Second Order Phase Transition in Anisotropic Lattice Gauge Theories with Extra Dimensions

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Outline

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3. A second order phase transition
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Introduction–Motivation

The idea is to take the extra dimension(s) seriously and to try and understand how *quantum* effects of four–dimensional physics decouple from the *quantum* effects of a higher dimensional theory. This decoupling could occur through a second order phase transition.

Introduction–Motivation

Kaluza–Klein, Randall–Sundrum and deconstruction make a commitment on the Lorentz transformations of the fields. Compact lattice gauge fields allow us to postpone this commitment. They allow for different combinations at strong than at weak coupling.

The lattice action in $D = d_{\parallel} + d_{\perp}$

$$S = \sum_{n} \left[\beta \sum_{\mu<\nu\leq d_{\parallel}} (1 - \mathrm{Re}\, U_{\mu\nu}(n)) + \beta' \sum_{d_{\parallel}<\mu<\nu<D} (1 - \mathrm{Re}\, U_{\mu\nu}(n)) \right]$$

$$+ S_{\mathrm{matter}}$$

The effective action

Concentrate on the pure gauge case. Then

$$Z[J] = e^{W[J]} = \int \left[\prod_{\text{links}} dV_l \frac{d\alpha_l}{2\pi \mathrm{i}}\right] e^{-S[V]-\alpha\cdot V+\sum_{\text{links}} w(\alpha)+J\cdot V}$$

and the effective action is obtained through a Legendre transform

$$\Gamma[V,\alpha] = S[V] + \sum_{\mathrm{l}\in\text{links}} (\alpha_l \cdot V_l - w(\alpha_l))$$

where

$$e^{w(\alpha_l)} \equiv \int \mathscr{D} U_l e^{\alpha_l U_l + \alpha_l^\dagger U_l^\dagger}$$

The effective potential

Look for uniform solutions : $V_l = v$ for $l \in d_{\|}$, $V_l = v'$ for $l \in d_{\perp}$. Thus $\alpha_l = \alpha$, for $l \in d_{\|}$ and $\alpha_l = \alpha'$ for $l \in d_{\perp}$.

$$
\begin{aligned}
V_{\rm eff}(v, v', \alpha, \alpha') = \\
& N^D \left\{ \beta \frac{d_{\|}(d_{\|} - 1)}{2}(1 - v^4) + \right. \\
& \beta' \frac{(D - d_{\|})(D - d_{\|} - 1)}{2}(1 - v'^4) + \\
& \left. \beta' \frac{(D - d_{\|})(D + d_{\|} - 1)}{2}(1 - v^2 v'^2) \right\} + \\
& N^D \left\{ d_{\|}\alpha \cdot v + (D - d_{\|})\alpha' \cdot v' \right\} + \\
& N^D \left\{ d_{\|} w(\alpha) + (D - d_{\|}) w(\alpha') \right\}
\end{aligned}
$$

Compact lattice gauge fields and a possible instability

For isotropic theories only the terms

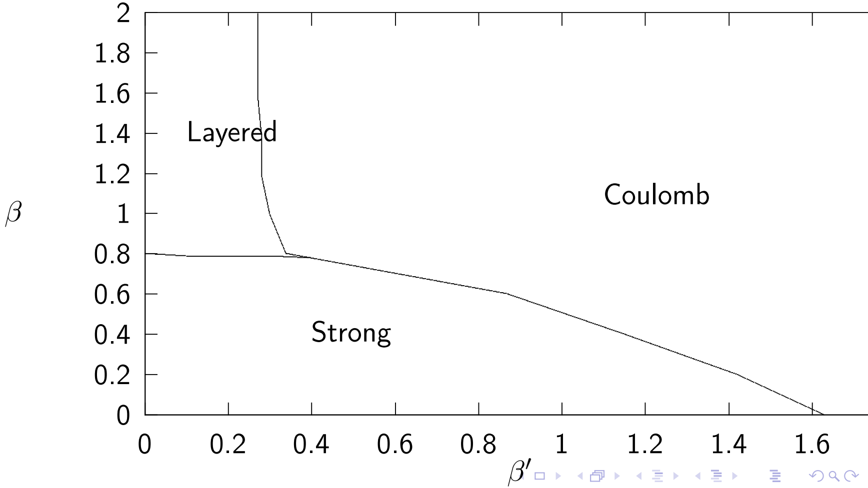
$$\left\{d_{\parallel}\alpha\cdot v+(D-d_{\parallel})\alpha'\cdot v'\right\}+\left\{d_{\parallel}w(\alpha)+(D-d_{\parallel})w(\alpha')\right\}$$

can give rise to quadratic terms-and these are always positive : The confining phase is always a minimum of the effective potential.
For anisotropic theories, the term

$$\beta'\frac{(D-d_{\parallel})(D+d_{\parallel}-1)}{2}(1-v^2v'^2)$$

can contribute with a **minus** sign to the quadratic terms–and, thus, destabilize the confining phase. For $D=5, d_{\parallel}=4$ this is what happens. For this to occur, $v\neq 0$ should be a valid solution-i.e. the layer should be in the Coulomb phase!

A typical phase diagram



- Lattice gauge theories typically display a confining phase, that is always a minimum of the effective potential. If there can appear a minimum at weak coupling (Coulomb phase) the transition is necessarily first order. But anisotropy can destabilize the confining phase through *second* order phase transition, by turning the minimum at the origin into a maximum.

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- The low energy effective field theory will be valid, if $1/\sqrt{\sigma} > l_{\mathrm{string}}$. Since the gauge anomalies must be cancelled, the layers, even thick, are BPS states. These are new D-brane configurations whose properties could be accessible to a lattice analysis.