Constraints for the QCD phase diagram from imaginary chemical potential

Owe Philipsen

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

- Taking imaginary μ more seriously, study in its own right
- Triple, critical and tri-critical structures at $\mu = i\frac{\pi T}{3}$
- Implications for the QCD phase diagram at real μ
- Everything here standard staggered, Nt=4

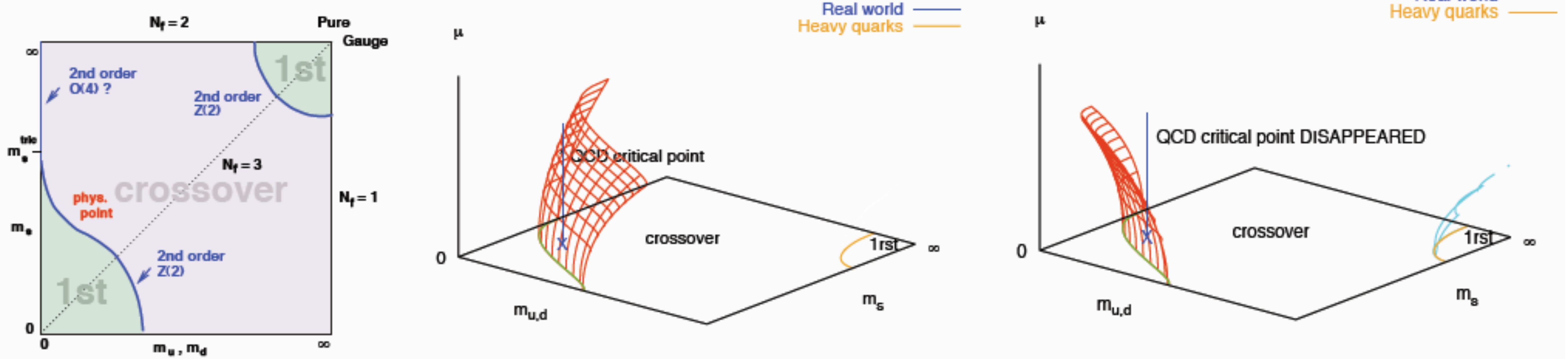
with Ph. de Forcrand (ETH/CERN), arXiv:1004.3144

The sign problem and its work-arounds

- QCD phase diagram at finite baryon density largely unknown
- Approximate results:
  Reweighting, Taylor expansion, imaginary $\mu = i\mu_i$ + analytic continuation
- Only valid for $\mu/T \lesssim 1$

→ Here: imaginary chem. pot. without continuation:
no sign problem, no approximations

Finite density: critical lines $\longrightarrow$ critical surfaces



$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$

$c_1 > 0$ $\qquad$ $c_1 < 0$

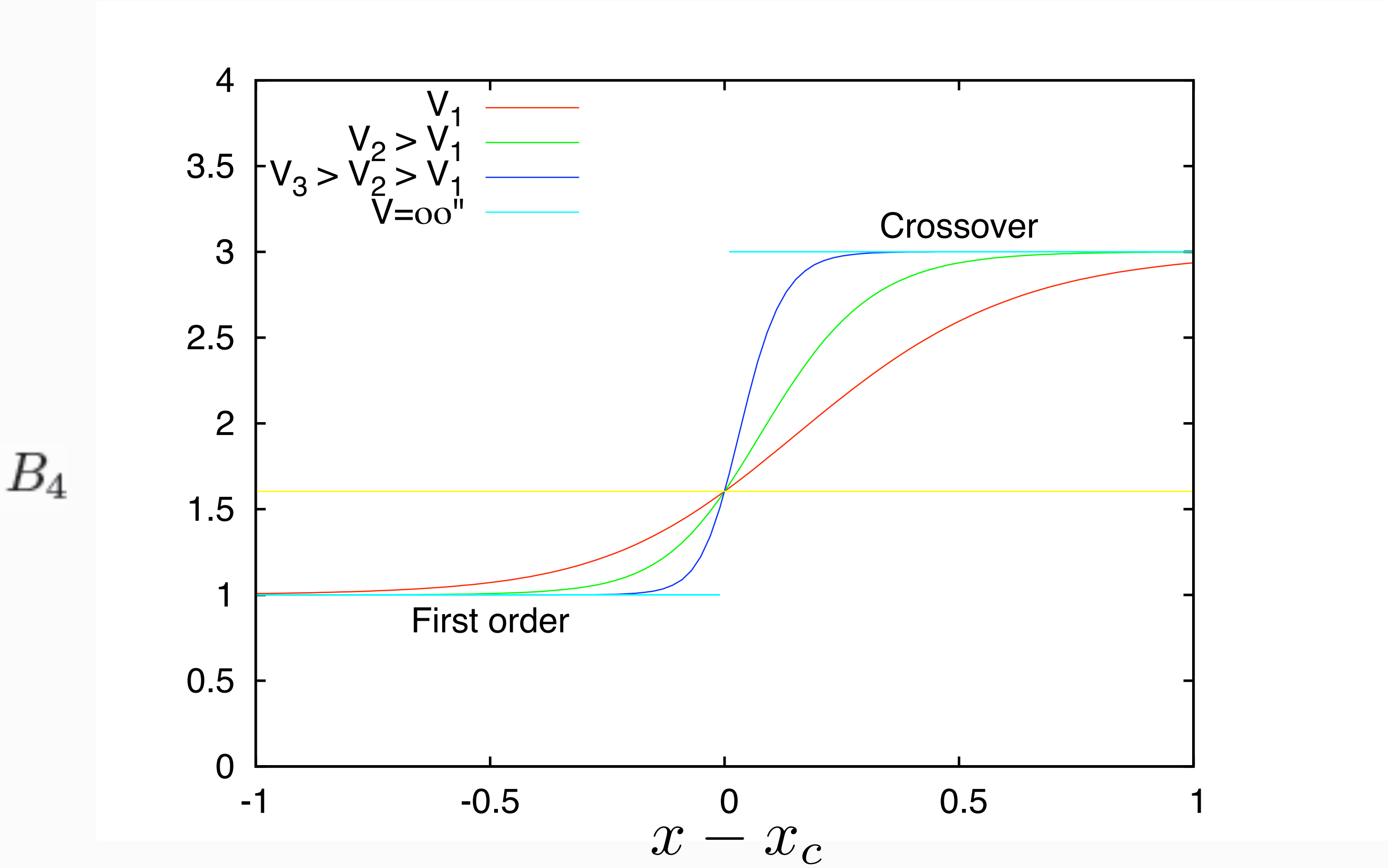
On coarse lattices: right scenario, no chiral critical point de Forcrand, O.P. 08

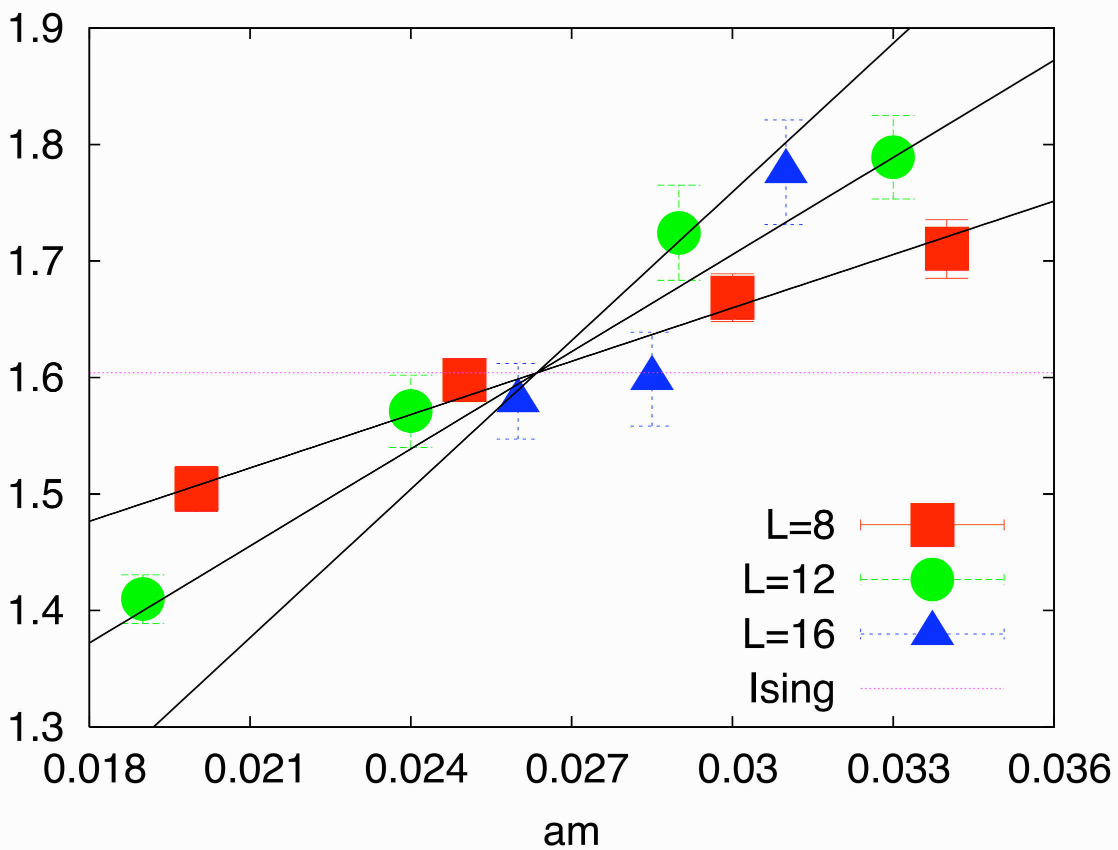
Nf=3: $$\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3)\left(\frac{\mu}{\pi T}\right)^2 - 47(20)\left(\frac{\mu}{\pi T}\right)^4 - \ldots$$

How to identify the order of the phase transition

$$B_4(\bar\psi\psi) \equiv \frac{\langle(\delta\bar\psi\psi)^4\rangle}{\langle(\delta\bar\psi\psi)^2\rangle^2} \xrightarrow{V\to\infty} \begin{cases} 1.604 & \text{3d Ising} \\ 1 & \text{first-order} \\ 3 & \text{crossover} \end{cases}$$

$\mu = 0:$ $$B_4(m,L) = 1.604 + bL^{1/\nu}(m - m_0^c), \quad \nu = 0.63$$





parameter along phase boundary, $T = T_c(x)$

QCD at complex μ: general properties

$$Z(V,\mu,T) = \mathrm{Tr}\left(\mathrm{e}^{-(\hat{H}-\mu\hat{Q})/T}\right); \quad \mu=\mu_r+\mathrm{i}\mu_i; \quad \bar{\mu}=\mu/T$$

exact symmetries: μ-reflection and μ_i-periodicity

Roberge, Weiss

$$Z(\bar{\mu}) = Z(-\bar{\mu}), \qquad Z(\bar{\mu}_r,\bar{\mu}_i) = Z(\bar{\mu}_r,\bar{\mu}_i + 2\pi/N_c)$$

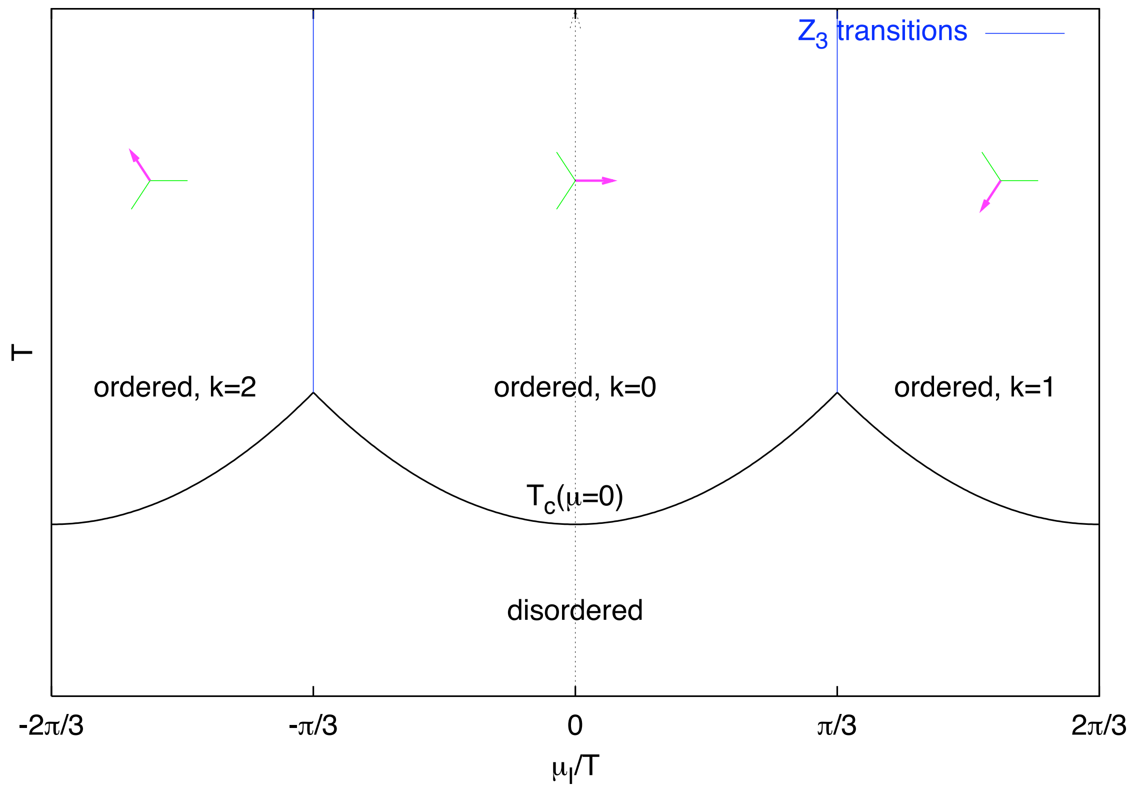
Imaginary μ phase diagram:

Z(3)-transitions: $\bar{\mu}_i^c = \frac{2\pi}{3}\left(n+\frac{1}{2}\right)$

1rst order for high T, crossover for low T

analytic continuation within:

$|\mu|/T \leq \pi/3 \Rightarrow \mu_B \lesssim 550\mathrm{MeV}$



So far:

$$\langle O\rangle = \sum_n^N c_n \bar{\mu}_i^{2n} \Rightarrow \mu_i \longrightarrow -\mathrm{i}\mu_i$$

QCD at complex μ: general properties

$$Z(V,\mu,T) = \mathrm{Tr}\left(\mathrm{e}^{-(\hat{H}-\mu\hat{Q})/T}\right); \quad \mu = \mu_r + \mathrm{i}\mu_i; \quad \bar{\mu} = \mu/T$$

exact symmetries: μ-reflection and μ_i-periodicity   Roberge, Weiss

$$Z(\bar{\mu}) = Z(-\bar{\mu}), \qquad Z(\bar{\mu}_r, \bar{\mu}_i) = Z(\bar{\mu}_r, \bar{\mu}_i + 2\pi/N_c)$$

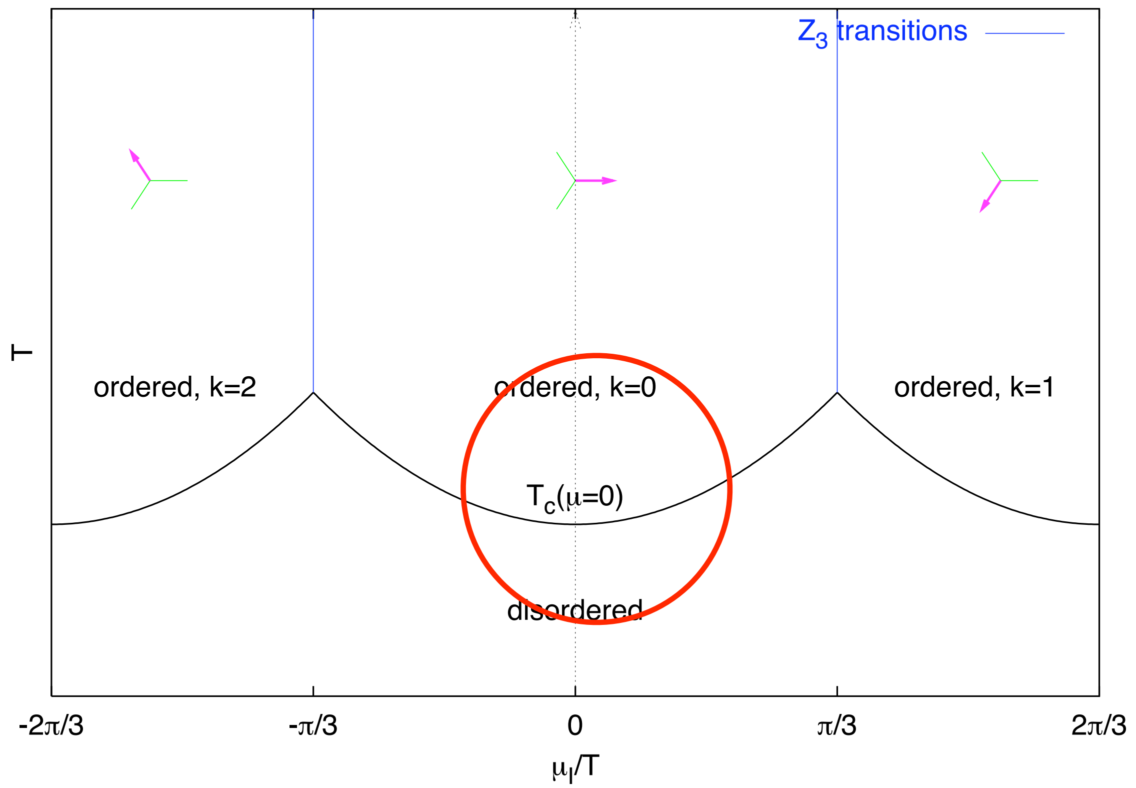
Imaginary μ phase diagram:

Z(3)-transitions: $\bar{\mu}_i^c = \frac{2\pi}{3}\left(n + \frac{1}{2}\right)$

1rst order for high T, crossover for low T

analytic continuation within:

$|\mu|/T \leq \pi/3 \Rightarrow \mu_B \lesssim 550\mathrm{MeV}$



So far: $$\langle O \rangle = \sum_n^N c_n \bar{\mu}_i^{2n} \Rightarrow \mu_i \longrightarrow -\mathrm{i}\mu_i$$ chiral/deconf. transition

QCD at complex μ: general properties

$$Z(V,\mu,T) = \mathrm{Tr}\left(\mathrm{e}^{-(\hat{H}-\mu\hat{Q})/T}\right); \quad \mu = \mu_r + \mathrm{i}\mu_i; \quad \bar{\mu} = \mu/T$$

exact symmetries: μ-reflection and μ_i-periodicity — Roberge, Weiss

$$Z(\bar{\mu}) = Z(-\bar{\mu}), \qquad Z(\bar{\mu}_r, \bar{\mu}_i) = Z(\bar{\mu}_r, \bar{\mu}_i + 2\pi/N_c)$$

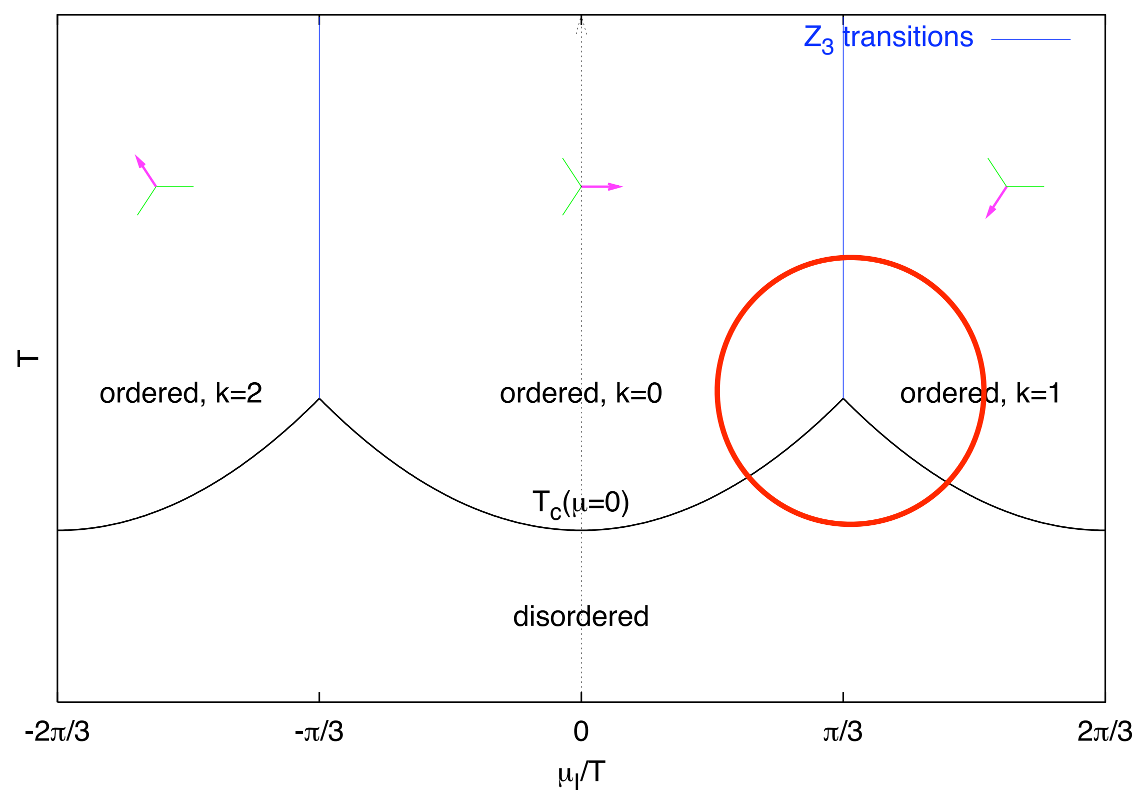
Imaginary μ phase diagram:

Z(3)-transitions: $\bar{\mu}_i^c = \frac{2\pi}{3}\left(n + \frac{1}{2}\right)$

1rst order for high T, crossover for low T

analytic continuation within:

$|\mu|/T \leq \pi/3 \Rightarrow \mu_B \lesssim 550\mathrm{MeV}$



So far: $\langle O \rangle = \sum_n^N c_n \bar{\mu}_i^{2n} \Rightarrow \mu_i \longrightarrow -\mathrm{i}\mu_i$

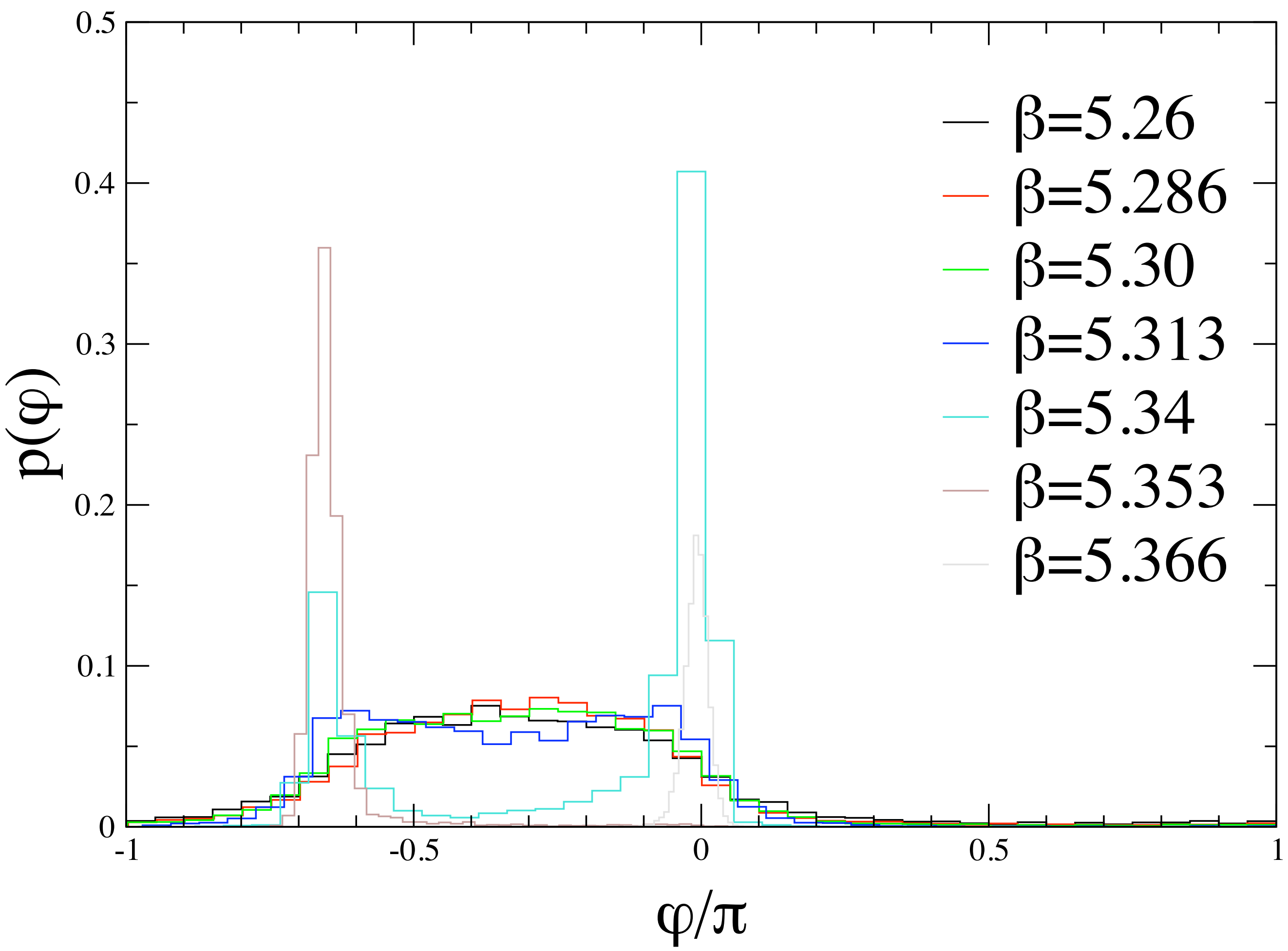
Now: endpoint/junction of Z(3) transition

The Z(3) transition numerically

Nf=2: de Forcrand, O.P. 02

Nf=4: D'Elia, Lombardo 03

Sectors characterised by phase of Polyakov loop: $\langle L(x)\rangle = |\langle L(x)\rangle|\, \mathrm{e}^{i\varphi}$



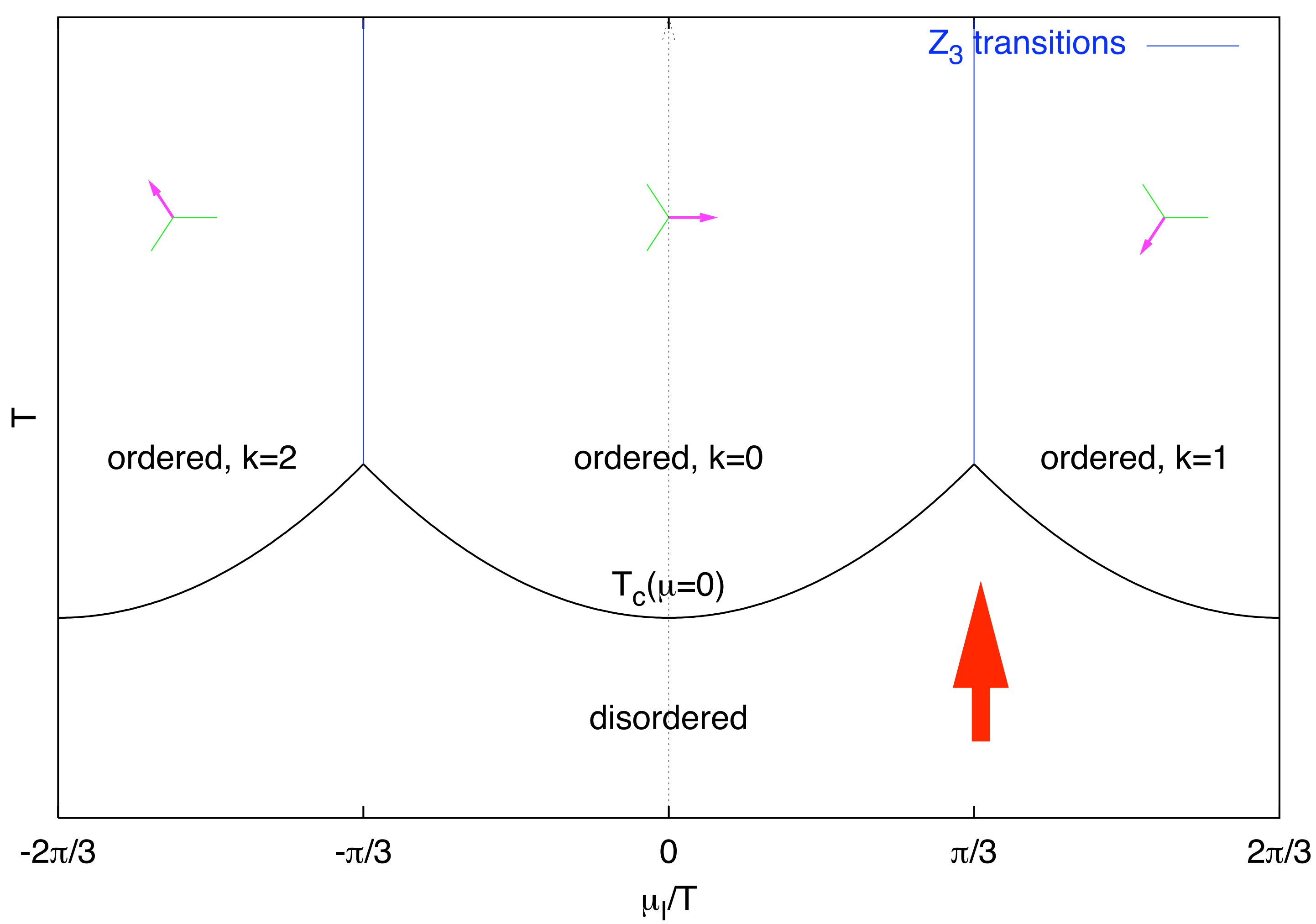
Low T: crossover   High T: first order p.t.

The nature of the Z(3) end points

Nf=4: D'Elia, Di Renzo, Lombardo 07 Nf=2: D'Elia, Sanfilippo 09 Here: Nf=3

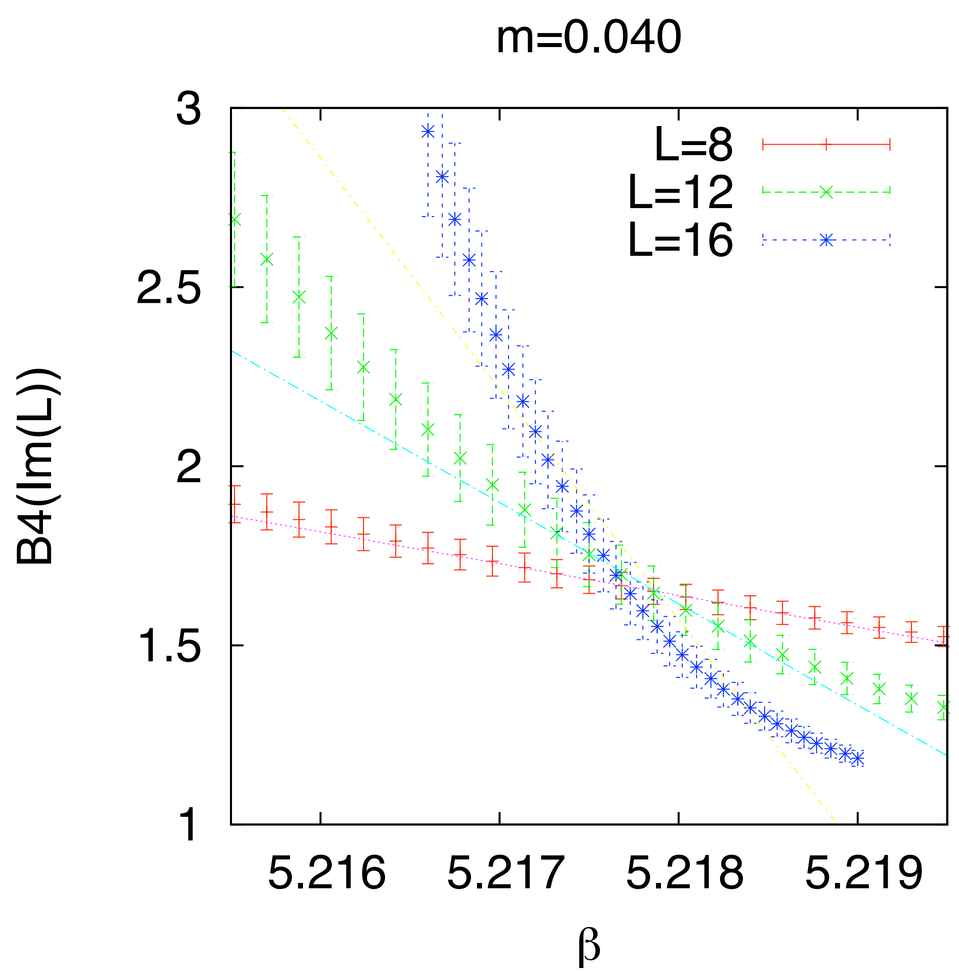
Strategy: fix $\frac{\mu_i}{T} = \frac{\pi}{3}, \pi$, measure Im(L), order parameter at $\frac{\mu_i}{T} = \pi$

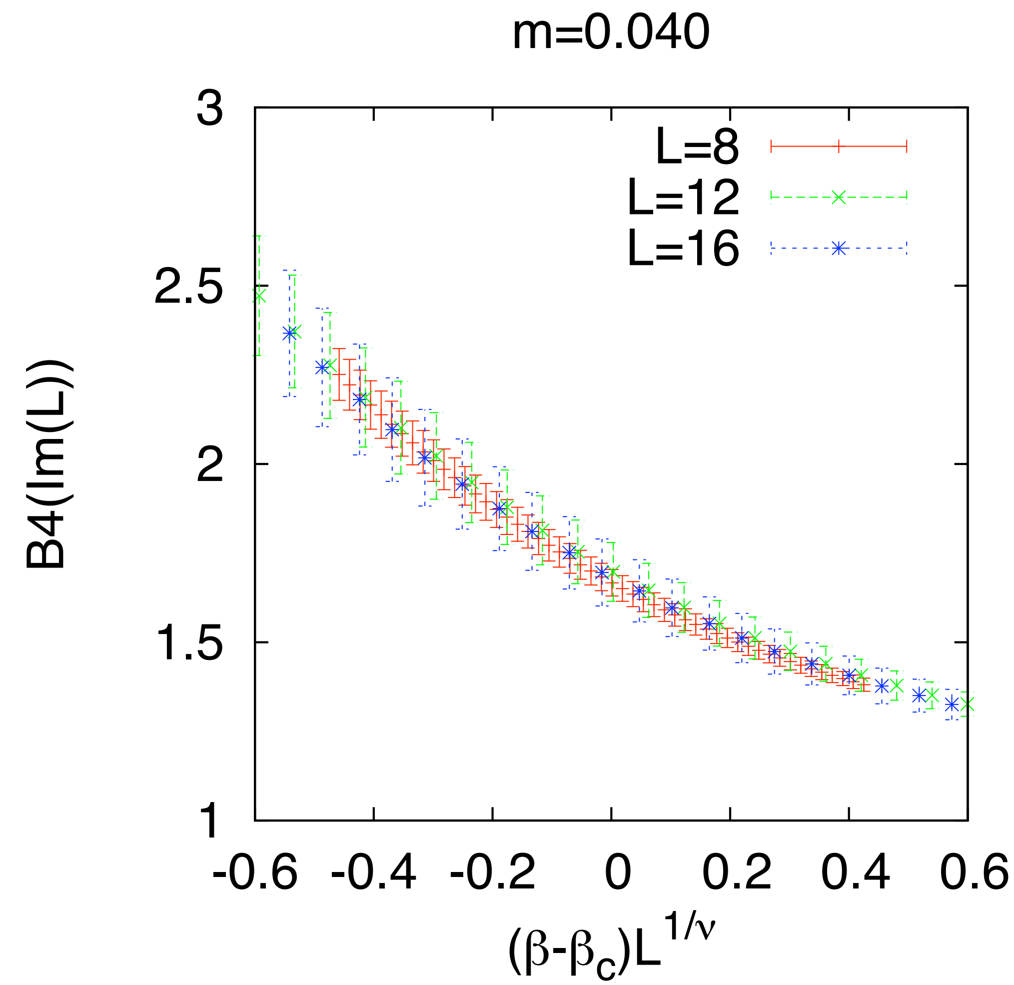
determine order of Z(3) branch/end point as function of m



$$B_4 = \frac{\langle \delta \mathrm{Im}(L)^4 \rangle}{\langle \delta \mathrm{Im}(L)^2 \rangle^2}$$

Results:



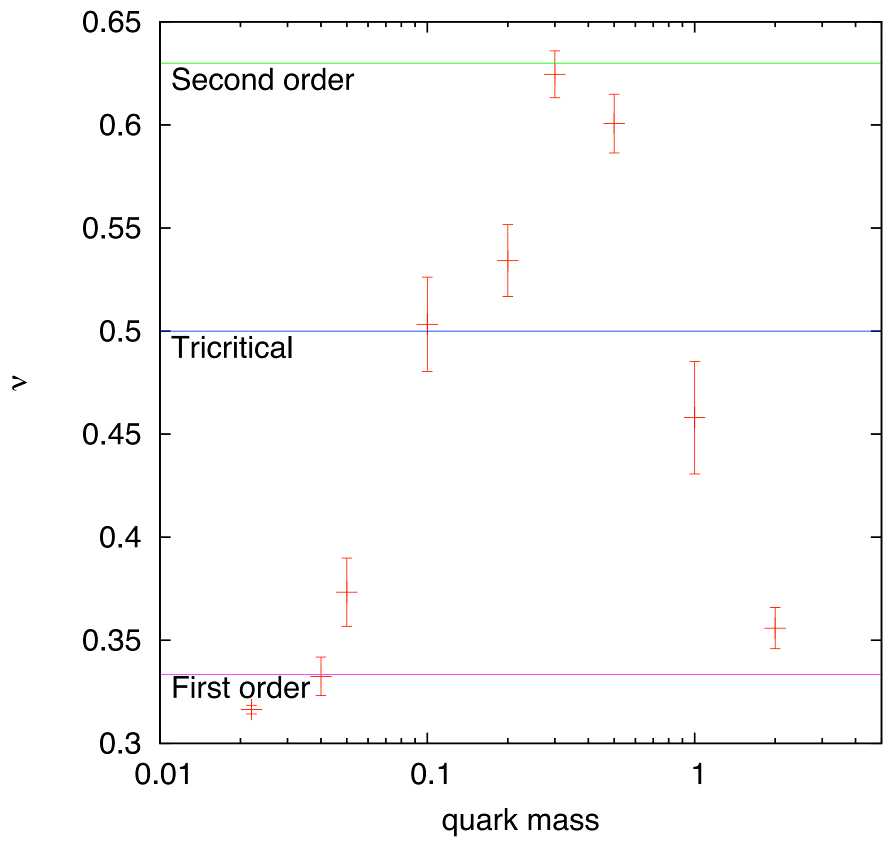


$\nu = 0.33$

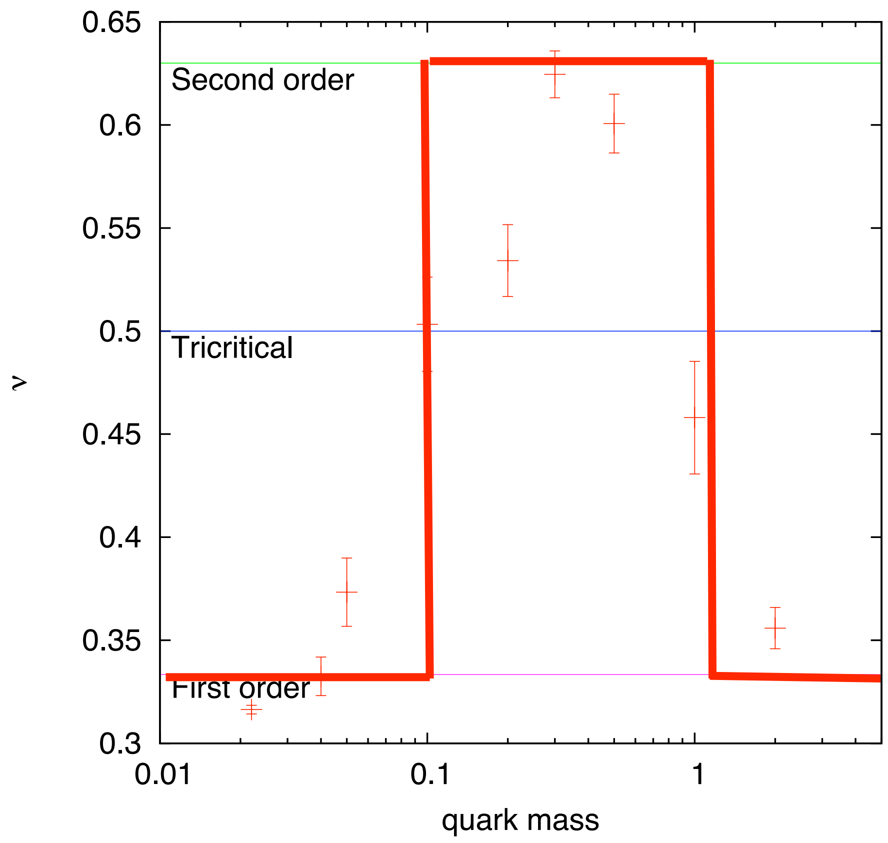
$$B_4(\beta, L) = B_4(\beta_c, \infty) + C_1(\beta - \beta_c)L^{1/\nu} + C_2(\beta - \beta_c)^2 L^{2/\nu} \dots$$

B4 at intersection has large finite size corrections (well known), ν more stable

$\nu = 0.33, 0.5, 0.63$ for 1st order, tri-critical, 3d Ising scaling

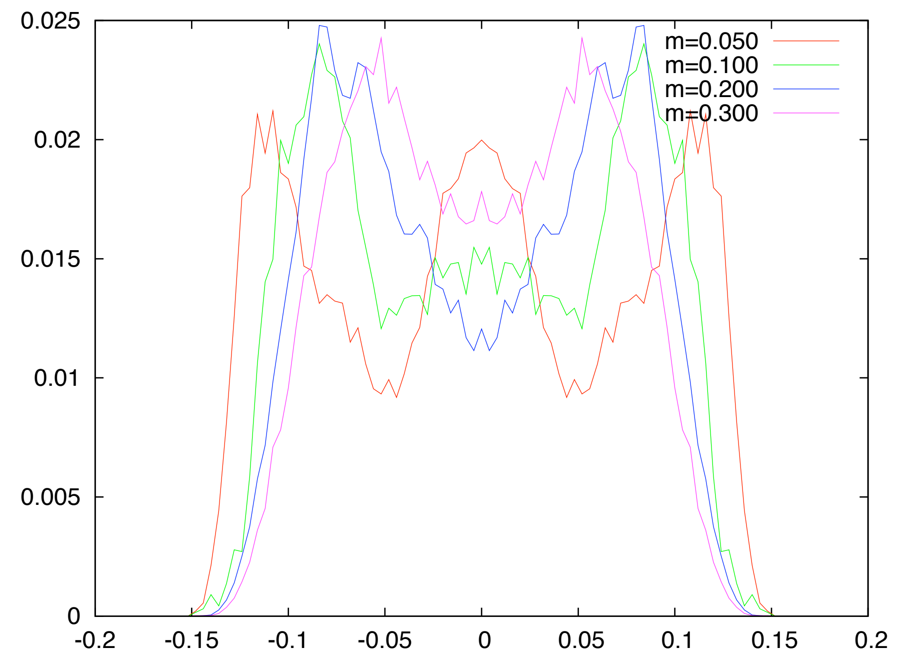


$\nu = 0.33, 0.5, 0.63$ for 1st order, tri-critical, 3d Ising scaling



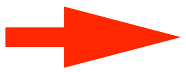
On infinite volume, this becomes a step function, smoothness due to finite L

Details of RW-point: distribution of Im(L)



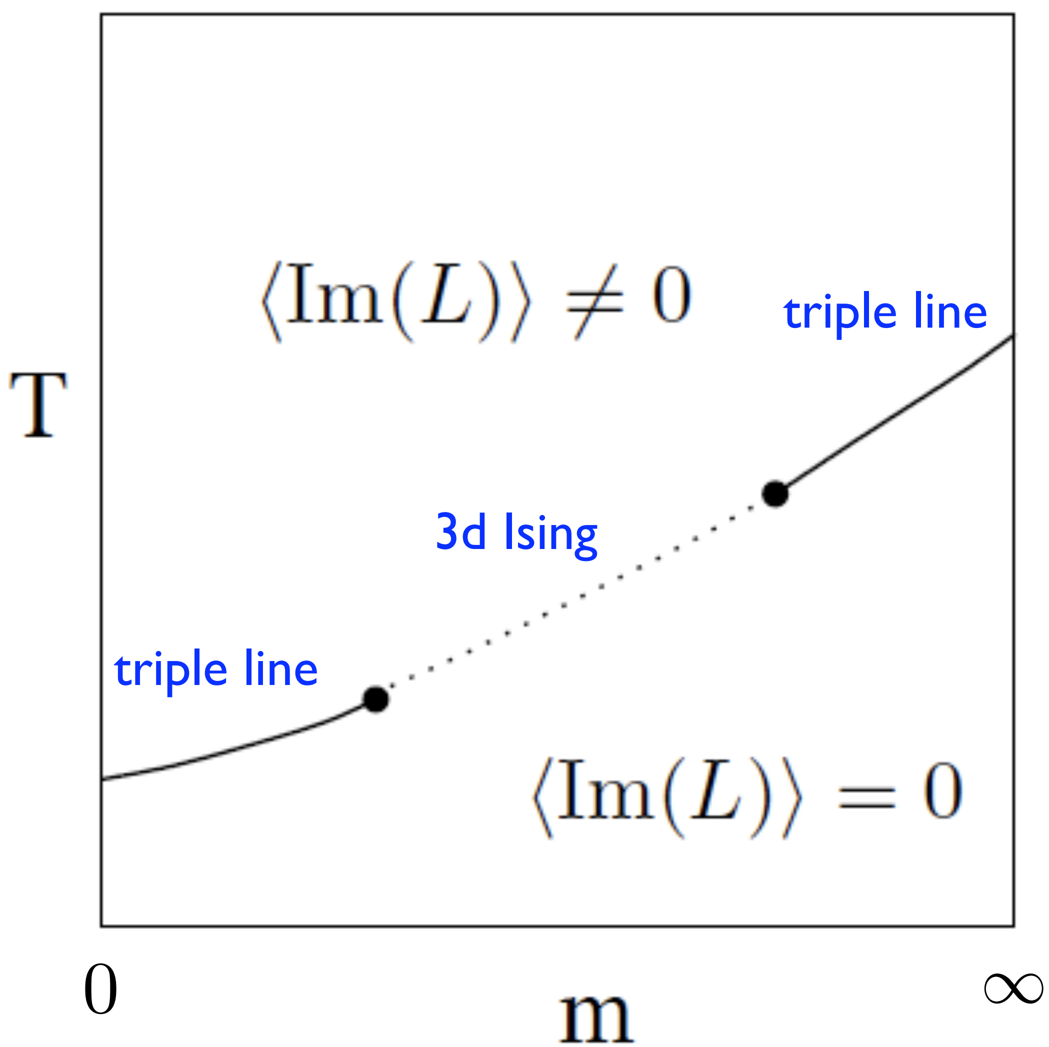
Small+large masses: three-state coexistence   triple point

Intermediate masses: middle peak disappears   Ising distribution in magn. direction



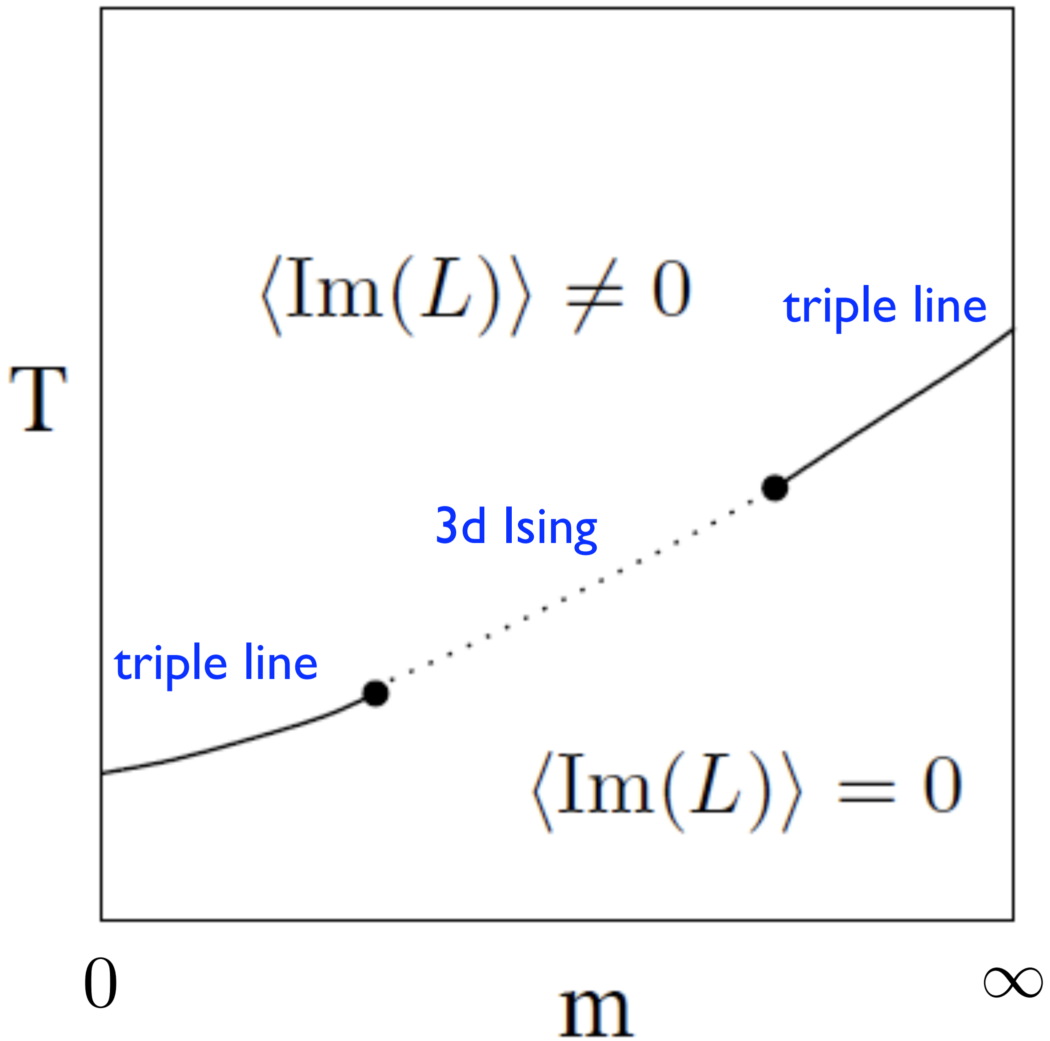
tri-critical point in between!

Phase diagram at $\mu = i\frac{\pi T}{3}$



Nf=2, light and intermediate masses: 1st and 3d Ising behaviour D'Elia, Sanfilippo 09

Phase diagram at $\mu = i\frac{\pi T}{3}$

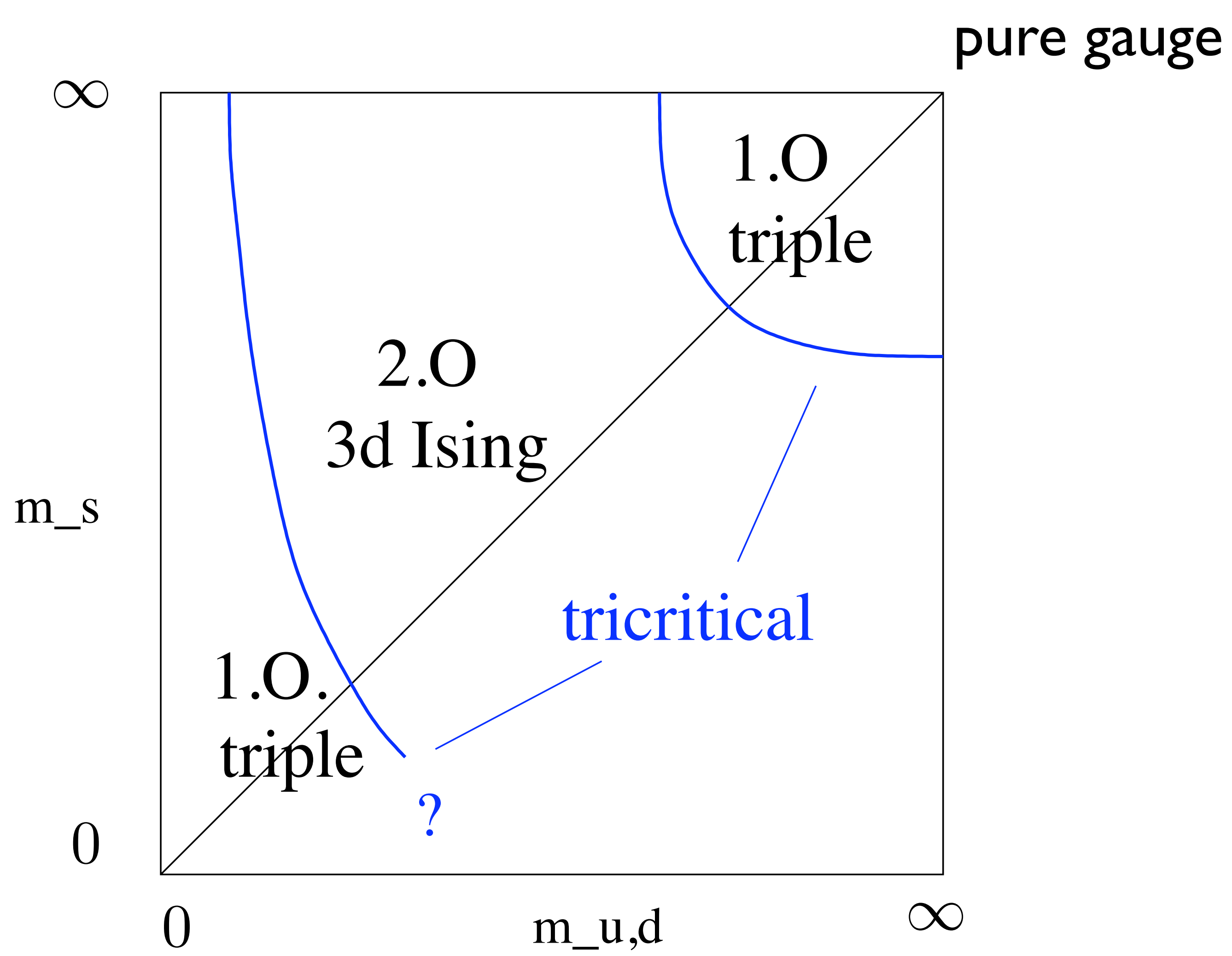


Cut-off effects?

-location of lines, tric. points strongly affected
-qualitative structure stable, universality!
(up to tric. points merging or on boundary?)

Nf=2, light and intermediate masses: 1st and 3d Ising behaviour   D'Elia, Sanfilippo 09

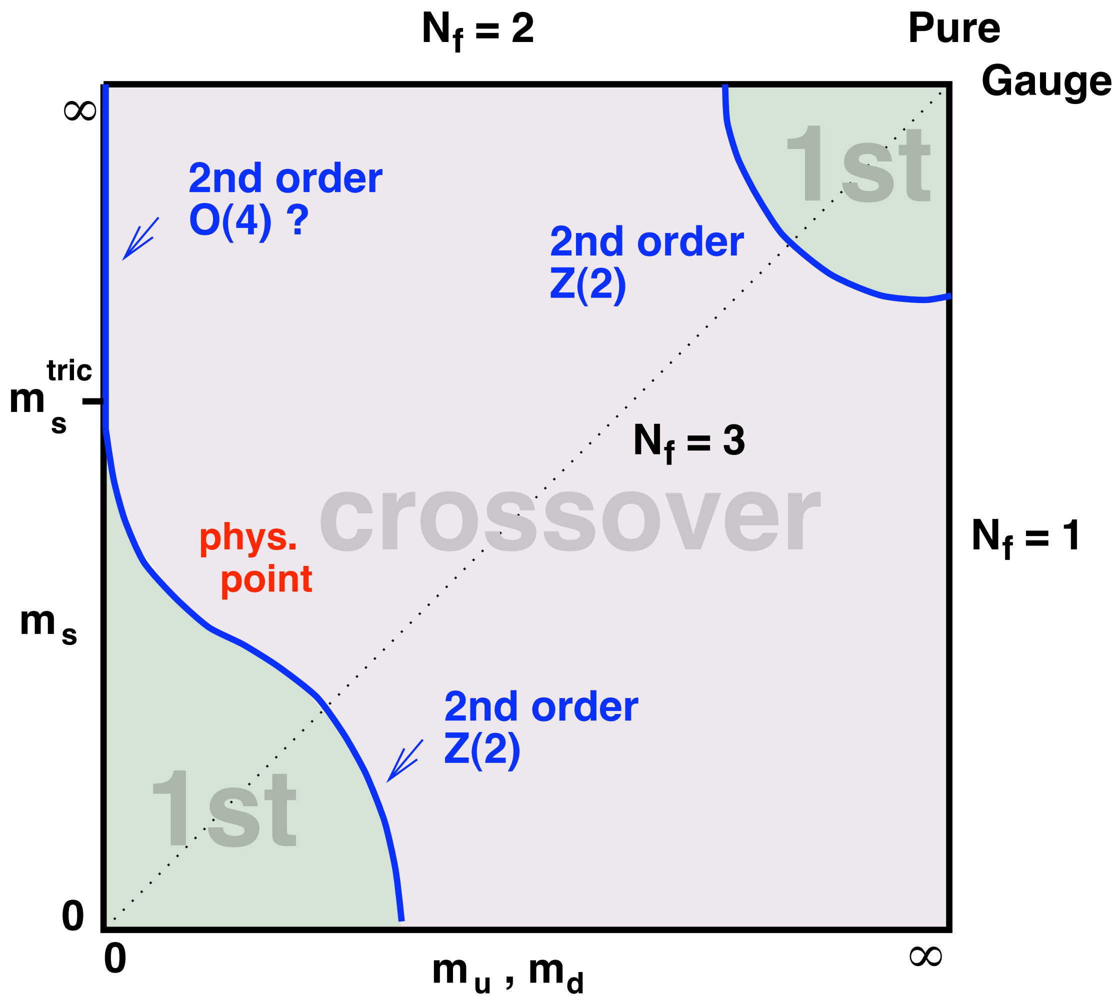
Generalisation: nature of the Z(3) endpoint for Nf=2+1

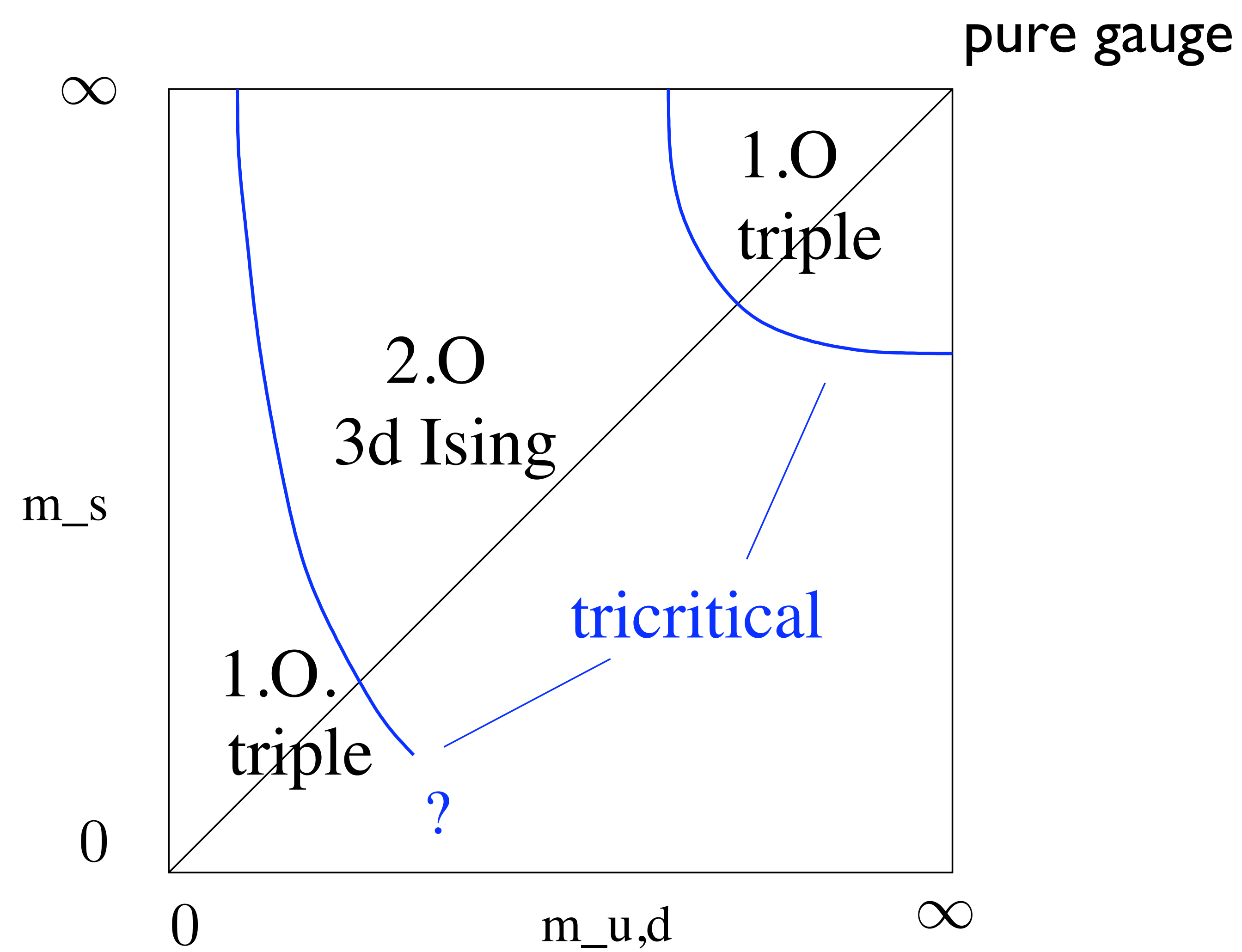


-Diagram computable with standard Monte Carlo, continuum limit feasible!

-Benchmarks for PNJL, chiral models etc.

Connection with zero and real μ





$$\mu = 0$$

$$\mu = i\frac{\pi T}{3}$$

-Connection computable with standard Monte Carlo!

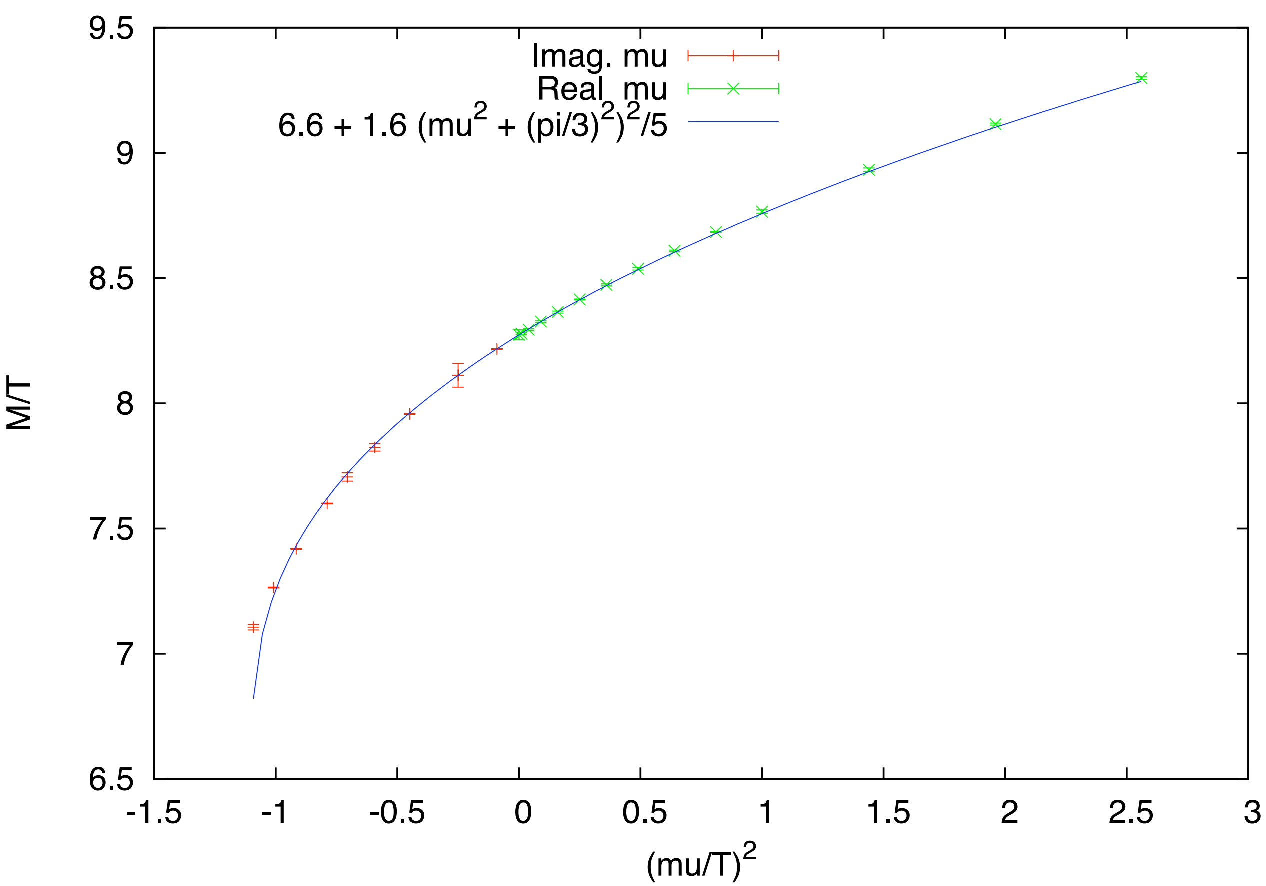
-Here: heavy quarks in eff. theory

$m \to \infty$: QCD$\to$ theory of Polyakov lines $\to$ universality class of 3d 3-state Potts model

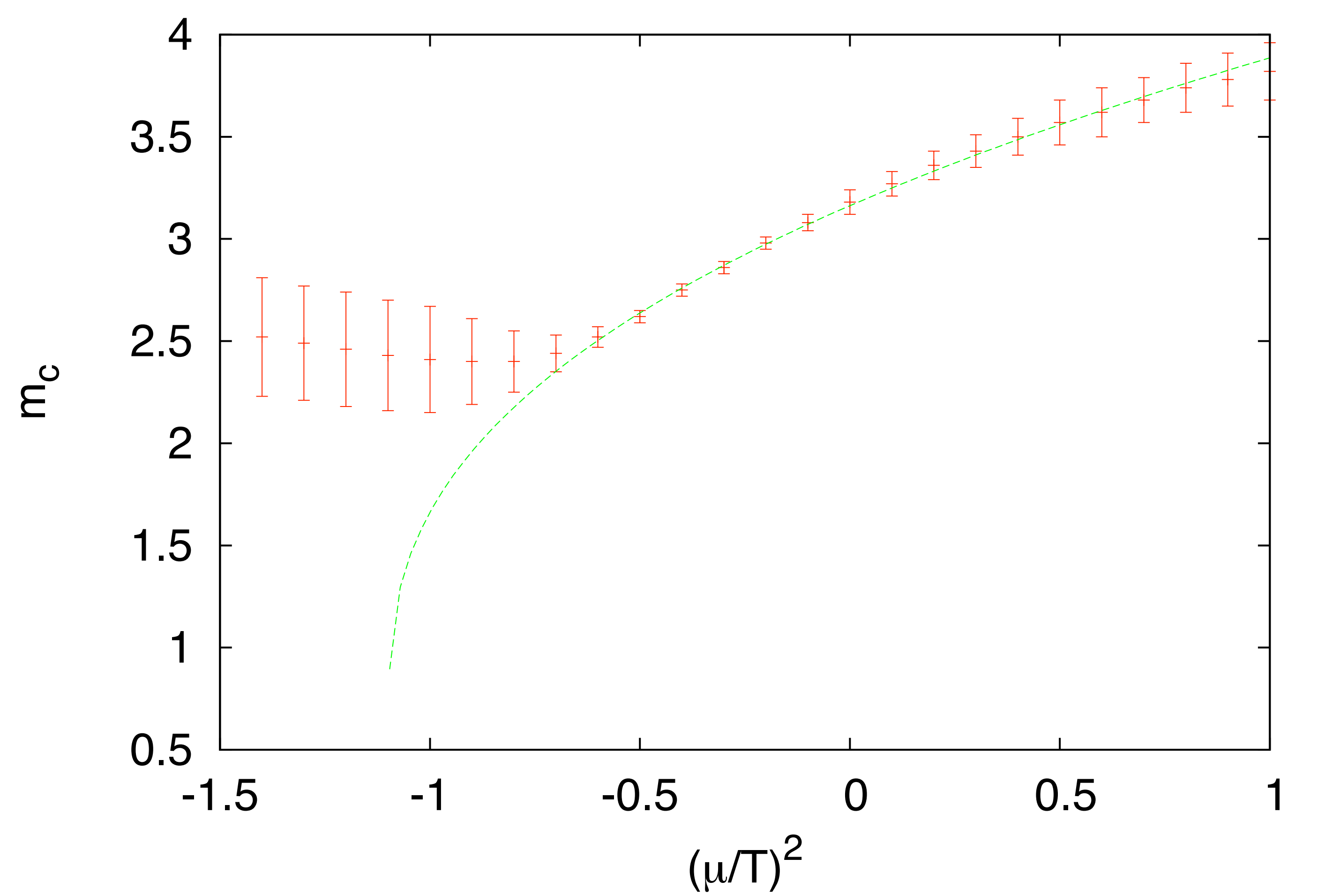
(3d Ising, Z(2))

small μ/T: sign problem mild, doable for real μ!

Potts, Monte Carlo:
de Forcrand, Kim, Kratochvila, Takaishi 05



QCD, Nt=1, strong coupling series:
Langelage, O.P. 09

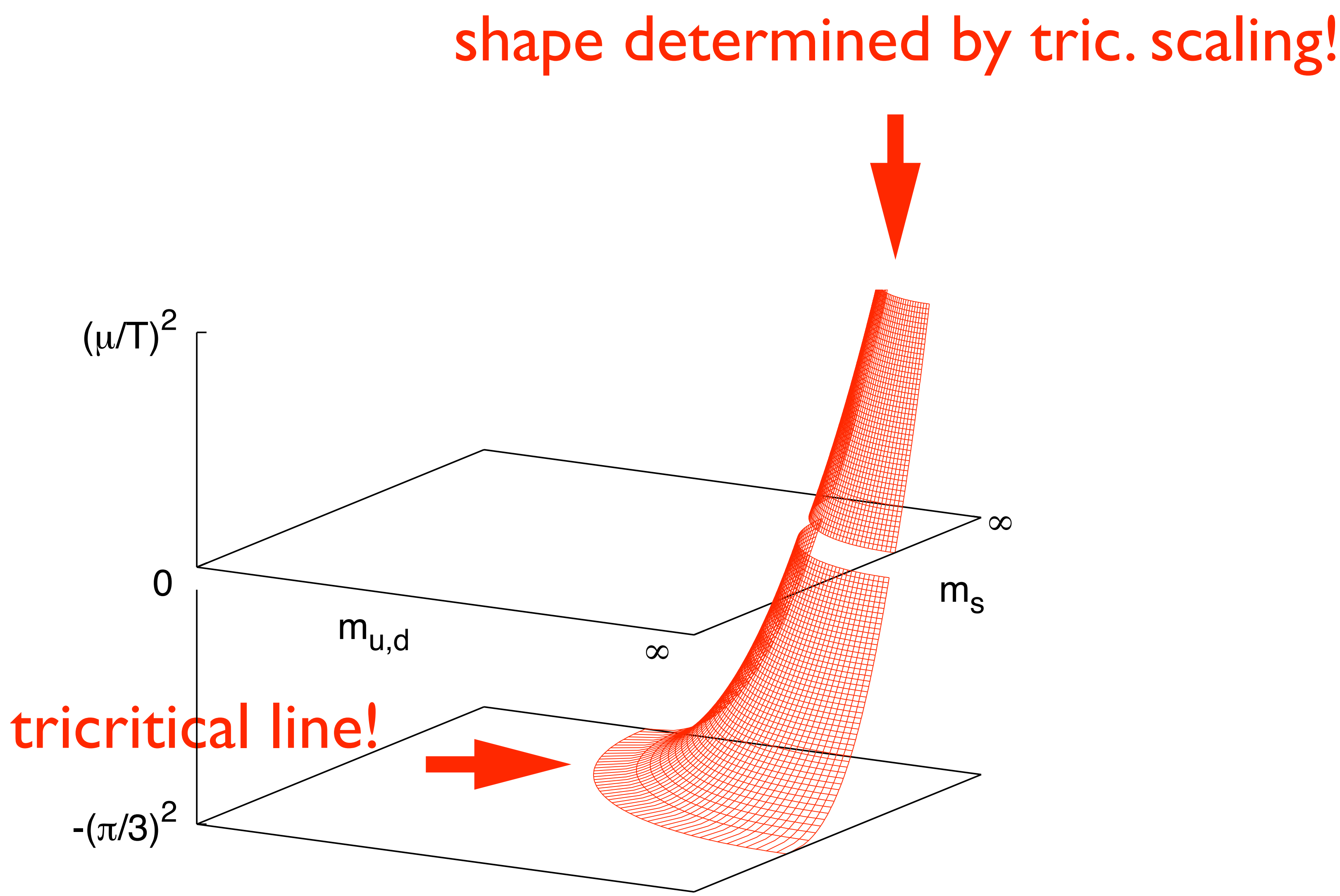


tri-critical scaling:

$$\frac{m_c}{T}(\mu^2) = \frac{m_{tric}}{T} + K\left[\left(\frac{\pi}{3}\right)^2 + \left(\frac{\mu}{T}\right)^2\right]^{2/5}$$

exponent universal

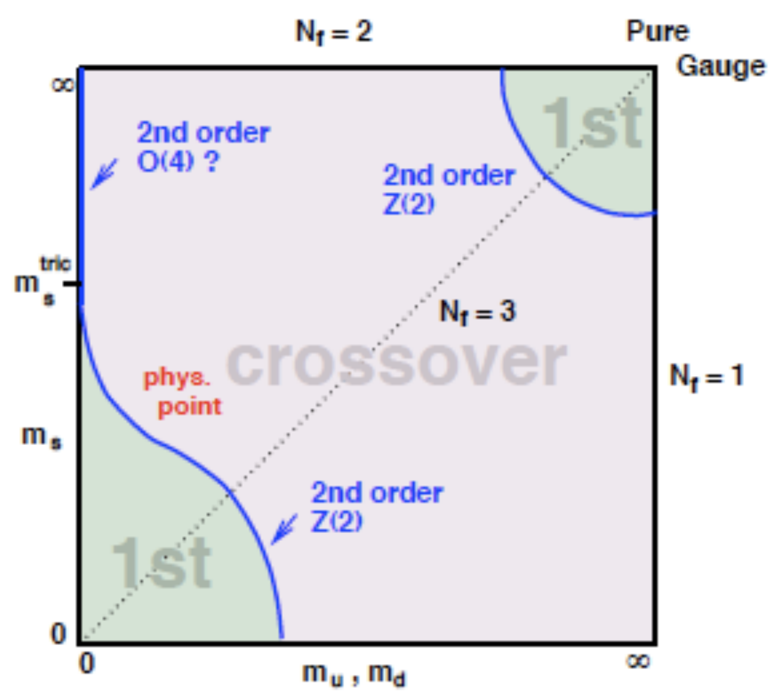
Deconfinement critical surface for heavy quarks

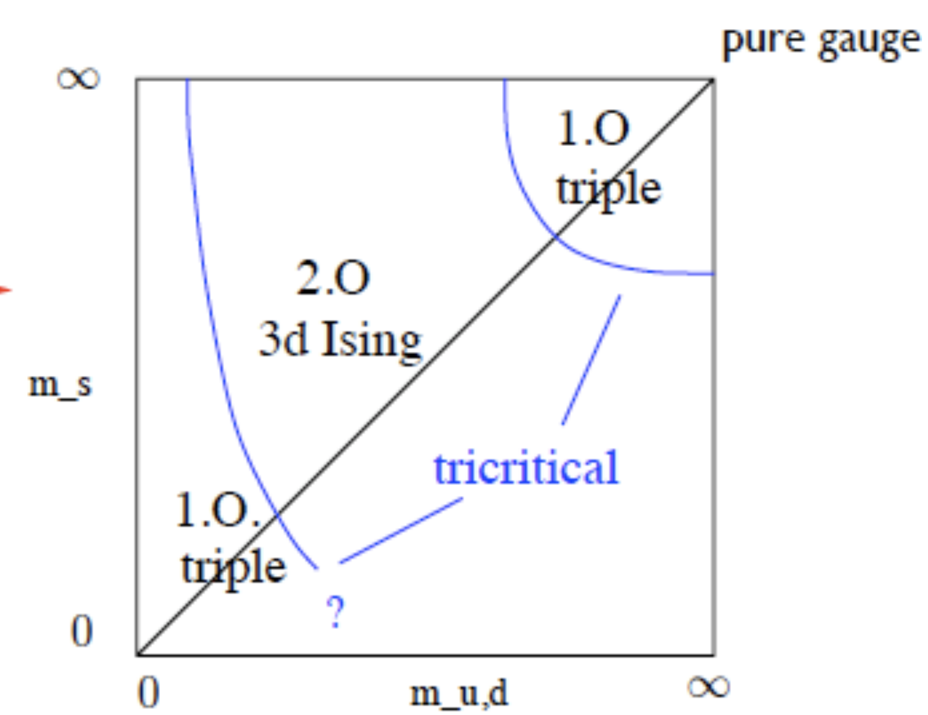


Chiral critical surface?

Check for tri-critical scaling possible by extensive simulations

Know now:





$$m_c(\mu = 0) \qquad < \qquad m_{tric}(\mu = i\pi T/3)$$

Consistent with 1st order region shrinking with real chemical potential!

Independent confirmation of earlier simulations!

Conclusions

- Z(3) transition at imaginary chem. pot. connects with chiral/deconf. transition
- Deconfinement and chiral critical surfaces end in tri-critical lines
- Curvature of deconfinement critical surface determined by tri-critical scaling!
- Chiral critical surface: to be checked, but first order shrinking for real chem. pot.
- Physical QCD: no chiral critical point for moderate chemical potentials!