\rm b}(\mathbf{y})\gamma_{\rm k}\zeta_{\rm l}(\mathbf{z}))\right\rangle$$



Implementation: 2-point functions in the static theory

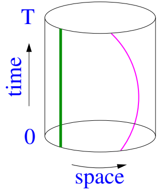
Boundary to current correlators

$$f_{\mathrm{A}}^{\mathrm{stat}}(x_0) \quad = \quad -\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}} \left\langle (A_{\mathrm{I}}^{\mathrm{stat}})_0(x)\,(\bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_5\zeta_{\mathrm{l}}(\mathbf{z}))\right\rangle$$



and boundary to boundary correlator

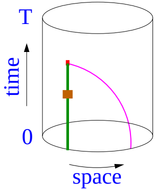
$$f_1^{\mathrm{stat}} = -\frac{a^{12}}{2L^6}\sum_{\mathbf{y},\mathbf{z},\mathbf{y}',\mathbf{z}'} \left\langle (\bar{\zeta}'_{\mathrm{h}}(\mathbf{y}')\gamma_5\zeta'_{\mathrm{l}}(\mathbf{z}'))\,(\bar{\zeta}_{\mathrm{h}}(\mathbf{y})\gamma_5\zeta_{\mathrm{l}}(\mathbf{z}))\right\rangle$$



Implementation: 2-point functions at the $1/m$ order

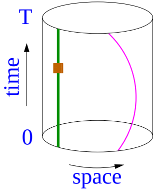
Boundary to current correlators

$$f_{\rm A}^{\rm kin}(x_0) = -\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z},u}\left\langle \mathrm{A}_0^{\rm stat}(x)\, O^{\rm kin}(u)\, (\bar\zeta_{\rm h}(\mathbf{y})\gamma_5\zeta_{\rm l}(\mathbf{z}))\right\rangle$$



Boundary to boundary correlator

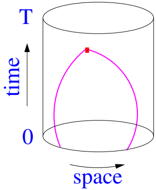
$$f_1^{\rm kin} = -\frac{a^{12}}{2L^6}\sum_{\mathbf{y},\mathbf{z},\mathbf{y}',\mathbf{z}',u}\left\langle (\bar\zeta'_{\rm h}(\mathbf{y}')\gamma_5\zeta'_{\rm l}(\mathbf{z}'))\, O^{\rm kin}(u)\, (\bar\zeta_{\rm h}(\mathbf{y})\gamma_5\zeta_{\rm l}(\mathbf{z}))\right\rangle$$

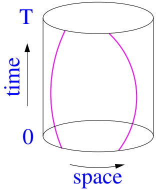


And the same for $f_{\rm A}^{\rm spin}$, $f_1^{\rm spin}$

F_B, including $1/m$ corrections

From the current-to-boundary f_A and the boundary-to-boundary f_1 correlators





Build an observables related to the decay constant :

$$\Phi_2^{\rm QCD} = \ln\left(\frac{-f_{\rm A}(x_0)}{\sqrt{f_1}}\right) \quad \stackrel{L\gg 1}{\longrightarrow} \quad \ln\left(\frac{1}{2} F_{\rm B}\sqrt{m_{\rm B} L^3}\right)$$

At the $1/m$ oder of HQET

$$\Phi_2^{\rm HQET} = \ln Z_{\rm A}^{\rm HQET} + \ln\left(\frac{-f_{\rm A}^{\rm stat}}{\sqrt{f_1^{\rm stat}}}\right) + \underbrace{c_{\rm A}^{HQET}\frac{f_{\delta \rm A}^{\rm stat}}{f_{\rm A}^{\rm stat}} + \omega_{\rm kin}\left(\frac{f_{\rm A}^{\rm kin}}{f_{\rm A}^{\rm stat}} - \frac{1}{2}\frac{f_1^{\rm kin}}{f_1^{\rm stat}}\right) + \omega_{\rm spin}\left(\frac{f_{\rm A}^{\rm spin}}{f_{\rm A}^{\rm stat}} - \frac{1}{2}\frac{f_1^{\rm spin}}{f_1^{\rm stat}}\right)}_{1/m}$$

The observables (II)

$$
\begin{array}{llllllll}
\Phi_1 = & L\Gamma^{\rm P} = & L\Gamma^{\rm stat} & + L m_{\rm bare}^{\rm HQET} & + c_{\rm A}^{\rm HQET} L\Gamma_{\delta \rm A}^{\rm stat} & + \omega_{\rm kin} L E^{\rm kin} & + \omega_{\rm spin} L E^{\rm spin} \\
\Phi_2 = & \ln\left(\frac{-f_{\rm A}}{\sqrt{f_1}}\right) = & \zeta_{\rm A} & + \ln Z_{\rm A}^{\rm HQET} & + c_{\rm A}^{\rm HQET} \rho_{\delta A} & + \omega_{\rm kin} \Psi^{\rm kin} & + \omega_{\rm spin} \Psi^{\rm spin} \\
\Phi_3 = & R_{\rm A} = & R_{\rm A}^{\rm stat} & & + c_{\rm A}^{\rm HQET} R_{\delta \rm A} & + \omega_{\rm kin} R_{\rm A}^{\rm kin} & + \omega_{\rm spin} R_{\rm A}^{\rm spin} \\
\Phi_4 = & \frac{1}{4}(R_1^{\rm P} + 3R_1^{\rm V}) = & R_1^{\rm stat} & & & + \omega_{\rm kin} R_1^{\rm kin} & \\
\Phi_5 = & \frac{3}{4}\ln\left(\frac{f_1}{k_1}\right) = & & & & & \omega_{\rm spin} \frac{f_1^{\rm spin}}{f_1^{\rm stat}}
\end{array}
$$

We define the 5 dimensional vectors Φ, η, ω and a 5 by 5 matrix ϕ

$$\Phi(L, m_{\rm q}) = \lim_{a \to 0} \left[\phi(L, a)\, \omega(m_{\rm q}, a) + \eta(L, a) \right]$$

HQET parameters at the $1/m$ order

We compute $\omega(m_{\mathrm{q}}, a)$ from the matching in L_1

$$\Phi^{\mathrm{QCD}}(L_1, m_{\mathrm{q}}) = \phi(L_1, a)\,\omega(m_{\mathrm{q}}, a) + \eta(L_1, a)$$

then the observables in L_2, from

$$\Phi(L_2, m_{\mathrm{q}}) = \lim_{a\to 0}\left[\phi(L_2, a)\,\left(\phi^{-1}(L_1, a)\,\Phi^{\mathrm{QCD}}(L_1, m_{\mathrm{q}}) - \eta(L_1, a)\right) + \eta(L_2, a)\right]$$

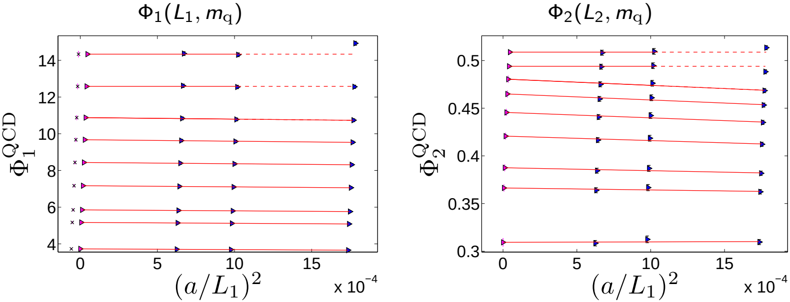
The HQET parameters needed for a large volume simultation are given by

$$\omega(m_{\mathrm{q}}, a) = \phi^{-1}(L_2, a)\Phi(L_2, m_{\mathrm{q}}) - \eta(L_2, a)$$

A last step : interpolate these results to the lattice spacing used in the large volume simulation (CLS)

Results

Continuum extrapolation of the QCD observables



The RGI quark masses M are such that $z = L_1 M \in (4, 6, 7, 9, 11, 13, 15, 18, 21)$

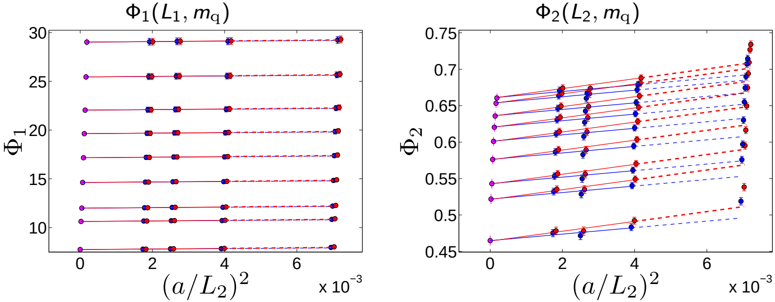
$L_1/r_0 = 0.9 \Rightarrow L_1 \sim 0.45$ fm

$L_1/a = 40, 32, 24(20)$

$\beta = 6.638, 6.4574, 6.2483$

Results

Continuum extrapolation of the static observables in L_2



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$L_1/r_0 = 0.9 \Rightarrow L_1 \sim 0.45\ \mathrm{fm}$

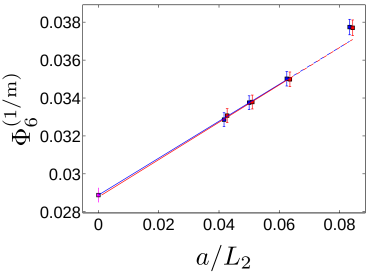
$L_1/a = 12, 10, 8$ $\qquad$ $L_1/a = 24, 20, 16$

$\beta = 5.758, 5.619, 5.4689$ $\qquad$ Point with $L_2/a = 32, L_1/a = 16$ will be added in the near future

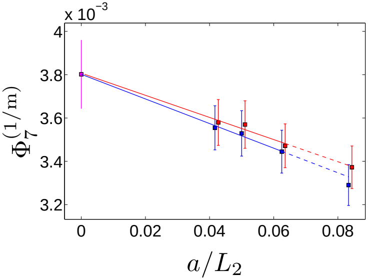
Results

Continuum extrapolation of the $1/m$ observables in L_2

$\Phi_4(L_1, m_q)$



$\Phi_5(L_2, m_q)$



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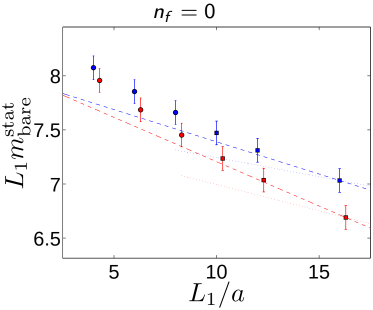
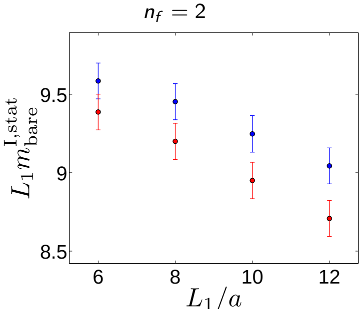
$\beta = ..., ...$ $\qquad$ Point with $L_2/a = 32, L_1/a = 16$ will be added in the near future

Results

Example of static parameter: $m_{\mathrm{bare}}^{\mathrm{stat}}$





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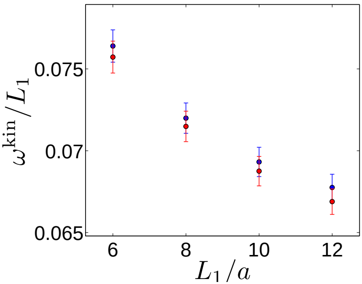
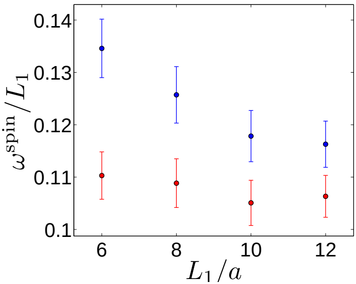
Example of $1/m$ parameters





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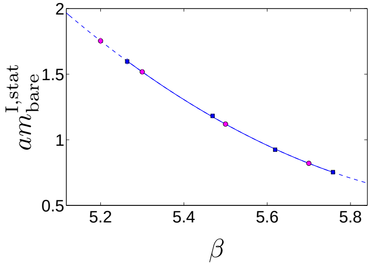
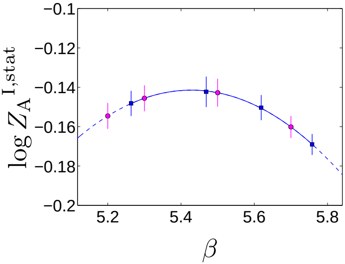
Results

Interpolation at the desired β values

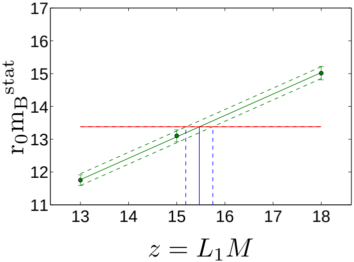




from $\beta = \ldots$ to $\beta = 5.2, 5.3, 5.5, 5.7$

Results

Interpolation at the physical mass, in the static approximation



The RGI quark masses M are such that $z = L_1 M \in (13, 15, 18)$

Conclusion - Status of the project

- $n_f = 0$
  - ▷ b-quark mass (Alpha '06)

$$n_f = 0 \qquad m_{\rm b}(m_{\rm b}) = \underbrace{4.350(64)}_{\rm static}\ {\rm GeV}\ \underbrace{-0.049(29)}_{{\rm O}(\Lambda^2/m_{\rm b})}\ {\rm GeV} + \underbrace{{\rm O}(\Lambda^3/m_{\rm b}^2)}$$

  - ▷ I. HQET parameters (Alpha '10)
  - ▷ II. Spectoscopy in the quenched approximation (Alpha '10)
  - ▷ III. $f_{\rm B_s}$:in preparation
- $n_f = 2$
  - ▷ HQET parameter : almost finished
  - ▷ Large volume part: preliminary results (1 lattice spacing)

VERY PRELIMINARY $m_{\rm b}(m_{\rm b})^{\rm stat} = 4.255(25)(50)(??)$ $\qquad m_{\rm b}(m_{\rm b})^{\rm HQET} = 4.276(25)(50)(??)$

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