Poisson Statistics in the High Temperature QCD Dirac Spectrum

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Introduction

Statistics of low Dirac eigenvalues in QCD

$T < T_c$ ε-regime

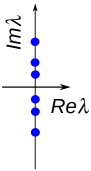
- Dirac eigenvalues obey random matrix statistics
  - Low energy effective σ-model calculation
  - Lattice (numerical verification)

$T > T_c$ $V \gg \frac{1}{T_c^3}$

- No analytic information
- Dirac operator is just a fluctuating “random matrix”

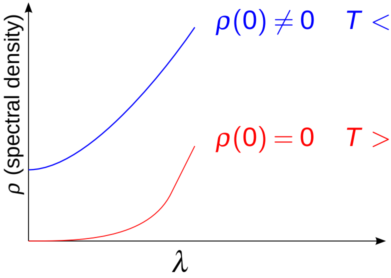
- Continuum symmetries:

$$\left.\begin{array}{l}\{\gamma_5, D\} = 0 \\ D^\dagger = -D\end{array}\right\} \Rightarrow \quad D = \begin{pmatrix} 0 & iC \\ iC^\dagger & 0 \end{pmatrix}$$



- $\Rightarrow$ Spectrum imaginary, symmetric w.r.t. real axis.

- Transition around $T_c \approx 200\text{MeV}$



$\rho(0) \neq 0 \quad T < T_c$ hadronic phase

$\rho(0) = 0 \quad T > T_c$ quark-gluon plasma

Two extreme possibilities

Random matrix statistics

Statistically independent eigenvalues (generalized Poisson)

Two extreme possibilities — lattice results so far

Random matrix statistics

- ⊗ Edge RMT (Farchioni et al. PRD 2000; Damgaard et al. NPB 2000)
- √ Bulk RMT (Pullirsch, Rabitsch, Wettig, Markum, PLB 1998)
- ? Typical fluctuations can mix eigenmodes.
  (Gavai, Gupta, Lacaze, PRD 2008)
- Eigenmodes delocalized.

Statistically independent eigenvalues (generalized Poisson)

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Statistically independent eigenvalues (generalized Poisson)

- Spectral density contains all statistical information.
- Typical fluctuations cannot mix eigenmodes.
- √ Eigenmodes localized. (Garcia-Garcia, Osborn, PRD 2007)

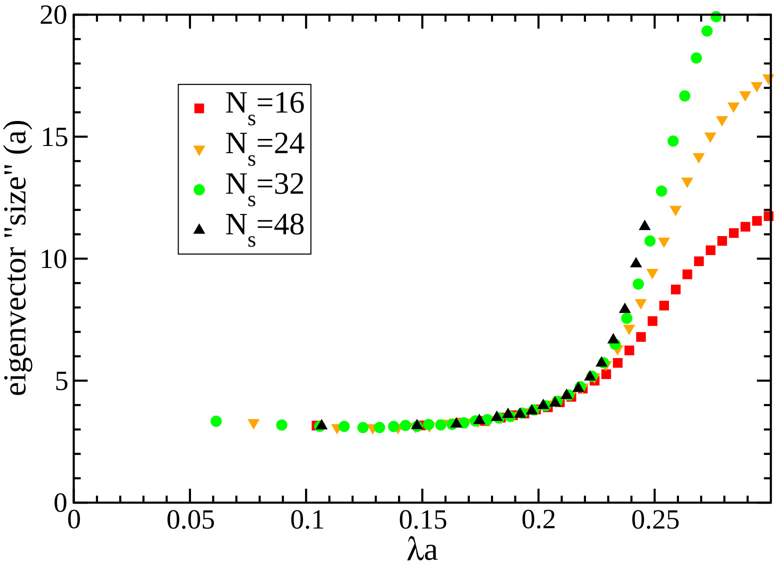
Lattice simulations — this work

(TGK, PRL 2010; TGK and Pittler, arXiv:1006.1205)

- SU(2) quenched
- Physical ($P > 0$) Polyakov sector selected “by hand”
- $T = 2.6T_c$
- $N_t = 4, 6 \quad N_s = 12 - 48$
- Dirac operator: overlap, staggered.
- Overlap and staggered give qualitatively similar results.

Average spatial extension of eigenmodes

staggered, $N_t = 4$



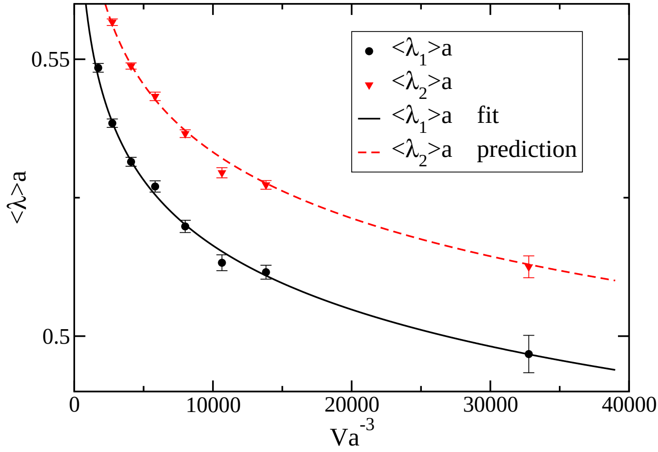
3-vol. dependence of average lowest two eigenvalues

overlap, $N_t = 4$

Generalized Poisson with power-law spectral density $\Rightarrow$

$$\langle\lambda_1\rangle = (CV\mu)^{-\mu}\cdot\Gamma(1+\mu) \quad \text{Fit } C,\mu$$

$$\langle\lambda_2\rangle = (CV\mu)^{-\mu}\cdot\Gamma(2+\mu) \quad \text{No free parameter!}$$

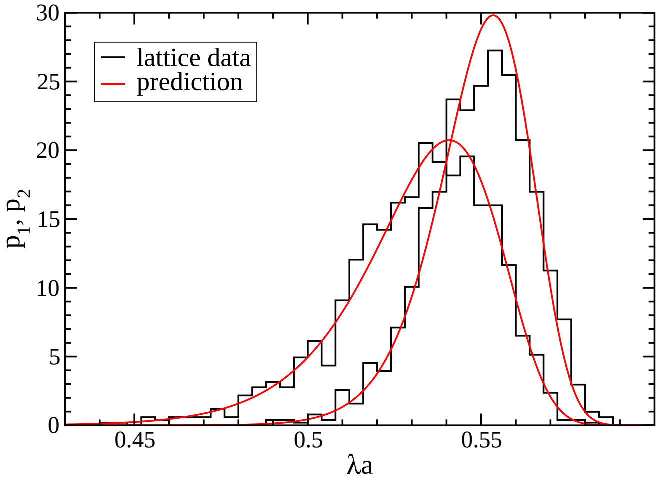


Distribution of two smallest eigenvalues

overlap, $N_t = 4$, $N_s = 16$

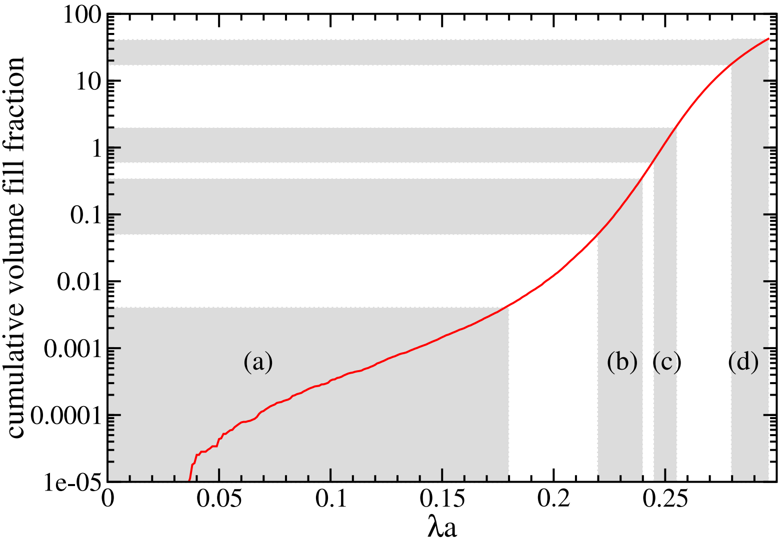
No free parameter!

$p_1(\lambda)$ and $p_2(\lambda)$ contain the already fitted parameters μ, C.



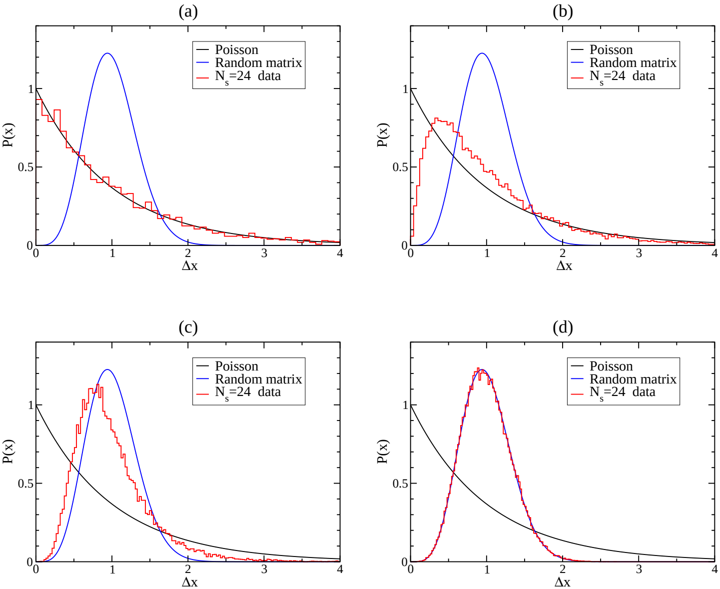
Cumulative volume fill fraction

staggered, $N_t = 4$, $N_s = 24$



Unfolded level spacing distribution

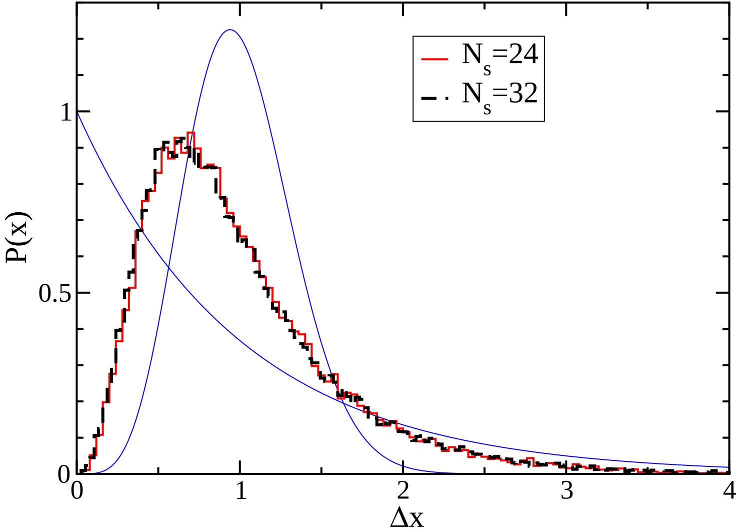
staggered, $N_t = 4$, $N_s = 24$



Scaling with spatial volume

staggered, $N_t = 4$

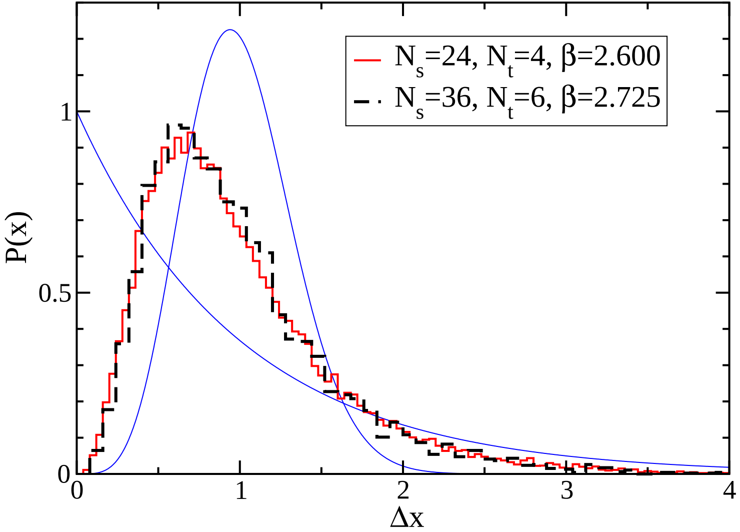
Eigenvalues 10-20 for $N_s = 24$ Eigenvalues 24-47 for $N_s = 32$



Continuum limit

staggered, $N_t = 4$ and $N_t = 6$; same temperature and physical volume

Eigenvalues 10-20 for both $N_t = 4$ and $N_t = 6$.



Conclusions

- At $T > T_c$ lowest Dirac eigenvalues uncorrelated (Poisson)
- These modes are localized (scale $\approx 1/T_c$)
- Number of Poisson modes scales with physical volume.
  - Maybe localized on physical gauge field objects
  - Definitely not instantons!
- Fermion b.c. essential:
  Poisson modes absent for periodic b.c.
- Upper part of the spectrum:
  transition to bulk RMT statistics and delocalized modes