Hadron properties at finite temperature and density with two flavors of Wilson fermion

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Collaboration with

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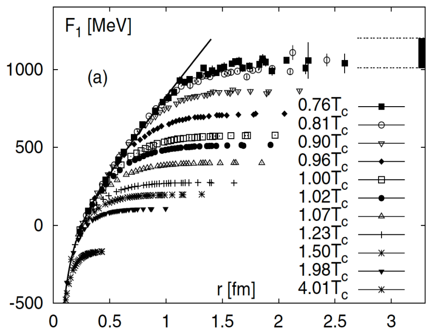
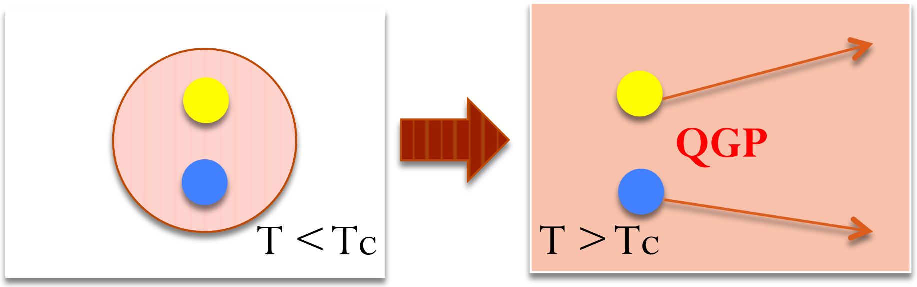
Hadrons as a probe of environment

- Study finite temperature & density system by detecting changes of hadron properties
  - **As a probe of deconfinement** **J/ψ suppression**

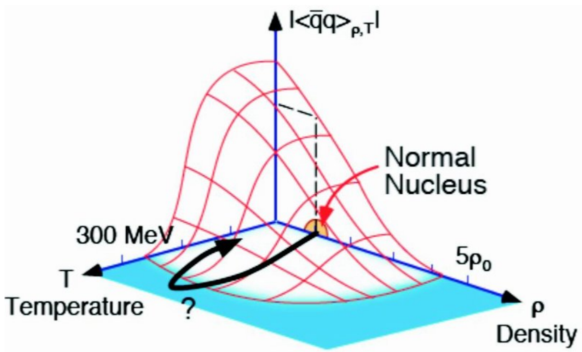


Temperature dependence of $\bar{q}q$ potential
(O.Kaczmarek, F.Zantow, PRD71, 114510 (2005))



J/ψ … used for detector of deconfinement

  - **As a probe of chiral symmetry restoration**



Taken from http://niham.nipne.ro/rp9/

- Light vector mesons (ρ , ω , ϕ …)
  … detector for chiral symmetry restoration
  Mass modification, width broadening
  → indication of (partial) chiral symmetry restoration

We need correct knowledge of hadrons at finite temperature and density

- We study screening masses of mesons (PS and V) at finite temperature and density in lattice QCD.

… screening mass reflects the "mass" of mesons.

Note) If the single particle picture is satisfied,
screening mass is the mass of a meson for a neutral meson in continuum limit.

1) screening masses at finite temperature (μ =0)

2) screening masses at finite density by Taylor expansion method (preliminary)

- We use the configurations of two-flavor Wilson fermion generated by WHOT-QCD collaboration.

… So far, dynamical calculation of screening mass has been performed by staggered fermion (QCD-TARO collaboration, RBC-Bielefeld Collaboration,…)

To see (or reduce) the lattice artifact by the choice of lattice fermions,
calculation by dynamical Wilson fermion is important.

※ **Numerical calculation was performed on RIKEN Integrated Cluster of Clusters system. (RICC)**

- Gauge configurations along the lines of constant physics (m_{PS}/m_V constant)

  → Accurate calculation can be performed in the wide range of temperature.

  - Action: RG improved gauge action & clover-improved Wilson quark action
  - Lattice size & quark masses: $16^3\times4$, m_{PS}/m_V=0.65, 0.80
  - Temperature: 0.82-4.02 (m_{PS}/m_V=0.65), 0.76-3.01 (m_{PS}/m_V=0.80)
  - Number of configurations: 100 confs.

m_{PS}/m_V=0.65

β	K	T/Tpc	Traj.
1.50	0.150290	0.82(3)	5000
1.60	0.150030	0.86(3)	5000
1.70	0.148086	0.94(3)	5000
1.75	0.146763	1.00(4)	5000
1.80	0.145127	1.07(4)	5000
1.85	0.143502	1.18(4)	5000
1.90	0.141849	1.32(5)	5000
1.95	0.140472	1.48(5)	5000
2.00	0.139411	1.67(6)	5000
2.10	0.137833	2.09(7)	5000
2.20	0.136596	2.59(9)	5000
2.30	0.135492	3.22(12)	5000
2.40	0.134453	4.02(15)	5000

m_{PS}/m_V=0.80

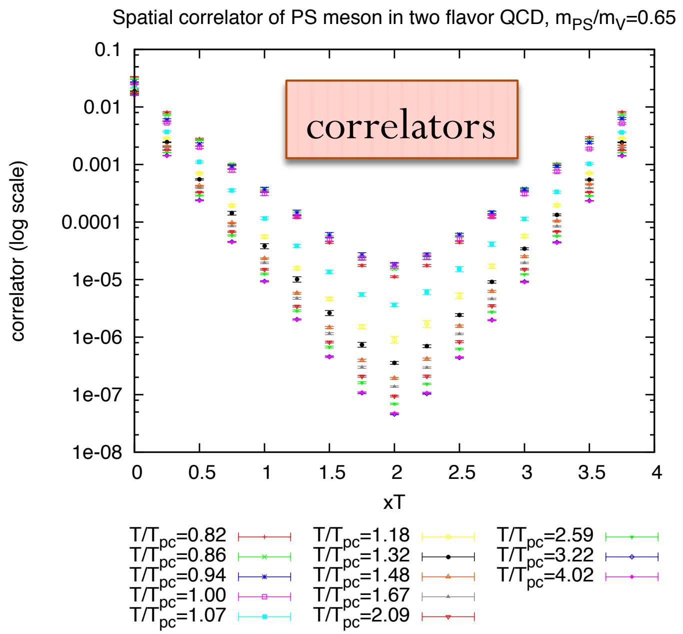
β	K	T/Tpc	Traj.
1.50	0.143480	0.76(4)	5500
1.60	0.143749	0.80(4)	6000
1.70	0.142871	0.84(4)	6000
1.80	0.141139	0.93(5)	6000
1.85	0.140070	0.99(5)	6000
1.90	0.138817	1.08(5)	6000
1.95	0.137716	1.20(6)	6000
2.00	0.136931	1.35(7)	5000
2.10	0.135860	1.69(8)	5000
2.20	0.135010	2.07(10)	5000
2.30	0.134194	2.51(13)	5000
2.40	0.133395	3.01(15)	5000

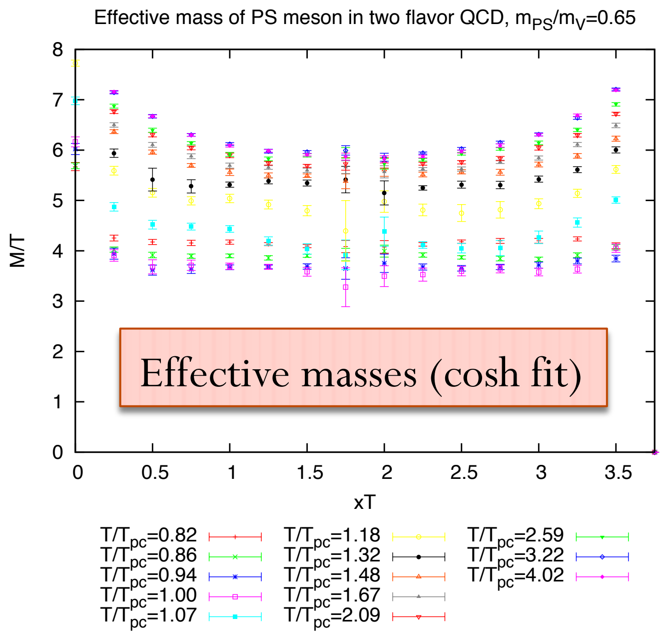
1) Finite temperature (μ=0)

We measure spatial correlators of mesons (M: meson operator):

$$G(x) \equiv \sum_{y,z,t} \langle M(x,y,z,t) M(0,0,0,0)^\dagger \rangle \quad (M(x,y,z,t) \equiv \bar{q}(x,y,z,t)\Gamma q(x,y,z,t))$$

Fitting to the functional form, $G(x) = A(e^{-\hat{M}\hat{x}} + e^{-\hat{M}(L_x - \hat{x})})$, we obtain the meson screening mass $\hat{M}$.

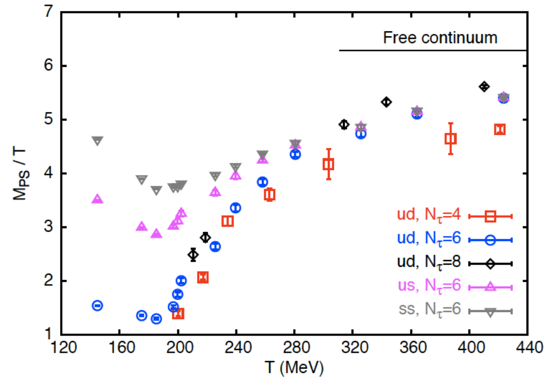




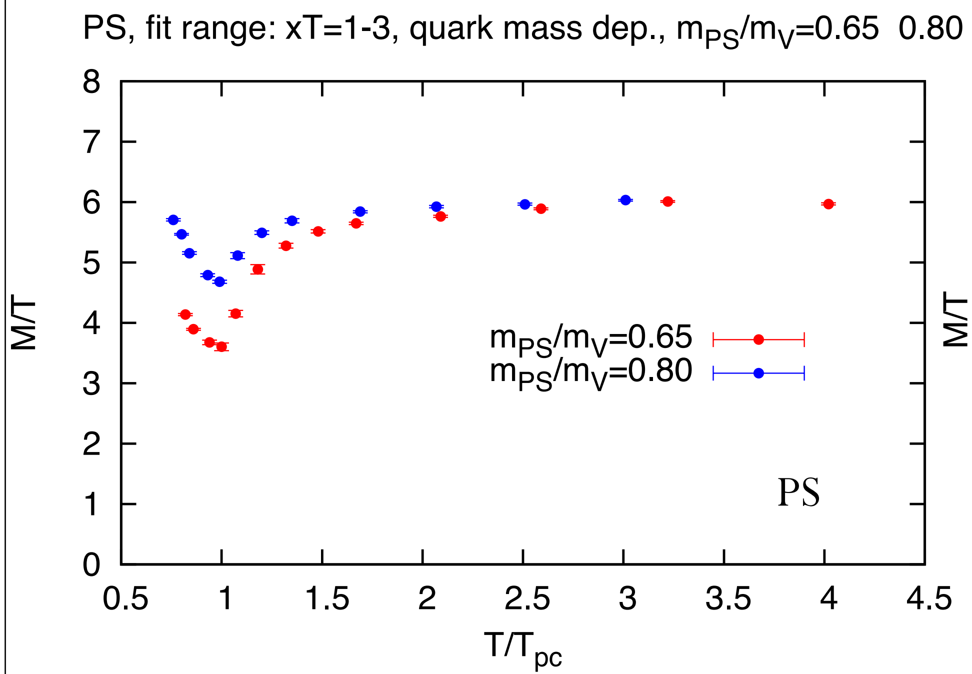
… Signals are clean at all temperatures

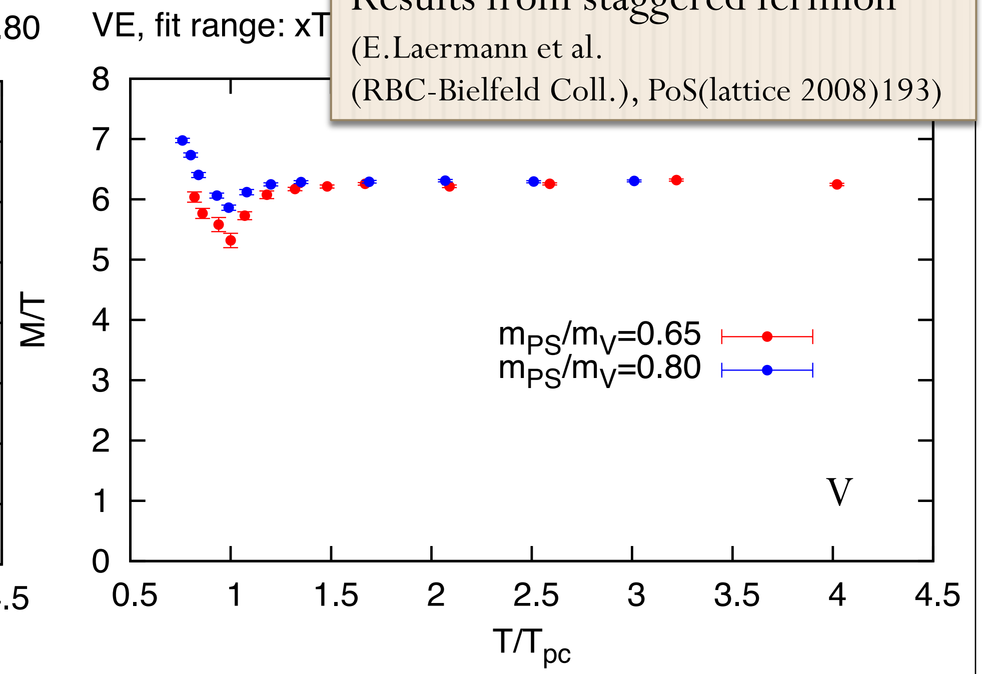
Results

- Temperature dependence of meson mass



Results from staggered fermion
(E.Laermann et al.
(RBC-Bielfeld Coll.), PoS(lattice 2008)193)

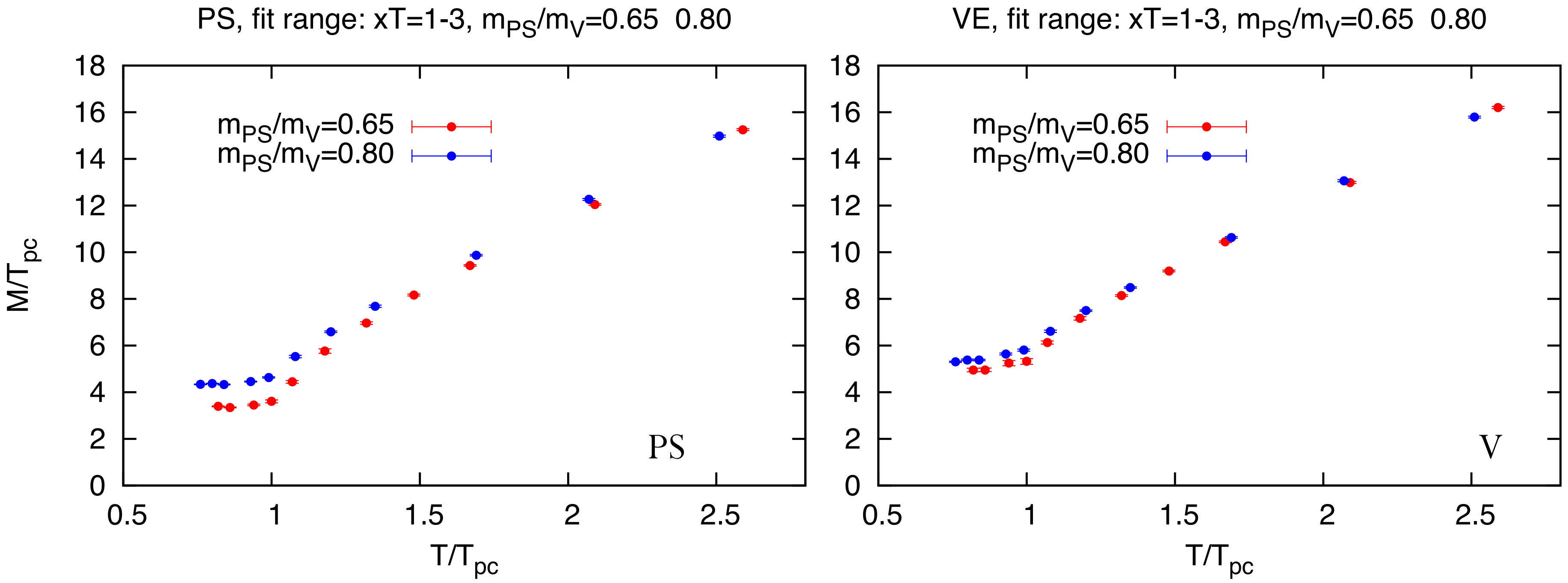




- There is a specific structure around Tc (in the plot of M/T)
- Meson masses become $2\pi T$ at high temperature
- Quark mass dependence of meson masses is larger in PS channel than V channel

Results

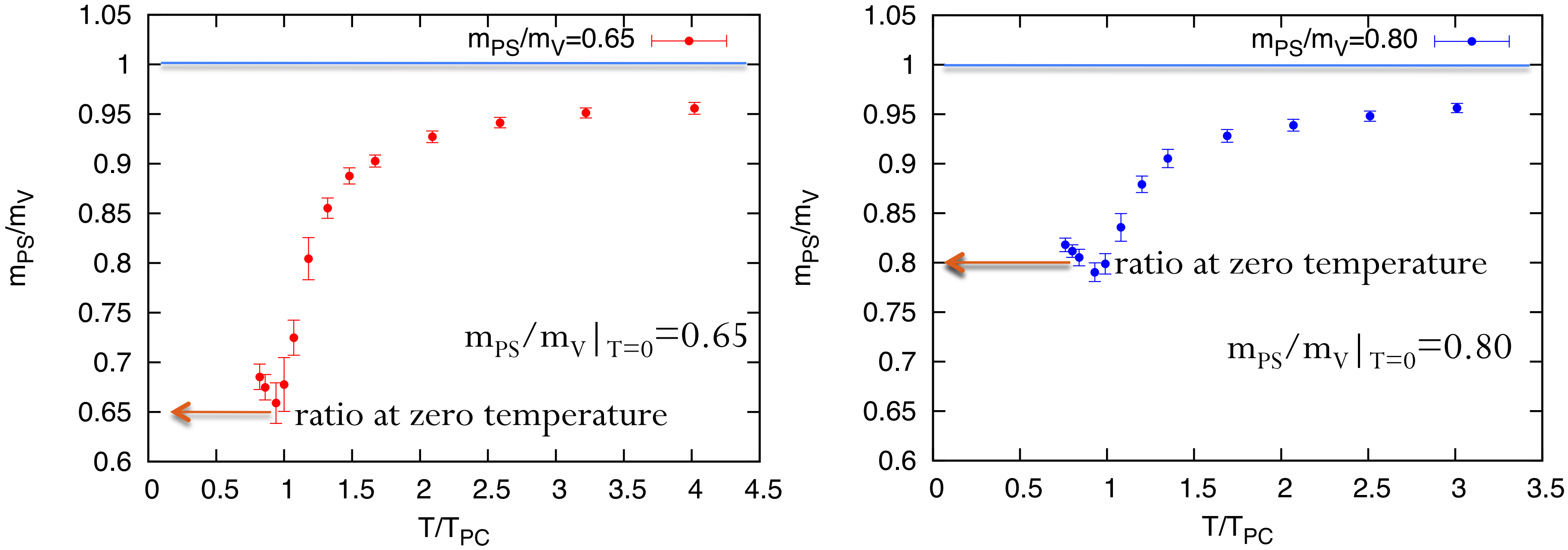
- Temperature dependence (M/Tpc).



- Meson screening mass increases very slowly below Tc, and rapidly above Tc.

Results

- Temperature dependence.



- At low temperature, the ratio is about 0.65 and 0.80 (ratio at T=0).
- Above Tc, the ratio quickly increases and approaches one.

→ Mesons are composed of a free quark and a free anti-quark,
each of which has the thermal mass πT at high temperature.

2) Finite density (preliminary)

- We calculate **second response of meson masses to the isoscalar chemical potential** in **two-flavor Wilson fermion by Taylor expansion method**

Taylor expansion method ref.) S.Choe et al., PRD65, 054501 (2002)…staggered

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}U e^{-S} (\det D(\mu))^2 \mathcal{O}}{\int \mathcal{D} e^{-S} (\det D(\mu))^2} \qquad (\mu \equiv \mu_u = \mu_d \ \ldots \text{ isoscalar chemical potential })$$

$$= \frac{\langle (\mathcal{O} + \dot{\mathcal{O}}\mu + \frac{1}{2}\ddot{\mathcal{O}}\mu^2 + O(\mu^3))(1 + \frac{\dot{\Delta}}{\Delta}\mu + \frac{\ddot{\Delta}}{\Delta}\mu^2 + O(\mu^3)) \rangle}{1 + \langle \frac{\dot{\Delta}}{\Delta} \rangle \mu + \frac{1}{2}\langle \frac{\ddot{\Delta}}{\Delta} \rangle \mu^2 + O(\mu^3)} \qquad (\Delta \equiv (\mathrm{detD}(\mu))^2|_{\mu=0} \ )$$

−We take $\mathcal{O}$ as the meson correlator G for isoscalar chemical potential:

$$G \equiv \mathrm{tr}(D^{-1}_{x0}(\mu)\Gamma D^{-1}_{0x}(\mu)\Gamma^\dagger) = \mathrm{tr}(D^{-1}_{x0}(\mu)\Gamma\gamma_5(D^{-1}(-\mu))^\dagger_{x0}\gamma_5\Gamma^\dagger)$$

$$\rightarrow \ \text{2nd order}: \langle \dot{G}\frac{\dot{\Delta}}{\Delta} \rangle + \frac{1}{2}\langle \ddot{G} \rangle + \frac{1}{2}\langle G\frac{\ddot{\Delta}}{\Delta} \rangle - \frac{1}{2}\langle G \rangle \langle \frac{\ddot{\Delta}}{\Delta} \rangle \qquad (\text{Note: } \langle \frac{\dot{\Delta}}{\Delta} \rangle = 0, \ \langle G\frac{\dot{\Delta}}{\Delta} \rangle = 0 \ )$$

Finite density

- **Leading:**

$$\langle G\rangle|_{\mu=0} = \langle \mathrm{tr}[D^{-1}_{x0}\Gamma\gamma_5(D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}]\rangle \ldots \text{ already shown}$$

- **Second derivative:**

$$\begin{aligned}
\frac{d^2}{d\mu^2}\mathrm{Re}\langle G\rangle|_{\mu=0} =& 4\langle \mathrm{Retr}[(D^{-1}\dot{D}D^{-1}\dot{D}D^{-1})_{x0}\Gamma\gamma_5(D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}]\rangle \\
&- 2\langle \mathrm{Retr}[(D^{-1}\ddot{D}D^{-1})_{x0}\Gamma\gamma_5(D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}]\rangle \\
&- 2\langle \mathrm{Retr}[(D^{-1}\dot{D}D^{-1})_{x0}\Gamma\gamma_5(D^{-1}\dot{D}D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}]\rangle \\
&+ 8\langle \mathrm{Imtr}[(D^{-1}\dot{D}D^{-1})_{x0}\Gamma\gamma_5(D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}]\cdot \mathrm{ImTr}(D^{-1}\dot{D})\rangle \\
&+ 2\mathrm{Re}\{\langle \mathrm{tr}[D^{-1}_{x0}\Gamma\gamma_5(D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}](2(\mathrm{Tr}(D^{-1}\dot{D}))^2 - \mathrm{Tr}(D^{-1}\dot{D}D^{-1}\dot{D}) + \mathrm{Tr}(D^{-1}\ddot{D}))\rangle \\
&\quad - \langle \mathrm{tr}[D^{-1}_{x0}\Gamma\gamma_5(D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}]\rangle\langle 2(\mathrm{Tr}(D^{-1}\dot{D}))^2 - \mathrm{Tr}(D^{-1}\dot{D}D^{-1}\dot{D}) + \mathrm{Tr}(D^{-1}\ddot{D})\rangle\}
\end{aligned}$$

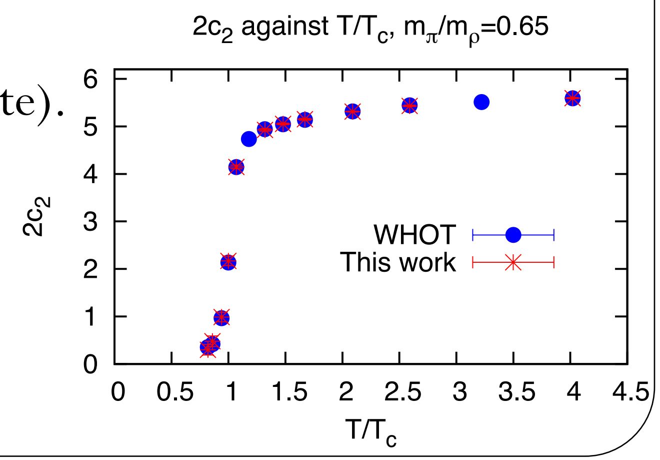
$$\frac{1}{2}\langle\ddot{G}\rangle \qquad \langle \dot{G}\frac{\dot{\Delta}}{\Delta}\rangle \qquad \frac{1}{2}\left(\langle G\frac{\ddot{\Delta}}{\Delta}\rangle - \langle G\rangle\langle\frac{\ddot{\Delta}}{\Delta}\rangle\right)$$

※ Tr denotes **trace including space-time coordinate**.

Noise method is adopted (to take trace for spatial coordinate).

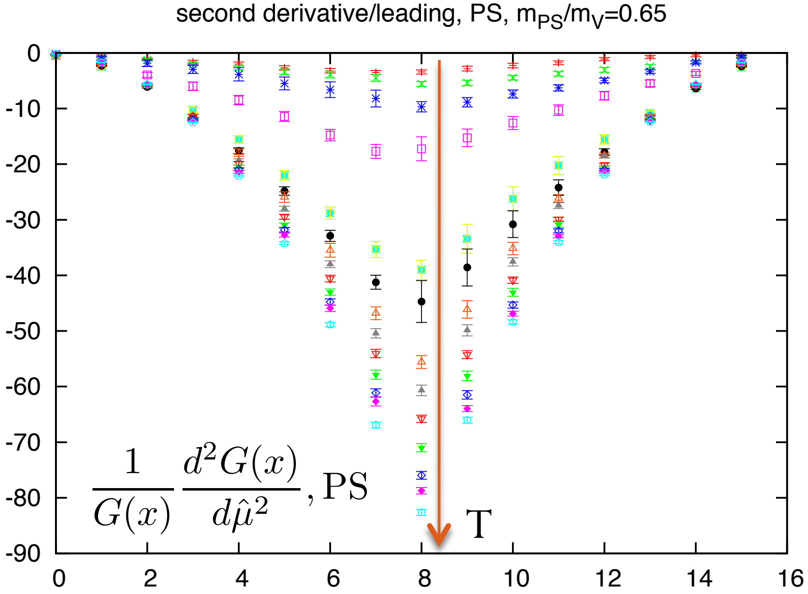
$$\mathrm{Tr}(A) \simeq \frac{1}{N_{\mathrm{noise}}}\sum_{i=1}^{N_{\mathrm{noise}}}\sum_{it,a,\alpha}^{N_t,3,4}\eta^{\dagger}_{i,it,a,\alpha}A\,\eta_{i,it,a,\alpha}$$

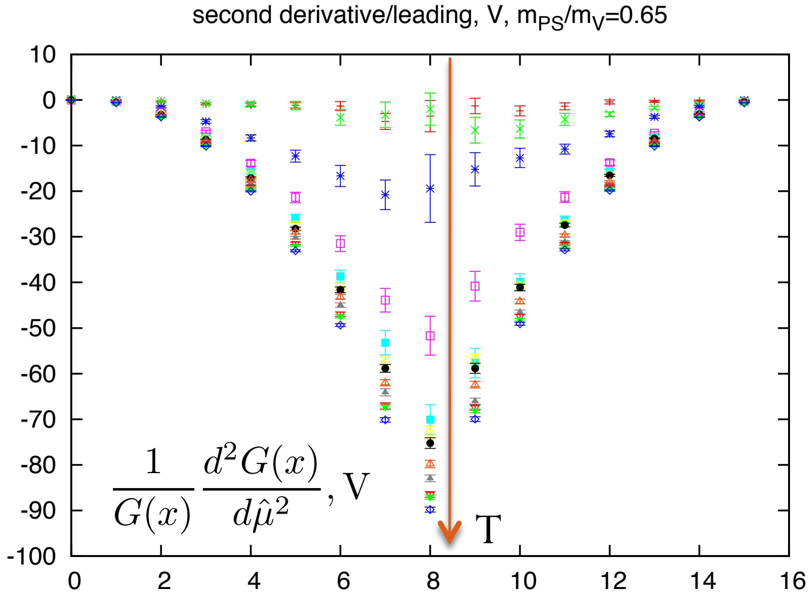
$$\frac{1}{N_{\mathrm{noise}}}\sum_{i=1}^{N_{\mathrm{noise}}}\eta(i,x)\eta^*(i,y) \simeq \delta_{x,y} \quad \ldots 100 \text{ noises, U(1)}$$



Second derivatives of correlators

Preliminary





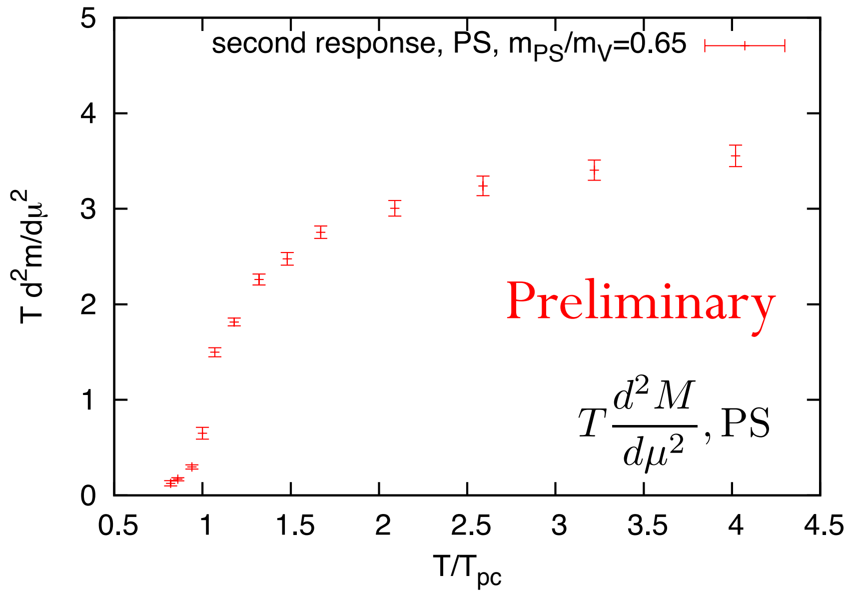
We fit the correlators by the following functional form:

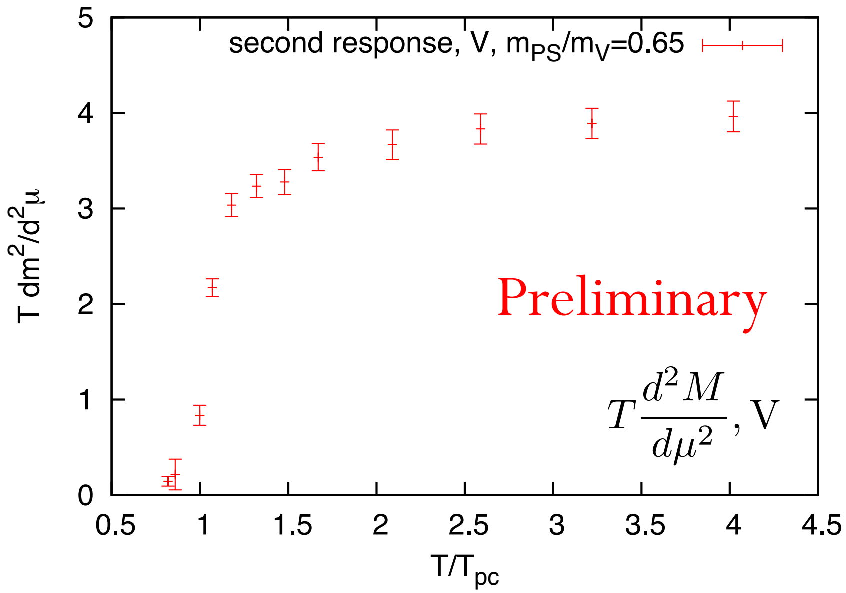
Leading order: $G(x) = A(e^{-\hat{M}\hat{x}} + e^{-\hat{M}(L_x-\hat{x})})$

2^{nd} order: $$\frac{1}{G(x)}\frac{d^2G(x)}{d\hat{\mu}^2} = \frac{1}{A}\frac{d^2A}{d\hat{\mu}^2} + \frac{d^2\hat{M}}{d\hat{\mu}^2}\left\{\left(\hat{x}-\frac{L_x}{2}\right)\tanh\left[\hat{M}\left(\hat{x}-\frac{L_x}{2}\right)\right]-\frac{L_x}{2}\right\}$$

$\rightarrow$ Second derivative of meson masses, $\frac{d^2\hat{M}}{d\hat{\mu}^2}(= N_t T\frac{d^2M}{d\mu^2})$,is obtained.

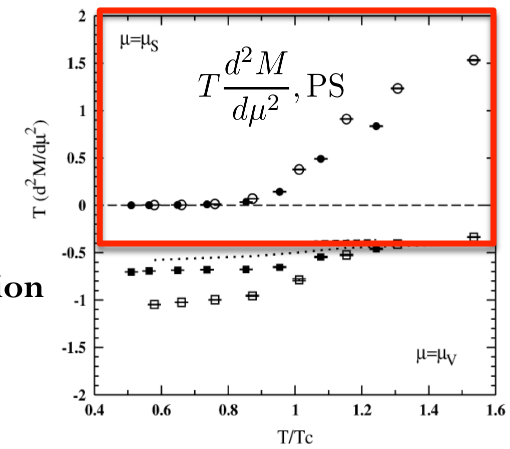
Second derivative of meson mass





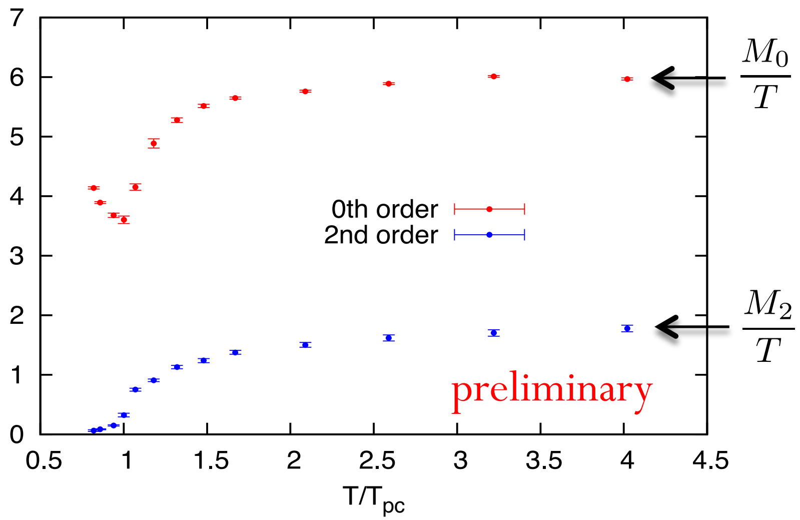
- **Second response is positive**
- **Becomes large after phase transition**

… Similar with the results from staggered fermion



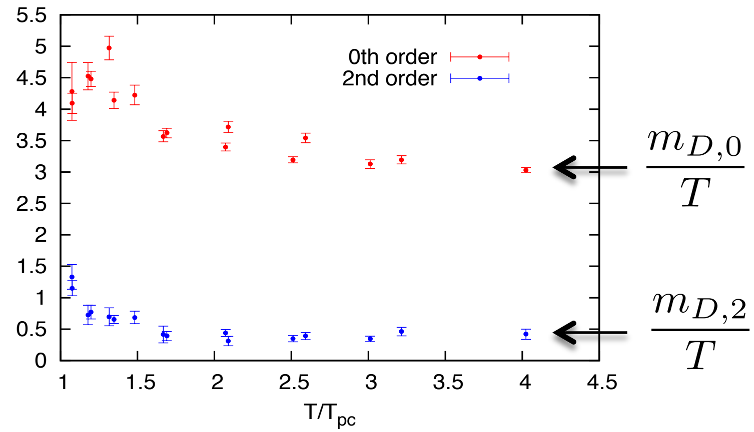
Results by staggered fermion
I.Pushkina et al.(QCD-TARO collab.)PLB609 (2005)

Comparison with gluon screening mass



$$\frac{M(\mu)}{T} = \frac{M_0}{T} + \frac{M_2}{T}\left(\frac{\mu}{T}\right)^2 + O(\mu^4)$$

cf) Second derivative of gluon screening mass
(Y.Maezawa et al., PRD**75**, 074501 (2007))



$$\frac{m_D(\mu)}{T} = \frac{m_{D,0}}{T} + \frac{m_{D,2}}{T}\left(\frac{\mu}{T}\right)^2 + O(\mu^4)$$

- **Behavior of screening masses are opposite between mesons and gluons:**

$\frac{M_0}{T}$ and $\frac{M_2}{T}$ $\rightarrow$ large, for T $\rightarrow$ large $\Leftrightarrow$ $\frac{m_{D,0}}{T}$ and $\frac{m_{D,2}}{T}$ $\rightarrow$ small, for T $\rightarrow$ large

- The ratio of 2nd order to 0th order is larger for mesons than gluons above Tc:

$$\frac{M_2}{M_0} > \frac{m_{D,2}}{m_{D,0}} \quad \text{above Tc}$$

(mesons…20-30% above Tc; gluons…about 10% above Tc)

(Note: quarks couple to μ directly $\Leftrightarrow$ gluons couple to μ only through quark loops)

Summary

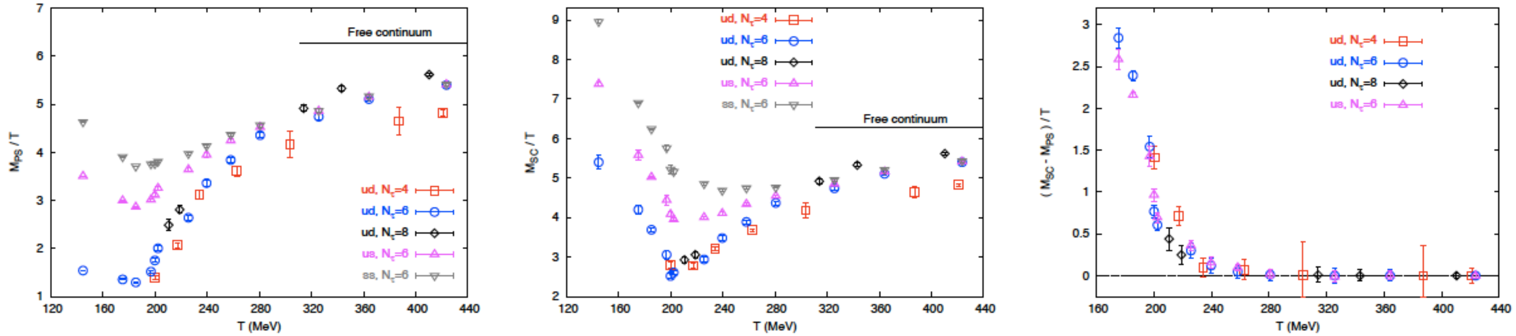
- We have studied meson screening masses (PS and V) at finite temperature and density in lattice QCD with two-flavor Wilson fermion generated by WHOT-QCD collaboration.

  (screening mass = mass for neutral mesons in cont. lim., if single particle picture is satisfied)
- Finite temperature, $\mu=0$:
  - Below and around Tc : meson masses increase very slowly.
  - Above Tc: increase rapidly and approach to $2\pi T$, where the mesons may become two free quarks.
- Finite μ (preliminary):
  - $T\frac{d^2M}{d\mu^2}$ is very small below Tc
  - $T\frac{d^2M}{d\mu^2}$ is positive and increases above Tc

    … consistent with the result from staggered fermion
  - Meson screening masses have qualitatively different behavior compared to gluon screening mass, which feels μ effect only through quark loops.

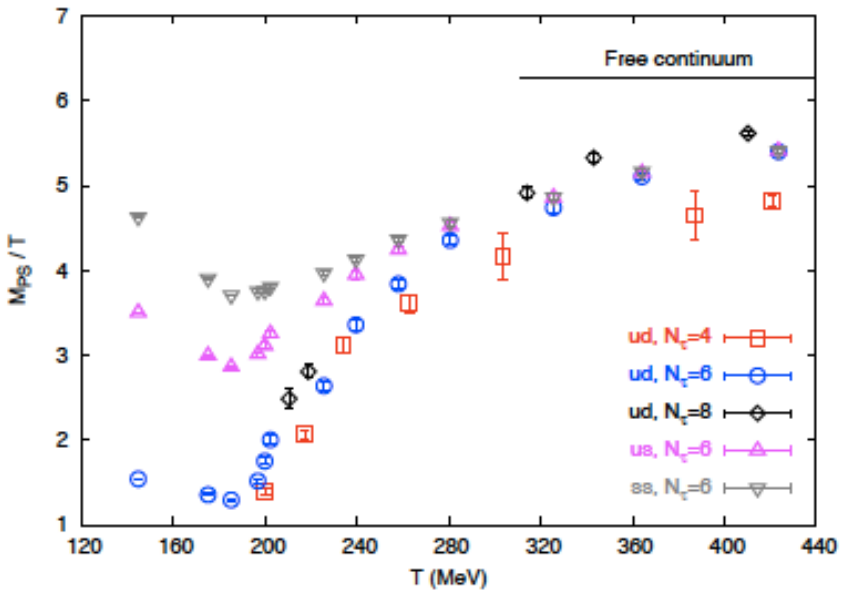
Backup slides

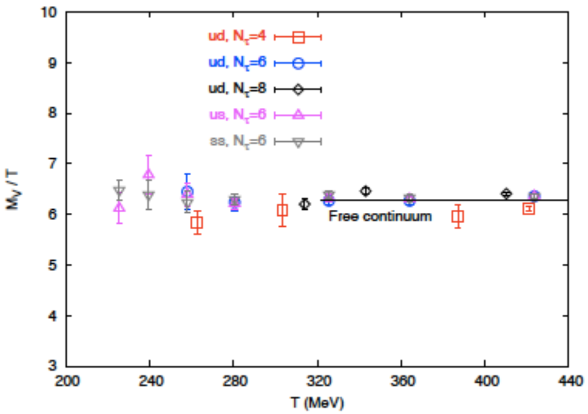
Pseudoscalar - Scalar (connected a_0/δ) sector

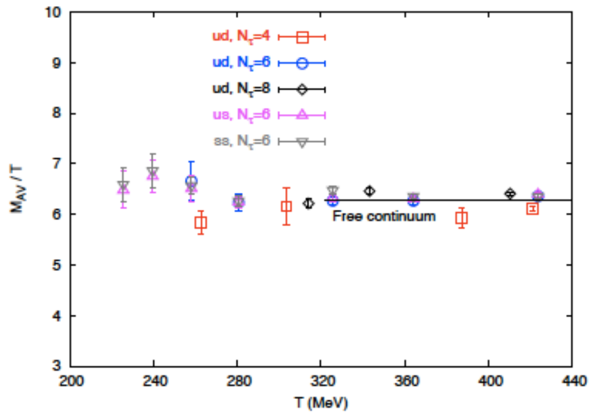


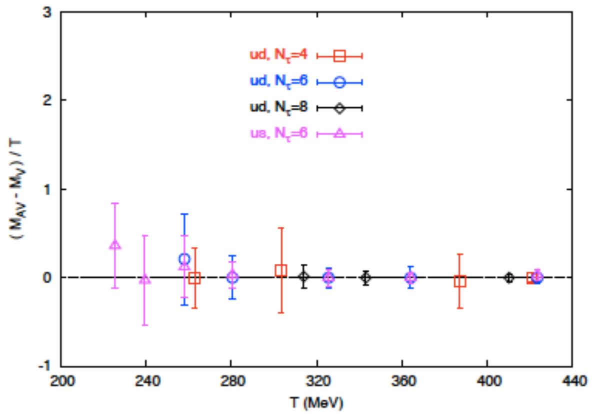
- all results at fixed $N_\sigma/N_\tau = 4$

ate up to $T \simeq 240$ MeV

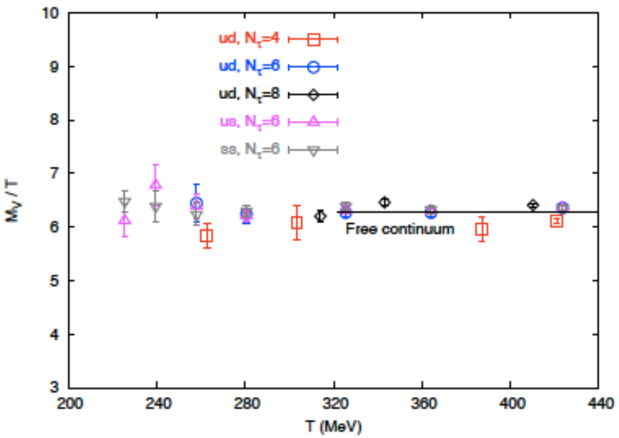


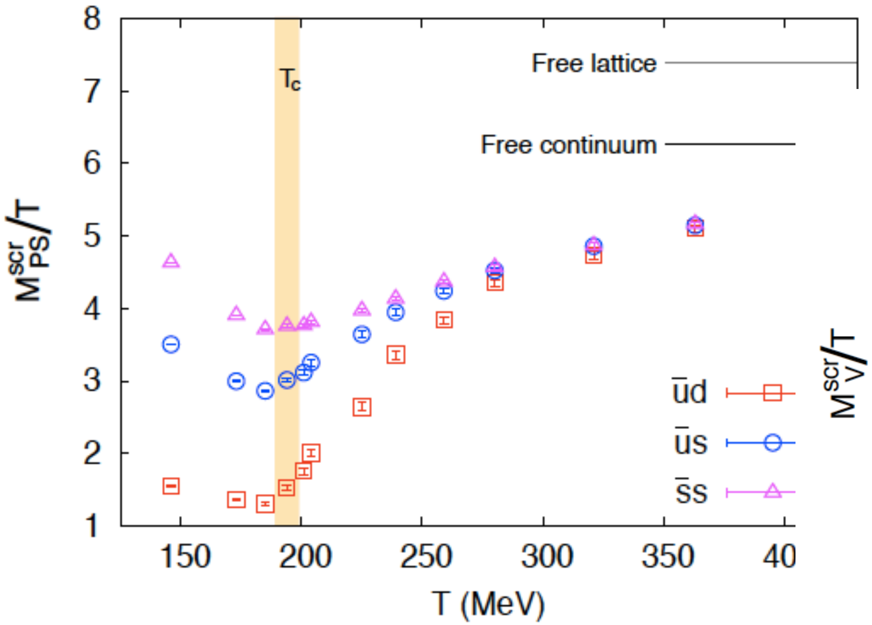




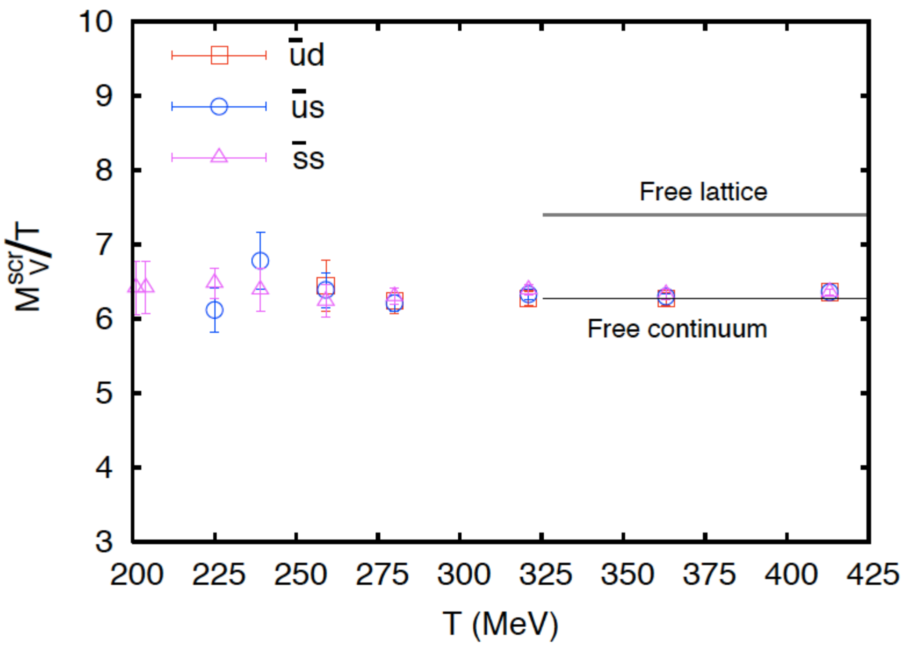


- all results at fixed $N_\sigma/N_\tau = 4$: coincidence with 2π is presumably accidental
- V_T and A_T appear degenerate even in the us channel at $T > T_c$



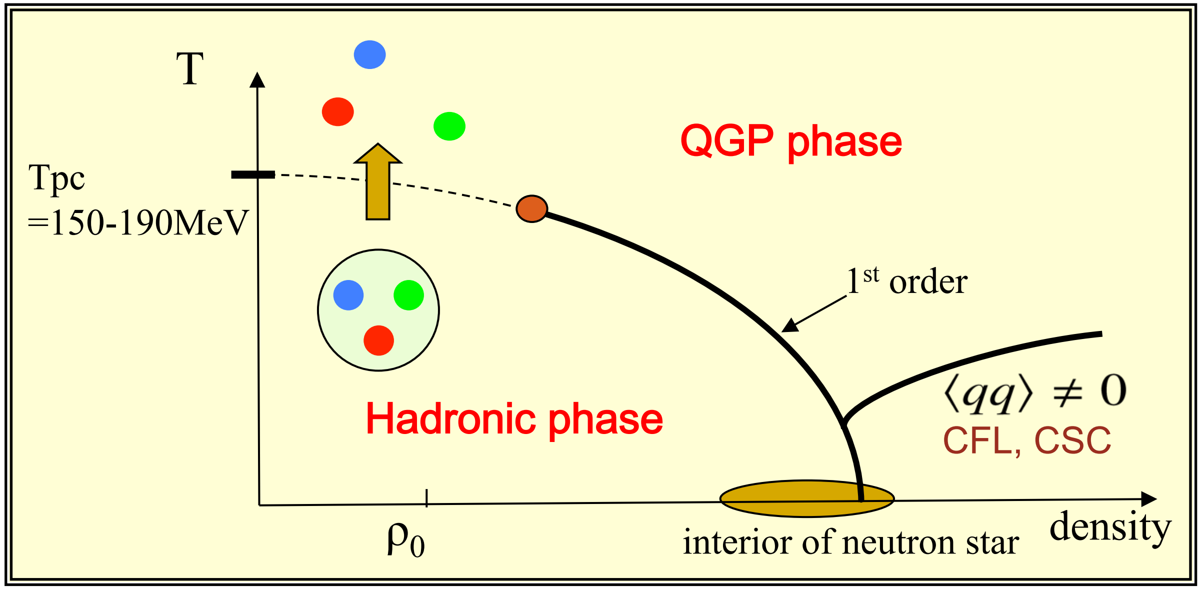


M^{scr}_{PS}/T
T (MeV)
T_c
Free lattice
Free continuum
$\bar{u}d$
$\bar{u}s$
$\bar{s}s$



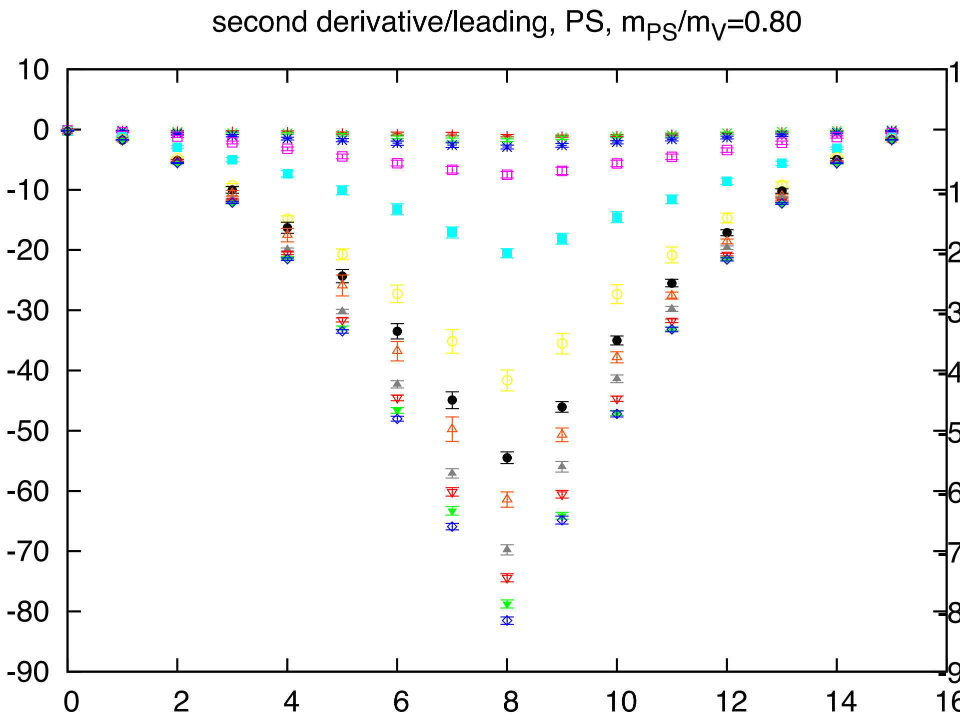
M^{scr}_{V}/T
T (MeV)
$\bar{u}d$
$\bar{u}s$
$\bar{s}s$
Free lattice
Free continuum

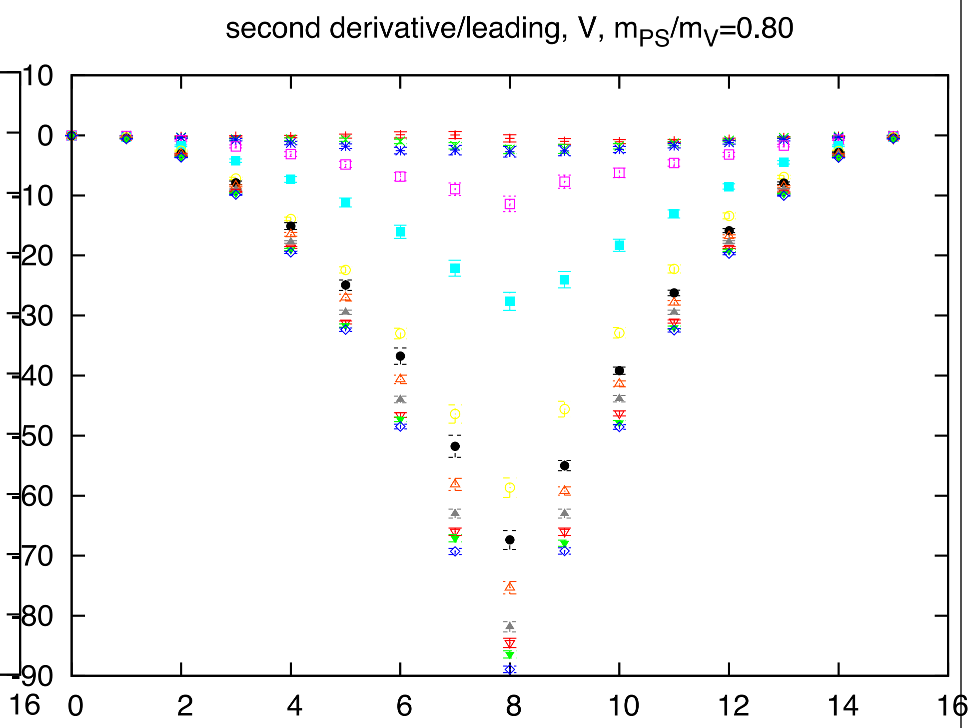
- Staggeredでは、PSとVで大きく異なる可能性があるか？…PSがWilsonと違っていたら、staggeredが正しい可能性が高い（？）。しかしVでは。。。むしろ、flavor symmetryがあり(staggeredはflavor sym.わるい）、(fourth root)^2を取らなくて済むWilsonが正しいか！？
- QCD TAROは、Laermannらの結果とだいぶ違う気がするが。。。PLB609(2005), Fig. 1
vector mesonはおなじ感じ。Piはmassが軽すぎる？


T
Tpc
=150-190MeV
QGP phase
1^{st} order
$\langle qq \rangle \neq 0$
CFL, CSC
Hadronic phase
ρ_0
interior of neutron star
density

Hadrons at finite temperature

- Screening masses in lattice QCD at finite temperature
  - Quenched Wilson
    QCD-TARO, …
  - Staggered fermion
    - Quenched…
    - Dynamical
      - 2 flavor
      - 2+1 flavor
  - Dynamical Wilson ← this study
    - Flavor symmetry ✓
    - Scaling property of phase transition ✓


second derivative/leading, PS, m_{PS}/m_V=0.80
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
0
2
4
6
8
10
12
14
16


second derivative/leading, V, m_{PS}/m_V=0.80
10
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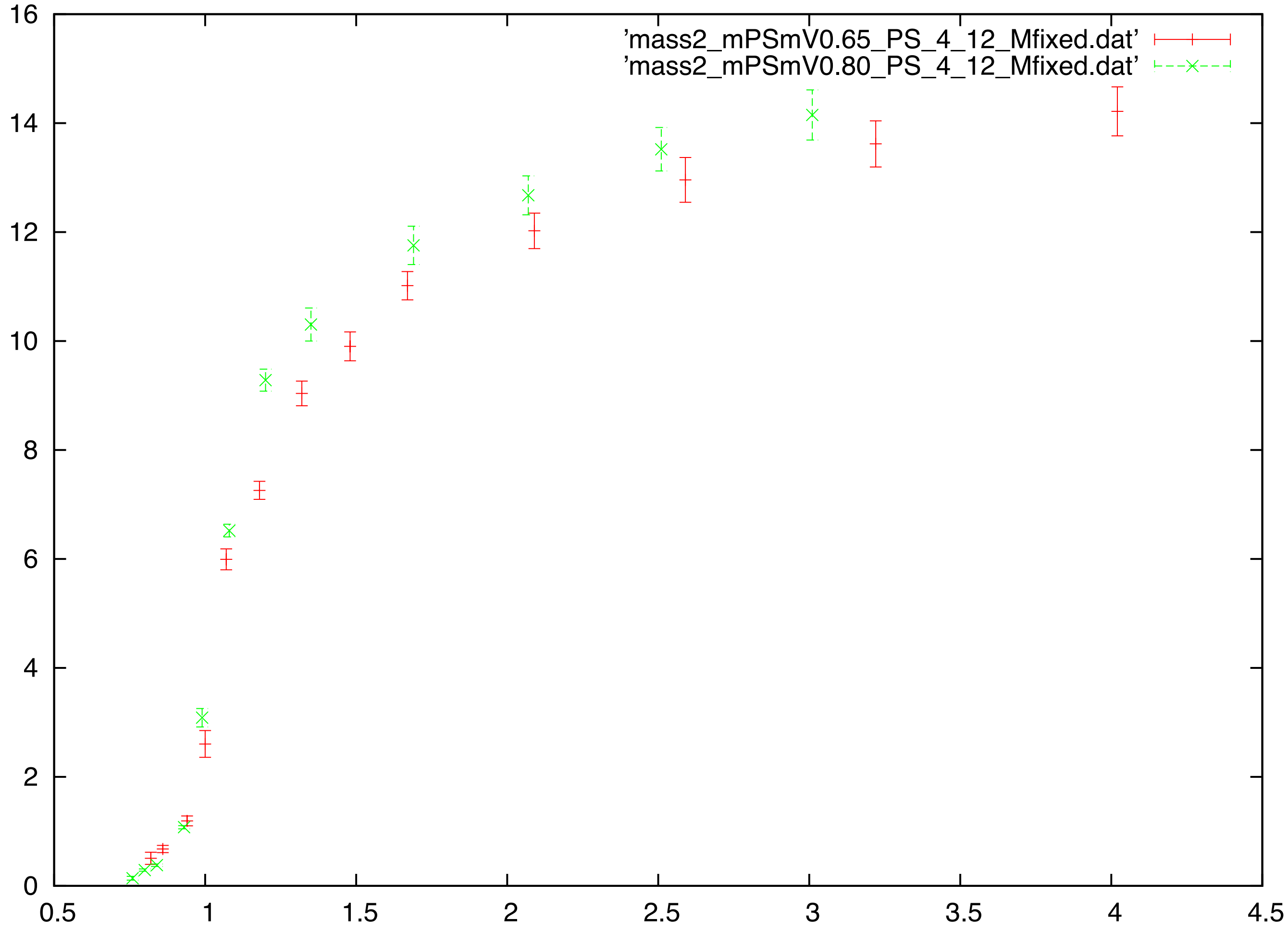
16

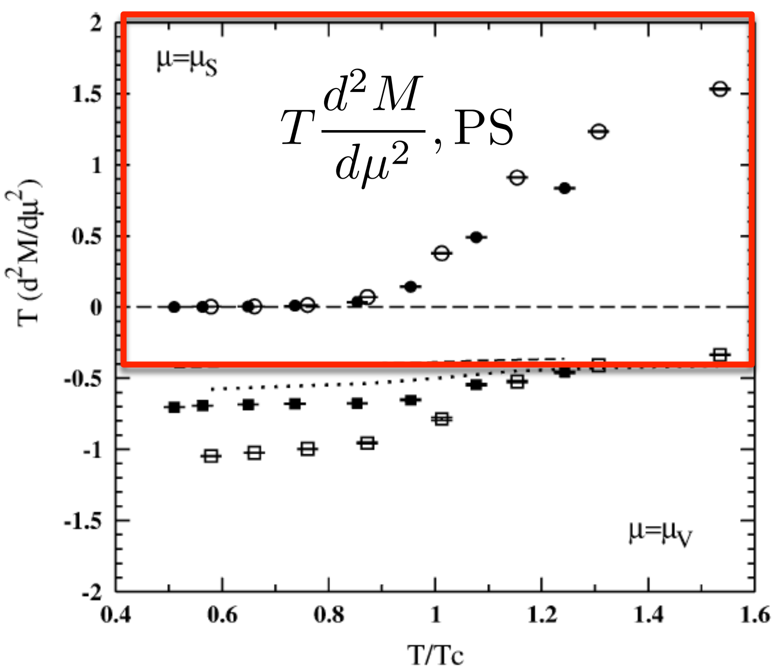


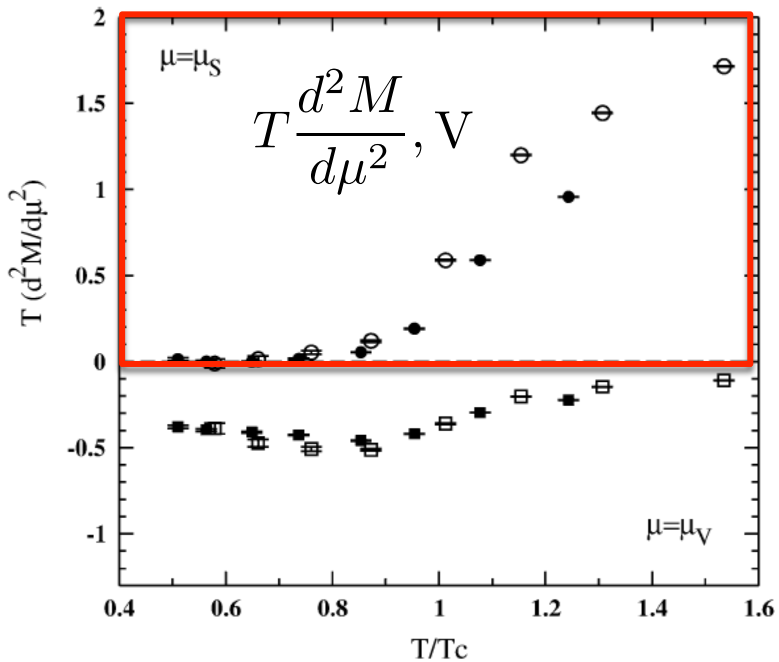
'mass2_mPSmV0.65_PS_4_12_Mfixed.dat'
'mass2_mPSmV0.80_PS_4_12_Mfixed.dat'
16
14
12
10
8
6
4
2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5

Comments

- Two flavor staggered:

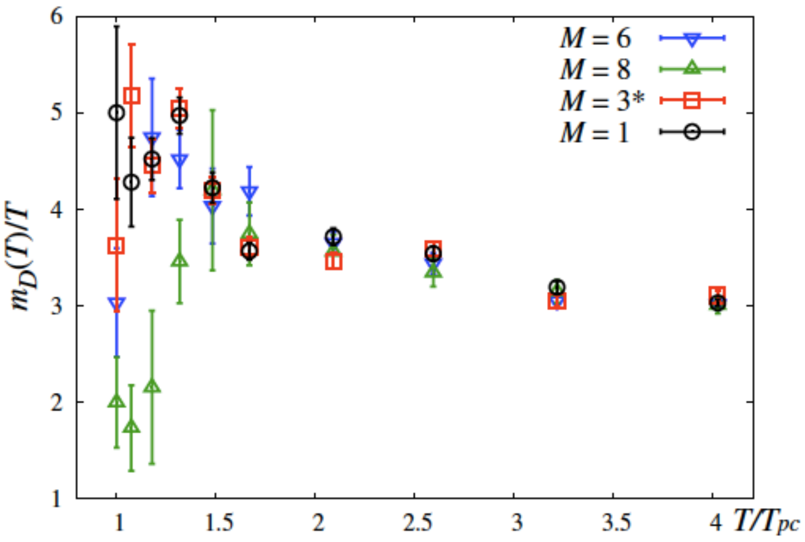
  I.Pushkina et al.(QCD-TARO collab.)PLB609 (2005)

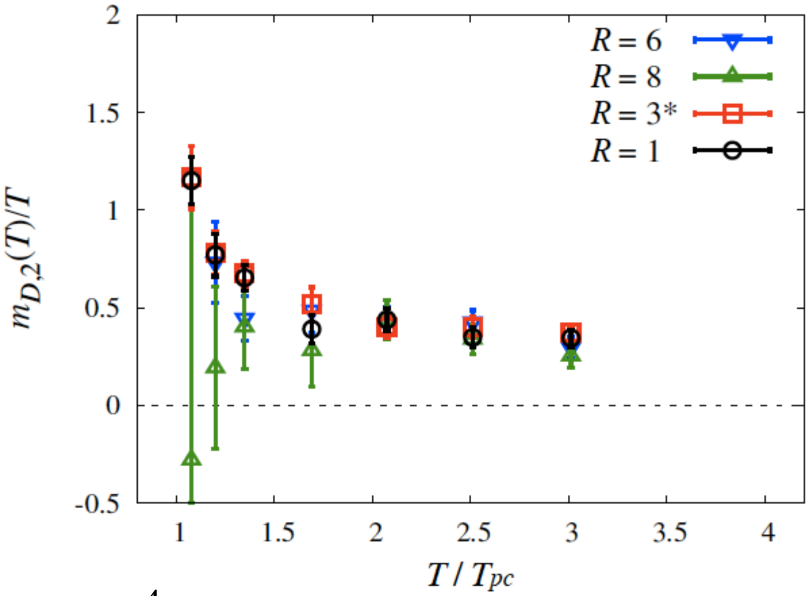




I.Pushkina et al.(QCD-TARO collab.)PLB609 (2005)

Qualitatively, we obtain the same tendency.





$$\frac{m_D(\mu)}{T} = \frac{m_{D,0}}{T} + m_{D,2}(\frac{\mu}{T})^2 + O(\mu^4)$$

4 0.5

$$\frac{M(\mu)}{T} = \frac{M|_{\mu=0}}{T} + \frac{1}{2}T \left.\frac{d^2M}{d\mu^2}\right|_{\mu=0} (\frac{\mu}{T})^2$$

6 $(T/T_{pc}=1.5)$ **1.3** $(T/T_{pc}=1.5)$

$$\frac{m_D(\mu)}{T} = \frac{m_{D,0}}{T} + m_{D,2}(\frac{\mu}{T})^2 + O(\mu^4)$$

$$\frac{M(\mu)}{T} = \frac{M|_{\mu=0}}{T} + \frac{1}{2}T \left.\frac{d^2M}{d\mu^2}\right|_{\mu=0} (\frac{\mu}{T})^2 + O(\mu^4)$$

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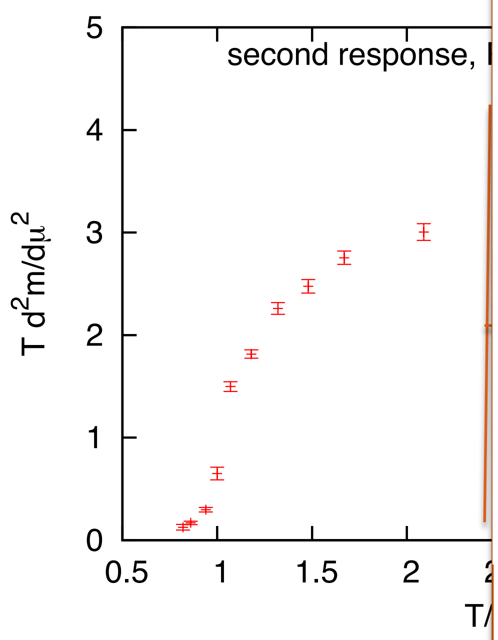

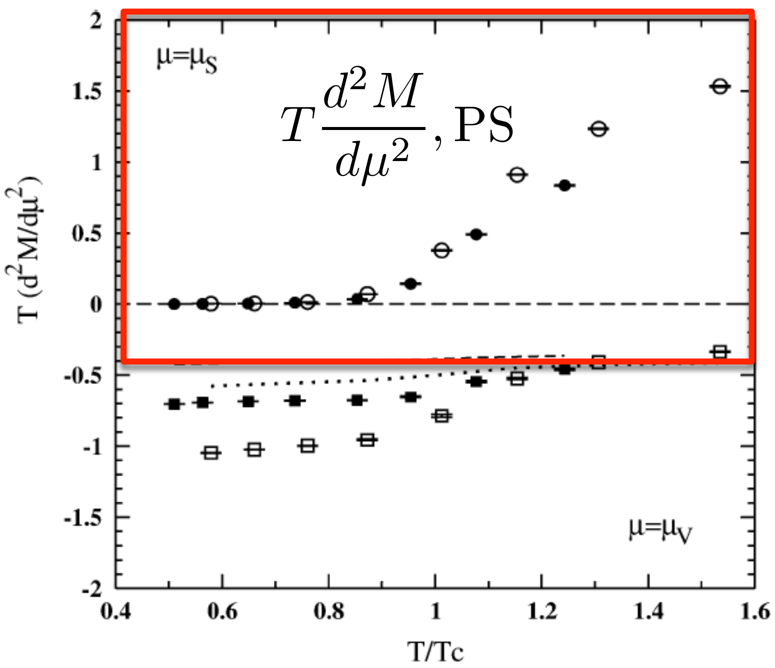

Results for staggered fermion

I.Pushkina et al.(QCD-TARO collab.)PLB609 (2005)

※V channel: same behavior as PS channel

on

Similar result obtained by staggered fermion