Conformal window on the lattice

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Lattice 2010

Outline

[Banks & Zaks 82]

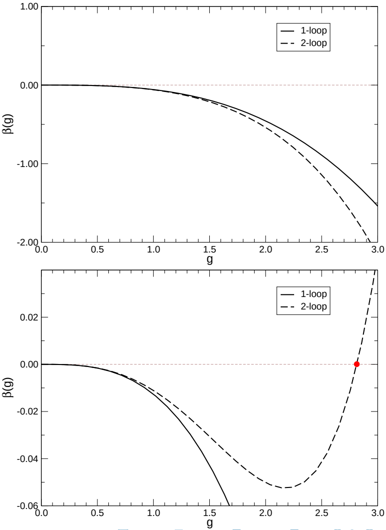
$$\beta(\bar{g}) = \mu \frac{d}{d\mu}\bar{g}(\mu) = -\beta_0 \bar{g}^3 - \beta_1 \bar{g}^5 + O(\bar{g}^7)$$

$$\begin{aligned} \beta_0 > 0 &\implies \text{asymptotic freedom}, n_f < n_f^{\mathrm{af}} \\ \beta(\bar{g}) < 0 &\implies \text{confining theories} \\ \beta(\bar{g}^*) = 0 &\implies \text{IR fixed point} \end{aligned}$$

conformal window: IR zero of the β function

$$\boxed{n_f^c < n_f < n_f^{\mathrm{af}}}$$

	$(4\pi)^2\beta_0$	$(4\pi)^4\beta_1$
SU(3), $n_f = 3$ fund	9	64
SU(3), $n_f = 12$ fund	3	-50
SU(2), $n_f = 2$ adj	2	-40



- existence of the IRFP is scheme-independent

$$\beta(\bar{g}) = \mu \frac{d}{d\mu} \bar{g}(\mu)$$

change of scheme: $\bar{g}' = \Phi(\bar{g}), \quad \Phi'(\bar{g}) > 0$

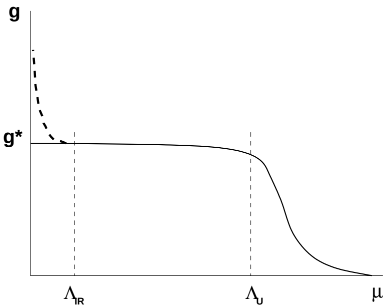
$$\boxed{\beta'(\bar{g}') = \frac{\partial}{\partial \bar{g}} \Phi(\bar{g}) \beta(\bar{g})}$$

universality of the first two coefficients

- β-function away from IRFP is not universal!

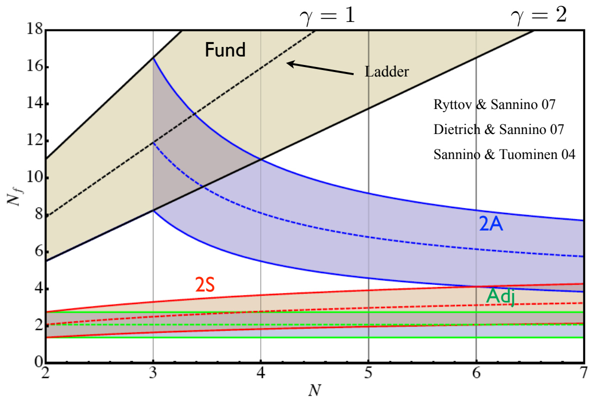
- $\bar{m}' = \bar{m} F(\bar{g})$

$$\boxed{\gamma' = \gamma + \beta \frac{\partial}{\partial \bar{g}} \log F}$$



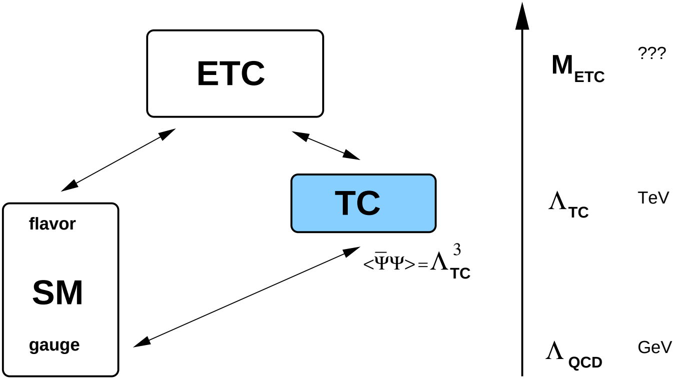
- asymptotic freedom at high-energy scale $\Lambda_{\rm UV}$
- scale invariance at large distances $\Longrightarrow$ no massive spectrum
- large-distance dynamics: anomalous dimensions at the IRFP
- nonperturbative IRFP: strong coupling, large anomalous dimensions
- scale invariance explicitly broken by a mass term/finite volume

SU(N) Phase Diagram

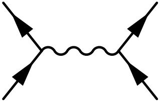


the edge of the window is relevant for phenomenology

[talk by Chivukula]



- ETC: mass term for the SM fermions, FCNC [talk by Chivukula]



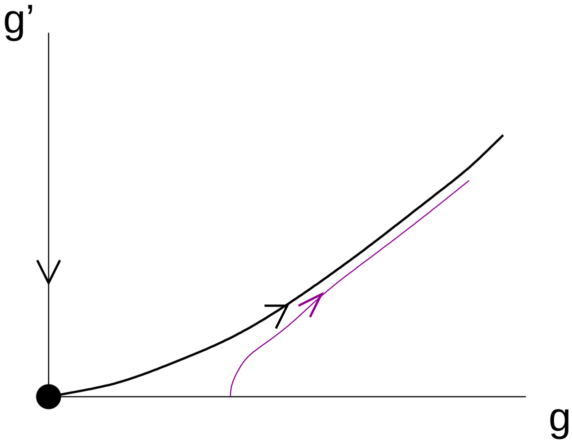
$$\longrightarrow \qquad \Delta\mathcal{L} \propto \frac{1}{M_{\mathrm{ETC}}^2}\langle\bar{\Psi}\Psi\rangle_{\mathrm{ETC}}\,\bar{\psi}\psi, \ \frac{1}{M_{\mathrm{ETC}}^2}\bar{\psi}\psi\ \bar{\psi}\psi$$

- running of $\langle\bar{\Psi}\Psi\rangle$: **(near) conformal IR behaviour**

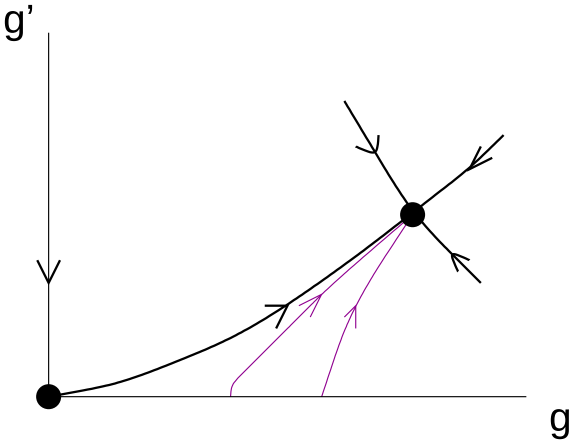
$$\boxed{\langle\bar{\Psi}\Psi\rangle_{\mathrm{ETC}} = \langle\bar{\Psi}\Psi\rangle_{\mathrm{TC}}\ \exp\left(\int_{\Lambda_{\mathrm{TC}}}^{M_{\mathrm{ETC}}} \frac{d\mu}{\mu}\,\gamma(\mu)\right)}$$

[holdom, lane, appelquist, luty, sannino, chivukula,...]

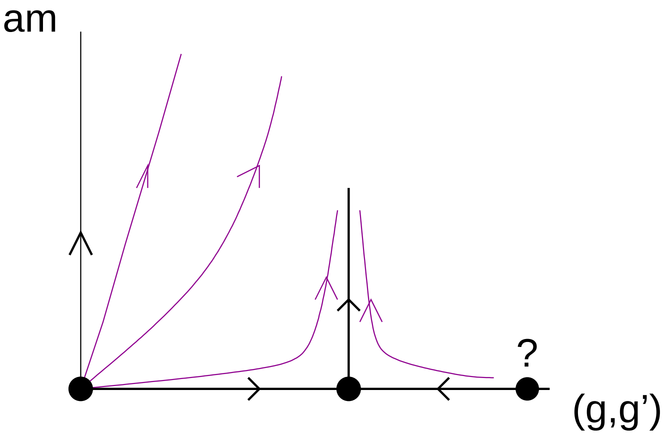
YM theory - g lattice bare coupling, g' space of irrelevant couplings



flow in the $m = 0$ plane - assuming only the mass is relevant in the IR



flow in the presence of a mass deformation



non-trivial UV fixed point? [Kaplan et al 09, Hasenfratz 10, talk by Pallante]

Using the linearized RGEs in the vicinity of the IRFP [DeGrand & Hasenfratz 09, Fodor et al 09, Idd et al 10]

- solution of RGEs for the two-point correlators

$$C_H(t; g, \hat{m}, a) = m^{2\Delta_H/y_m} \mathcal{F}(t\hat{m}^{1/y_m}) \sim \exp(-M_H t)$$

- scaling with the quark mass

$$\boxed{M_H \sim \xi^{-1} \sim m^{1/y_m}}$$

- finite-size scaling

$$M = L^{-1} F(x), \quad x = L^{y_m} m = (L/\xi)^{y_m}$$

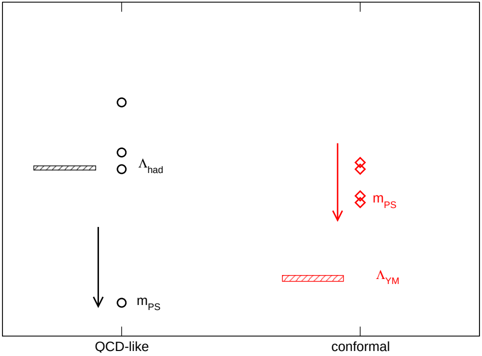
- scaling is characterized by the critical exponent $y_m = 1 + \gamma$

asymptotic behaviour: $m \to 0, L \to \infty$ (beware of $m \to 0$ at fixed L!!)

In the presence of a mass deformation (and infinite volume)

- no parametric separation between the PS and the other channels
- all masses/decay constants scale like m^{1/y_m} (but $M \sim F(0)/L$!!)

$\hookrightarrow$ very different spectrum from a QCD-like theory [Miransky 94]



Matching two lattices of size L/a and $L/(ab)$

$$O_k^{(n)}(\beta) = O_k^{(n-1)}(\beta')$$

Define bare step scaling function

$$\boxed{s_0(\beta, b) = \beta - \beta'}$$

fixed point

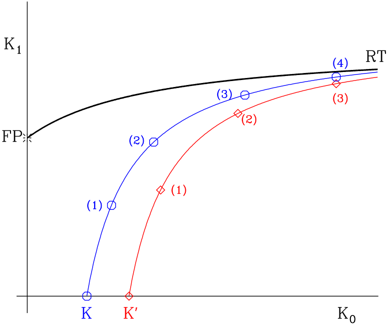
$$\boxed{s_0(\beta^*, b) = 0}$$

Matching m at fixed β

$$\begin{aligned} \xi_{\rm lat}(m) &= b\xi_{\rm lat}(m') \\ \frac{m'}{m} &= b^{y_m} \end{aligned}$$

Systematics in matching at strong coupling?

$$m = 0$$
$$\xi_{\rm lat}(\beta') = \xi_{\rm lat}(\beta)/b$$
$$a(\beta') = ba(\beta)$$



Schrödinger functional [ALPHA collaboration]

$$Z[\eta] = e^{-\Gamma[\eta]} = \int_{L\times L^3} \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\, \exp(-S[U,\psi,\bar{\psi}])$$

Dirichlet boundary conditions at $t = 0, L$, dependent on η

$$\boxed{\frac{k}{\bar{g}^2(L)} = \left.\frac{\partial\Gamma}{\partial\eta}\right|_{\eta=0}, \qquad \bar{g}^2 = g_0^2 + O(g_0^4)}$$

Lattice step scaling function

$$\Sigma(u, a/L) = \left.\bar{g}^2(bL)\right|_{\bar{g}^2(L)=u,m=0}$$

Step scaling function

$$\sigma(u) = \lim_{a\to 0} \Sigma(u, a/L)$$

fixed point:

$$\boxed{\sigma(u^*) = u^*}$$

Definition of the renormalized mass

$$\boxed{\partial_\mu (A_R)_\mu = 2\bar{m} P_R}$$

$$\begin{aligned} (A_R)_\mu(x) &= Z_A \bar{\psi}(x)\gamma_\mu\gamma_5\psi(x) \\ (P_R)(x) &= Z_P \bar{\psi}(x)\gamma_5\psi(x) \end{aligned}$$

Lattice step scaling function

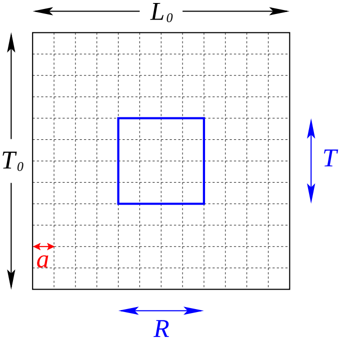
$$\begin{aligned} Z_P(g_0, L/a) &= c\,\frac{\sqrt{3f_1}}{f_P(L/2)} \\ \Sigma_P(u, a/L) &= \frac{Z_P(g_0, bL/a)}{Z_P(g_0, L/a)}, \quad \bar{g}^2(L) = u \end{aligned}$$

Step scaling function

$$\sigma_P(u) = \lim_{a\to 0} \Sigma_P(u, a/L)$$

[Bilgici et al 09, Fodor et al 09, talk by Etou]

$$g_w^2(L_0, R/L_0, a/L_0) = \frac{1}{k}(R/a)^2 \chi(R/a, L_0/a)$$



step scaling function

$$\Sigma_w(u, R/L_0, a/L_0) = g_w^2(bL_0, R/L_0, a/L_0),$$
$$u = g_w^2(L_0, R/L_0, a/L_0)$$

$$\sigma_w(u) = \lim_{a \to 0} \Sigma_w(u, R/L_0, a/L_0)$$

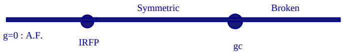
$O(a)$ improved

Numerical tools

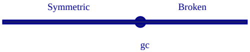
Phase diagram

[Deuzeman et al 09]

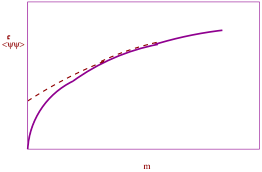
ANALOGY WITH STRONGLY COUPLED QED



QCD Conformal



QED

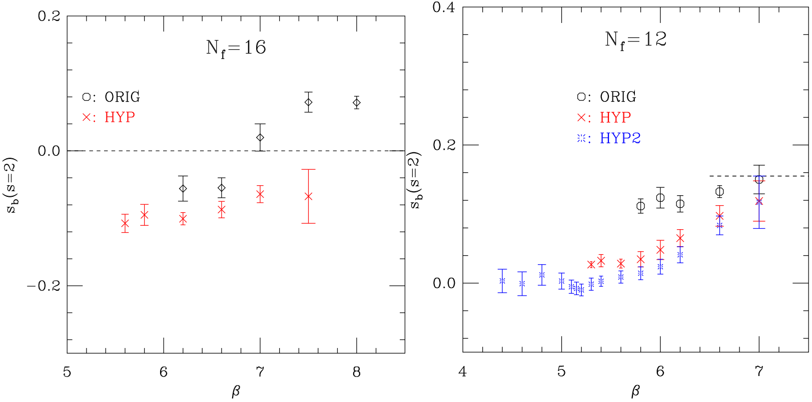


- At the critical point g_c
  $\langle\bar\psi\psi\rangle = am^{1/\delta}$
- In the symmetric phase
  Conformality perturbatively broken by Coulomb forces:
  $\langle\bar\psi\psi\rangle = am^d$, $d = \frac{3-\gamma}{1+\gamma}$
- Only in the free limit:
  $d = 1.$
- d measured in QED
  Kogut et al. 1985
  $1/\delta < d < 1$

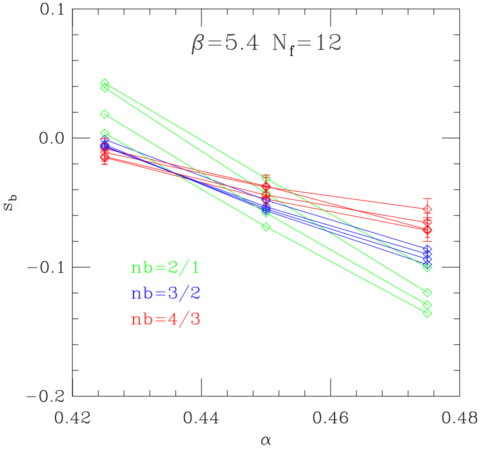
Lattice strategy:
–Contrast χ^2 of different fits –

- Symmetric Phase $\langle\bar\psi\psi\rangle = am^d$
- Broken Phase $\langle\bar\psi\psi\rangle = am + b$

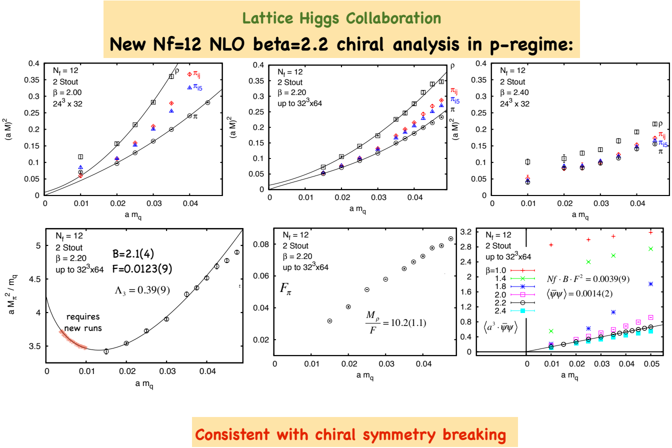
$n_f = 16, 12$ staggered, MCRG [Hasenfratz 10]



$n_f = 12$ staggered, MCRG [Hasenfratz 10]

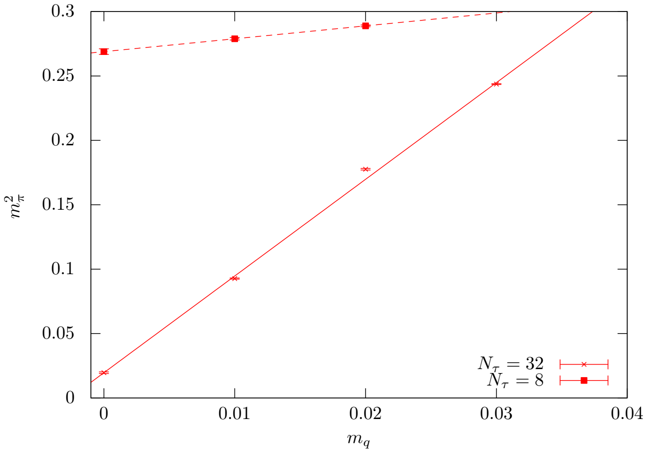


$n_f = 12$ staggered, spectrum [talks by Kuti]



+ eigenvalues distribution in agreement with RMT

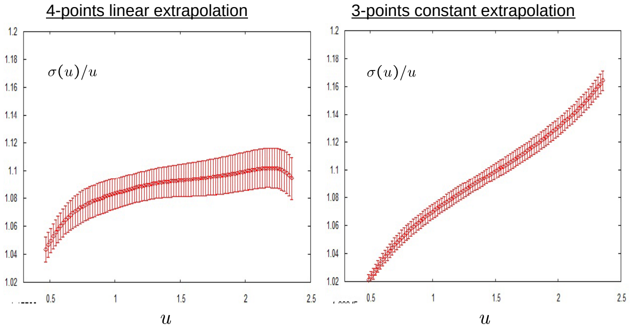
$n_f = 12$ staggered, spectrum [Jin & Mawhinney 09/10]



data consistent with broken chiral symmetry

$n_f = 12$ staggered, TPL scheme [talk by Itou]

There is a large systematic error.



There is no signal of the fixed point.
We should be study the large u region (beta<5) to find the fixed point
and
have to do some improvements (larger lattice size simulation).

$n_f = 12$ staggered, QQ scheme [talk by Holland]

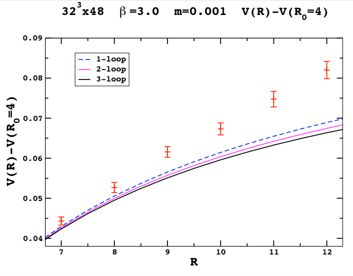
12 flavor QCD

Lattice Higgs Collaboration

running coupling QQ scheme $V(R) - V(R_0) = C_F \int_{R_0}^{R} \frac{\alpha_{qq}(R')}{R'^2} dR'$

measure $V(R)$ via simulations

$\alpha_{qq}(R_0 = 4) = 0.30$



run $\alpha_{qq}(R')$ via 1,2,3 loop pert thy

data run faster than pert thy

do not see IR fixed point

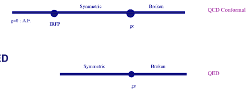
IRFP $\alpha_{qq}^{3-\mathrm{loop}} = 0.371$

Kieran Holland

Running coupling

$n_f = 12$ staggered, phase diagram [poster by Lombardo, talk by Pallante]

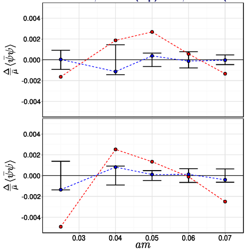
CHIRALLY SYMMETRIC PHASE : ANALOGIES WITH LATTICE QED



SIDE-BY-SIDE COMPARISON OF THE TWO SIMPLEST SCENARIOS at $\beta = 3.9$(top) and $\beta = 4.0$(bottom)

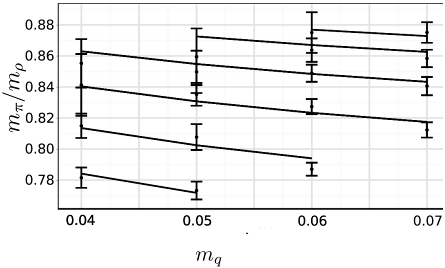
- $Y = bX^d$
  (χ^2/ndf) = 0.4–0.3,
  Chiral Symmetry :
  Consistent with data

- $Y = bX + K$
  (χ^2/ndf) = 3.9–4.0,
  Chiral Symmetry Breaking:
  Excluded within 2 σ



A. Deuzeman, E. Pallante, M.P. Lombardo

$n_f = 12$ staggered, phase diagram [poster by Lombardo, talk by Pallante]

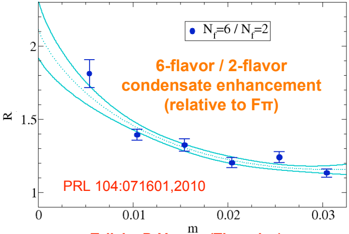


$g_L = 1.22 - 1.30$ from top to bottom

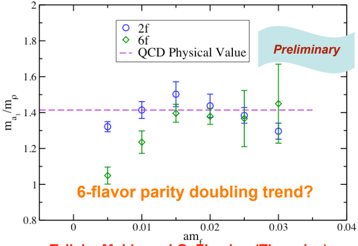
SPECTRUM AND CHIRAL SYMMETRY: SAME TREND AS IN SYMMETRIC QED, CONSISTENT WITH CHIRAL SYMMETRY

A. Deuzeman, E. Pallante, M.P. Lombardo

Lattice Strong Dynamics collaboration

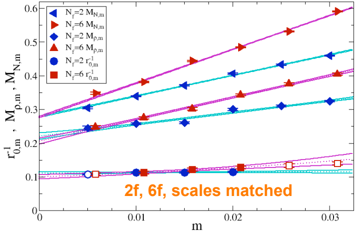


Talk by P. Vranas (Thursday)

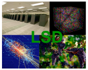


Talk by M. Lin and G. Fleming (Thursday)

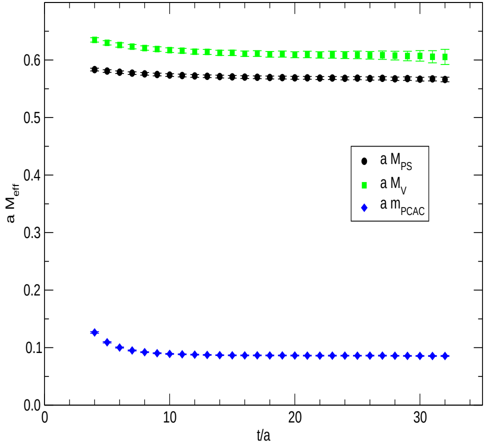
S parameter Talk by D. Schaich (Thursday)



- DWF ~ 1,000 configurations
- SU(3c), SU(2c) fundamental
- Small lattice spacing a ~ N_f matched
- V = 32^3 x 64 M_π L > 4
- Condensate enhancement
- Spectrum
- S parameter
- Eigenvalue spectrum
- LLNL BG/L ~ 300 million core hours

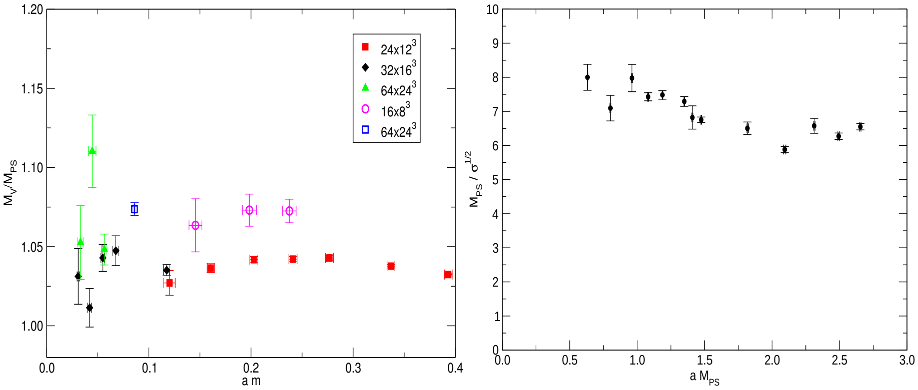


$n_f = 2$ Wilson, spectrum [Catterall et al 07-09, Idd et al 08-10, Hietanen et al 08]
$\beta = 2.10$, 64×24^3 lattice

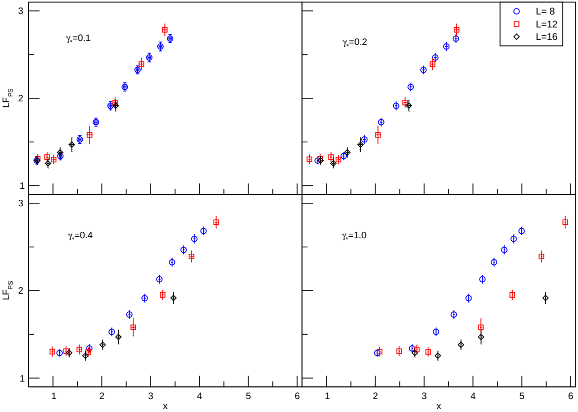


systematic errors on smaller volumes [talk by Kerrane]

$n_f = 2$ Wilson, spectrum [talks by Pica & Patella]

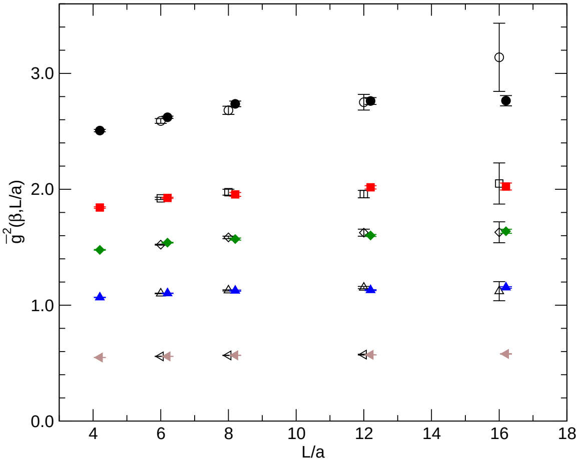


$n_f = 2$ Wilson, FSS spectrum [talk by Pica]

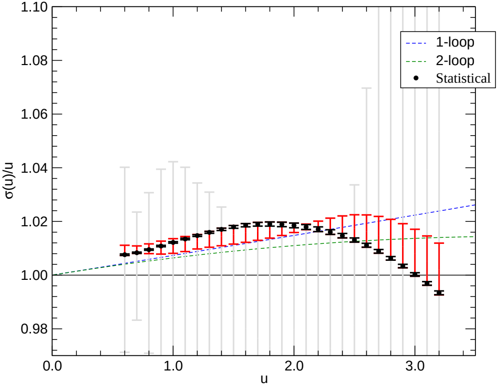


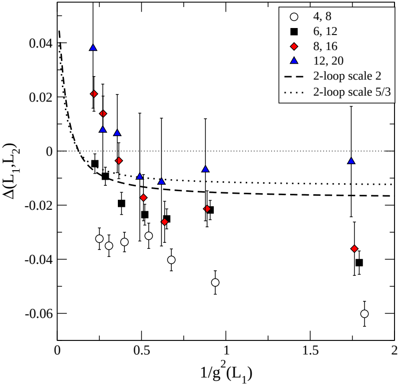
for several γ, $LF_{\rm PS} = F(L^{y_m} m)$; $\gamma < 0.5$ is favoured

$n_f = 2$ Wilson, Schrödinger functional coupling [Hietanen et al 08, Bursa et al 09]



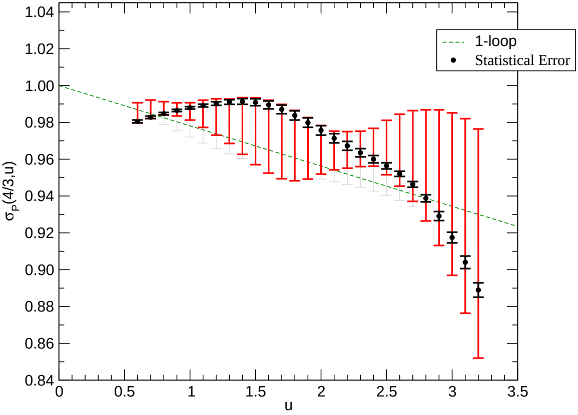
$n_f = 2$ Wilson, Schrödinger functional coupling [Hietanen et al 08, Bursa et al 09]





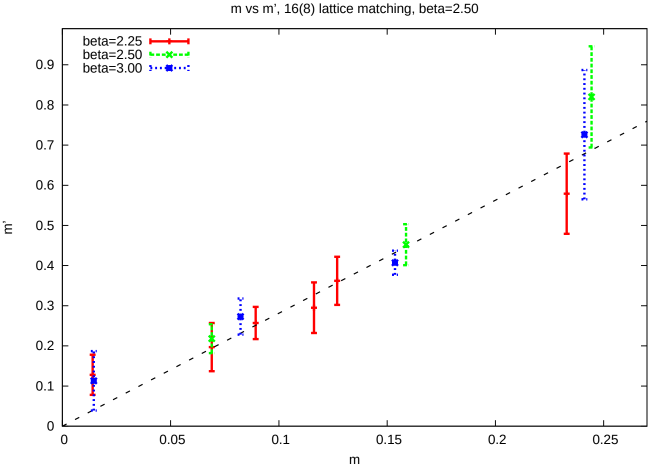
$$\Delta(L_1, L_2) = 1/g^2(L_2) - 1/g^2(L_1)$$

$n_f = 2$ Wilson, Schrödinger functional mass [Bursa et al 09]



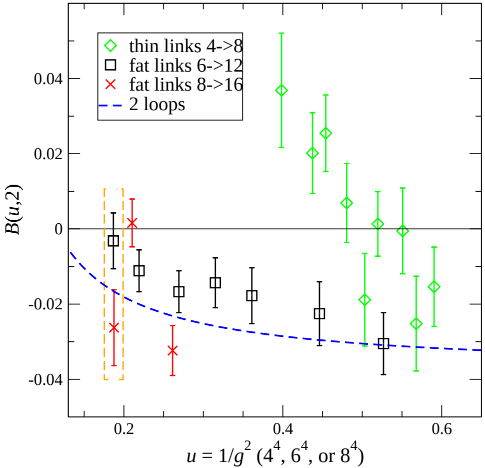
$$\log |\sigma_P(u, b)| = -\gamma \log b \implies 0.05 < \gamma < 0.56$$

$n_f = 2$ Wilson, MCRG mass matching - PRELIMINARY [talk by Keegan]

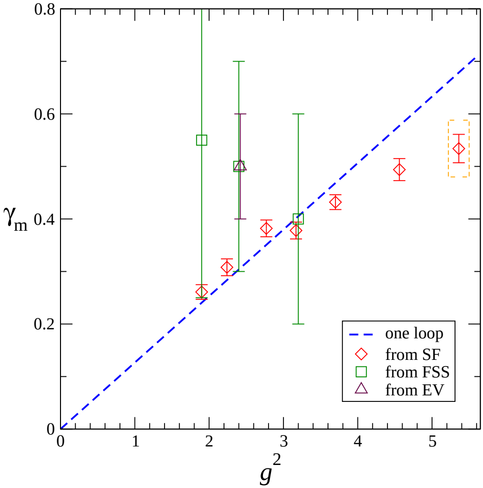


$$m' = b^{y_m} m \implies \gamma = 0.49(13)$$

$n_f = 2$ Wilson, SF coupling [talk by Svetitsky]



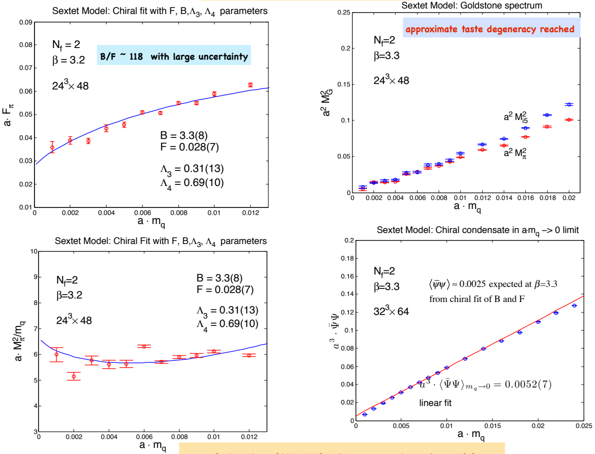
$n_f = 2$ Wilson, SF mass



$\gamma < 0.6$

$n_f = 2$ staggered [talk by Kuti]

Lattice Higgs Collaboration



Consistent with chiral symmetry breaking

$n_f = 2$ staggered [Kogut & Sinclair 10]
$N_t = 4, 6, 8$ - separated deconfinement and chiral transitions
β values for these transitions:

N_t	β_d	β_χ
4	5.40(1)	6.3(1)
6	5.54(1)	6.6(1)
8	5.66(1)	6.7(1)

Increase in β_χ from $N_t = 6$ tp $N_t = 8$ is very small
need to simulate at larger N_t and check how β_χ changes

Outlook

Towards phenomenology

- tools have been developed and tested - look for better tools
- preliminary results need to be put on solid ground
  - benchmark codes/analysis
  - control systematics: small masses & large volumes
  - improved actions [talks by Karavirta & Mykkänen]
- SU(2) $n_f = 2$ adjoint, consistent results
- more results needed for SU(3) $n_f = 12$ fundamental, $n_f = 2$ sextet
- all methods have systematics that "muddle" the answers
  more confident when different approaches yield consistent results
- anomalous dimensions are small - better control of systematics
- deformations away from conformality
  - $n_f < n_f^c$
  - mass deformation
  - 4fermi interactions
- approach the conformal window from the chirally broken phase
  LSD collaboration: deviations from QCD
- lattice results to be taken into account for model building
- talk to phenomenologists!

- analytical studies so far (*voodoo QCD*)
    - Schwinger-Dyson
    - AdS/CFT
    - other assumptions on the NP dynamics

- lattice provides first-principles results (w. systematic errors)

- Spectroscopy
    - masses and decay constants
    - finite-size scaling
    - eigenvalue distributions
- MCRG
    - two-lattice matching
- SF/TPL/QQ schemes
    - running coupling
    - running mass
- Phase diagram

- field theory on a lattice

$$a, \quad \phi(x), \quad \{K_\alpha\}$$

- integrate out UV $\longrightarrow$ blocked lattice

$$a' = ba, \quad \phi'(x')$$

- constant physics

$$\int \mathcal{D}\phi' \, \exp[-S'(\phi')] = \int \mathcal{D}\phi \, \exp[-S(\phi)]$$

- flow in the space of couplings

$$S(\phi) = \sum_\alpha K_\alpha O_\alpha(\phi) \mapsto S'(\phi') = \sum_\alpha K'_\alpha O_\alpha(\phi')$$

- RG trajectory

$$\begin{array}{ccc} \{K_\alpha\} & \mapsto & \{K'_\alpha\} \mapsto \{K''_\alpha\} \\ \xi_{\rm lat} & \mapsto & \xi_{\rm lat}/b \ldots \end{array}$$

RG transformation

$$K'_i = R_i(K)$$

fixed point

$$K^*_i = R_i(K^*)$$

linearized flow equations in a *neighbourhood* of K^*

$$\delta K'_i = R_{ij}\,\delta K_j , \quad R_{ij} = \left.\frac{\partial R_i}{\partial K_j}\right|_{K^*}$$

eigenvalues and eigenvectors of R_{ij}

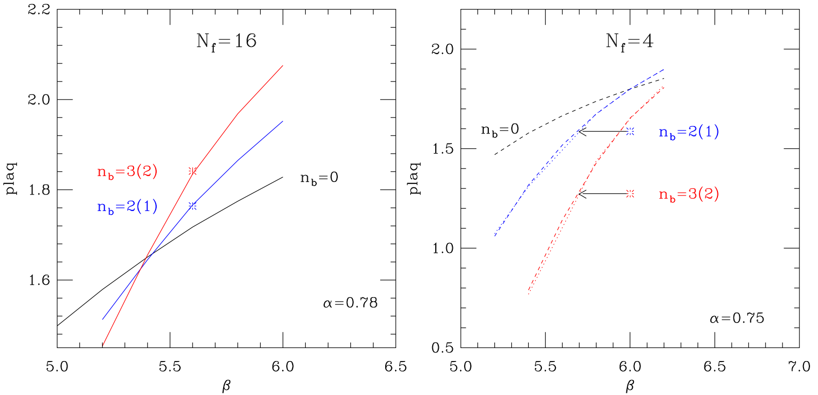
$$u'_i = b^{y_i}\,u_i$$

y_i critical exponents

$y_i > 0$: relevant coupling at the fixed point

(for $n_f < 4$ see other talks!!)

$n_f = 16$ staggered, evidence for IRFP from MCRG [Hasenfratz 09]

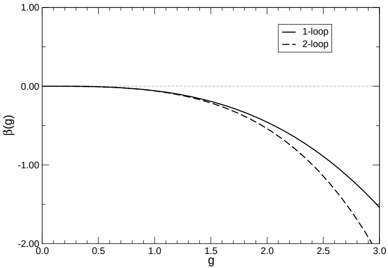


SU(N) gauge + n_f massless fermions in representation R

$$\beta(\bar{g}) = \mu \frac{d}{d\mu}\bar{g}(\mu) = -\beta_0 \bar{g}^3 - \beta_1 \bar{g}^5 + O(\bar{g}^7)$$

$$\beta_0 = \frac{1}{(4\pi)^2}\frac{11}{3}C_2(A)\left[1 - \frac{4}{11}x_f\right]$$

$$\boxed{x_f = \frac{T_R n_f}{C_2(A)}}$$



asymptotic freedom: $\beta_0 > 0$

$$\boxed{x_f < \frac{11}{4} \iff n_f < n_f^{\mathrm{af}}}$$

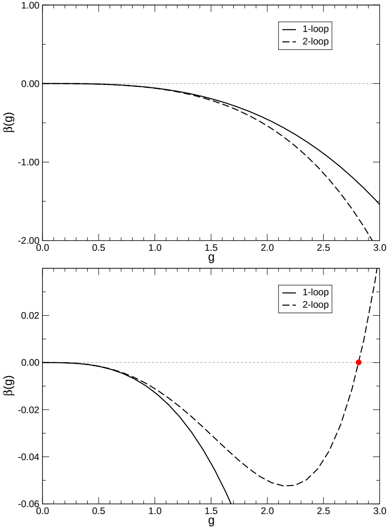
$$\beta(\bar{g}) = \mu \frac{d}{d\mu}\bar{g}(\mu) = -\beta_0 \bar{g}^3 - \beta_1 \bar{g}^5 + O(\bar{g}^7)$$

$$\beta_1 = \frac{1}{(4\pi)^4}\frac{34}{3} C_2(A)^2 \left[1 - F_1(R) x_f\right]$$

$$\boxed{F_1(R) = \frac{1}{34}\left[20 + 12\frac{C_2(R)}{C_2(A)}\right]}$$

$$\beta_1 < 0 \Longleftrightarrow 1 < F_1(R) x_f$$

	$(4\pi)^2\beta_0$	$(4\pi)^4\beta_1$
SU(3), $n_f = 3$ fund	9	64
SU(3), $n_f = 12$ fund	3	-50
SU(2), $n_f = 2$ adj	2	-40



- UV Gaussian fixed point at $g = 0$ if $n_f < n_f^{\mathrm{af}}$
- IR fixed point at g^* if $n_f > n_f^c$

$$g^{*2} = -\frac{\beta_0}{\beta_1} = -(4\pi)^2 \frac{11}{34} \frac{1}{C_2(A)} \frac{1 - \frac{4}{11} x_f}{1 - F_1(R) x_f}$$

- conformal window

$$\boxed{n_f^c < n_f < n_f^{\mathrm{af}}}$$

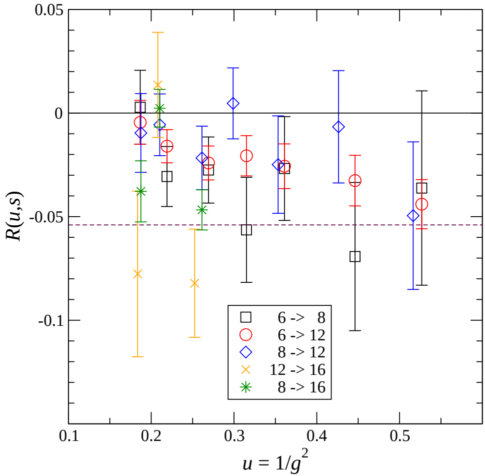
- for fundamental fermions:

$$x_f^c = \frac{17}{10 + 3\frac{N^2-1}{N^2}}, \quad n_f^c = \frac{34N}{10 + 3\frac{N^2-1}{N^2}}$$

- for adjoint fermions:

$$x_f^c \equiv n_f^c = 34/32$$

$n_f = 2$ Wilson, SF coupling [DeGrand et al 10]



Schrödinger functional [ALPHA collaboration]

$$Z[\eta] = e^{-\Gamma[\eta]} = \int_{L\times L^3} \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\, \exp(-S[U,\psi,\bar{\psi}])$$

Dirichlet boundary conditions at $t = 0, L$, dependent on η

$$\boxed{\left.\frac{\partial\Gamma}{\partial\eta}\right|_{\eta=0} = \frac{k}{\bar{g}^2}, \qquad \bar{g}^2 = g_0^2 + O(g_0^4)}$$

Lattice step scaling function:

$$\Sigma(u, a/L) = \bar{g}^2(bL)\Big|_{\bar{g}^2(L)=u, m=0}$$

Step scaling function:

$$\sigma(u) = \lim_{a\to 0} \Sigma(u, a/L)$$

$$\beta(\sqrt{\sigma(u)}) = \beta(\sqrt{u})\sqrt{\frac{u}{\sigma(u)}}\sigma'(u)$$