An application of the variational analysis to calculate the meson spectral functions

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Plan of this talk

- Introduction
- Spectral functions via variational analysis
- Numerical results
  - **Test in the free quark case**
  - **Results at zero temperature**
  - **Results at finite temperature**
- Conclusions

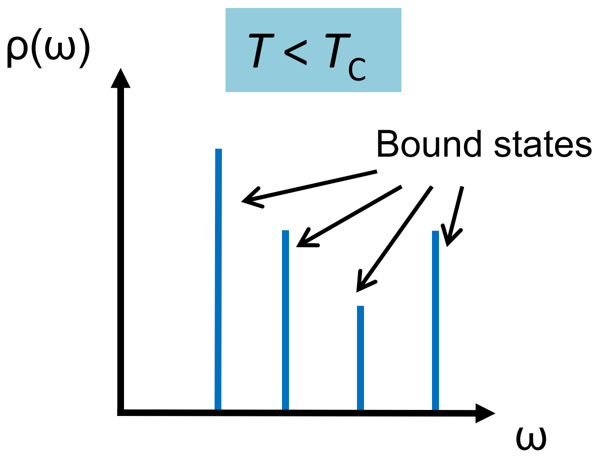
Introduction : motivation

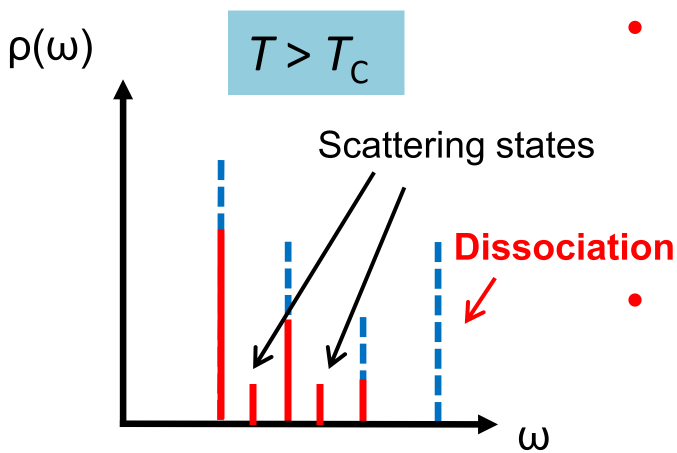
- Charmonia dissociation in Quark Gluon Plasma (QGP)
  - **Sequential J/Ψ suppression**
    - : Suppression of J/Ψ is one of the important signals of QGP formation in heavy ion collisions such as RHIC and LHC. T. Matsui and H. Satz (1986)
    - : Ψ' → J/Ψ 10%, χ_c → J/Ψ 30% L. Antoniazzi et al. [E705 Collaboration] (1993)
    - : Dissociation of excited states and P-wave states is also important.
- Meson spectral function (SPF) at finite temperature
  - **has information of the in-medium meson properties.**
- Current lattice studies e.g. A. Jackovac et al (2007)
  - **calculate SPFs with Maximum Entropy Method (MEM)**
  - **S-wave states (η_c, J/Ψ) seem to survive up to 1.5 T_C but may dissolve at very high temperature.**
  - **P-wave states (χ_c) seem to dissolve just above T_C.**
  - **Excited states have NOT investigated well yet.**

It is necessary to check the results and also investigate excited states with the other methods.

Introduction : our approach

spectral function





- On a finite volume lattice
  - SPF consists of discrete spectra only
- Below T_c (the critical temperature)
  - There are some peaks corresponding to bound states.
- Above T_c
  - Bound states' peaks should exist if they still survive but the value of corresponding SPF can change.
  - Some scattering states may appear.
  - If a bound state dissolves, corresponding value of SPF should become zero.
- **We investigate temperature dependence of SPFs**
  - Not whole shape of SPFs but just the value of SPFs corresponding to each discrete spectra is needed.
  - Excited states should be investigated, too.
- **Variational analysis**
  - can extract the properties of some low-lying states.
  - is well-suited for discrete spectra.

Definition of SPFs

- Meson correlator

$$C(t) \equiv \sum_{\vec{x}} \langle O^\Gamma(\vec{x},t) O^\Gamma(\vec{0},0) \rangle$$

  - Meson operator

$$O^\Gamma(\vec{x},t) \equiv \bar{q}(\vec{x},t)\Gamma q(\vec{x},t)$$

$$\Gamma = \begin{cases} \gamma_5 & \text{Pseudo scalar (Ps)} \\ \gamma_i & \text{Vector (Ve)} \\ \mathbf{1} & \text{Scalar (Sc)} \\ \gamma_5\gamma_i & \text{Axial vector (Av)} \end{cases}$$

$i = 1, 2, 3$

- SPF

$$C(t) = \sum_k \rho(m_k) \frac{\cosh[m_k(t - N_t/2)]}{\sinh[m_k N_t/2]}$$

$$\rho(m_k) \equiv (2\pi)^3 \sum_{m,n} \langle E_m, \vec{p}_m | O^\Gamma | E_n, \vec{p}_n \rangle \langle E_n, \vec{p}_n | O^{\Gamma\dagger} | E_m, \vec{p}_m \rangle$$
$$\times \left(1 - e^{-(E_n - E_m)/T}\right) \frac{e^{-E_m/T}}{Z} \delta^{(3)}(\vec{p}_n - \vec{p}_m) \delta(M_k - E_n + E_m)$$

SPFs via variational analysis

- Smeared meson operator

$$O_i^\Gamma(\vec{x},t) \equiv \sum_{\vec{y},\vec{z}} \omega_i(\vec{y})\omega_i(\vec{z})\bar{q}(\vec{x}+\vec{y},t)\Gamma q(\vec{x}+\vec{z},t)$$

A_0	A_1	A_2	A_3	A_4	A_5	A_6
∞	0.02	0.05	0.10	0.15	0.20	0.25

↑ point operator

  - **Gaussian smearing function**

$$\omega_i(\vec{x}) \equiv e^{-A_i||\vec{x}||^2} \qquad i = 1, 2, \cdots, N_{\mathrm{state}}$$

: approximate number of states

- Effective mass
  - **correlator matrix**

$$C(t) \equiv \left[\sum_{\vec{x}} \langle O_i^\Gamma(\vec{x},t) O_j^\Gamma(\vec{0},0)^\dagger\rangle\right]_{i,j=1}^{N_{\mathrm{state}}}$$

  - **generalized eigenproblem**

$$C(t)\boldsymbol{v}_k = \lambda_k(t,t_0)C(t_0)\boldsymbol{v}_k$$

→

$$\lambda_k(t,t_0) = \frac{\cosh[m_k(t-N_t/2)]}{\cosh[m_k(t_0-N_t/2)]}$$

- SPF

$$\Lambda(t,t_0) \equiv \mathrm{diag}[\lambda_1(t,t_0), \lambda_2(t,t_0), \cdots, \lambda_{N_{\mathrm{state}}}(t,t_0)]$$

$$V \equiv [\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_{N_{\mathrm{state}}}]$$

$$C(t) = C(t_0)V\Lambda(t,t_0)V^{-1}$$

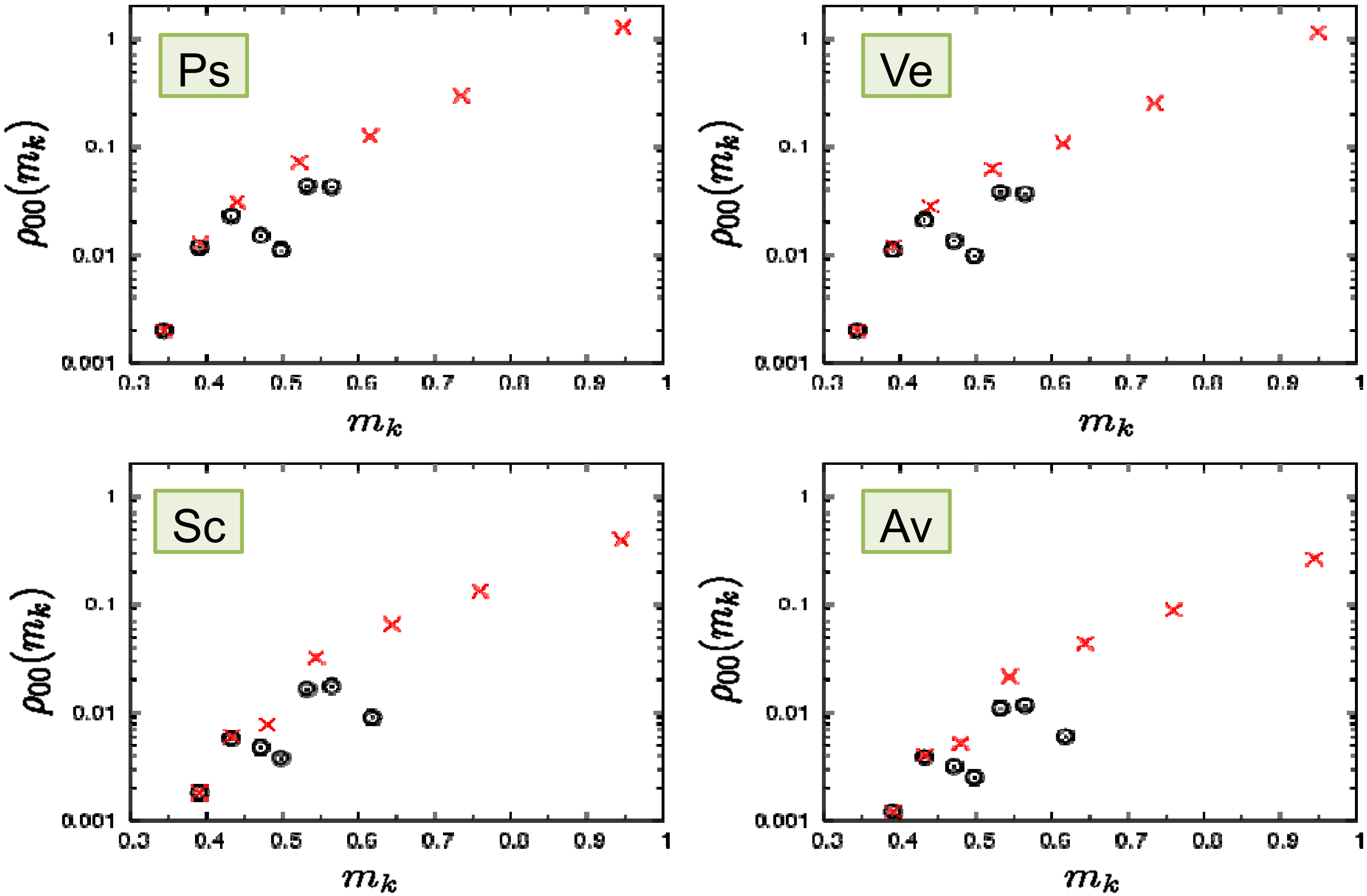
  - **point-point component**

$$C_{00}(t) = \sum_k \rho_{00}(m_k)\frac{\cosh[m_k(t-N_t/2)]}{\sinh[m_kN_t/2]}$$

→

$$\rho_{00}(m_k) = (C(t_0)V)_{0k}V_{k0}^{-1}\frac{\sinh[m_kN_t/2]}{\cosh[m_k(t_0-N_t/2)]}$$

Numerical results (1) : Test in the free quark case



$20^3 \times 32$ lattice

$N_{state} = 7$

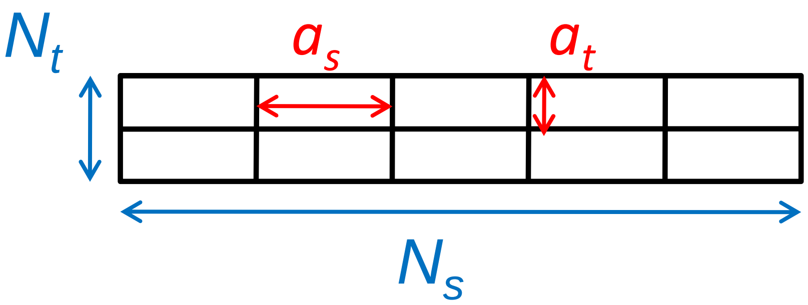
○ analytic solution for Wilson quarks

✕ variational analysis

Both m_k and ρ_{00} of variational analysis data are consistent with their analytic solutions up to the second low-lying state.

Lattice setup

- Action
  - Standard plaquette gauge action
  - $O(a)$-improved Wilson fermion action
  - Quenched approximation



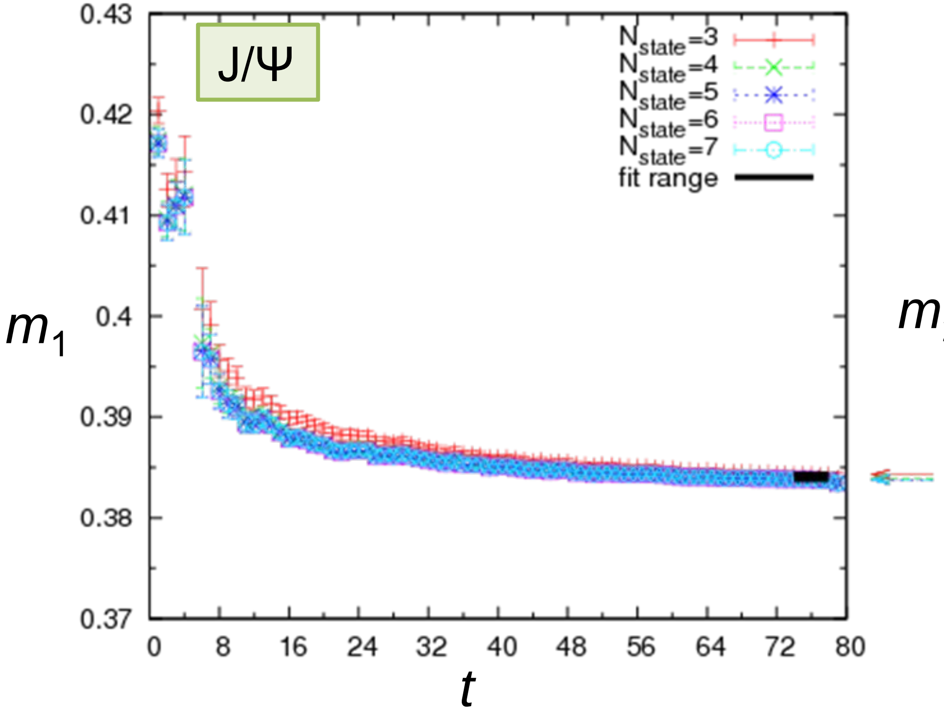
- Lattice
  - Anisotropic lattice : anisotropy $a_s/a_t = 4$
  - $a_s = 0.0970(5)$ fm ($a^{-1}_s = 2.030(13)$ GeV)
  - $N_s = 20$
  - $N_t = 160$ (zero temperature),
    32 ($0.88T_c$), 26 ($1.1T_c$), 20 ($1.4T_c$)

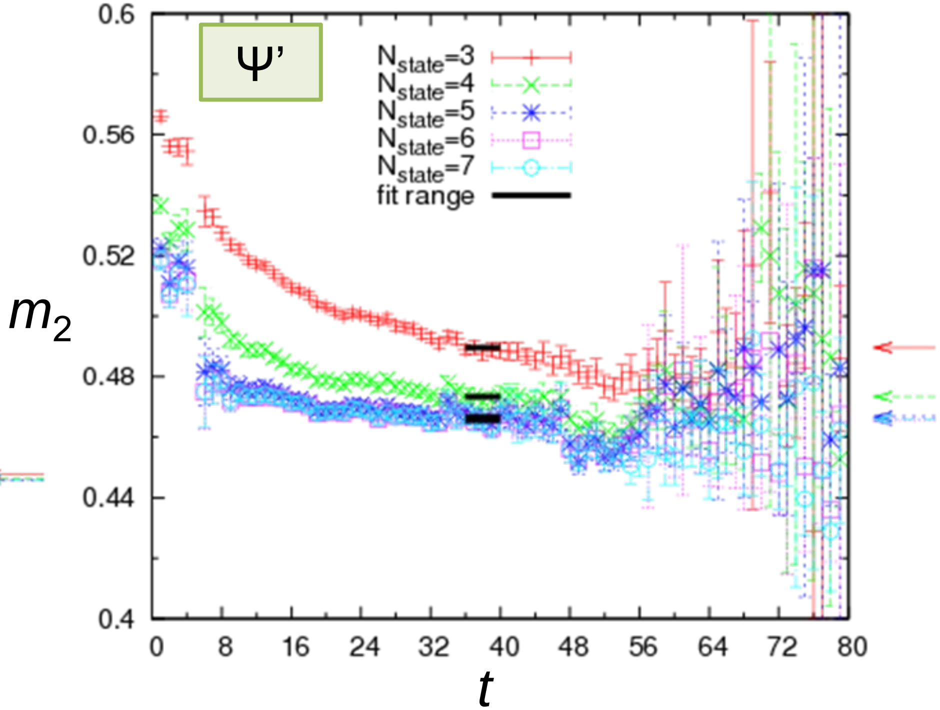
Temperature is changed by means of changing N_t

- Number of gauge configurations
  - for zero temperature : 299
  - for finite temperature : 800
- Coulomb gauge fixing

Numerical results (2) : at zero temperature 1

Effective mass (Ve channel)





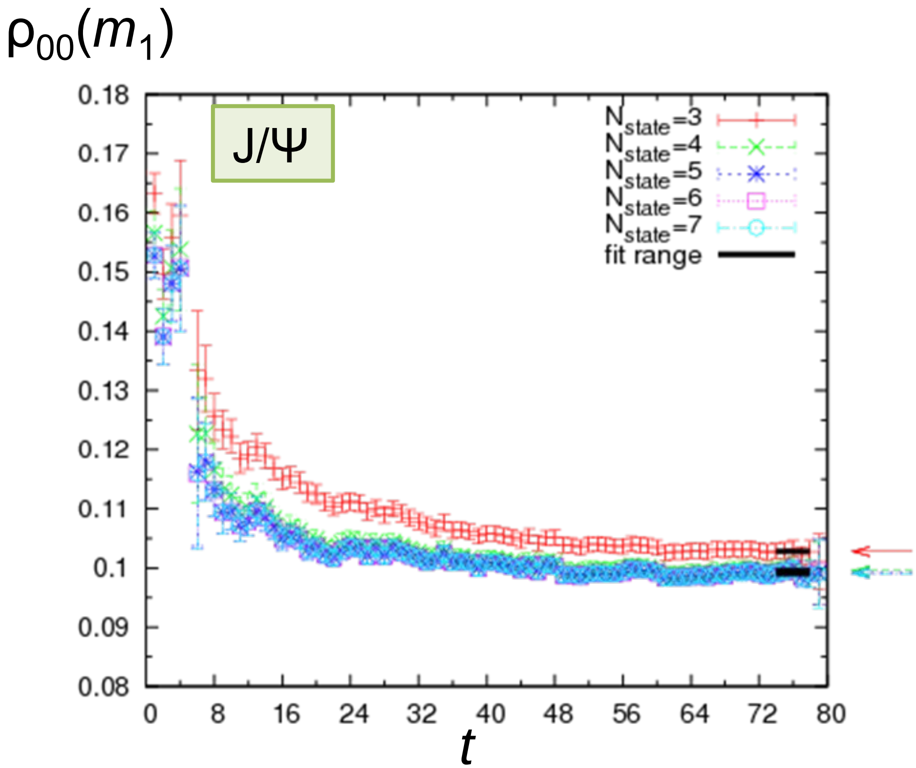
The difference of colors indicates the difference of N_{state}.

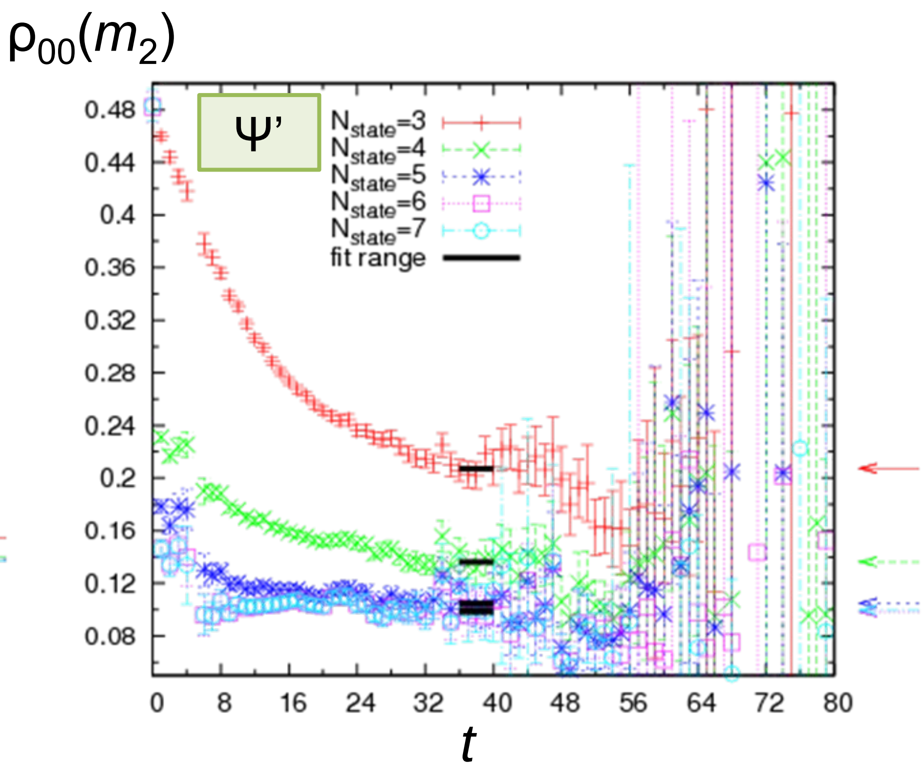
Arrows indicate fitted value with fit range shown by black bars.

N_{state} = 5,6,7 data converge on almost the same value for both the ground state and the excited state.

Numerical results (2) : at zero temperature 2

SPF (Ve channel)





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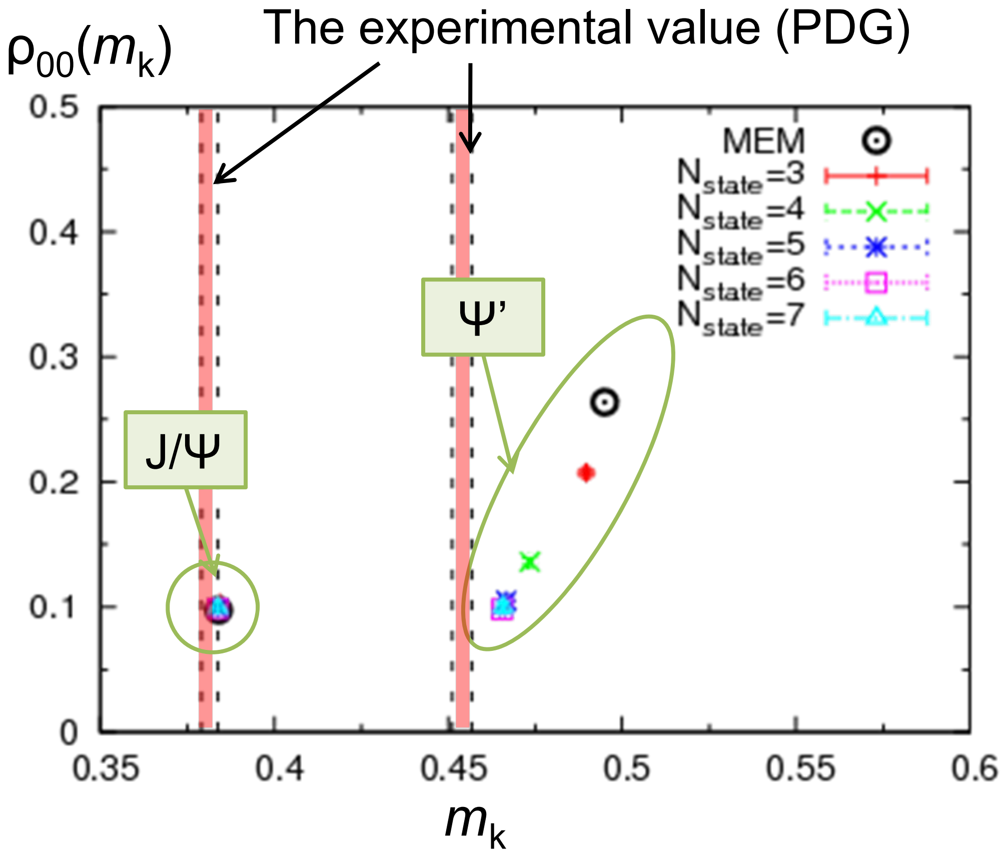
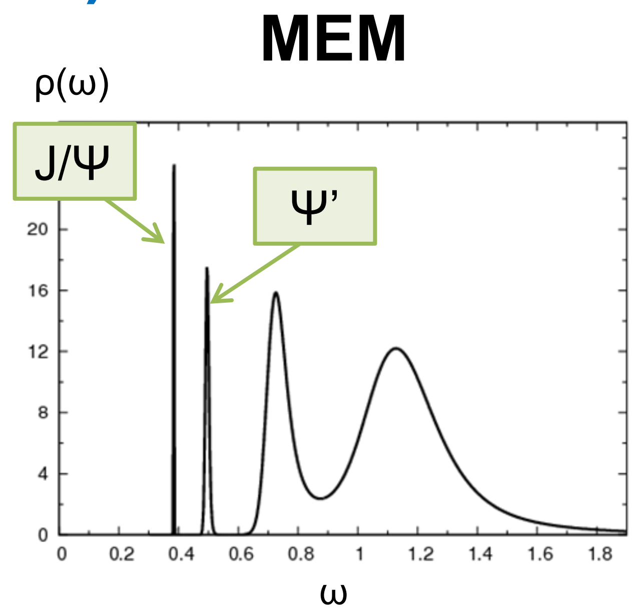
Numerical results (2) : at zero temperature 3

Comparison with MEM (Ve channel)





$m_k \leftarrow$ the position of each peak
$\rho_{00}(m_k) \leftarrow$ the area of each peak

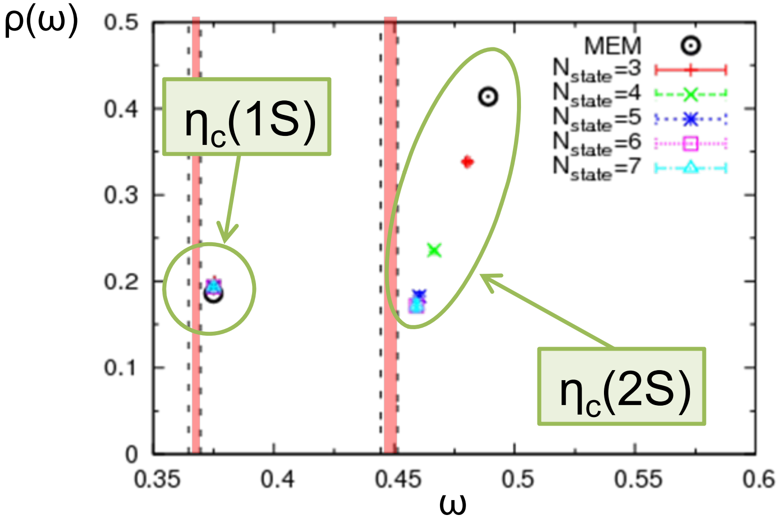
Variational analysis : fitted value

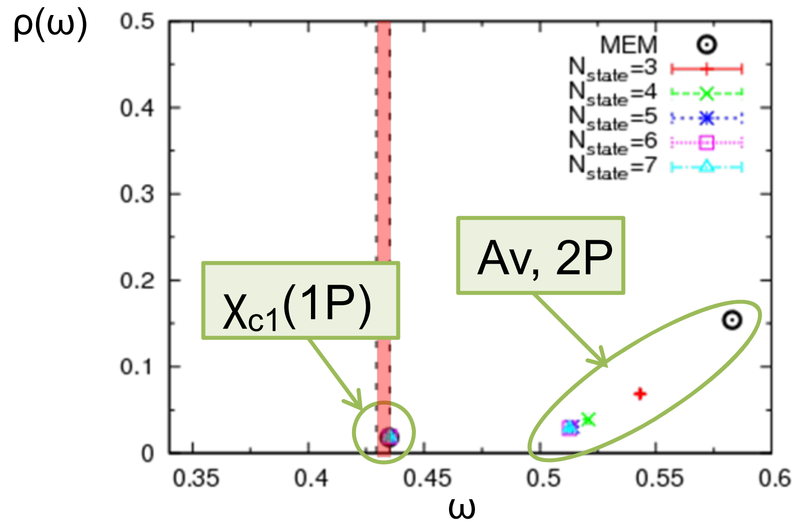
All data corresponding to the ground state converge on almost the same point.

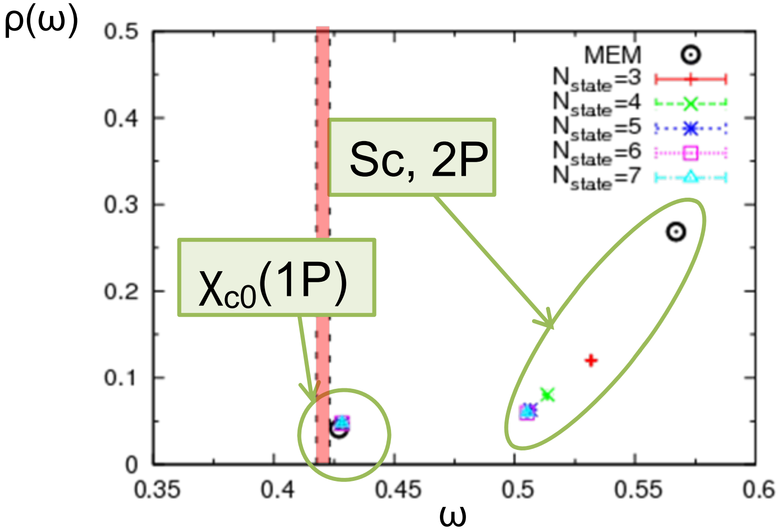
The data corresponding to the first excited state converge as N_{state} increases.

Numerical results (2) : at zero temperature 4

Comparison with MEM (Ps, Sc, Av channel)







All data corresponding to the ground state converge on almost the same point.

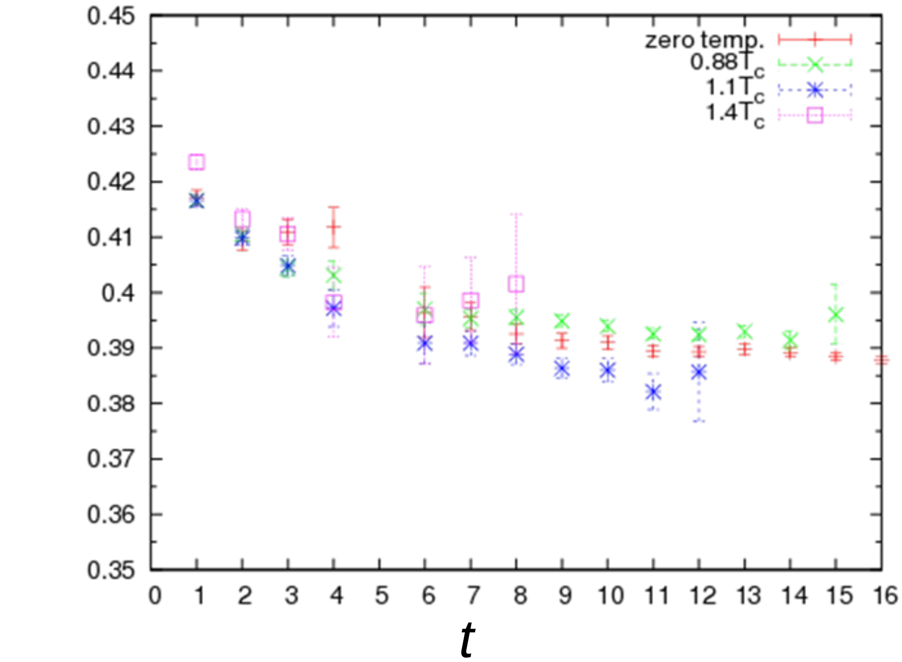
The data corresponding to the first excited state converge as N_{state} increases.

Numerical results (3) : at finite temperature 1

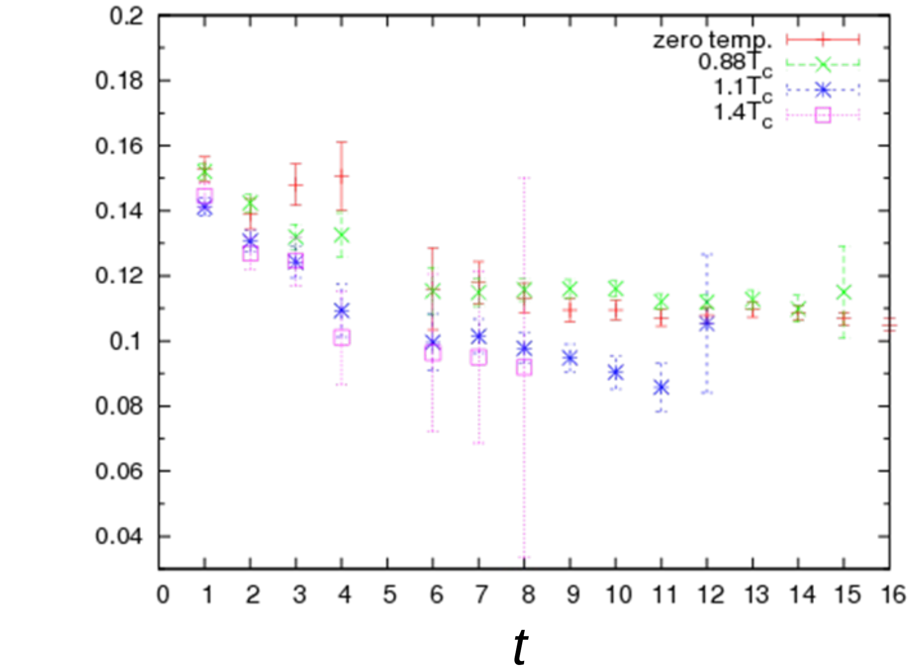
Temperature dependence (Ve channel, ground state)

$N_{state} = 7$

Effective mass



SPF



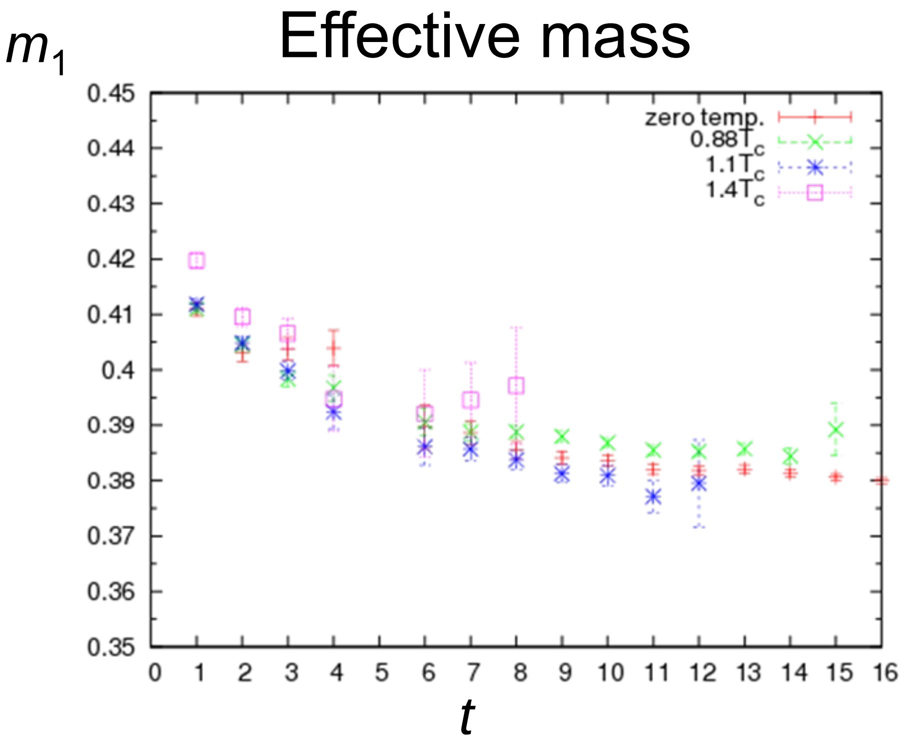
The difference of colors indicates the difference of temperature (0-1.4T_c).

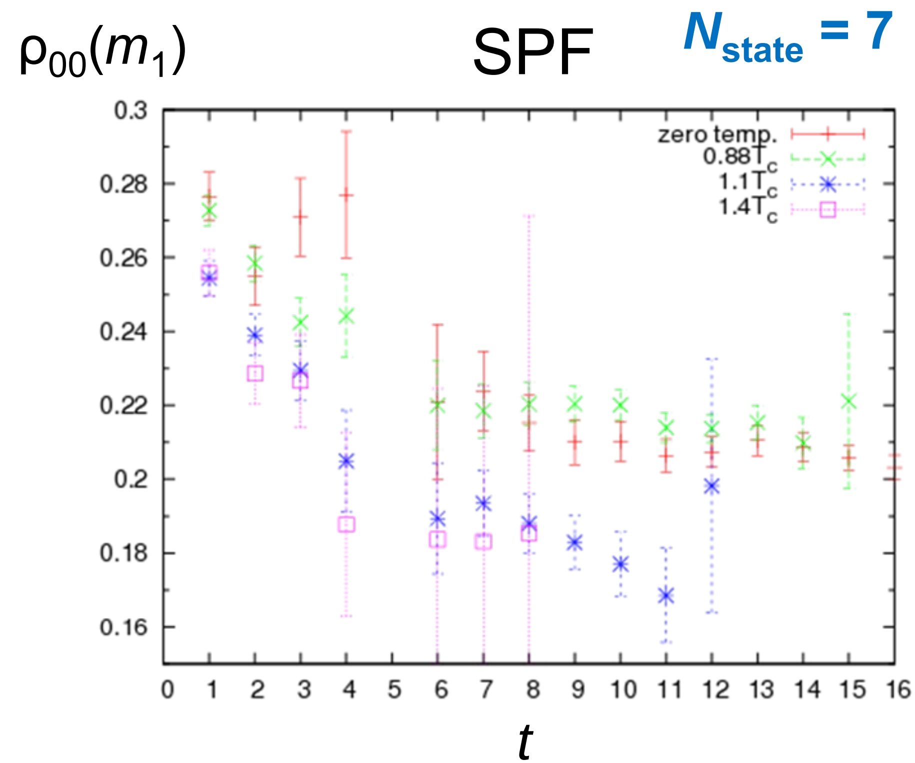
There seem to be no clear temperature dependence for the effective masses.

The value of SPF may change but does NOT become zero.

Numerical results (3) : at finite temperature 2

Temperature dependence (Ps channel, ground state)





The difference of colors indicates the difference of temperature (0-1.4T_c).

There seem to be no clear temperature dependence for the effective masses.

The value of SPF may change but does NOT become zero.

Conclusions

- We calculate meson SPFs with variational analysis.
- At zero temperature,
  - we can extract SPFs corresponding to both the ground state and the first excited state well for Ps, Ve, Sc and Av channel.
  - we show that the value of SPF corresponding to the first excited state can be improved by increasing $N_{state.}$
- At finite temperature,
  - there seem to be no clear temperature dependence for the effective masses of both the ground state and the first excited state of S-wave up to 1.4 T_c.
  - the value of SPF corresponding to the ground state and first excited state of S-wave may change but does NOT become zero up to 1.4 T_c.
  - there is no clear evidence of dissociation for the S-wave ground state up to1.4T_c.
  - more accurate data quality is needed to investigate SPF at higher temperature region and for P-wave states.