Mass Anomalous Dimension and Running of the Coupling in SU(2) with Six Fundamental Fermions

Thomas Pickup

University of Oxford

Lattice 2010
17/6/10

Francis Bursa, Luigi Del Debbio,
Liam Keegan and Claudio Pica

Motivation

- Most work on (near-)conformal theories have concentrated on $SU(3)$ with fundamental fermions or those with higher representations
- $SU(2)$ with fundamental fermions requires fewer fermions to be conformal than $SU(3)$
- Combined with the small size of the representation this can have a small naïve S parameter
- Six fermions places us between a number of predictions for the lower end of the conformal window:
  - Loss of Asymptotic Freedom: 11
  - Ladder Approximation: 7.86
  - All Orders Beta Function: 7.33 / 5.5
  - UV / IR degrees of Freedom: 4.74

Walking Technicolor

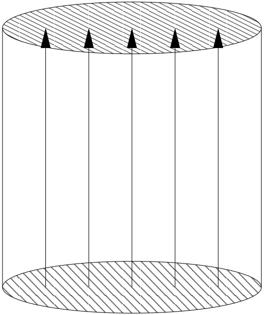
- The original Technicolor theories have a tension between larger fermion masses and small FCNCs
- Walking Technicolor circumvents this by having the underlying gauge theory run slowly between the TC and ETC scales
- This results in a large value for $\langle \bar{\Psi}\Psi \rangle_{ETC}$ avoiding the tension

$$\langle \bar{\Psi}\Psi \rangle_{ETC} \sim \langle \bar{\Psi}\Psi \rangle_{ETC} \left(\frac{M_{ETC}}{\Lambda_{TC}} \right)^{\gamma}$$

- A viable theory must not only run slowly but must have a large anomalous dimension

Schrödinger Functional

- The SF formalism replaces the (anti-)periodic boundary conditions of standard lattice formulations with Dirichlet boundaries in the time-like direction



$$U(x,k)|_{t=0} = exp[\eta\tau_3 a/iL],$$
$$U(x,k)|_{t=L} = exp[(\pi-\eta)\tau_3 a/iL]$$

$$P_+\psi = 0,\ \overline{\psi}P_- = 0 \ \text{ at } t = 0\,,$$
$$P_-\psi = 0,\ \overline{\psi}P_+ = 0 \ \text{ at } t = L$$

with
$P_\pm = 1/2(1 \pm \gamma_0)$

- The periodicity is maintained in the space-like directions
- Classically this is equivalent to a background Chromoelectric Flux

SF Coupling

- We can use the effective action S of the theory to define a coupling
- We measure the expectation of derivative of S as this a better observable

$$\bar{g}^2 = k \left\langle \frac{\partial S}{\partial \eta} \right\rangle^{-1}$$

- Where k is chosen so $\bar{g}^2 = g_0^2$ to first order in perturbation theory

$$k = -24 \frac{L^2}{a^2} \sin\left(\frac{a^2}{L^2} (\pi - 2\eta) \right)$$

- $\bar{g}^2$ is deined on the scale of the Lattice size
- We can probe different scales by varying the size of the lattice

Step Scaling

- We can define a step scaling function Σ that measures the changing in the coupling after a scaling of size s

$$\Sigma(u, s, a/L) = \bar{g}^2(g_0, sL/a)|_{\bar{g}^2(g_0, L/a)=u}$$

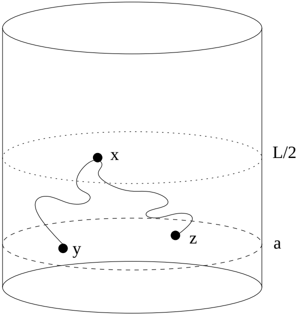
- By simulating at multiple values of L we can calculate Σ at different values of a/L allowing us to form a continuum extrapolation

$$\sigma(u, s) = \lim_{a/L \to 0} \Sigma(u, s, a/L)$$

- The continuum step scaling function can be directly related to the β function

$$-2 \log s = \int_u^{\sigma(u,s)} \frac{dx}{\sqrt{x}\beta(\sqrt{x})}$$

Mass



- We use the PCAC relation to define the physical quark mass and set it to zero by tuning the bare quark mass

$$m_{PCAC}(x) = \frac{\frac{1}{2}(\partial_0 + \partial_0^*) f_A(x)}{2 f_P(x)}$$

- The Schrödinger Functional Boundary conditions suppress fermionic zero modes allowing us to simulate at $m_q = 0$

- All correlators can be measured from either boundary giving a non-trivial check of the mass

Mass Renormalisation

- The correlators f_P, f_A are the pseudoscalar and axial currents defined by:

$$f_P(x) = -1/12 \int d^3y\, d^3z\, \left\langle \bar{\psi}(x)\gamma_5\tau^a\psi(x)\bar{\zeta}(y)\gamma_5\tau^a\zeta(z)\right\rangle,$$

$$f_A(x) = -1/12 \int d^3y\, d^3z\, \left\langle \bar{\psi}(x)\gamma_0\gamma_5\tau^a\psi(x)\bar{\zeta}(y)\gamma_5\tau^a\zeta(z)\right\rangle$$

- From these we can calculate the pseudoscalar renormalisation constant Z_P allowing us to calculate the running of the quark mass

$$Z_P = \sqrt{3f_1}/f_P(L/2)$$

$$f_1 = \frac{-1}{12L^6} \int d^3u\, d^3v\, d^3y\, d^3z\, \left\langle \bar{\zeta}'(u)\gamma_5\tau^a\zeta'(v)\bar{\zeta}(y)\gamma_5\tau^a\zeta(z)\right\rangle$$

- Z_P requires boundary fields set to Identity and must be calculated on different lattices from $\bar{g}^2$

Mass Scaling

- We can define a step scaling function for the mass in the same way as for the coupling

$$\Sigma_P(u, s, a/L) = \left. \frac{Z_P(g_0, sL/a)}{Z_P(g_0, L/a)} \right|_{\bar{g}^2(g_0, L/a) = u}$$

- Again we use the raw data to construct a continuum limit

$$\sigma_P(u, s) = \lim_{a \to 0} \Sigma_P(u, s, a/L)$$

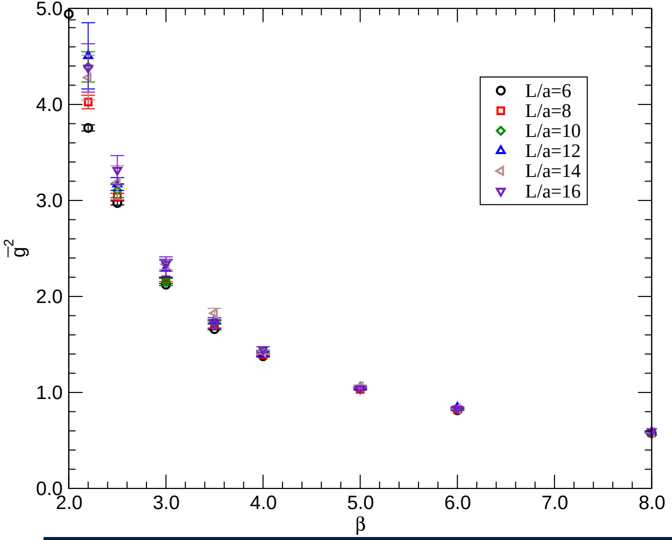
- We can use this to form an estimator for γ^*

$$\hat{\gamma}(u) = -\frac{\log |\sigma_P(u, s)|}{\log |s|}$$

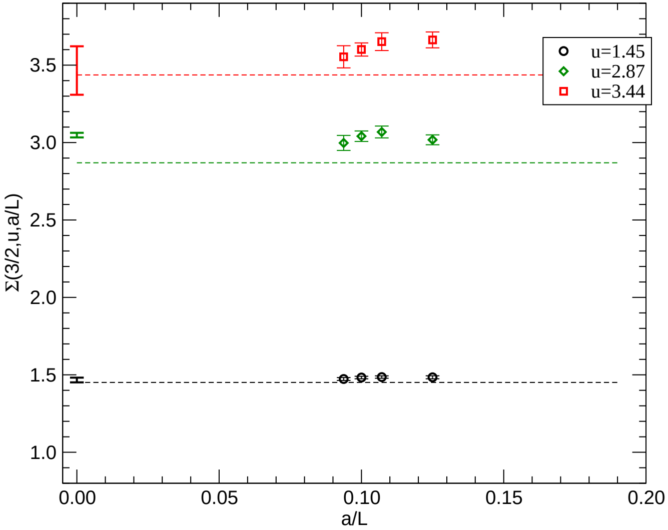
Simulation Details

- We use unimproved Wilson fermions and an RHMC with 4 pseudofermions
- κ_c is determined from lattices with unit boundary conditions
- We simulate on lattices of size $L = 6, 8, 10, 12, 14, 16$ for a range of bare couplings between $\beta = 2$ and $\beta = 8$
- We interpolate in L to produce results for $L = 9, 9\frac{1}{3}, 10\frac{2}{3}, 15$
- This gives us multiple pairs of lattices with $s = 3/2$ to allow for a continuum limit

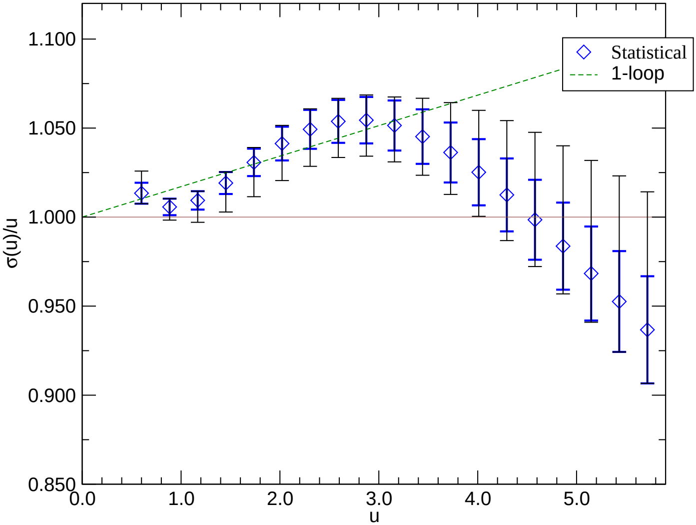
Coupling Results



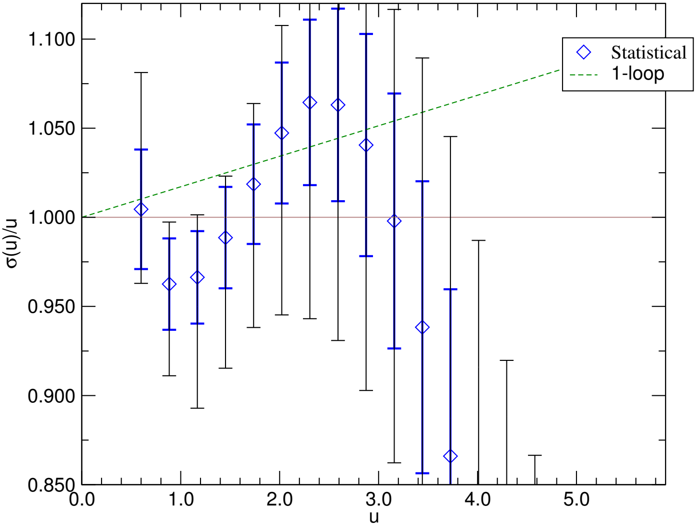
Continuum Extrapolation



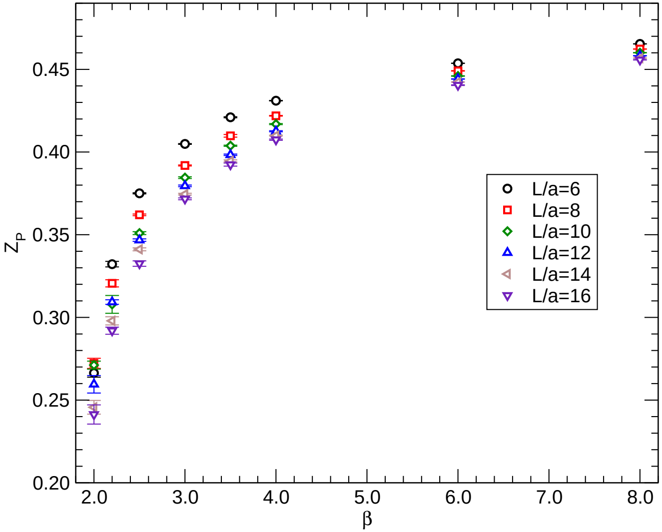
Constant Extrapolation Scaling



Linear Extrapolation Scaling

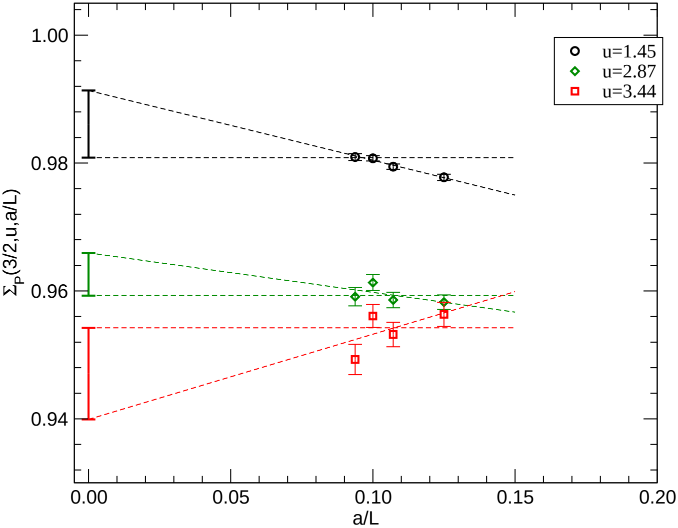


Z_P Results

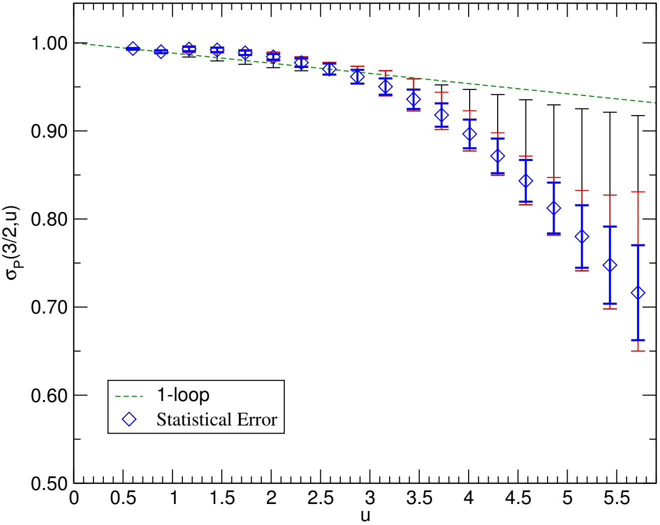


0.45
0.40
0.35
0.30
0.25
0.20
Z_P
2.0
3.0
4.0
5.0
6.0
7.0
8.0
β
L/a=6
L/a=8
L/a=10
L/a=12
L/a=14
L/a=16

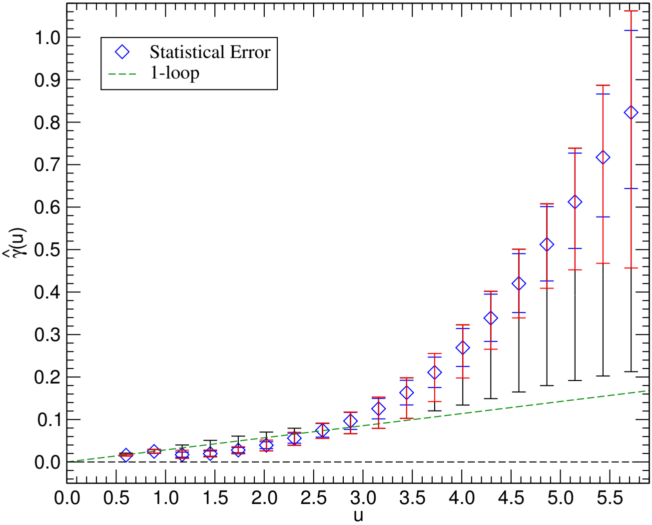
Z_P Continuum Extrapolation



Mass Scaling



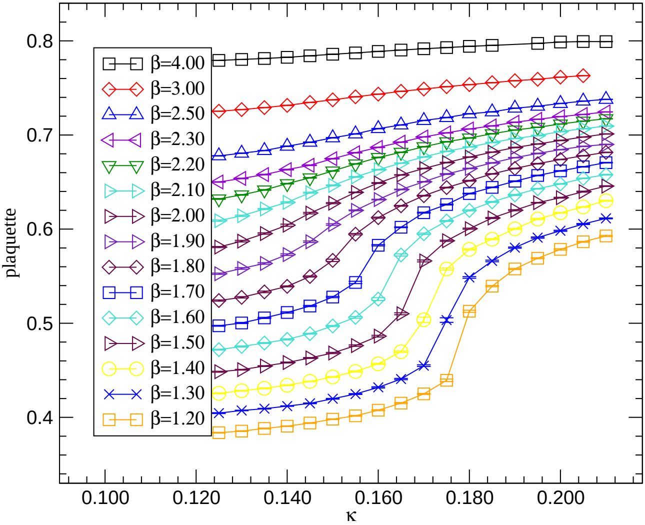
Mass Anomalous Dimension



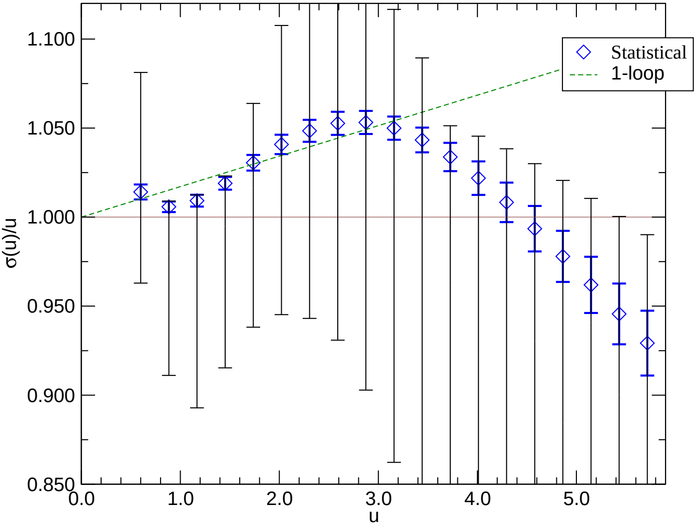
Summary

- The coupling runs very slowly across the range investigated
- It is hard to conclusively argue for the existence of an IRFP
- Further simulation is required to pin down the correct continuum extrapolation
- The anomalous dimension is better determined but exact values are limited by the uncertainty in the fixed point
- The data favours a low value of the anomalous dimension except at the extreme range of the coupling

Thermal Scan



Total Error



Statistical
1-loop
1.100
1.050
1.000
0.950
0.900
0.850
σ(u)/u
0.0
1.0
2.0
3.0
4.0
5.0
u