# Light quark physics

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# Introduction

Kaon physics is becoming a precision science, and averages are necessary as input to phenomenology.

I will cover:

- light quark masses
- $K \to \pi\ell\nu$ form factors
- kaon mixing: $B_K$
- $K \to \pi\pi$ matrix elements

# www.latticeaverages.org

New web page with most recent updates for lattice averages based on JL, E Lunghi, R S Van de Water, Phys Rev D 81 034503 (2010) [arXiv:0910.2928].

Includes light and heavy quark physics quantities, mostly weak-matrix elements for flavor physics.
Criteria:

- Only quantities that are documented on the arXiv (including proceedings) or in publications are included. No numbers pulled from slides at various conferences!
- Only quantities that include complete statistical and systematic error budgets are included in the averages.
- Averages only include 3 (or 4) flavor numbers. It is difficult to assess the error due to quenching the strange quark, and the $\sim$percent level precision for some averages is approaching the size that one would expect for this effect.
- Rooted staggered results are included.

Thanks to my collaborators Ruth and Enrico for all of their hard work producing averages!

# Treatment of correlations

We don't have a complete correlation matrix between various lattice calculations. We combine lattice errors with the following assumptions:

- Whenever a source of error is at all correlated between two lattice calculations, we assign the degree-of-correlation a value of $100\%$.
- This assumption is conservative, and will lead to an overestimate in the total error of the averages.
- It still takes better advantage of the available results then assigning the smallest systematic error of any of the individual lattice calculations appearing in the average.

For example, statistical errors of results derived from the same ensemble of configurations are treated as $100\%$ correlated. A perturbative matching calculation between common schemes (for $B_K$) is treated as $100\%$ correlated, since this leads to the same renormalization factor.

# PDG prescription

We adopt the PDG prescription to combine several measurements whose spread is wider than what is expected from the quoted errors.

The error on the average is rescaled by the square root of the minimum of the chi-square per degree of freedom:

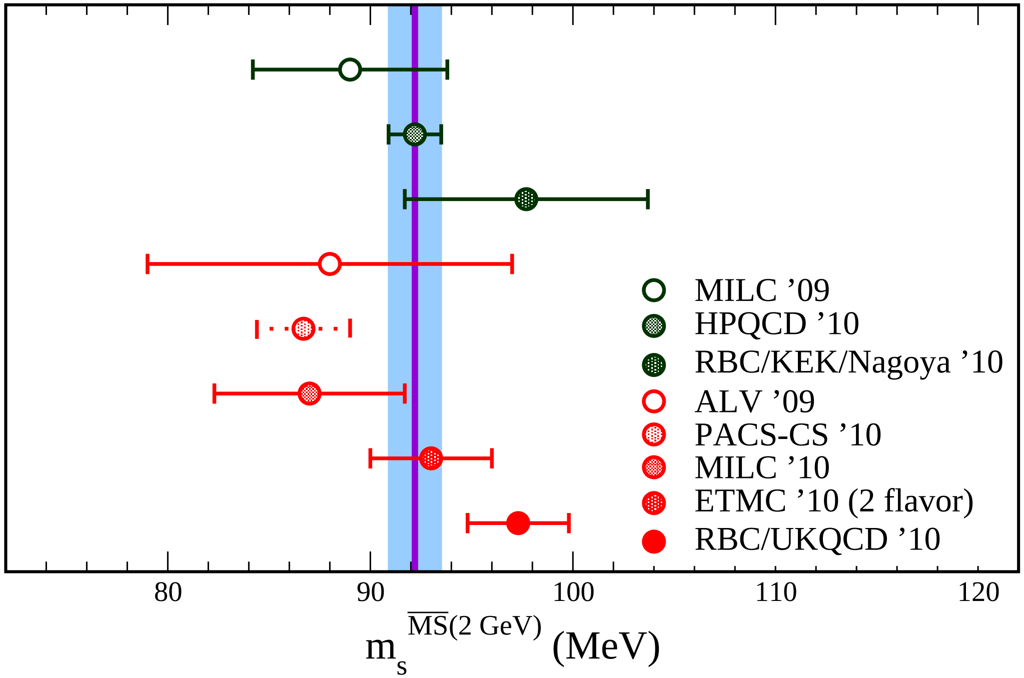
$$\sqrt{\sum (x_i - x_{\rm avg})(C^{-1})_{ij}(x_j - x_{\rm avg})/(n-1)} \qquad (1)$$

# For this talk

Quantities entering the “official” averages are shown in dark green with total error bars. The average is the cyan band.

Quantities not included in averages are shown in red. This includes new, but not yet documented results, and 2-flavor results, for comparison.

Quantities not including a full systematic error budget have dotted error bars.

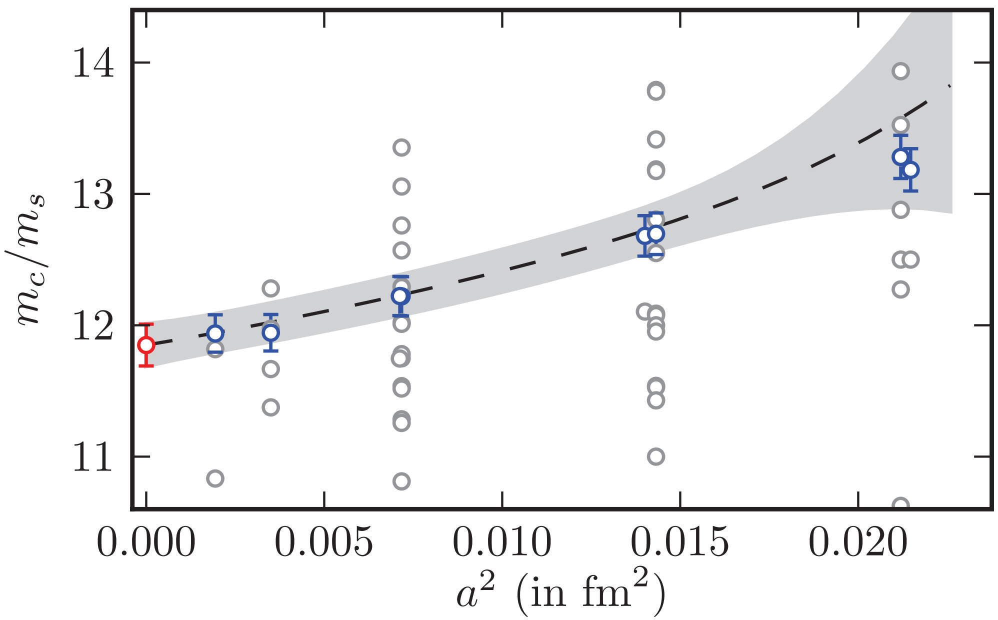


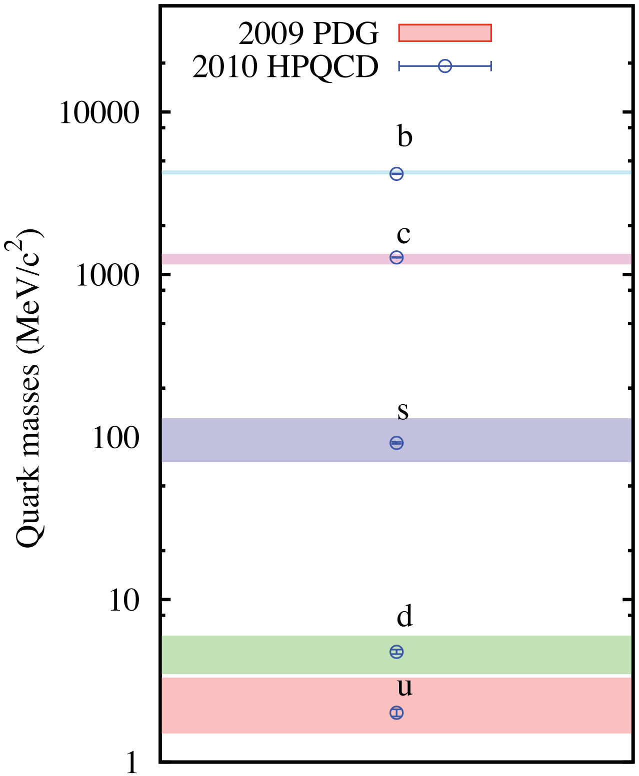
# The simulations

| Group | $N_f$ | action | $a$(fm) | $m_\pi L$ | $m_\pi^{\min}$ (MeV) sea/val |
|---|---|---|---|---|---|
| ETMC | 2 | Twisted Mass | $0.05$-$0.10$ fm | $\gg 1$ | $280/280$ |
| MILC | 2+1 | (Asqtad) staggered | $0.045$-$0.12$ fm | $> 4$ | $250/180$ |
| RBC/UKQCD | 2+1 | Domain Wall | $0.085$-$0.11$ fm | $> 4$ | $290/210$ |
| JLQCD | 2+1 | Overlap | $0.11$ fm | $\geq 2.7$ | $310/310$ |
| PACS-CS | 2+1 | Clover | $0.09$ fm | $\geq 2.0$ | $140/140$ |
| BMW | 2+1 | Clover | $0.065$-$0.125$ fm | $\geq 4$ | $190/190$ |
| ALV | 2+1 | DW on MILC | $0.06$-$0.12$ fm | $> 3.5$ | $250/210$ |
| HPQCD | 2+1 | HISQ on MILC | $0.045$-$0.15$ fm | $\geq 3.7$ | $360/310$ |

In staggered simulations, the sea pion mass quoted is the rms value. The valence pion mass quoted is the taste-goldstone.

# HPQCD




HPQCD uses ratio of $m_c/m_s$ "to cascade the accuracy of the heavy quark mass down to the light quarks." Very fine MILC lattices, down to $0.045$ fm, and the HISQ formalism for valence quarks, allow a precise determination of this ratio.

# RBC/UKQCD

Talk by Chris Kelly

Results at two lattice spacings down to $0.085$ fm.

Now using non-exceptional momenta in NPR: RI/SMOM schemes.

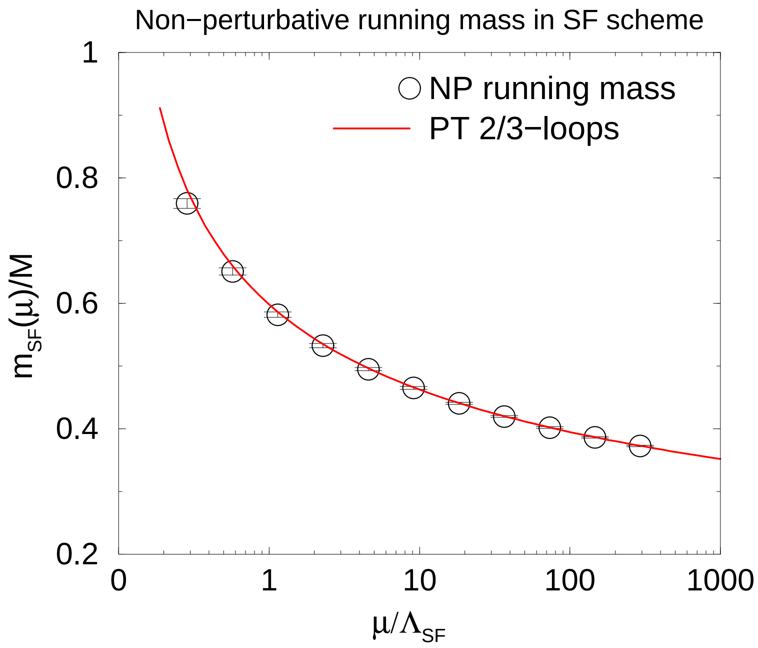
Used volume sources for NPR to improve statistical error.

Investigated several SMOM schemes for better estimate of the truncation error.

$$m_{ud}^{\overline{\text{MS}}}(\mu = 2\ \text{GeV}) = 3.65(20)_{\text{stat}}(13)_{\text{sys}}(8)_{\text{ren}}\ \text{MeV}, \tag{2}$$

$$m_{s}^{\overline{\text{MS}}}(\mu = 2\ \text{GeV}) = 97.3(1.4)_{\text{stat}}(0.2)_{\text{sys}}(2.1)_{\text{ren}}\ \text{MeV}, \tag{3}$$

# PACS-CS



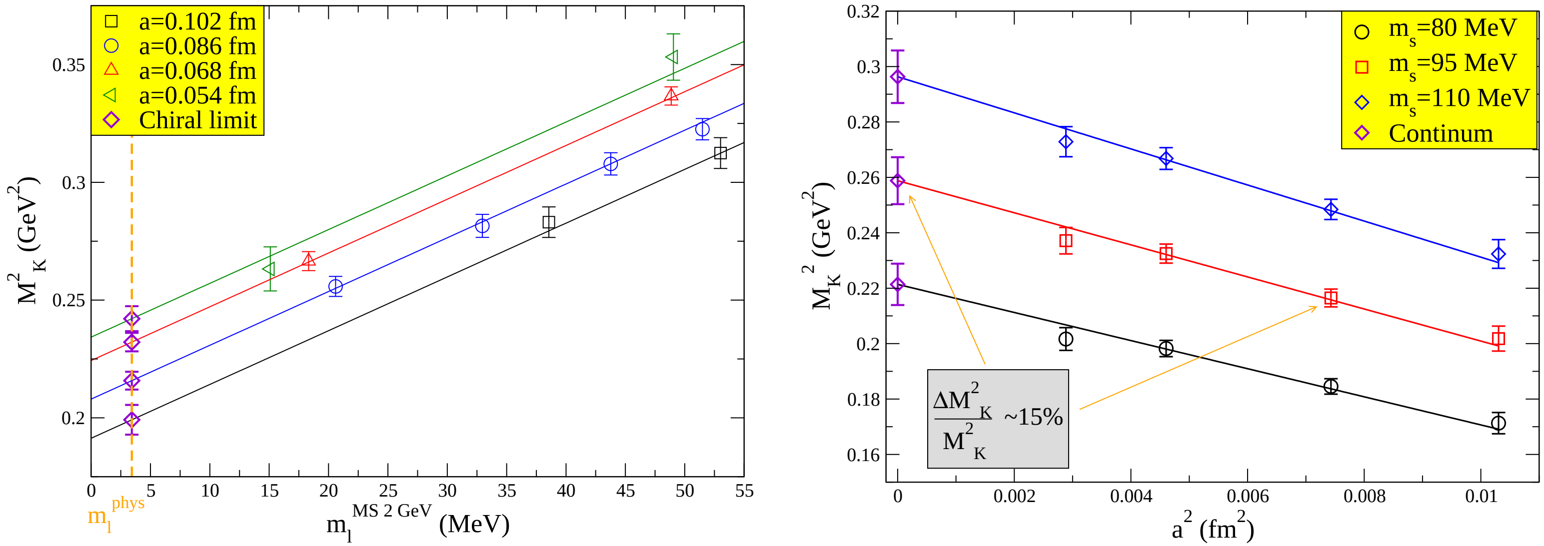

$$m_{ud}^{\overline{\text{MS}}}(\mu = 2\ \text{GeV}) = 2.78(27)\ \text{MeV}, \quad m_s^{\overline{\text{MS}}}(\mu = 2\ \text{GeV}) = 86.7(2.3)\ \text{MeV},$$

Errors include: statistical errors, systematic error due to reweighting to the physical quark masses, and systematic error in $Z_m$.

Errors do not include: continuum extrapolation, finite volume effects.

# ETMC

Talk by Francesco Sanfilippo




Various chiral extrapolation formulas are tried, $SU(2)$, $SU(3)$.
The quark masses are extracted from chiral/continuum extrapolated $m_K$, $m_\pi$ and also $m_{s\bar{s}}$.
Four lattice spacings are used, and non-perturbative matching to the RI-MOM scheme is performed.

# MILC

Update over previous year's analysis with complete Asqtad ensemble set. (Talk by Claude Bernard)

Two-loop $SU(3)$ chiral perturbation theory to perform extrapolations. Cross-checked with two-loop $SU(2)$ chiral perturbation theory. Staggered effects included through one-loop.

Only finest three lattice spacings are used ($0.09$-$0.045$ fm) so that one-loop staggering effects appear at same order in power counting as two-loop continuum terms.

$$m_{ud}^{\overline{\text{MS}}}(\mu = 2\text{ GeV}) = 3.17(1)(7)(16)(0)\text{ MeV}, \quad m_s^{\overline{\text{MS}}}(\mu = 2\text{ GeV}) = 87.0(2)(15)(44)(1)\text{ MeV},$$

where errors are: statistical, lattice systematic, perturbative matching, E+M.

$$m_s/m_{ud} = 27.46(4)(16)(4), \quad m_u/m_d = 0.432(1)(7)(39),$$

where errors are: statistical, lattice systematic, E+M.

# RBC/KEK/Nagoya

Included non-compact quenched QED. (Talk by Taku Izubuchi)

Used $SU(2)$ heavy kaon chiral perturbation theory, including electromagnetic effects.

Two volumes, one lattice spacing.

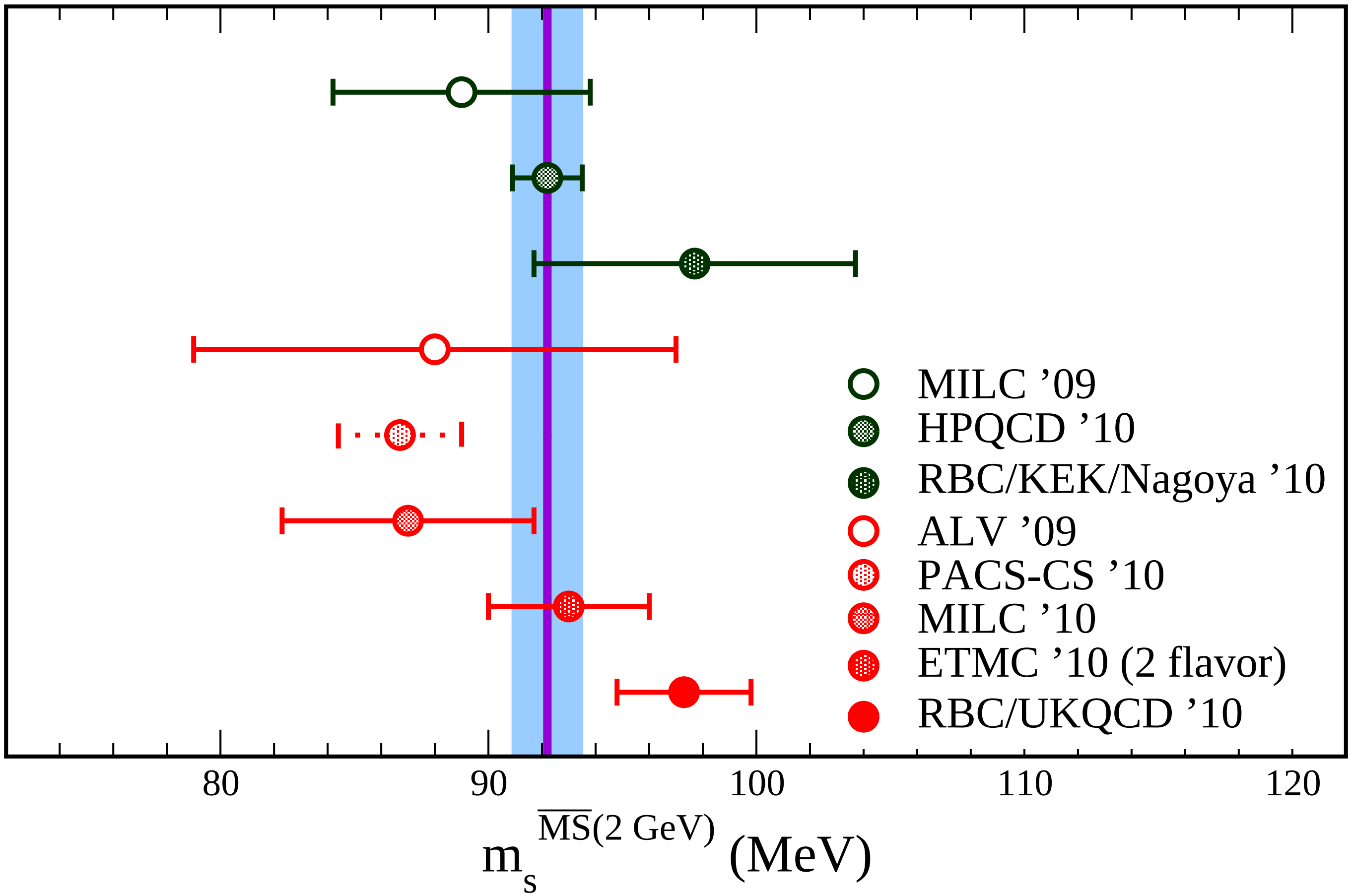
$Z_m$ is taken from RBC/UKQCD calculation using non-exceptional momenta in the RI/SMOM scheme.

$$m_{ud}^{\overline{\text{MS}}}(\mu = 2 \text{ GeV}) = 3.44 \pm 0.12 \pm 0.24 \text{ MeV}, \quad m_s^{\overline{\text{MS}}}(\mu = 2 \text{ GeV}) = 97.7 \pm 2.9 \pm 5.2 \text{ MeV},$$

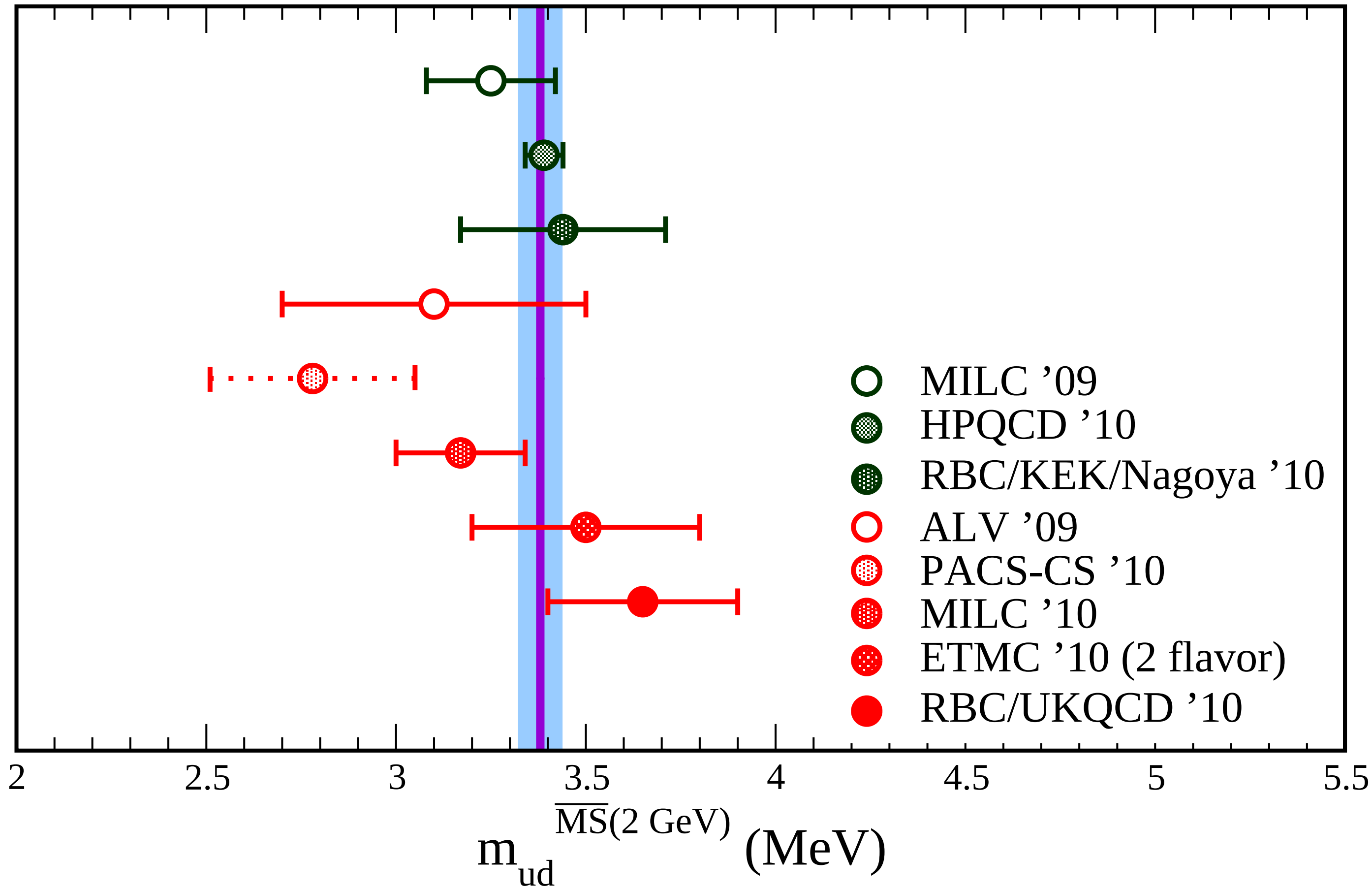
$$m_s/m_{ud} = 28.34 \pm 0.28 \pm 1.61, \quad m_u/m_d = 0.5238 \pm 0.0093 \pm 0.0533.$$

where errors are: statistical, total systematic.

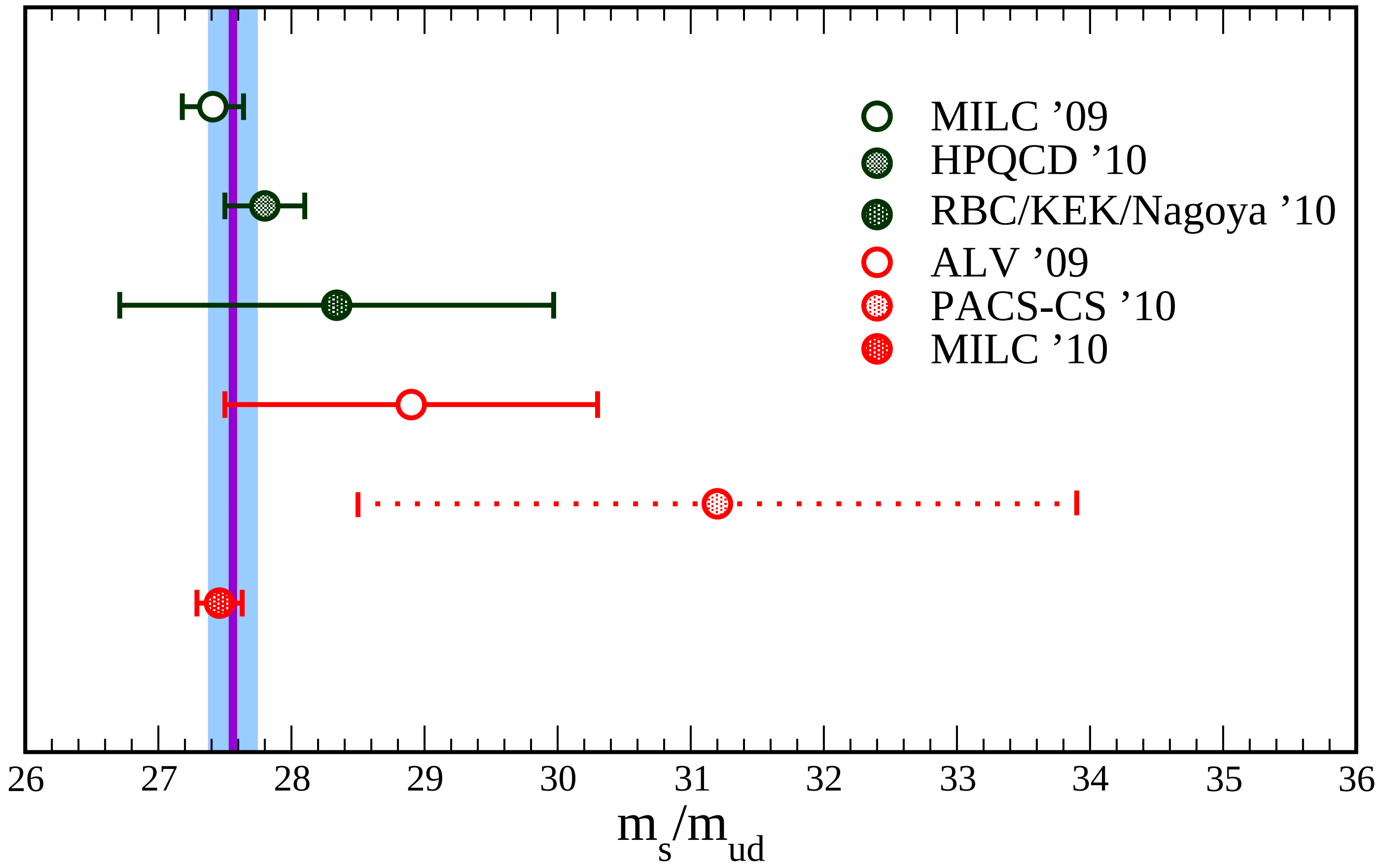
# Strange quark mass



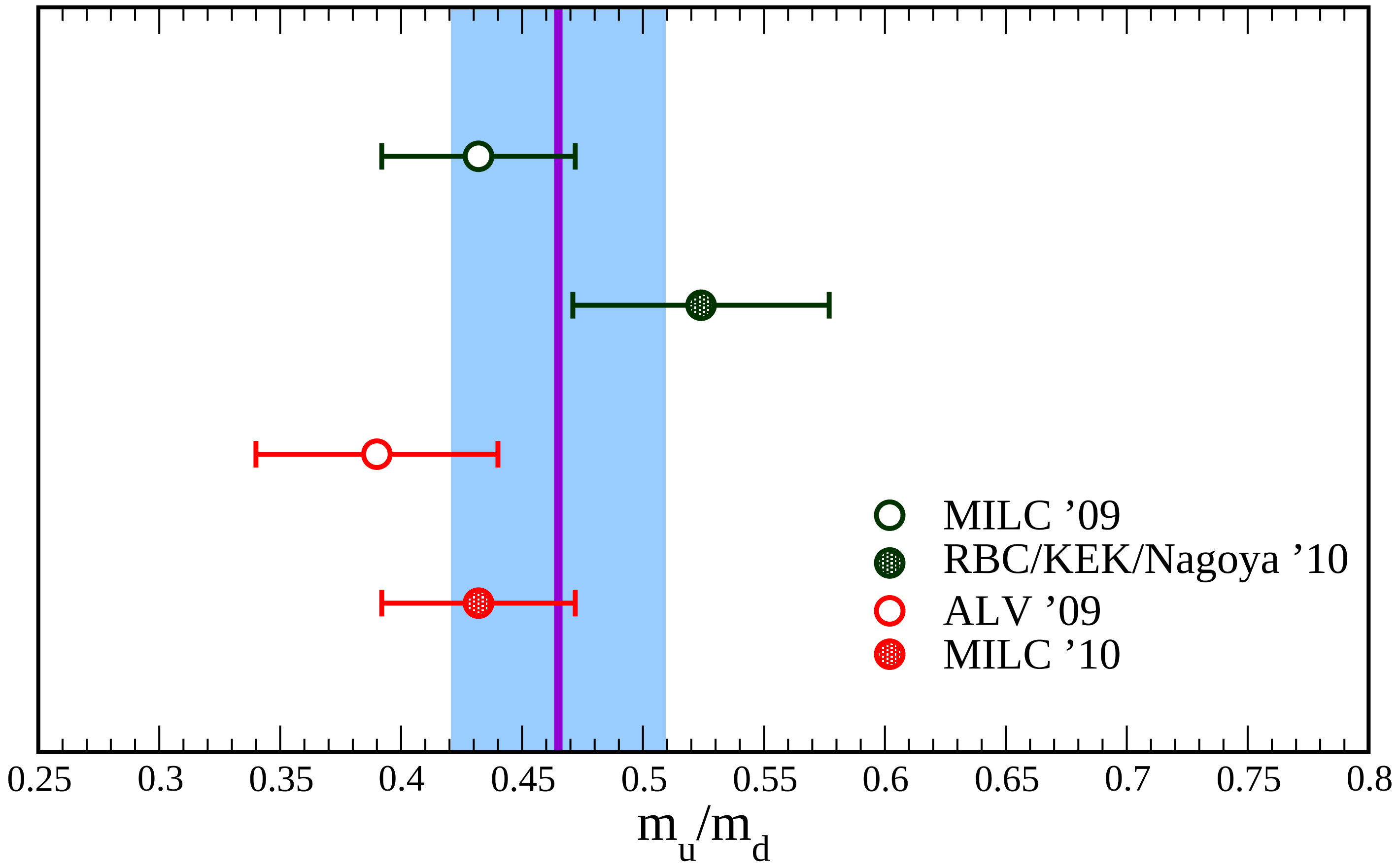
# Light-quark mass



# Quark mass ratio

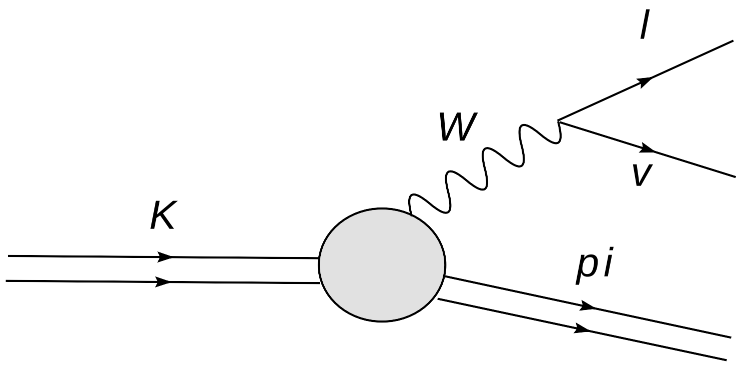


# Light quark mass ratio




Errors inflated by $\sim 1.4$ due to somewhat low confidence level. Still $\sim 10\sigma$ from zero.

# $K \to \pi \ell \nu$




$$\Gamma_{K\ell 3} = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{\mathrm{EW}} (|V_{us}| f_+^{K^0\pi^-}(0))^2 I_{K\ell}(1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU(2)}}^{K\pi})^2, \quad (4)$$

where $S_{EW} = 1.0232(3)$ is the short-distance electroweak correction, $C_K$ is a Clebsch-Gordan coefficient, $f_+^{K^0\pi^-}(0)$ is the form factor at zero momentum transfer, and $I_{K\ell}$ is a phase-space integral that is sensitive to the momentum dependence of the form factors. The quantities $\delta_{\mathrm{EM}}^{K\ell}$ and $\delta_{\mathrm{SU(2)}}^{K\pi}$ are long-distance EM corrections and isospin breaking corrections, respectively.

# RBC/UKQCD

New calculation with twisted boundary conditions to directly simulate at $q^2 = 0$ (arXiv:1004.0886).

No interpolation in momentum transfer necessary.

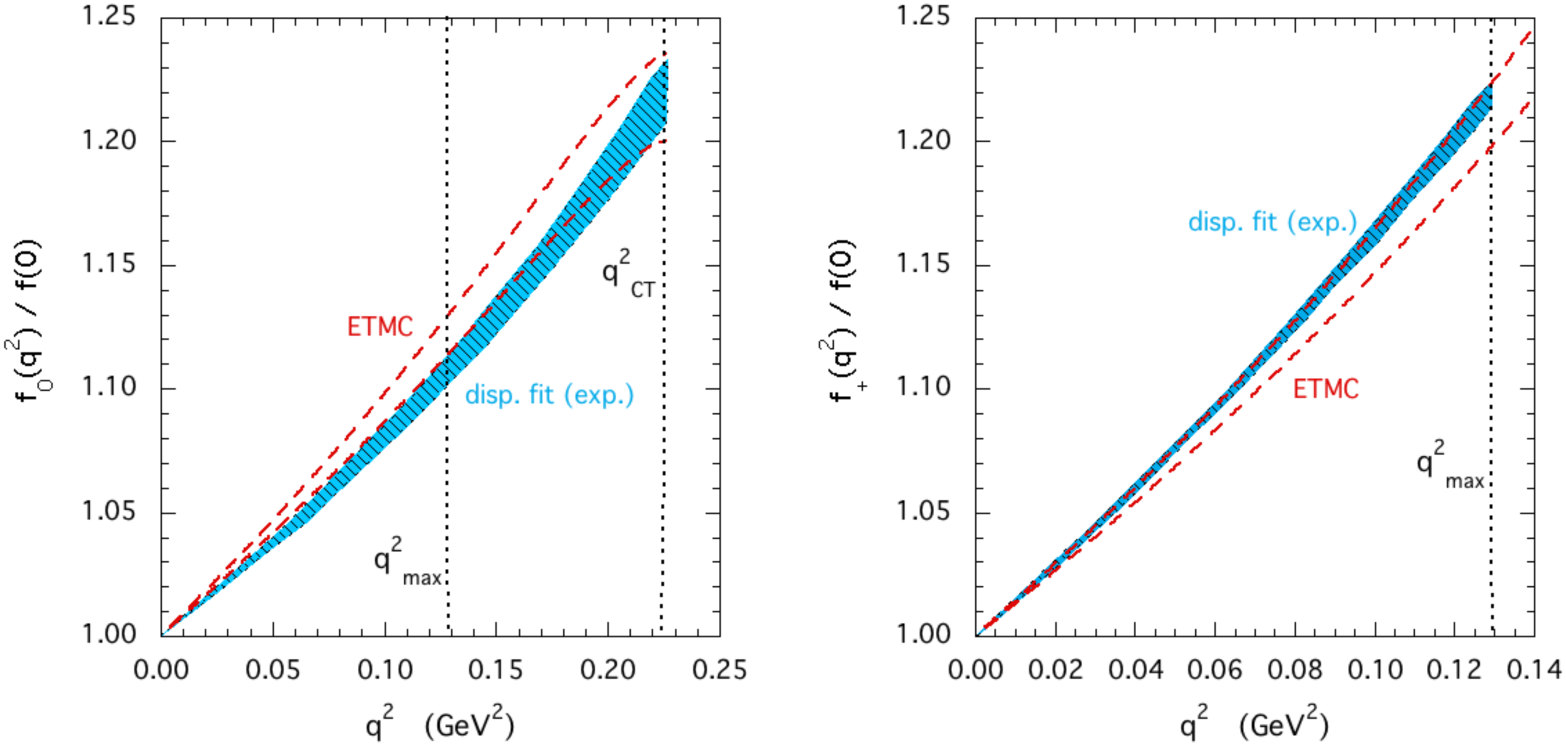
New estimation of chiral extrapolation errors, and slightly different choice of extrapolation method, where (analytic) NNLO terms that do not obey the Ademollo-Gatto theorem are included. Such terms are possible if $f_\pi$ is used in the NLO expression instead of $f_0$. (When reordering the series to use $f_\pi$ at NLO, analytic terms that do not respect the mass interchange symmetry of AG can appear at NNLO.) RBC/UKQCD quote:

$$f_+^{K\pi}(0) = 0.9599(34)(^{+31}_{-43})(14) \tag{5}$$

where the first error is statistical, the second is due to the chiral extrapolation, and the third is an estimate of discretization effects.

# ETMC

Talk by Lorenzo Orifici




Cyan bands include complete error budgets for form factor shape dependence, as compared to experiment. Note that the calculation here is 2 flavor, but there is an estimate for the error due to quenching the strange quark using $\chi$PT through NLO.

# FNAL/MILC

No preliminary results yet, but work is in progress. (Talk by Elvira Gamiz)

Uses method developed by HPQCD for $D \to K\ell\nu$ to get result for $f_+^{K\pi}(0)$

$$f_+(0) = f_0(0) = \frac{m_s - m_q}{m_K^2 - m_\pi^2} \langle \pi | S | K \rangle |_{q^2=0} \tag{6}$$

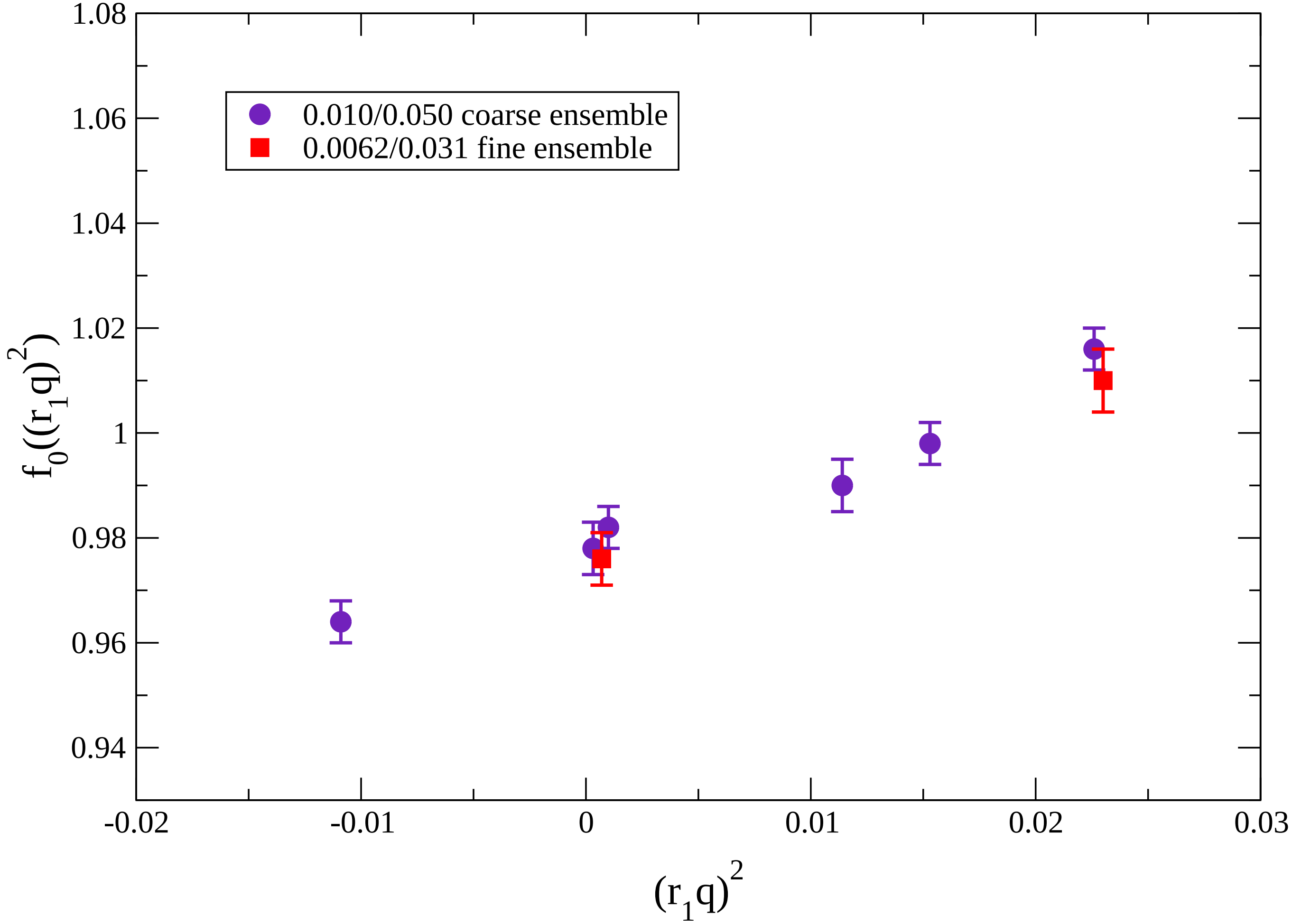
No renormalization is required.

Avoids the use of non-local vector currents. It does not require multiple three-point correlators to form various double ratios.

Disadvantage: One only gets $f_0(q^2)$ for $q^2 \neq 0$, but this is still sufficient to determine $|V_{us}|$.

# Early look

Comparison of coarse (a=0.12 fm) and fine (a=0.09 fm) ensembles



# JLQCD

Talk by Takashi Kaneko

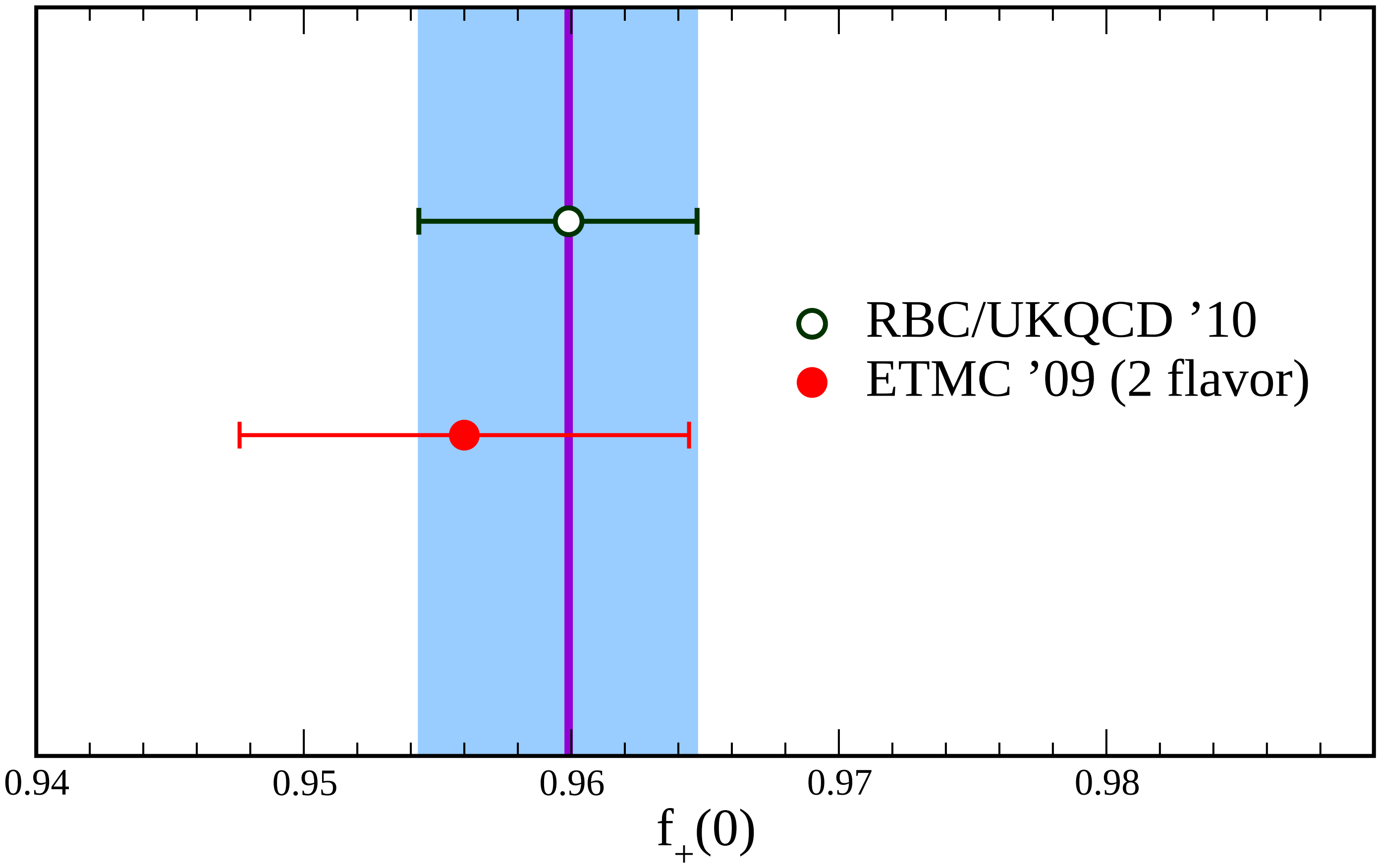
$2+1$ flavor overlap calculation with one lattice spacing and somewhat small ($1.7$ fm) volume.

The $q^2$ dependence is modeled using polynomial, free-pole term, free-pole +polynomial in order to interpolate to $q^2 = 0$.

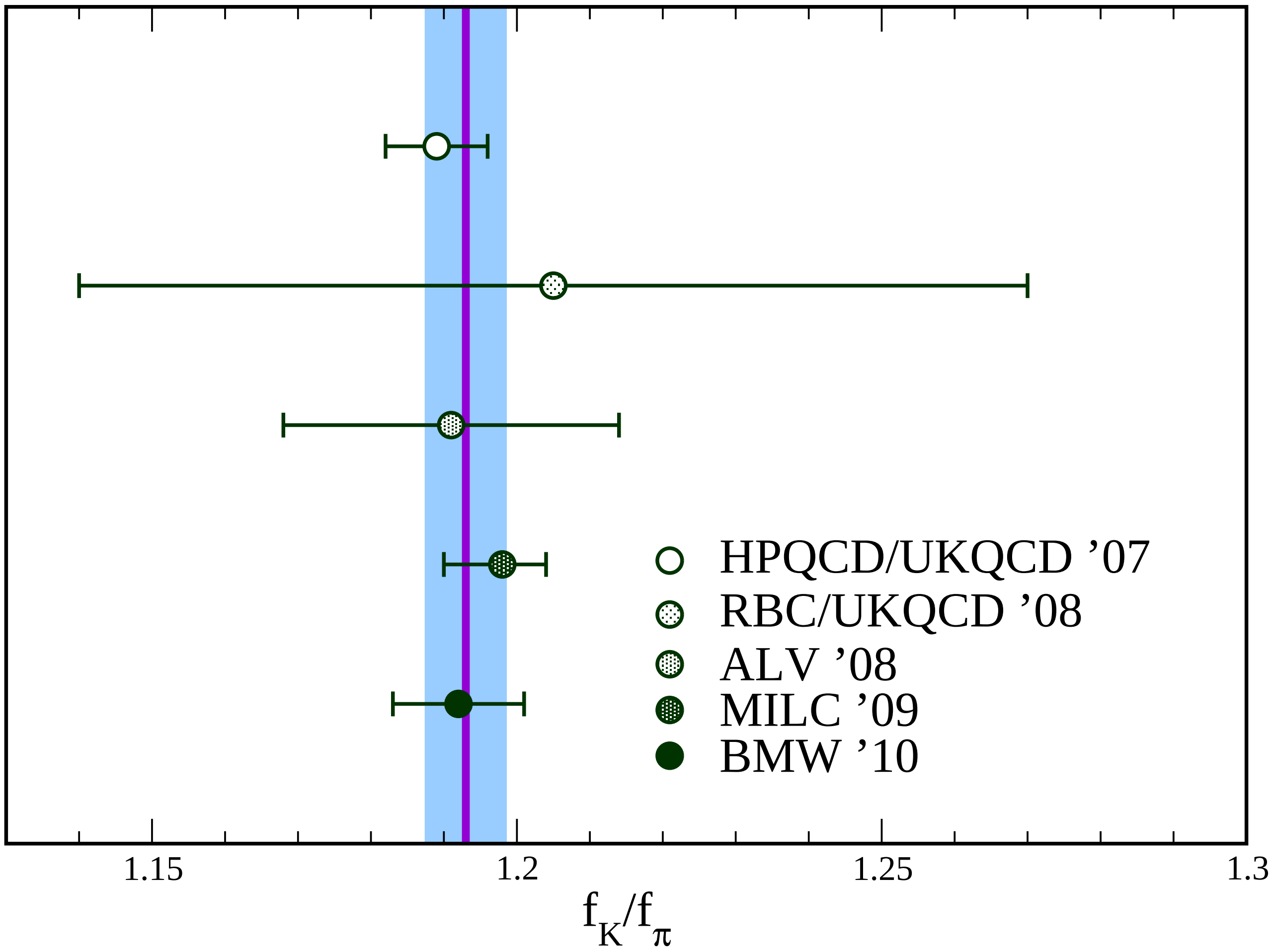
Curvature terms in form factor shape dependence reasonably consistent with experiment.

Work with twisted boundary conditions and larger volumes is in progress.

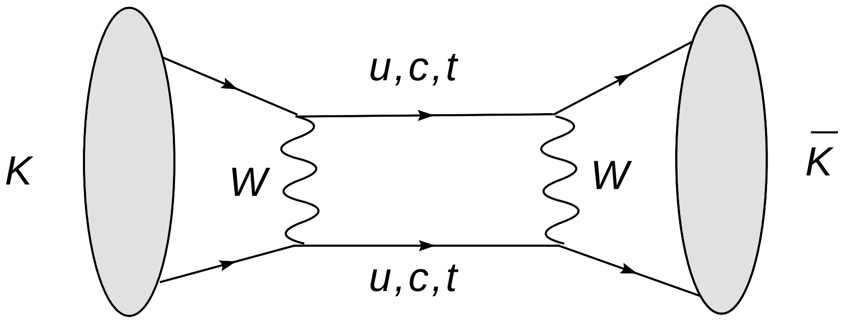
# $K \to \pi \ell \nu$



# $f_K/f_\pi$


HPQCD/UKQCD ’07
RBC/UKQCD ’08
ALV ’08
MILC ’09
BMW ’10
1.15
1.2
1.25
1.3
$f_K/f_\pi$

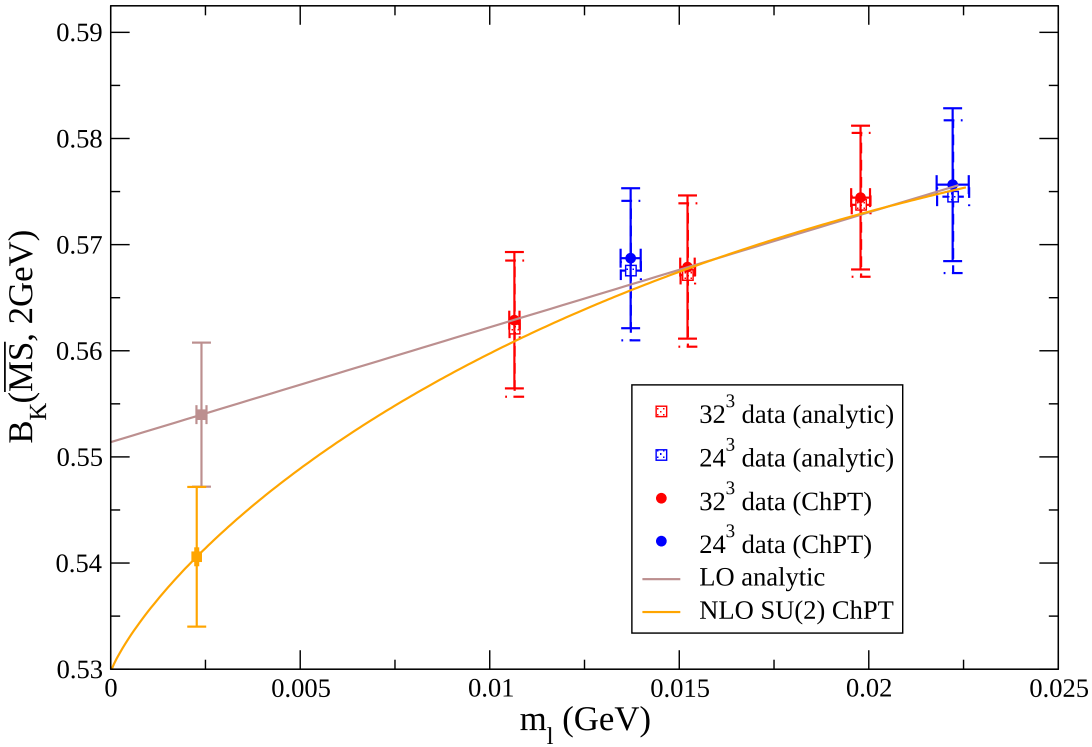
# $B_K$




$$|\epsilon_K| = C_\epsilon \kappa_\epsilon B_K A^2 \overline{\eta} \{-\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1-\overline{\rho})\}$$

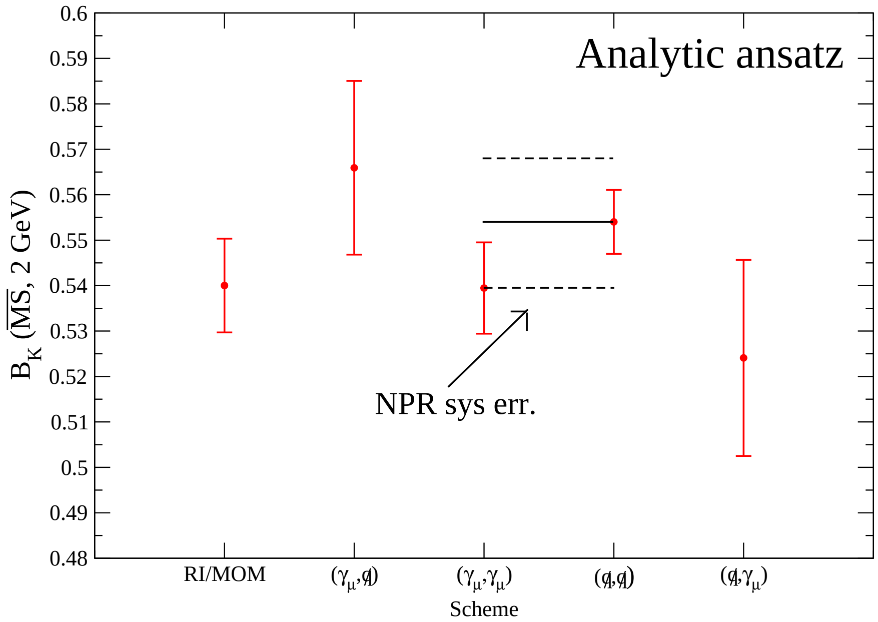
where $C_\epsilon$ is a collection of experimentally determined parameters, $\kappa_\epsilon$ represents long-distance corrections and a correction due to the fact that $\phi_\epsilon \neq 45$ degrees, the $\eta_i S_0$ are perturbative coefficients, the terms in blue are CKM matrix elements in Wolfenstein parameterization.

# RBC/UKQCD




New treatment of chiral extrapolation, where $SU(2)$ chiral extrapolation result is averaged with linear extrapolation result. Motivated by absence of curvature in lattice data, and the tendency for the $SU(2)$ fit to undershoot $f_\pi$. (Talk by Chris Kelly)

# RBC/UKQCD




Multiple RI-SMOM schemes with non-exceptional momenta are used to determine the matching factor. Different schemes have different one-loop truncation errors, so the perturbative matching error is reduced by taking an average over results from different schemes.

$$B_K(\overline{MS}, 2\ \text{GeV}) = 0.546(7)(16)(3)(14) \tag{7}$$

where errors are: statistical, chiral extrapolation, finite volume, renormalization.

# ETMC

Uses a mixed-action (Frezzotti-Rossi, JHEP 2004) at three lattice spacings down to $0.07$ fm. (Talk by Petros Dimopoulos)

Valence action is Osterwalder-Seiler on the $N_f = 2$ ETMC configurations.

$$\widehat{B}_K = 0.733(29)(16) \tag{8}$$

where the first error contains statistics, chiral extrapolation/fit and matching, and the second error is due to the different assumptions of $\mathcal{O}(a^2p^2)$ dependence in the RI-MOM scheme matching factor.

Additional 4 bag parameters needed for beyond Standard Model operators were computed with errors $5-10\%$. First calculation beyond $N_f = 0$.

Calculation with $2+1+1$ flavors has already started.

# SBW

Mixed action approach (posters by Taegil Bae, Jangho Kim, Yong-Chull Jang, Boram Yoon):

HYP-smeared staggered fermions on MILC Asqtad lattices using 4 lattice spacings down to $0.045$ fm.

One-loop perturbative matching.

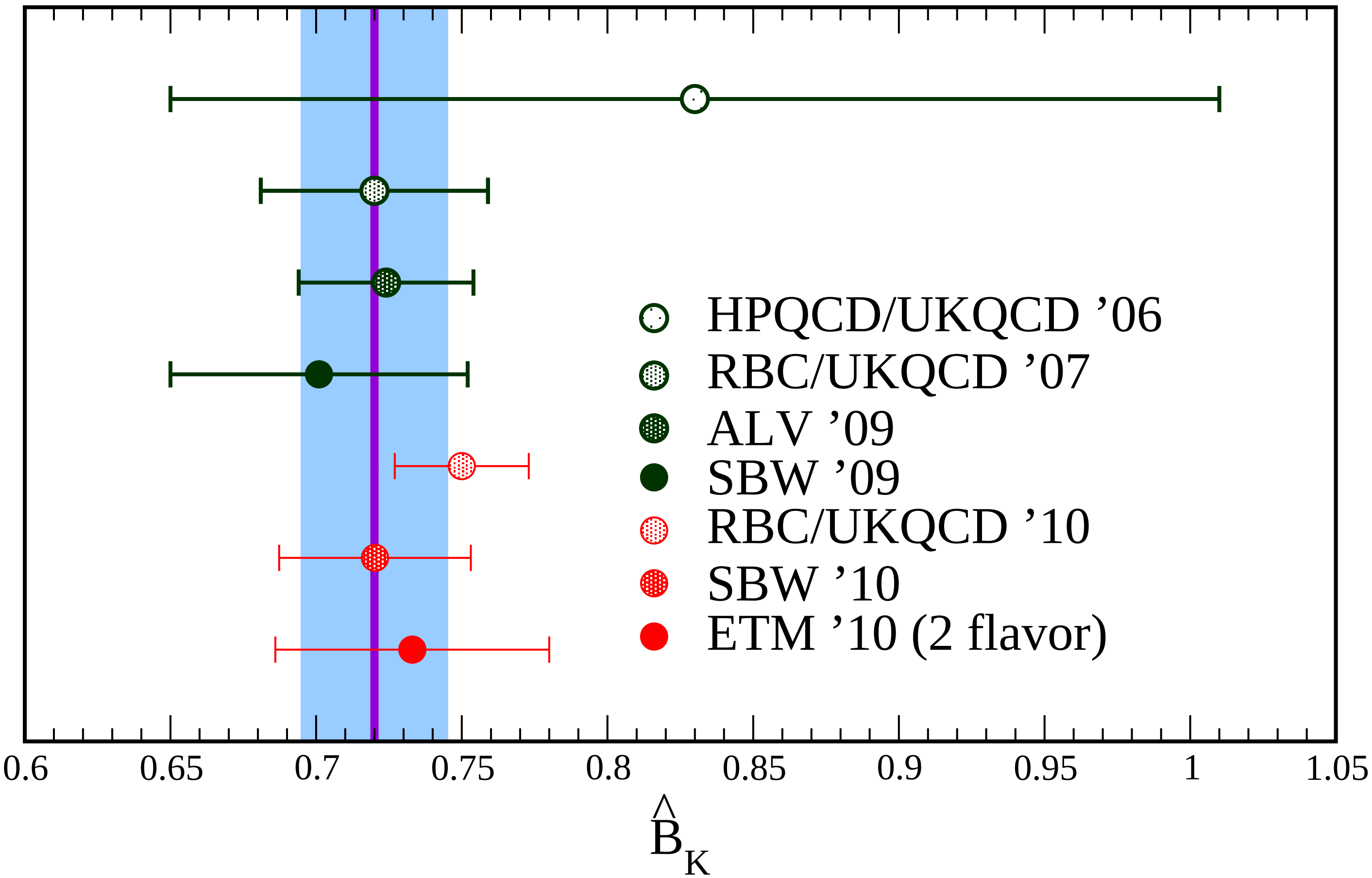
$SU(2)$ chiral perturbation theory is used. This provides much simpler extrapolation formulas than in the $SU(3)$ case, where many new staggered parameters enter.

Preliminary results:

$$\widehat{B}_K = 0.720(10)(33) \tag{9}$$

where errors are statistical and the sum of systematic errors in quadrature.

# $B_K$


HPQCD/UKQCD '06
RBC/UKQCD '07
ALV '09
SBW '09
RBC/UKQCD '10
SBW '10
ETM '10 (2 flavor)
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1
1.05
$\hat{B}_K$

# Approaches to $K \to \pi\pi$ matrix elements

Maiani-Testa no-go theorem tells us that we cannot extract physical matrix elements from Euclidean correlation functions with multi-hadron states.

1) Difficulties simulating at physical kinematics for $K \to \pi\pi$ matrix elements avoided by using Lellouch-Lüscher finite volume method. This is still costly. Most straightforward implementation requires a large (6 fm) box, momentum insertion, and physical light quark masses.

2) The indirect method constructs $K \to \pi\pi$ matrix elements using the low energy constants of $\chi$PT obtained from calculating simpler quantities (like $K \to 0$ and $K \to \pi$) on the lattice. Shown in hep-lat/0306035 that all LEC's through next-to-leading order could be obtained from "simple" lattice quantities. Concerns about the slow convergence of the chiral expansion at the physical kaon mass.

# Different approach

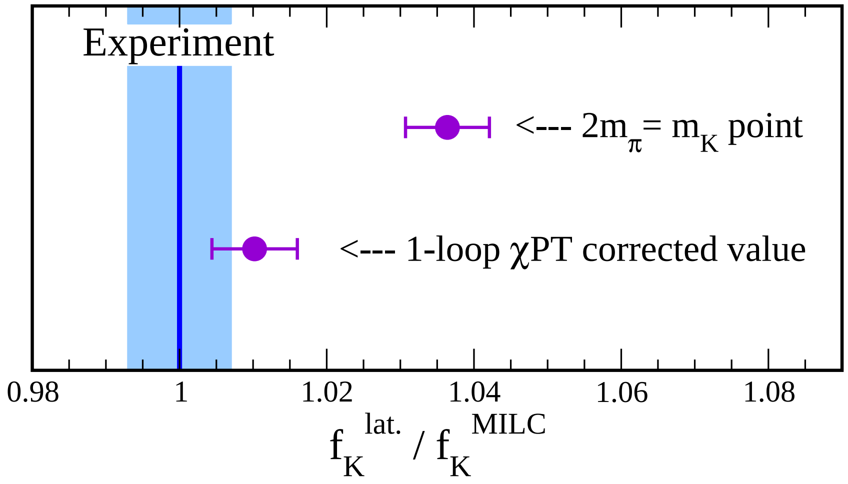
Poster by Dan Coumbe, JL, Matthew Lightman, Ruth Van de Water

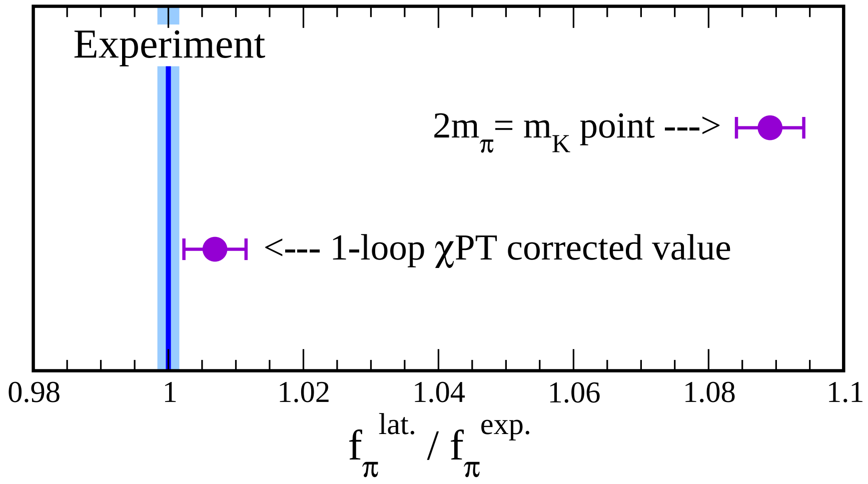
Bypass Maiani-Testa theorem by simulating with both pions at rest. Set the quark masses so that $m_K = m_K^{\rm phys}$ and $m_\pi = 1/2 m_K^{\rm phys}$. This requires an interpolation in quark mass (plus an extrapolation to the continuum).

Then correct this unphysical kinematics point (the "2-$\pi$" point) using fixed order $SU(3)$ $\chi$PT. The low energy constants needed for this correction can be obtained from simpler quantities, like $f_K$, $K - \overline{K}$ and $K \to \pi$.

Since the kaon is tuned to its physical value, terms involving only kaons are correct to all orders in the $SU(3)$ chiral expansion. $10$-$30\%$ precision of NLO $SU(3)$ $\chi$PT now appears in small correction factor, rather than entire amplitude.

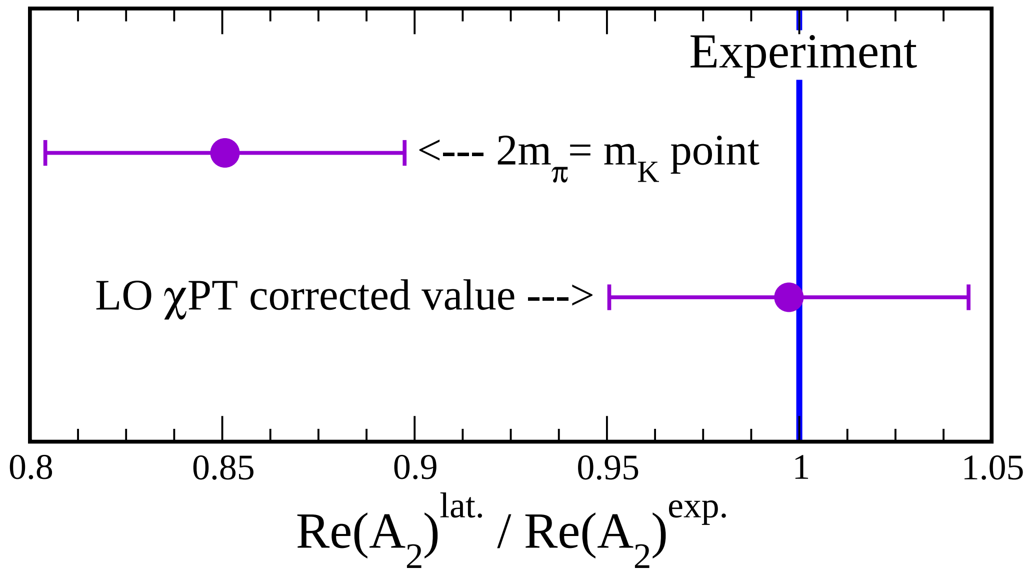
# Proof of concept: $f_{\pi,K}$



# Proof of concept for Re$(A_2)$

$$\langle \pi^+\pi^- | O^{(27,1),(3/2)} | K^0 \rangle_{\mathrm{LO}} = -\frac{4i\alpha_{27}}{f_K f_\pi^2}(m_K^2 - m_\pi^2), \tag{10}$$



# Estimated error budget for Re$(A_2)$

| uncertainty | Re$(A_2)$ |
|---|---|
| statistics | 4.7% |
| continuum extrapolation | 4% |
| chiral truncation | 9% |
| uncertainties in leading order LEC's | 4% |
| finite volume errors | few percent |
| matching factor | 3.4% |
| scale uncertainty | 3% |
| Wilson coefficient | few percent |
| total | less than $20\%$ |

Will improve significantly with 1-loop $\chi$PT correction factor and full data set.

# RBC/UKQCD

Direct approach of Lellouch-Lüscher. (talk by Matthew Lightman)

Calculation on $32^3 \times 64 \times 32$ (DSDR) domain wall fermion ensembles, with $a^{-1} = 1.4$ GeV and $4.5$ fm box.

To give the pions momentum without having to fit excited states, twisted boundary conditions are used (Kim and Christ, Lattice 2002 [hep-lat/0210003], Sachrajda and Villadoro hep-lat/0411033).

# RBC/UKQCD Re$(A_2)$

| uncertainty | Re$(A_2)$ |
|---|---|
| statistics | $4\%$ |
| finite lattice spacing | $15\%$ |
| finite volume errors | $4\%$ |
| Masses not physical | ? |
| Partial quenching effect | $2\%$ |
| operator renormalization | few percent |
| Wilson coefficient | few percent |
| total | $\sim 15\%$ |

Similar errors expected for Im$(A_2)$.

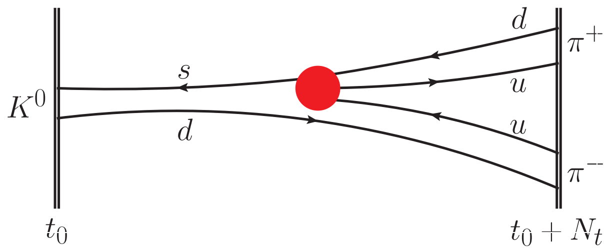
# Complications of $\Delta I = 1/2$ channel

1) Power divergences. These can be handled by a vacuum subtraction, as shown by RBC in the quenched approximation.

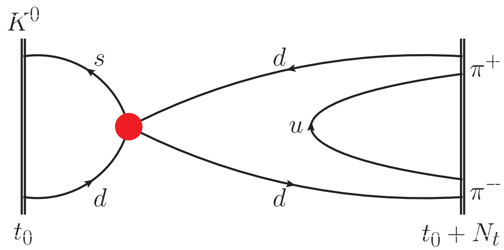
2) Enhanced finite volume effects. Can be controlled by using the unitary points (requires a not-so-severe tuning of the valence pion mass in the mixed-action case).

3) Disconnected graph. Requires brute force computing. Contributes at NLO in the chiral expansion, so nominally sub-leading.

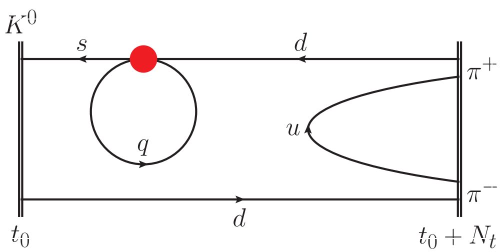
# Diagrams for $\Delta I = 1/2$ channel



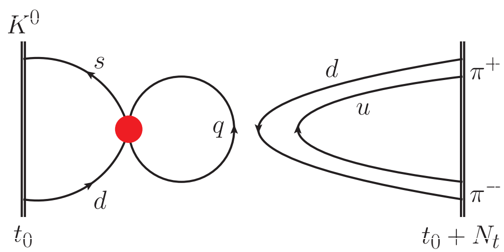

(a)




(b)



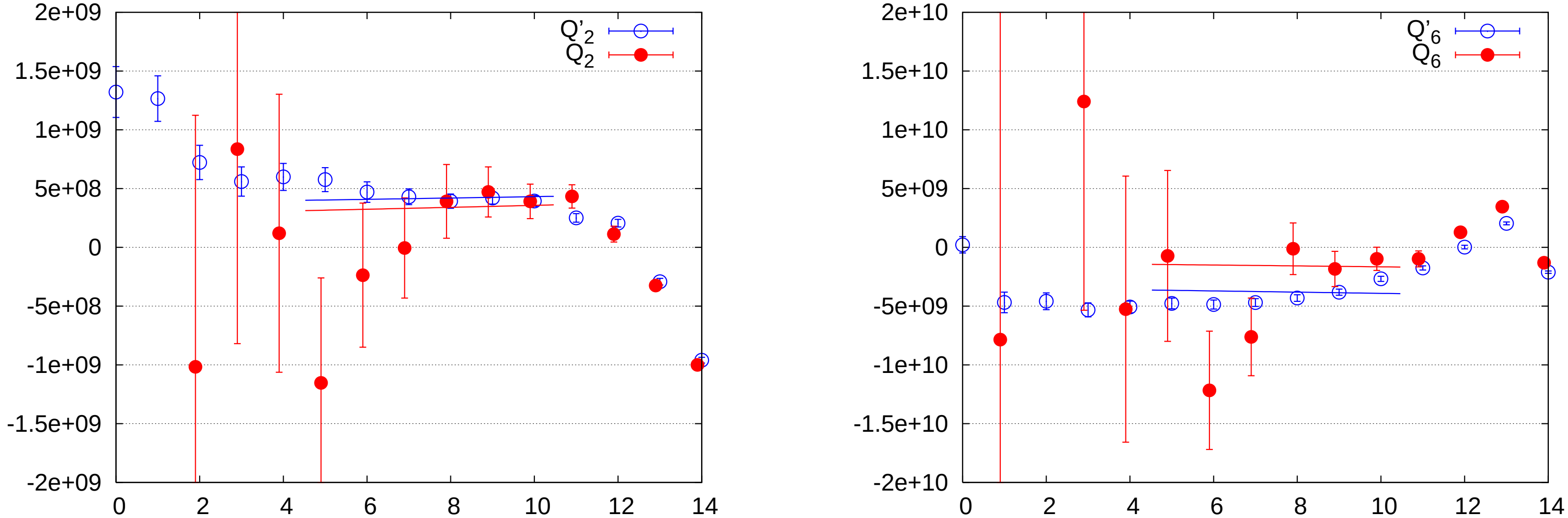

(c)




(d)

# Signals for $K \to \pi\pi$

Talk by Qi Liu




Signals for $K \to \pi\pi$ matrix elements at zero momentum for $Q_2$ [relevant for Re$(A_0)$] and $Q_6$ [relevant for Im$(A_0)$]. Filled symbols include disconnected diagrams and open symbols do not. Propagators were inverted on each time slice ($T = 32$) for 400 configurations.

# Results

Results at unphysical pion mass $m_\pi = 420$ MeV and zero momentum:

| Re$(A_0)_{\rm no\ discon}$ | Re$(A_0)$ | Im$(A_0)_{\rm no\ discon}$ | Im$(A_0)$ |
|---|---|---|---|
| $38.7(2.1) \times 10^{-8}$ | $30(8) \times 10^{-8}$ | $-63.1(5.3) \times 10^{-12}$ | $-29(22) \times 10^{-12}$ |

At non-zero momentum, barely a signal, even without fully disconnected diagram.

Improvements expected by going to larger volumes, better inverter algorithms (?), and a much bigger machine!

# Conclusions

Quark masses from many different formulations and renormalization methods are converging after many years.

Simplest kaon physics quantities are now precision calculations, and results are in good agreement. Averages are necessary!

Difficult quantities like $K \to \pi\pi$ in the $\Delta I = 3/2$ channel are now within reach. The $\Delta I = 1/2$ channel presents a greater difficulty, but is likely attainable in the next few years.

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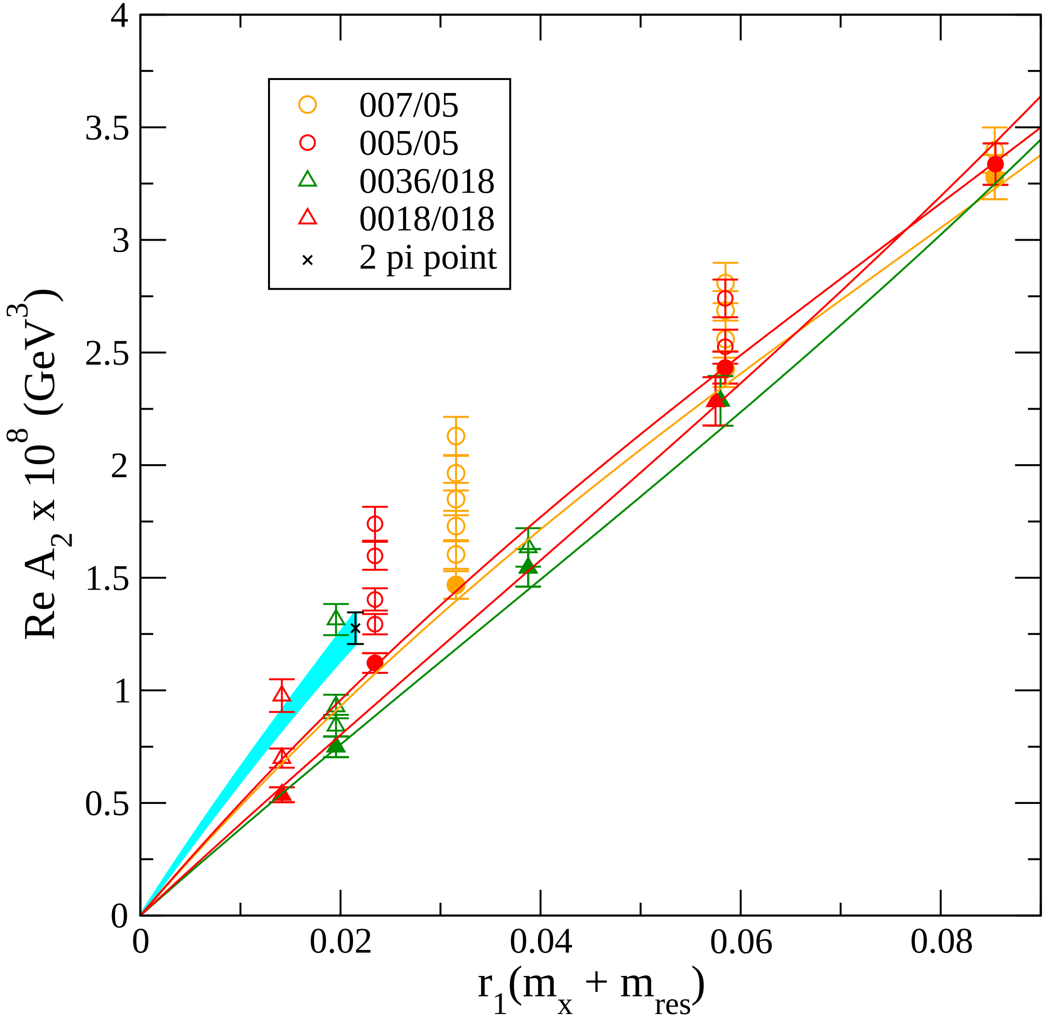
Thank you for staying to the end, and have a safe journey!

# Backup Slides

# $K \to \pi\pi$, $\Delta I = 3/2$, $(27, 1)$



# Motivation

$2.4\sigma$ tension between lattice $B_K$ value and preferred value from CKM fit with $B_K$ omitted: $\widehat{B}_K = 0.725 \pm 0.027$ versus $(\widehat{B}_K)_{\rm fit} = 0.98 \pm 0.10$.