Heavy flavour dynamics from lattice QCD

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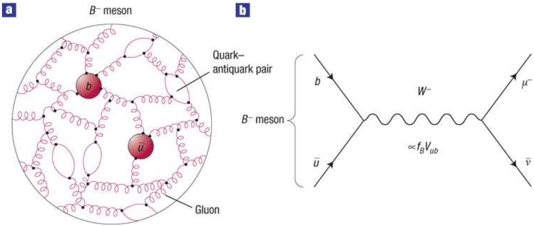
(Lattice) QCD and the weak interaction

New Physics effects expected in the quark flavour sector, because most extensions of the Standard Model contain

- new CP-violating phases
- new quark flavour-changing interactions



Changes of quark flavour inside a hadron are weak interaction processes

$\longrightarrow$ Due to confinement, QCD corrections to the decay rate are significant

$\longrightarrow$ Non-perturbative QCD effects typically absorbed into hadronic matrix elements such as decay constants, form factors and bag parameters

$\Rightarrow$ A task for lattice QCD

The CKM matrix . . .

. . . encodes the mixing between quark flavours under weak interactions

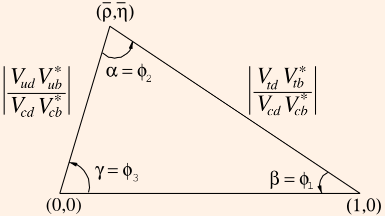
$$\underbrace{\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}}_{\text{weak int.}} = V_{\mathrm{CKM}} \underbrace{\begin{pmatrix} d \\ s \\ b \end{pmatrix}}_{\text{strong int.}} \qquad V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Wolfenstein parametrization of the CKM matrix

- Empirically, matrix elements are largest among the diagonal
  → hierarchy gets explicit by expansion in powers of $|V_{us}| = \lambda \simeq 0.22$
- $\exists$ unitarity relations such as $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$
  → V_{CKM} represented as unitarity triangle in the complex (ρ, η)–plane

up to $O(\lambda^4)$:

$$\begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



Impact of LQCD on precision heavy flavour physics

Heavy quark sector constrains UT: angles & sides are related to hadronic matrix elements of $\mathcal{H}_{\text{weak}}^{(\text{eff})}$, corresponding to mesonic decays/transitions

$$\Delta m_d \propto F_{B_d}^2 \widehat{B}_{B_d} |V_{td} V_{tb}^*|^2 \qquad \frac{\Delta m_s}{\Delta m_d} = \xi^2 \frac{m_{B_s}}{m_{B_d}} \frac{|V_{ts}|^2}{|V_{td}|^2} \qquad \xi = F_{B_s}\sqrt{\widehat{B}_{B_s}} \Big/ F_{B_d}\sqrt{\widehat{B}_{B_d}}$$

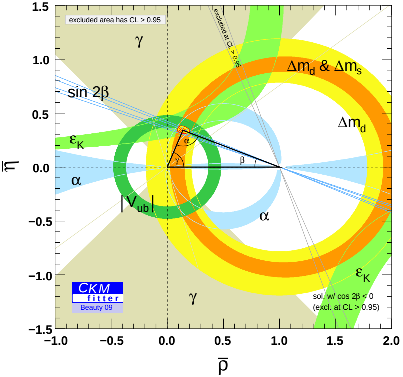
- $\exists$ large number of experimental data from heavy flavour-factories (CLEO, BaBar, Belle, LHCb, ...)
- Inputs of theory and predominantly LQCD computations needed to
  - interpret results of experimental measurements
  - determine / pin down heavy quark masses & CKM matrix elements
  - overconstrain unitarity relations $\leftrightarrow$ unveiling New Physics effects

$\mathbf{V_{ud}}$ $\pi \to \ell\nu$	$\mathbf{V_{us}}$ $K \to \ell\nu$ $K \to \pi\ell\nu$	$\mathbf{V_{ub}}$ $B \to \pi\ell\nu$
$\mathbf{V_{cd}}$ $D \to \ell\nu$ $D \to \pi\ell\nu$	$\mathbf{V_{cs}}$ $D_s \to \ell\nu$ $D \to K\ell\nu$	$\mathbf{V_{cb}}$ $B \to D\ell\nu$ $B \to D^*\ell\nu$
$\mathbf{V_{td}}$ $B_d \leftrightarrow \overline{B}_d$	$\mathbf{V_{ts}}$ $B_s \leftrightarrow \overline{B}_s$	$\mathbf{V_{tb}}$

"Gold-plated" lattice processes

- 1 hadron in the initial state, 0 or 1 hadron in the final state
- stable hadrons (or narrow, far from theshold)
- controlled χ-extrapolation

Impact of LQCD on precision heavy flavour physics



- Constrain apex $(\bar{\rho}, \bar{\eta})$ as precisely as possible by independent processes
- Theory & Exp. sufficiently precise
  $\Rightarrow$ New Physics $=$ inconsistent $(\bar{\rho}, \bar{\eta})$
- LQCD inputs from the heavy sector:
  - B-meson decays & mixing: F_B, B_B
  - $B \to D^{(*)}$ decays:
    $F(1), G(1) \hookrightarrow |V_{cb}|$
  - semi-leptonic B-meson decays:
    $f_+(q^2) \hookrightarrow |V_{ub}|$

What is the required precision for key contributions to phenomenology ?

- Experiments reach few-% level, even $\leqslant 5\%$ $\Rightarrow$ theory error dominates
  $\Delta m_{d,s}$: $< 1\%$ [PDG,CDF], $\mathcal{B}(D_{(s)} \to \mu\nu)$: $\leqslant 4\%$ [CLEO-c], $\mathcal{B}(B \to D^* \ell\nu)$: 1.5% [HFAG]
- Lattice calculations with an accuracy of $O(5\%)$ or better required
  $\rightarrow$ incl. *all* systematics (unquenching, extrapolations, renormalization, . . .)
- Verification/Agreement of results using different formulations crucial !

Light sea quark configurations in use

[in current studies of heavy quark physics]

Quenched approximation ($N_f = 0$)

- No dynamical fermions, not suitable for phenomenology
- Still useful test laboratory, e.g., to understand methodologies etc.

Two-flavour QCD ($N_f = 2$)

- NP'ly $O(a)$ improved Wilson ($=$ clover) action
  - ▶ algorithmic progress (e.g., "Hasenbusch trick" and M. Lüscher's DD-HMC) render simulations competitive in the chiral regime
  - ▶ ALPHA $\in$ Coordinated Lattice Simulations $=$ European team effort
  - ▶ Regensburg (QCDSF)
- Twisted mass Wilson (with tree-level Symanzik-improved glue)
  - ▶ $O(a)$ improved by tuning to maximal twist; keep an exact χ-symmetry at the price of breaking part of the flavour symmetries and parity
  - ▶ ETMC
- Stout-smeared, chirally improved (with 1-loop improved LW glue)
  - ▶ BGR

Light sea quark configurations in use

[in current studies of heavy quark physics]

Three-flavour QCD ($N_f = 2 + 1$)

- MILC ensembles of AsqTad-improved staggered quarks (with LW-improved glue)
  - ▶ computationally "cheap", permit simulations within the chiral regime
  - ▶ debated rooting prescription $\left[\det{}^{(4)}(D_{st} + m)\right]^{\frac{1}{4}} \equiv \det{}^{(1)}(\gamma_\mu D_\mu + m)$, but effects seem to disappear in the CL; results agree with experiment
  - ▶ MILC & FNAL, HPQCD
- Domain wall fermions (with Iwasaki gauge action)
  - ▶ chirality preserving (realized as 5th dim. $L_s = \infty$)
  - ▶ RBC & UKQCD
- NP'ly $O(a)$ improved Wilson (with Iwasaki gauge action)
  - ▶ PACS-CS

Four-flavour QCD ($N_f = 2 + 1 + 1$)

$\longrightarrow$ in progress by ETMC & planned/started by other groups [Talk by G. Herzoida]

Light *valence* quarks usually discretized in the same way as the sea

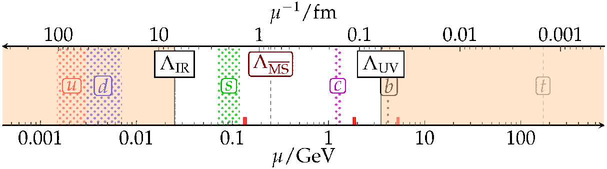
Challenge of LHQP: The multi-scale problem

Predictivity in a quantum field theory relies upon a large scale ratio

interaction range $\ll$ physical length scales

momentum cutoff $\gg$ physical mass scales : $\Lambda_{\mathrm{cut}} \sim a^{-1} \gg E_i, m_j$

This is a challenge in QCD, which has many physical scales:



hierarchy of disparate physical scales to be covered:

$$\Lambda_{\mathrm{IR}} = L^{-1} \ll m_\pi, \ldots, m_D, m_B \ll a^{-1} = \Lambda_{\mathrm{UV}}$$

$\downarrow$ $\downarrow$

$$\left\{ O(e^{-Lm_\pi}) \Rightarrow L \gtrsim \frac{4}{m_\pi} \sim 6\,\mathrm{fm} \right\} \curvearrowright \; L/a \gtrsim 120 \; \curvearrowleft \left\{ am_D \lesssim \frac{1}{2} \Rightarrow a \approx 0.05\,\mathrm{fm} \right\}$$

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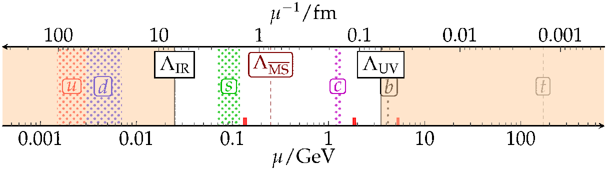

$\Rightarrow$ Difficult to satisfy simultaneously, clever technologies are required

- charm just doable, but lattice artefacts may be substantial (see later)
- given the today's computing resources, it seems impossible to work directly with relativistic b-quarks (i.e. resolving its propagation) on the currently simulated lattices
- the b-quark scale ($m_b/m_c \sim 4$) has to be separated from the others in a theoretically sound way before simulating the theory

Heavy quark formalisms in use

Lattice heavy quark physics has to deal with the presence of

strong lattice artefacts : $am_c \lesssim 1 \qquad am_b > 1$

Heavy quarks introduced as valence quarks $=$ "Partially quenched" setting

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Relativistic formulations → mainly for D-physics applications

- Wilson-like quarks: clover or TM, $am_c \leqslant 1/2 \ll 1$ desirable
  - $O\left[(am_c)^2\right]$ discretization effects ALPHA, ETMC
- Fermilab approach: relativistic clover action with HQET interpretation [El-Khadra, Kronfeld & Mackenzie, 1997]
  - $O\left[\alpha_s(\Lambda_{QCD}/m_Q), (\Lambda_{QCD}/m_Q)^2\right]$ errors
  - variants = RHQ actions → NP'ly tuned parameters, $O\left[(a\mathbf{p})^2\right]$ errors [Aoki et al., 2001; Christ et al., 2006]
  - adopted for charm & beauty FNAL & MILC, PACS-CS, RBC & UKQCD
- HISQ: goes beyond $O(a^2)$ tree-level improvement of AsqTad
  - perturbative Symanzik-improvement / smearing of the gauge fields $\Rightarrow$ no tree-level $O\left[(am_Q)^4, \alpha_s(am_Q)^2\right]$ errors to leading order in v/c
  - 1-loop taste-changing interactions reduced by a factor ~ 3
  - now also being tried towards the bottom region HPQCD

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Heavy quarks introduced as valence quarks = "Partially quenched" setting

Non-relativistic / effective field theory strategies → B-physics applications

- NRQCD: discretized non-relativistic expansion of the continuum $\mathcal{L}_D$
  - improved through $O\left(1/m_Q^2, a^2\right)$ and leading relativistic $O\left(1/m_Q^3\right)$
  - $O\left[\alpha_s^n/(am_Q)\right]$ divergences — HPQCD
- Static approximation = Leading-order HQET (ETMC)
  - HQET-guided extrapolations of fully relativistic simulations in the charm regime, turning into interpolations if the static limit is known
  - also in conjunction with finite-volume / finite-size scaling techniques
  - INFN-TOV, ALPHA, ETMC
- HQET for the b-quark: systematic expansion in Λ_{QCD}/m_b
  - NP fine-tuning of parameters to $O(1/m_b)$ & impr. statistical precision
  - connect different volumes iteratively with "step scaling functions"
  - ALPHA

Summary of heavy quark physics calculations

group	a [fm]	$m_\pi^{(\min)}$ [MeV]	q	Q
		$N_f = 2$		
ETMC	0.05, 0.065, 0.085, 0.10	270	TM	static / TM
Regensburg	0.08	170	clover	clover
ALPHA	0.08, 0.07, 0.05	250	clover	static + $1/m$
		$N_f = 2 + 1$		
FNAL & MILC I	0.09, 0.12, 0.15	230	AsqTad	Fermilab
FNAL & MILC II	0.06, 0.09, 0.12, 0.15	230	AsqTad	Fermilab
HPQCD I	0.09, 0.12	260	AsqTad	NRQCD
HPQCD II	0.09, 0.12, 0.15	320	HISQ	NRQCD
HPQCD III	0.045, 0.06, 0.09, . . .	320	HISQ	HISQ
RBC & UKQCD	0.08, 0.11	330 (300)	DW	static / RHQ
PACS-CS	0.09	200	clover	RHQ
		$N_f = 2 + 1 + 1$		
ETMC	0.06, 0.079, 0.09	270 (230)	TM	Osterw.-Seiler

static $\equiv$ smeared static (HYP, APE)

[Update of C. Aubin's table @ Lattice 2009]

Outline

1. Heavy quark masses from Lattice QCD
   - Cutoff effects in the charm sector
   - c- and b-quark masses from current-current correlators
   - m_b via scaling laws in the heavy quark limit
2. Calculations of hadronic weak matrix elements
   - D-meson decay constants
   - B-meson decay constants
   - Semi-leptonic decay form factors
   - B-meson mixing parameters
   - $B^* \rightarrow B\pi$ coupling
3. Non-perturbative HQET in two-flavour QCD
   - Non-perturbative formulation of HQET
   - Strategy to determine HQET parameters at $O(1/m)$
   - First physical results in the two-flavour theory
4. Conclusions & Outlook

I will focus on a selection of most recent progress / results, however, not without some personal ”bias”. Therefore, sorry for omissions . . .

Heavy quark masses from Lattice QCD

- Cutoff effects in the charm sector
- c- and b-quark masses from current-current correlators
- m_b via scaling laws in the heavy quark limit

Cutoff effects in the charm sector ($N_f = 0$)

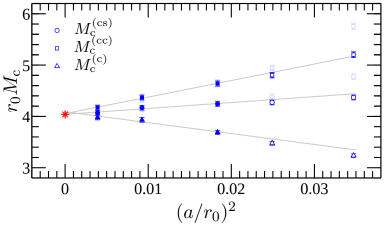
H. & Jüttner, JHEP0905(2009)101

Calculation of the charm quark's mass [Rolf & Sint, 2002]

- Physics input: bare charm mass in $\mathcal{L}_{QCD}$ s.th. m_{D_s}/F_K = experiment
- Additional complication in the charm sector:
  - ▶ $O(am_{q,c})$ cutoff effects become relevant, e.g., in the definition

$$M_c = Z_M\,[1 + (b_A - b_P)\,am_{q,c}]\,m_c = Z_M \frac{Z_m Z_P}{Z_A}\,m_{q,c}\,(1 + b_m\,am_{q,c})$$

$\rightarrow$ $O(am_{q,c})$ removed NP'ly [ALPHA Collaboration 2001 ($N_f = 0$) & 2010 ($N_f = 2$)]



large volume, $a \approx (0.09 - 0.03)$ fm

▶ $N_f = 0$, $r_0 = 0.5$ fm: $M_c = 1.60(3)$ GeV

$$\Rightarrow \quad \overline{m}_c^{\overline{MS}}(\overline{m}_c) = 1268(24)\ \text{MeV}$$

▶ $M_b \simeq 4M_c$

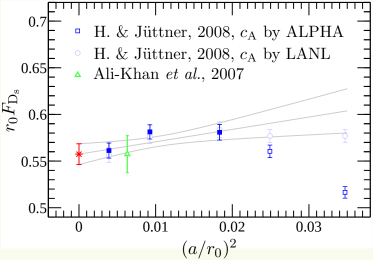
s.th. beauty is not yet accomodated

$\rightarrow$ for b-quarks, continuum limit $a \rightarrow 0$ can't be controlled in this way

$\Rightarrow$ *effective field theory* strategies needed

Cutoff effects in the charm sector ($N_f = 0$)

Warning from F_{D_s} : Lattice artefacts may be large for charm physics



- High-precision computation in $V = L^3 \times T$, $L \approx 2\,\mathrm{fm}$, $T = 2L$, $a \approx (0.09 - 0.03)\,\mathrm{fm}$ (!)
- $F_{D_s}^2 m_{D_s} = Z_A\, 2L^3 \left[\langle 0 | A_\mu^{cs} | D_s^+(\mathbf{p}=0) \rangle \right]^2$ from ground state dominance of SF CFs
- Controlling the continuum limit of charmed observables demands scaling study down to very fine lattice spacings ($a \lesssim 0.07\,\mathrm{fm}$)

Lesson from $N_f > 0$:

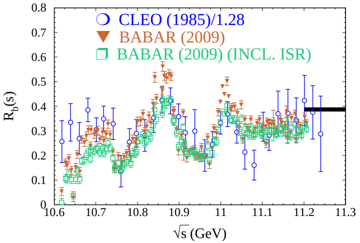
Symanzik programme works for charm, but $a < 0.08\,\mathrm{fm}$ seems mandatory

However, small lattice spacings are challenging:

Rapid slowing down of the gauge fields' topological modes with decreasing lattice spacings [Talks by M. Lüscher; F. Virotta]

m_c & m_b

Parametric inputs to many SM and Beyond SM calculations



Updated sum rule determination based on new 4-loop results & new BABAR data for b-quark production [Chetyrkin et al., PRD80(2009)074010]

→ By equating theoretically calculated and experimentally measured moments:

$$m_Q(\mu) = \frac{1}{2}\left[\frac{9Q_Q^2\bar{C}_n^{(\text{pert})}}{4\mathcal{M}_n^{(\text{exp})}}\right]^{\frac{1}{2n}}, \quad \mathcal{M}_n \propto \left(\frac{d}{dq^2}\right)^n \Pi_Q(q^2)\Big|_{q^2=0}$$

$\overline{m}_c^{\overline{MS}}(\overline{m}_c) = 1279(13)$ MeV, $\overline{m}_b^{\overline{MS}}(\overline{m}_b) = 4163(16)$ MeV; consistent with NP methods?

m_c

- LQCD can contribute to further reduce the error budget for the rare decay branching ratio $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ by precisely computing m_c

m_b

- Tensions between inclusive & exclusive determinations of $|V_{cb}|$, $|V_{ub}|$

[R. van de Water @ Lattice 2009]

- Extraction of / UT constraint via $|V_{ub}|$ from inclusive decays extremely sensitive to the input value for m_b

  $\Rightarrow$ accurate unquenched determinations required

m_c & m_b from current-current correlators

HPQCD, McNeile et al., arXiv:1004.4285

In the spirit of the previous method, heavy quark masses are extracted via dispersion relations by comparing perturbative zero-momentum moments of current-current correlators (available to 4-loop by the Karlsruhe group) with lattice data in place of experimental data for $\sigma(e^+e^- \to \text{hadrons})$

$$G(t) = a^6 \sum_{\mathbf{x}} (am_{0h})^2 \langle 0 | j_5(\mathbf{x}, t)\, j_5(0,0) | 0 \rangle \qquad j_5 = \overline{\psi}_h \gamma_5 \psi_h$$

is finite and unrenormalized as $a \to 0$ (PCAC), and g_n from continuum PT:

$$G_n \equiv \sum_t (t/a)^n G(t) = \frac{g_n(\alpha_{\overline{MS}}(\mu), \mu/m_h)}{(am_h(\mu))^{n-4}} + O\big((am_h)^m\big) \qquad n \geqslant 4$$

Reduced moments to suppress lattice artefacts and tuning errors in am_{0h}:

$$R_n \equiv \begin{cases} G_4/G_4^{(0)} & \text{for } n = 4 \\ \dfrac{am_{\eta_h}}{2am_{0h}} \left(G_n/G_n^{(0)}\right)^{1/(n-4)} & \text{for } n \geqslant 6 \end{cases} \quad \leftrightarrow \quad \begin{cases} \text{continuum quantities,} \\ m_{\eta_c}^{(\exp)}, m_{\eta_b}^{(\exp)} \text{ as input} \end{cases}$$

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New simulation / Analysis features compared to 2008:

- MILC $N_f = 2+1$ sea, HISQ for valence c- and very close-to b-quarks
- Finer lattice resolutions: $a = (0.06, 0.045)$ fm
- New 3rd order PT for R_{10}, variety of masses around m_c
- Sophisticated fitting techniques
  - simultaneous constrained, Bayesian fits to all parameter sets (specified by a, am_{0h}), with priors for a large # of parameters
  - applied to the ansatz for the cutoff effects modelled according to

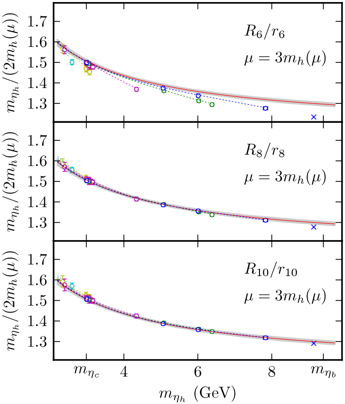
$$R_n(\mu, m_{\eta_h}, a, N_{am}) \equiv R_n^{\text{cont}} \Big/ \left[1 + \sum_{i=1}^{N_{am}} \sum_{j=0}^{N_z} c_{ij}^{(n)} \left(\frac{am_{\eta_h}}{2}\right)^{2i} \left(\frac{2\Lambda}{m_{\eta_h}}\right)^{j}\right]$$

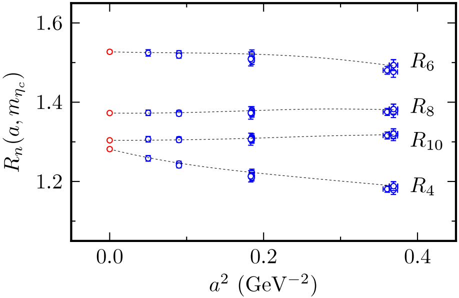
  - $0.3 \lesssim am_{\eta_h}/2 \lesssim 1.1$ & tiny statistical errors

    $\Rightarrow$ decent fits only when $N_{am} > 10 - 20$ & restricting $am_{\eta_h} \leqslant 1.95$!

m_c & m_b from current-current correlators

HPQCD, McNeile et al., arXiv:1004.4285





Cutoff effects decrease with n, but n should be small enough for PT to be applicable

- One presumes that 1.) the Symanzik expansion is a convergent expansion and 2.) that it is still useful up to $am_h \approx 1$ $\rightarrow$ too optimistic ?
- As final results, incl. all systematics, HPQCD quotes:

$$\overline{m}_c^{\overline{\rm MS}}(\overline{m}_c, N_f = 4) = 1.273(6)\,\text{GeV}\,,\; \overline{m}_b^{\overline{\rm MS}}(\overline{m}_b, N_f = 5) = 4.164(23)\,\text{GeV}$$

m_b via scaling laws in the heavy quark limit

ETMC, Blossier at al., JHEP1004(2010)049

Determine B-physics parameters by extrapolating ratios of heavy-light meson masses & decay constants obtained around m_c to the m_b−region, employing scaling laws in the heavy-quark limit

- For many years:
  Conventional extrapolations of charm data to the bottom-scale based on heavy quark scaling laws

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- For many years:
  Conventional extrapolations of charm data to the bottom-scale based on heavy quark scaling laws
- New method proposed:
  1.) Interpolation of proper *ratios* between the charm region and their (known) static limits to a sequence of reference quark masses $\overline{m}_h^{(i)}$ towards m_b
  2.) Mapping of simulation data of observables in the charm region to the B-scale $m_B^{(\text{exp})}$, by multiplying them with these ratios

$$\mathcal{Y}(x, \lambda, \overline{m}_l) \sim \lambda^{-1} \frac{\mathcal{O}_{hl}(1/x, \overline{m}_l)}{\mathcal{O}_{hl}(1/\lambda x, \overline{m}_l)} \frac{\mathcal{Z}(\ln \lambda x)}{\mathcal{Z}(\ln x)} \qquad x = 1/\overline{m}_h \,, \; \lambda = \frac{x^{(n-1)}}{x^{(n)}} > 1$$

where further logarithmic terms must be included and $\mathcal{O}_{hl}^{\text{QCD}} = \mathcal{Z}\, \mathcal{O}_{hl}^{\text{HQET}}$

$\Rightarrow \qquad \lim_{x \to 0} \mathcal{Y}(x, \lambda, \overline{m}_l) = 1 \qquad \mathcal{Z}:$ PT'ly known

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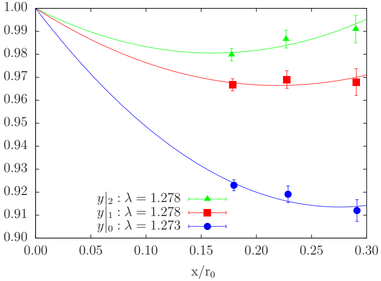
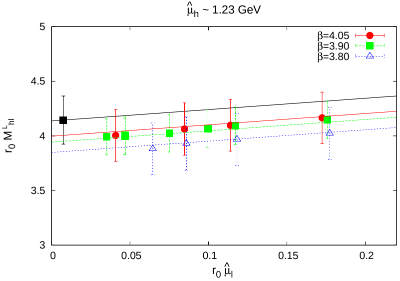
ETMC, Blossier at al., JHEP1004(2010)049





Results for $N_f = 2$ maximally twisted mass Wilson fermions:

$\overline{m}_b^{\overline{MS}}(\overline{m}_b) = 4.63(27)$ GeV $F_B = 194(16)$ MeV $F_{B_s} = 235(12)$ MeV

- $O_{hl} = m_{hl}$: heavy-light meson mass $\hookrightarrow$ computation of $\overline{m}_b$
  $O_{hl} = F_{hl}$: heavy-light decay constant $\hookrightarrow$ computation of F_B, F_{B_s}
- Error budget: $\sim 50\%$ from $O_{hl}(\overline{m}_h^{(1)})$, $\sim 50\%$ from y–ratios
- Authors expect this method to have smaller errors than free extrapolations with heavy quark scaling laws

m_c & m_b

Further work to determine heavy quark masses reported at the conference

- Preliminary $N_f = 2$ result by ETMC: [Talk by F. Sanfilippo]

  $\overline{m}_c^{\overline{MS}}(\overline{m}_c) = 1.275(35)\ \mathrm{GeV}$ RI-MOM renormalization & continuum limit

- The c-quark mass from charm current-current correlators in TM QCD

  [ETMC, talk by M. Petschlies]

- The b-quark mass from lattice NRQCD (using PT and simulation data)

  [Poster by C. Monahan]

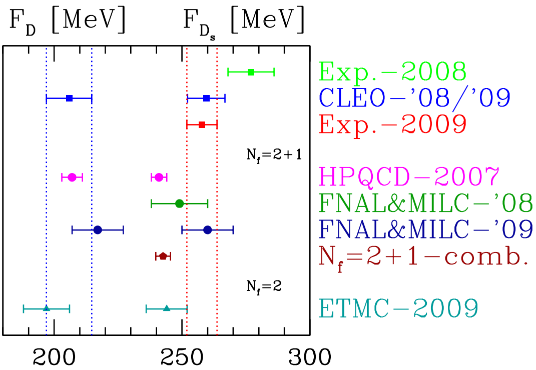
Calculations of hadronic weak matrix elements

- D-meson decay constants
- B-meson decay constants
- Semi-leptonic decay form factors
- B-meson mixing parameters
- $B^* \to B\pi$ coupling

F_D & F_{D_s} — Test of LQCD techniques

$$\mathcal{B}\left(D_s^+ \to \ell^+ \bar{\nu}\right) = \frac{G_F^2\, m_\ell^2\, m_{D_s^+}}{8\pi}\left(1 - \frac{m_\ell^2}{m_{D_s^+}^2}\right)^2 F_{D_s}^2\, |V_{cs}|^2 \qquad \ell^+ = \mu^+, \tau^+$$

- Measuring the branching ratio, experiment yields $F_{D_s}^2\, |V_{cs}|^2$ s.th. assuming CKM unitarity $|V_{ud}| = |V_{cs}| + O(\lambda^4)$, one can compare F_{D_s} with $\langle 0 \,|\, \bar{s}\, \gamma_\mu \gamma_5\, c \,|\, D_s(p) \,\rangle = i\, F_{D_s} p_\mu$ from LQCD
- $F_D \leftrightarrow V_{cd}$, but F_{D_s} needs no chiral extrapolation in the valence sector



Among the possible explanations for the discrepancy between experiment and lattice:

- ▶ Experimental issues ?
- ▶ Systematic effect, e.g., discret. error missed ?
- ▶ Tension = Hint of new physics in the flavour sector ?

F_D & F_{D_s} — The "F_{D_s} puzzle" revisited

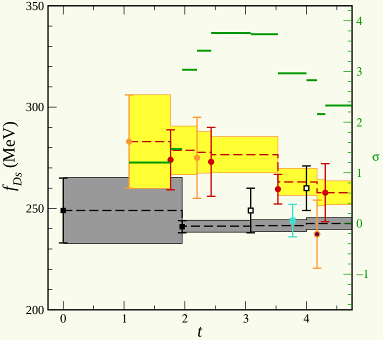
Discrepancy rose to $3.8\,\sigma$ in 2007 w.r.t. HPQCD's result, using $N_f = 2+1$ HISQ valence quarks on rooted staggered MILC sea

(based on $a = 0.15, 0.12, 0.09$ fm, but consistent with adding $a \approx 0.06, 0.045$ fm)

$F_D = 207(4)$ MeV $\quad F_{D_s} = 241(3)$ MeV $\quad$ combined χ & continuum extrap.

Tracing the discrepancy's history

[compilation by A. Kronfeld, arXiv:0912.0543]



- new meas. by CLEO 01/09: -0.8σ
- FNAL & MILC's update 2009 after re-analysis of $r_1 F_\pi$: -0.13σ
- HFAG's interpretation of the BaBar measurement: -0.67σ
- new meas. by CLEO 10/09: $+0.1\sigma$

$\Rightarrow$ The tension moved down to 2.3σ

F_D & F_{D_s} — The "F_{D_s} puzzle" revisited

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$F_D = 207(4)\,\text{MeV}$ $\quad F_{D_s} = 241(3)\,\text{MeV}$ $\quad$ combined χ & continuum extrap.

Influence of the lattice scale setting by r_1 :

$r_1^2\,F(r_1) \stackrel{!}{=} 1 \qquad F(r) = dV/dr \qquad r_1 = 0.321(5)\,\text{fm}$ from $\Upsilon\ 2S - 1S$ splitting

(uncertainty on r_1 dominates the error budget of F_{D_s})

New scale determination, combining r_1–results from Υ, D_s mass splittings (via HISQ) and F_{η_s} with MILC's r_1/a [HPQCD, Davies et al, PRD81(2010)034506]

$r_1 = 0.3133(23)\,\text{fm} \quad \Rightarrow \quad 1.6\,\sigma$ discrepancy with CLEO-2009

$\Rightarrow$ Given the high statistical accuracy of the calculations, it's even more important to carefully assess the overall error incl. all systematics

F_D & F_{D_s} — The "F_{D_s} puzzle" revisited

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$F_D = 207(4)\,\text{MeV}$ $\quad F_{D_s} = 241(3)\,\text{MeV}$ $\quad$ combined χ & continuum extrap.

Preliminary $N_f = 2+1+1$ results by ETMC:

[Talk by C. Urbach]

$F_D = 204(3)\,\text{MeV}$ $\quad F_{D_s} = 251(3)\,\text{MeV}$ $\quad F_{D_s}/F_D = 1.230(6)$

- Wilson twisted mass fermions at maximal twist; $a = (0.079, 0.060)\,\text{fm}$
- Mixed action approach: Osterwalder-Seiler quarks in the valence sector
- Extrapolation of $F_{D_s}\sqrt{m_{D_s}}$ to the physical point employing SU(2) HMχPT, where terms proportional to $a^2 m_{D_s}^2, 1/m_{D_s}$ are included
- Error is purely statistical, systematics not yet accounted for

F_D & F_{D_s} — The "F_{D_s} puzzle" revisited

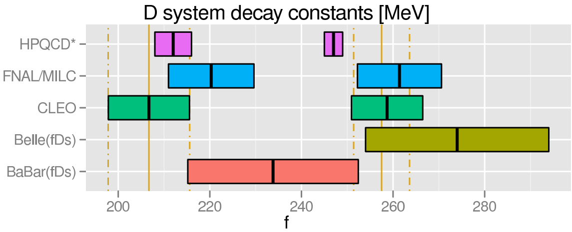
Discrepancy rose to $3.8\,\sigma$ in 2007 w.r.t. HPQCD's result, using $N_f = 2+1$ HISQ valence quarks on rooted staggered MILC sea
(based on $a = 0.15, 0.12, 0.09\,$fm, but consistent with adding $a \approx 0.06, 0.045\,$fm)

$F_D = 207(4)$ MeV $\quad F_{D_s} = 241(3)$ MeV $\quad$ combined χ & continuum extrap.

Update of $N_f = 2+1$ results by FNAL & MILC: [Talk by J. Simone]

$F_D = 220(8)(5)$ MeV $\quad F_{D_s} = 261(8)(5)$ MeV $\quad F_{D_s}/F_D = 1.19(1)(2)$

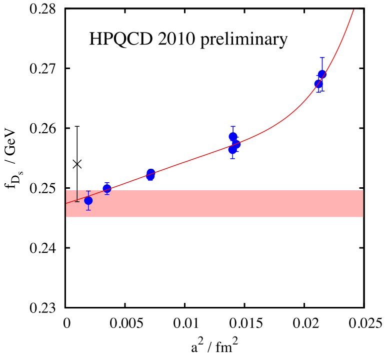
- First error from statistics & discretization, where extrapolation function incl. terms (with priors on coefficients) modelling heavy & light cutoff effects
- Second error $=$ combined other systematic error sources (taken in quadrature)



F_D & F_{D_s} — The "F_{D_s} puzzle" revisited

Update of $N_f = 2 + 1$ results by HPQCD: [Talk by E. Follana]

$$F_{D_s} = 247(2)\ \mathrm{MeV}$$



Some simulation / analysis features:

- Finer lattices: $a = (0.06, 0.045)$ fm
- Accounts for scale re-determination
- Bayesian simultaneous fits
- Further new HISQ formalism studies:
  - hyperfine splitting
  - quark mass ratios $\overline{m}_c/\overline{m}_s \hookrightarrow \overline{m}_s$
  - HISQ with $m_h \rightarrow m_b \hookrightarrow F_{B_{(s)}}$
  - heavy-light current-current CFs

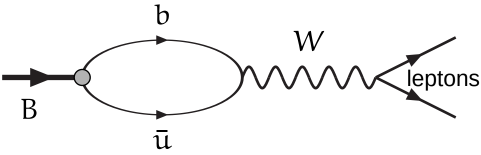
  [Talk by J. Koponen]

"Puzzle" seems to disappear:

No conclusive evidence for New Physics in the charm quark sector yet, but the $D_{(s)}$ leptonic decays will continue to help constraining SM extensions

F_B & F_{B_s}

- F_B
  - $\underbrace{\mathcal{B}(B^- \to \tau^- \bar{\nu}_\tau)}_{\text{experiment}} \propto |V_{ub}|^2 \underbrace{F_B^2}_{\text{lattice}}$
  - Process is sensitive probe of charged Higgs boson effects



- F_{B_s}
  - Relevant for CKM analysis & BSM effects in $B_s \to \mu^+\mu^-$ (decay will be measured at LHCb)

F_B & F_{B_s}

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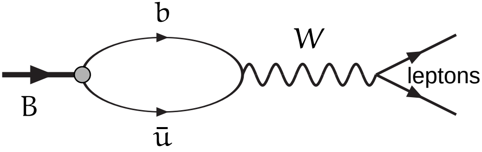

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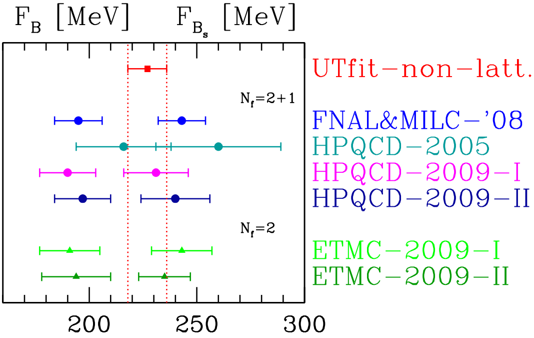


Direct SM meas. by Belle '06:
$F_B = 229^{+36}_{-31}(\text{stat})^{+34}_{-37}(\text{syst})$
$\to$ few-% at super-B factories ?

1.9σ deviation of exp. determ. from LQCD (using $|V_{ub}|$ exclusive from the lattice)

Goal of lattice computations:

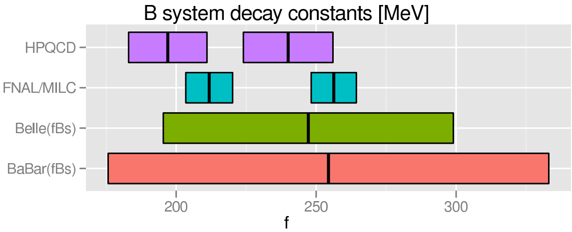
$O(10\%) \to O(3\%)$ errors; better control of a– and mass effects, NP renormalization

F_B & F_{B_s}

Update of $N_f = 2+1$ results by FNAL & MILC: [Talk by J. Simone]

$F_B = 212(6)(6)$ MeV $F_{B_s} = 256(6)(6)$ MeV $F_{B_s}/F_B = 1.21(1)(2)$

- $a \approx (0.09, 0.12, 0.15)$ fm MILC sea; partially quenched staggered χPT fits
- Combination of perturbative & NP renormalization
- First error from statistics & discretization, where extrapolation function incl. terms (with priors on coefficients) modelling heavy & light cutoff effects
- Second error = combined other systematic error sources (taken in quadrature)

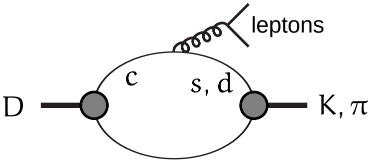


- Experimental branching ratios & (excl. & incl.) average for $|V_{ub}|$ to extract F_{B_s}

[Rosner & Stone, arXiv:1002.1655]

D-meson semi-leptonic decay form factors

- Independent determination of $|V_{cs}|, |V_{cd}|$; holds $|V_{ud}| \approx |V_{cs}|$ actually ?
  - $|V_{cs}|$ consistent with CKM unitarity requirement at the $O(10\%)$ level, but this is not stringent enough for precision CKM physics



- Differential rate for the decay
  $D \to \pi \ell \nu_\ell$ for massless leptons

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{192\pi^3 m_D^3}\left[\left(m_D^2 + m_\pi^2 - q^2\right)^2 - 4m_D^2 m_\pi^2\right]^{\frac{3}{2}} |f_+(q^2)|^2 \, |V_{cd}|^2$$

- Thus, either
  - $\Gamma^{(\text{exp})}$ & LQCD $\hookrightarrow$ $|V_{cd}|$

  or
  - $\Gamma^{(\text{exp})}$ & CKM unitarity $\hookrightarrow$ test of LQCD

D-meson semi-leptonic decay form factors

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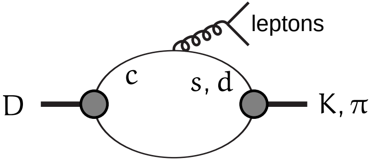

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- Thus, either
  - $\Gamma^{(\mathrm{exp})}$ & LQCD $\hookrightarrow$ $|V_{cd}|$

  or
  - $\Gamma^{(\mathrm{exp})}$ & CKM unitarity $\hookrightarrow$ test of LQCD
- Also of interest w.r.t. the F_{D_s} tension: Not obvious how to reconcile it with BSM physics, since SM leptonic D_s decay occurs at tree-level, though models with a charged Higgs or leptoquark could do but would lead to signals in $D_s \to K\ell\bar{\nu}_\ell$ decays [Dobrescu & Kronfeld, Kronfeld, 2008]

D-meson semi-leptonic decay form factors

HPQCD, Na et al., arXiv:0910.3919 (Lattice 2009)
Talk by H. Na

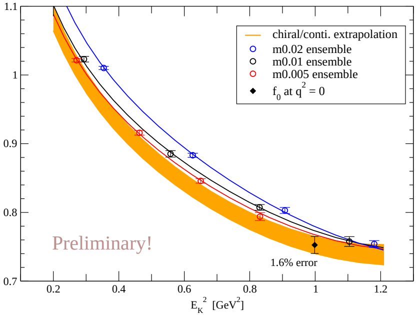
$D \to K$ form factor with HISQ charm & light quarks

- $N_f = 2+1$ $a \approx (0.09, 0.12)$ fm MILC sea, HISQ for valence light & c-quarks
  $\Rightarrow$ $f_0(q^2), f_+(0)$ from scalar current via PCVC, without operator matching:

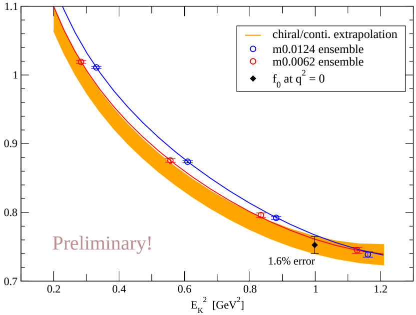
$$q^\mu \langle V_\mu^{lat} \rangle Z = (m_c - m_q) \langle S^{lat} \rangle \qquad f_0(q^2) = \frac{m_c - m_q}{m_D^2 - m_\pi^2} \langle S \rangle \; , \; f_+(0) = f_0(0)$$

- Bayesian fits of 3- & 2-pt. functions and of chiral & continuum extrapolations

$f_0^{D\to K}$: coarse lattice



$f_0^{D\to K}$: fine lattice



Note: At $E_K^2 \approx 1\,\text{GeV}^2$ $(q^2 = 0)$ applicability of χPT appears questionable

D-meson semi-leptonic decay form factors

HPQCD, Na et al., arXiv:0910.3919 (Lattice 2009)
Talk by H. Na

$D \rightarrow K$ form factor with HISQ charm & light quarks

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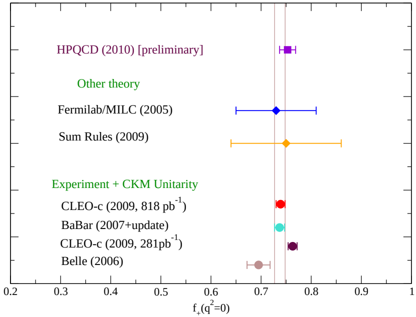
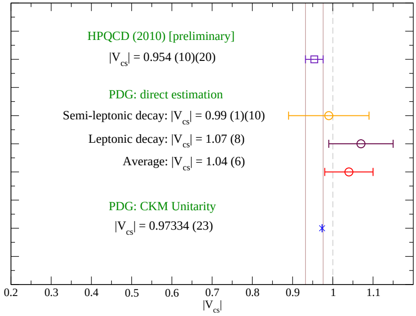
Preliminary result with full error budget:

$f_+(q^2 = 0) = 0.753(12)(10)$ [(stat)(syst)] $|V_{cs}| = 0.954(10)(20)$ [(exp)(lat)]





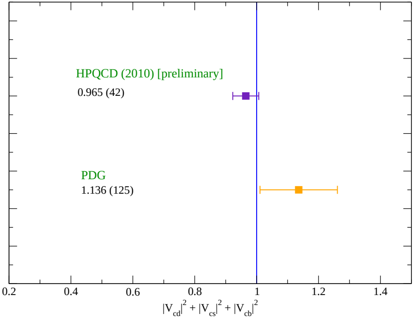
D-meson semi-leptonic decay form factors

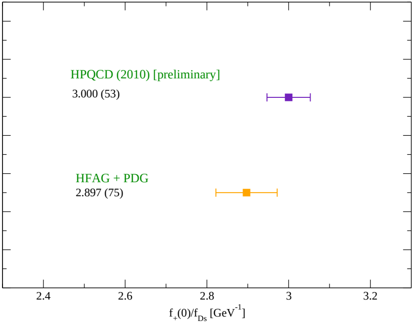
HPQCD, Na et al., arXiv:0910.3919 (Lattice 2009)
Talk by H. Na

$D \to K$ form factor with HISQ charm & light quarks

Unitarity check of 2nd row

$f_+(0)/F_{D_s}$





Future plans:

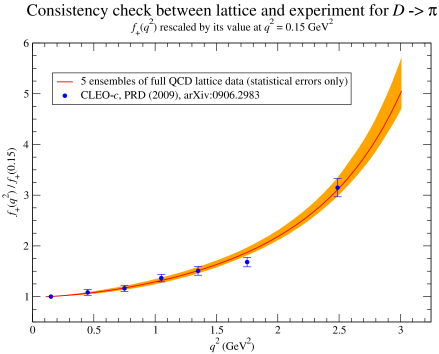
- $D \to \pi$ FF using the same method
- D semi-leptonic decay via the vector current with fully NP operator matching

D-meson semi-leptonic decay form factors

Status of $D \to \pi$ for $N_f = 2+1$ from FNAL & MILC: [Talk by E. Gamiz]

- $a \approx (0.09, 0.12)$ fm MILC ensembles, quadrupled statistics, Fermilab heavy quarks
- Overall normalization due to $Z_{j_{ab}} = \rho_{j_{ab}} [Z_{V_{aa}} Z_{V_{bb}}]^{1/2}$ "blinded"
- Combined chiral (excluding $\sqrt{2}\, E_\pi/(4\pi F_\pi) > 1$) & continuum extrapolation
- Comparison of the shape of the form factor to CLEO-c
  ($\to f_+(q^2)/f_+(0.15\,\mathrm{GeV}^2)$ to remove blinding factor from f_+ and $|V_{cd}|$ from CLEO)

$\Rightarrow$ Statistical error ($\sim 5\%$ for $f_+(0.15\,\mathrm{GeV}^2)$) and agreement are much better, but analysis of systematics has to be awaited



D-meson semi-leptonic decay form factors

Preliminary $N_f = 2$ results by ETMC:

[Talk by S. Di Vita]

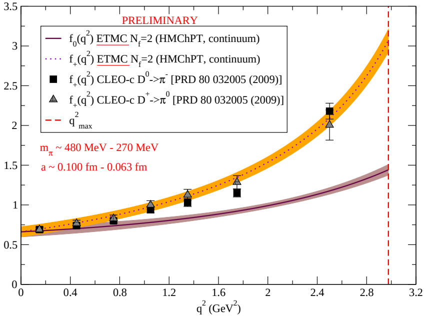
- $a \approx (0.1, 0.079, 0.063)$ fm, $m_\pi \approx (500 - 270)$ MeV, controlled finite-size effects
- Ratios of 3- and 2-point functions s.th. Z–factors cancel
- Only slight interpolation necessary to bring the simulated c- and s-quark masses to their physical values before any chiral extrapolation
- Extrapolation to the physical point by combined fits to HMχPT formulae, down to $q^2 = 0$, adding allowed LO $O(a^2)$ discretization effects to them

⇒ Good agreement of LQCD with exp. determinations in common q^2–range

Preliminary:

$$f^{D\to\pi}(0) = 0.66(6)_{stat}$$

$$f^{D\to K}(0) = 0.76(4)_{stat}$$



D-meson semi-leptonic decay form factors

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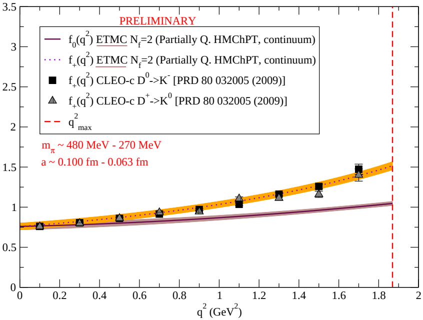
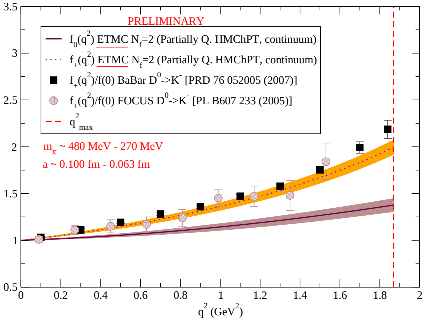
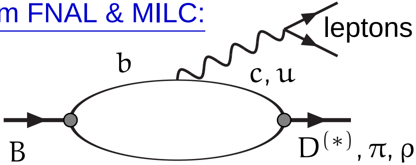




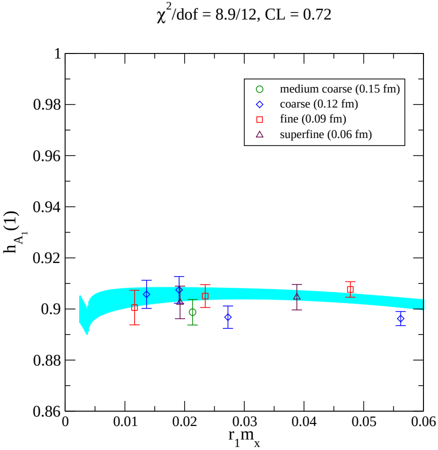
B-meson semi-leptonic decay form factors

Status of $B \to D^* \ell \nu_\ell$ for $N_f = 2+1$ from FNAL & MILC:

[Talk by A. Kronfeld]

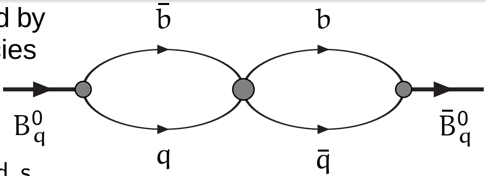


- Determination of $|V_{cb}|$, which normalizes the whole UT
- $\sim 2.3\sigma$ tension between inclusive and exclusive $|V_{cb}|$ (latter relying on $B \to D^* \ell \nu_\ell$ from FNAL & MILC 2008)
- Zero recoil $\Rightarrow$ just $F(1) \equiv h_A(1)$
- Double ratios of matrix elements: Cancellations of stat. errors and renormalization, left perturbative matching uncertainty small
- $a \approx (0.06 - 0.15)$ fm, quadrupled statistics
- $F_{\mathrm{blind}} F(1) = 0.8949(51)(88)(72)(93)(50)(30)$ (errors due to statistics, $g_{D^* D\pi}$, chiral extrapolation, HQ discretization errors, κ– tuning, perturbative matching)



B-meson mixing parameters

Apex of the UT triangle constrained by ratio of meson oscillation frequencies



$$\langle \overline{M} \,|\, \mathcal{O}_{\Delta M=2} \,|\, M \rangle = \frac{4}{3} m_M^2 F_M^2 B_M$$

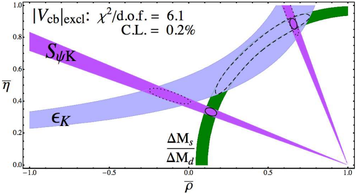
$$\langle 0 \,|\, \bar{b}\gamma_\mu\gamma_5 q \,|\, B_q \rangle = i\, p_\mu F_{B_q}\,,\ q = d, s$$

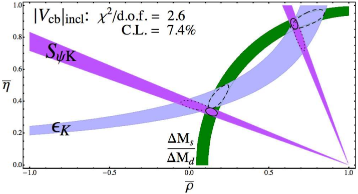
$$\Delta m_d \propto F_{B_d}^2 \widehat{B}_{B_d} |V_{td} V_{tb}^*|^2$$

$$\frac{\Delta m_s}{\Delta m_d} \propto \frac{F_{B_s}^2 \widehat{B}_{B_s}}{F_{B_d}^2 \widehat{B}_{B_d}} \frac{|V_{ts}|^2}{|V_{td}|^2} \equiv \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}$$

ξ : SU(3) breaking ratio

- If UT constraints from $\alpha, \gamma, |V_{ub}|$ are omitted, a $(2-3)\sigma$ tension between constraints from $\epsilon_K, \Delta m_s/\Delta m_d, \sin(2\beta)$ is observed [Lunghi & Soni, 2008]
- Degree of tension very sensitive to $|V_{cb}|$ [Laiho, Van De Water & Lunghi, 2009]
  $\rightarrow$ leave one input as free parameter & make prediction based on others





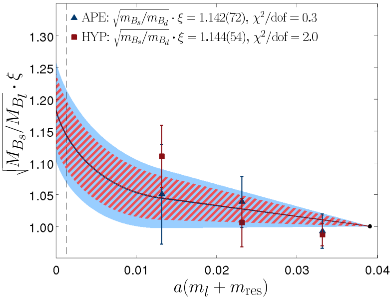
B-meson mixing parameters

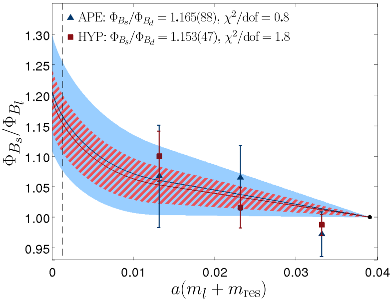
RBC & UKQCD, Albertus et al., arXiv:1001.2023
Talk by Y. Aoki

Feasibility study using $N_f = 2+1$ DW sea
and (APE & HYP) smeared static quarks

- $a \approx 0.11\,\mathrm{fm}$, m_π down to $\approx 430\,\mathrm{MeV}$
- $O(\alpha_s\, p a)$ improvement for the heavy-light decay constants
- NLO SU(2) HMχPT to extrapolate to the physical masses, which converges more rapidly if light valence and sea quark masses are sufficiently small

$$\frac{\Phi_{B_s}}{\Phi_{B_l}} = R_\Phi \left\{ 1 + \frac{1+3g^2_{B^*B\pi}}{(4\pi f)^2} \left(\frac{3}{4}\right) m_L^2 \ln\left(\frac{m_L^2}{\Lambda_\chi^2}\right) + C_l \frac{2Bm_l}{(4\pi f)^2} \right\}$$



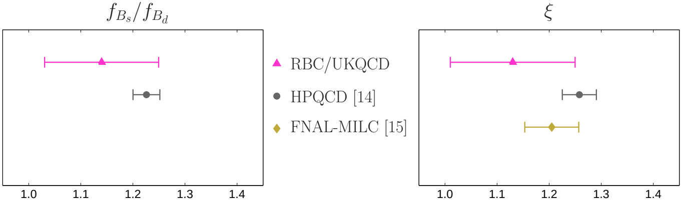


B-meson mixing parameters

RBC & UKQCD, Albertus et al., arXiv:1001.2023
Talk by Y. Aoki

Feasibility study using $N_f = 2+1$ DW sea and (APE & HYP) smeared static quarks

- $a \approx 0.11\,\mathrm{fm}$, m_π down to $\approx 430\,\mathrm{MeV}$
- $O(\alpha_s\, p a)$ improvement for the heavy-light decay constants



- Results including statistical and systematic uncertainties:

$$F_{B_s}/F_{B_d} = 1.15(12) \qquad \xi = F_{B_s}\sqrt{\widehat{B}_{B_s}} \Big/ F_{B_d}\sqrt{\widehat{B}_{B_d}} = 1.13(12)$$

(chiral extrapolation and discretization errors dominate; $g_{B^*B\pi} \hookrightarrow O(3\%)$)

- Extension to lighter d-quarks and larger volumes 24^3 ($a \approx 0.11\,\mathrm{fm}$) and 32^3 ($a \approx 0.08\,\mathrm{fm}$) under way

B-meson mixing parameters

Related work in progress reported at the conference

- B-physics study with $N_f = 2+1$ DW sea quarks and NP'ly tuned RHQ action for the heavy quarks by RBC & UKQCD

  [Talk by O. Witzel]

- Computation of $g_{B^*B\pi}$ with $N_f = 2+1$ DW sea and NP'ly tuned RHQ action for the heavy quarks by RBC & UKQCD

  [Talk P. Fritzsch]

- $B^0 - \bar{B}_q^0$ mixing calculation focusing on BSM contributions by FNAL & MILC

  [Talk C. Bouchard]

$g_{B^*B\pi}$

Matrix element for the *strong decay* $B^* \to B\pi$:

$$\langle B^0(p)\,\pi^+(q)\,|\,B^{*+}(p')\,\rangle \equiv -g_{B^*B\pi}(q^2)\, q_\mu \eta^\mu(p')(2\pi)^4\delta(p'-p-q)$$

Relevance

- Related to the coupling g of heavy-light meson χPT (HMχPT)

$$g \propto \lim_{m_b\to\infty,\, m_d\to 0} g_{B^*B\pi}$$

  $\to$ the only LEC at leading order in $1/m_{hl}$

- It constrains the chiral behaviour, e.g., of F_B, B_B and the $B \to \pi\ell\nu_\ell$ form factor
- LSZ-reduction of the pion and PCAC links $g_{B^*B\pi}$ in the static and chiral limits to the matrix element of the light axial current:

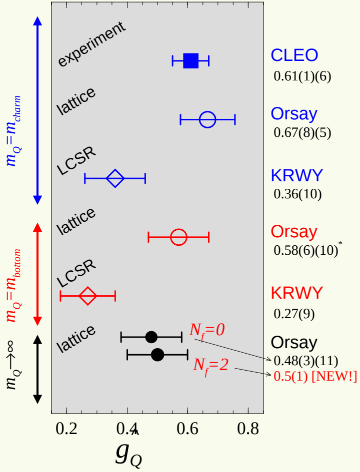
$$g_{B^*B\pi}(0) = -\frac{1}{F_\pi} F_1(0) \qquad F_1(0) = \langle\, B(p)\,|\,A_i(0)\,|\,B^*(p)\,\rangle$$

$g_{B^*B\pi}$

Matrix element for the *strong decay* $B^* \to B\pi$:

$$\langle B^0(p)\,\pi^+(q)\,|\,B^{*+}(p')\,\rangle \equiv -g_{B^*B\pi}(q^2)\,q_\mu\eta^\mu(p')(2\pi)^4\delta(p'-p-q)$$

Selection of previous results



$N_f = 0$ lattice
and light cone QCD sum rules results
[compilation by Bećirević et al. @ Lattice 2005]

$N_f = 2$ results:

- $g^{stat} = 0.516(5)_{stat}(31)_\chi(28)_{PT}(28)_a$
  [Ohki et al, 2008]
- $g^{stat} = 0.44(3)^{+0.07}_{-0.00}$
  [Bećirević et al. et al, 2009]

ALPHA Collaboration, Bulava, Donnellan, Simma & Sommer; talk by M. Donnellan

Static calculation — lattice 3-point functions pose technical challenges . . .

- In 3-point functions $C_3(t,t';q,p) = \langle \mathcal{O}_q(t)\mathcal{O}(t')\mathcal{O}_p^\dagger(0)\rangle$, two time separations t' and $t-t'$ have to be made large

$$\frac{C_3(t,t/2;p,p)}{C_2(t)} = \mathcal{M}(p,p) + O\left(e^{-(t/2)\Delta E}\right)$$

- 3-point function with summed insertion: [Maiani et al., 1987]

$$D(t;q,p) \equiv a\sum_{t'} C_3(t,t';q,p)$$

$$\Rightarrow \quad \partial_t \frac{D(t;q,p)}{\sqrt{C_2(t;p)C_2(t;q)}} = \mathcal{M}(q,p) + O\left(t\, e^{-t\Delta E}\right)$$

- Further computational details:
  - HYP static actions to avoid exponential decay of signal-to-noise in t
  - all-to-all light quark propagators (U(1) noise, full time dilution)
  - Smeared light quark fields to reduce excited state contamination

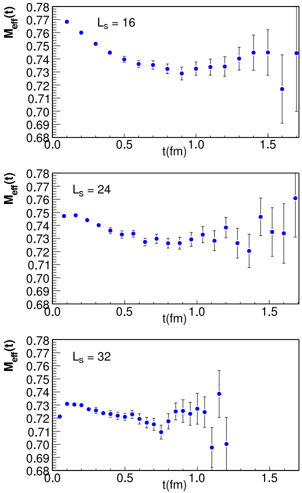
$g_{B^*B\pi}$

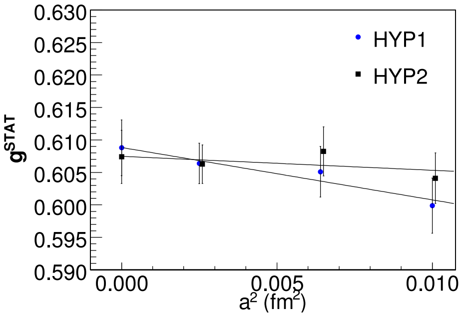
[Quenched test:]

$\overline{\text{A}}$LPHA Collaboration, Bulava, Donnellan, Simma & Sommer; talk by M. Donnellan

precision, plateaux & continuum limit

No discernible $a-$dependence
at this 0.5% level



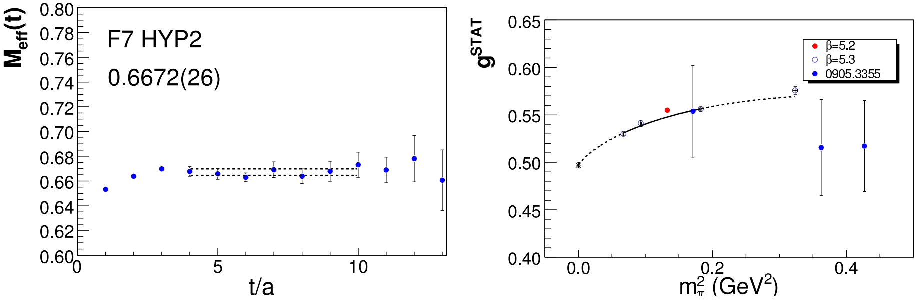


$g_{B^*B\pi}$

ALPHA Collaboration, Bulava, Donnellan, Simma & Sommer; talk by M. Donnellan

CLS based

$N_f = 2$ NP'ly improved Wilson: **preliminary**



- $\beta = 5.3$, $a \approx 0.07$ fm, $m_\pi \approx 250$ MeV [Scale setting preliminary; talk by B. Leder]
- Renormalization (NP Z_A) and κ_c adds a $\approx 0.5\%$ error [ALPHA Collaboration, 2007 & 2008]
- Chiral extrapolation linear in m_π^2 or via HMχPT formula [Fajfer & Kamenik, 2006]

$$g \;=\; g_0 \left\{ 1 - \frac{4g_0^2}{(4\pi f)^2}\, m_\pi^2 \ln^2(m_\pi) + c_0 m_\pi^2 \right\}$$

Non-perturbative HQET in two-flavour QCD

ALPHA Collaboration

B. Blossier, J. Bulava, M. Della Morte,
M. Donnellan, P. Fritzsch, N. Garron,
J. H., G.M. von Hippel, N. Tantalo,
H. Simma, R. Sommer

FlaviA net

- Non-perturbative formulation of HQET
- Strategy to determine HQET parameters at $O(1/m)$
- First physical results in the two-flavour theory

Scale, light quark masses from light sector:
F. Knechtli, B. Leder, S. Schaefer, F. Virotta

CLS based

Non-perturbative formulation of HQET

Action: $S_{\rm HQET}(x) = a^4 \sum_x \mathcal{L}_{\rm HQET}(x)$ for the b-quark (zero velocity HQET)

[Eichten, 1988; Eichten & Hill, 1990]

$$\begin{aligned}
\mathcal{L}_{\rm HQET}(x) &= \mathcal{L}_{\rm stat}(x) - \omega_{\rm kin}\mathcal{O}_{\rm kin}(x) - \omega_{\rm spin}\mathcal{O}_{\rm spin}(x) \\
\mathcal{L}_{\rm stat}(x) &= \overline{\psi}_h(x)\left[\, D_0 + m_{\rm bare}\,\right]\psi_h(x) \qquad \tfrac{1}{2}(1+\gamma_0)\psi_h(x) = \psi_h(x) \\
\mathcal{O}_{\rm kin}(x) &= \overline{\psi}_h(x)\,\mathbf{D}^2\,\psi_h(x)
\end{aligned}$$

$\rightarrow$ kinetic energy from heavy quark's residual motion

$$\mathcal{O}_{\rm spin}(x) = \overline{\psi}_h(x)\,\boldsymbol{\sigma}\cdot\mathbf{B}\,\psi_h(x)$$

$\rightarrow$ chromomagnetic interaction with the gluon field

Composite fields: axial current, related to the B-meson decay constant

$F_B\sqrt{m_B} = \langle\, B(\mathbf{p}=0)\,|\,A_0(0)\,|\,0\,\rangle$, where $A_0 = \overline{\psi}_l\gamma_0\gamma_5\psi_b \;\rightarrow\; A_0^{\rm HQET}$

$$\begin{aligned}
A_0^{\rm HQET}(x) &= Z_A^{\rm HQET}\left[A_0^{\rm stat}(x) + c_A^{\rm HQET}\,\delta A_0^{\rm stat}(x)\right] \\
A_0^{\rm stat}(x) &= \overline{\psi}_l(x)\gamma_0\gamma_5\psi_h(x) \\
\delta A_0^{\rm stat}(x) &= \overline{\psi}_l(x)\,\tfrac{1}{2}\left(\overleftarrow{\nabla}_i + \overleftarrow{\nabla}^*_i\right)\gamma_i\gamma_5\,\psi_h(x)
\end{aligned}$$

EVs = Functional integral representation at the quantum level:

$$\langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\varphi]\, O[\varphi]\, e^{-(S_{\rm rel}+S_{\rm HQET})} \qquad \mathcal{Z} = \int \mathcal{D}[\varphi]\, e^{-(S_{\rm rel}+S_{\rm HQET})}$$

Instead of including the NLO term in $1/m$ of $\mathcal{L}_{\rm HQET}$ in the action (as this theory wouldn't be renormalizable), the *FI weight* is expanded in a *power series* in $1/m$

$$\exp\{-S_{\rm HQET}\} =$$

$$\exp\left\{-a^4 \sum_x \mathcal{L}_{\rm stat}(x)\right\}$$
$$\times \left\{1 - a^4 \sum_x \mathcal{L}^{(1)}(x) + \tfrac{1}{2}\left[a^4 \sum_x \mathcal{L}^{(1)}(x)\right]^2 - a^4 \sum_x \mathcal{L}^{(2)}(x) + \ldots\right\}$$

$$\Rightarrow \quad \langle O \rangle \; = \; \frac{1}{\mathcal{Z}} \int \mathcal{D}[\varphi]\, e^{-S_{\rm rel}-a^4 \sum_x \mathcal{L}_{\rm stat}(x)}\, O \left\{1 - a^4 \sum_x \mathcal{L}^{(1)}(x) + \ldots\right\}$$

Important implications of this definition of HQET

- $1/m$ – terms appear only as *insertions* of local operators in CFs
  $\Rightarrow$ Power counting: Renormalizability at any given order in $1/m$
- $\Leftrightarrow$ Existence of the continuum limit with universality
- Effective theory $=$ Continuum asymptotic expansion in $1/m$ of QCD

Renormalization & Matching

Renormalization

- The mixing of operators of different dimension in $\mathcal{L}_{\rm HQET}$ induces power divergences [Maiani, Martinelli & Sachrajda, 1992]
  - $\rightarrow$ $\mathcal{L}_{\rm stat}$: linearly divergent additive mass renormalization δm originates from mixing of $\overline{\psi}_{\rm h} D_0 \psi_{\rm h}$ with $\overline{\psi}_{\rm h}\psi_{\rm h}$ $\Rightarrow$ $E^{\rm QCD}_{{\rm h},\bar{\rm h}} = E^{\rm stat}_{{\rm h},\bar{\rm h}}\big|_{\delta m=0} + m_{\rm bare}$

$$m_{\rm bare} = \delta m + m\,,\ \delta m = \frac{c(g_0)}{a} \sim e^{1/(2b_0 g_0^2)} \times \left\{c_1 g_0^2 + c_2 g_0^4 + \ldots\right\}$$

  - $\rightarrow$ PT: uncertainty $=$ truncation error $\sim e^{1/(2b_0 g_0^2)}\, c_{n+1}\, g_0^{2n+2} \stackrel{g_0\to 0}{\longrightarrow} \infty$!
  - $\Rightarrow$ Non-perturbative $c(g_0)$ needed, i.e., *NP renormalization of HQET (resp. fixing of its parameters) required for the continuum limit to exist*
- Power-law divergences even worse at the level of $1/m$ – corrections: $a^{-1} \rightarrow a^{-2}$ (e.g., δm picks up a contribution $a^{-2}\omega_{\rm kin}$)

Matching

- The finite parts of renormalization constants must be fixed s.th. the effective theory describes the underlying theory, QCD
- Proper conditions for these must be imposed from QCD with finite $m_{\rm b}$

Mass dependence at leading order in $1/m$

The rôle of perturbative anomalous dimensions

Consider matrix elements of composite fields involving b-quarks as, e.g., obtained from a QCD correlation function of the heavy-light axial current

$$\begin{aligned} C_{AA}^{QCD}(x_0) &= Z_A^2 a^3 \sum_{\mathbf{x}} \langle A_0(x)(A_0)^\dagger(0)\rangle_{QCD} \\ \left[\Phi^{QCD}\right]^2 &\equiv F_B^2\, m_B = \left|\langle B\,|\,Z_A A_0\,|\,0\rangle\right|^2 \\ &= \lim_{x_0\to\infty}\left[2\exp\{x_0\, m_B^{eff}(x_0)\}\, C_{AA}^{QCD}(x_0)\right] \end{aligned}$$

- B-meson state dominates spectral representation of C_{AA}^{QCD} at large x_0
- $Z_A(g_0)$ fixed by chiral Ward identities, renormalization scale independent

In the static approximation this translates into

$$\left[\Phi(\mu)\right]^2 = \left|\langle B\,|\,Z_A^{stat} A_0^{stat}\,|\,0\rangle\right|^2 = \lim_{x_0\to\infty}\left[2\exp\{x_0\, E_{stat}^{eff}(x_0)\}\, C_{AA}^{stat}(x_0)\right]$$

- $\mu-$dependence in $Z_A(g_0, a\mu) = 1 + g_0^2\,[B_0 - \gamma_0 \ln(a\mu)] + O(g_0^4)$
- Better alternative: work with the RGI opertator $(A_{RGI}^{stat})_0$

How does one get from $\Phi_{\rm RGI} = Z^{\rm stat}_{\rm A,RGI} \langle\, \mathrm{B} \,|\, A_0^{\rm stat} \,|\, 0 \,\rangle$ to $F_{\rm B}$?

Generic structure of the HQET-expansion of QCD matrix elements

$$\Phi = \langle\, \mathrm{B} \,|\, A_0 \,|\, 0 \,\rangle : \quad \Phi^{\rm QCD} \equiv F_{\rm B}\sqrt{m_{\rm B}} = \underbrace{C_{\rm PS}\,(M_{\rm b}/\Lambda)}_{\text{conversion function} \;\Leftarrow\; \text{renormalization}} \times \underbrace{\Phi_{\rm RGI}}_{\text{RGI matrix element in effective theory}} + \mathrm{O}\,(1/M_{\rm b})$$

- In HQET: Absence of chiral symmetry as it is met in (massless) QCD implies a scale dependence $\Phi^{\rm stat}(\mu) \equiv Z^{\rm stat}_{\rm A}(\mu)\,\langle\, \mathrm{B} \,|\, A_0^{\rm stat} \,|\, 0 \,\rangle$
- $M_{\rm b}$ = scale & scheme independent (RG-invariant) b-quark mass

Choosing a convenient scale ($\mu = m_\star = \overline{m}(m_\star)$, $g_\star = \bar{g}(m_\star)$), C_{PS} can be parametrized in terms of RG invariants Λ, M:

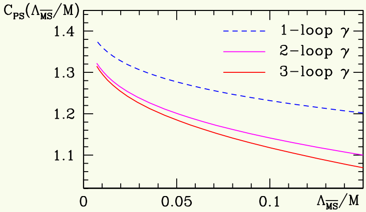
$$\Phi^{QCD} = C_{PS}(M/\Lambda) \times \Phi_{RGI} \;, \quad C_{PS}(M/\Lambda) = \exp\left\{ \int^{g_\star(\frac{M}{\Lambda})} dx \, \frac{\gamma^{match}(x)}{\beta(x)} \right\}$$

To evaluate C_{PS}, insert $\gamma^{match}(g_\star) \overset{g_\star \to 0}{\sim} -\gamma_0 g_\star^2 - \gamma_1^{match} g_\star^4 - \gamma_2^{match} g_\star^6 + \ldots$

$\Rightarrow$ leading large-mass behaviour via $\left.\frac{M}{\Phi}\frac{\partial \Phi}{\partial M}\right|_\Lambda = \left.\frac{M}{C_{PS}}\frac{\partial C_{PS}}{\partial M}\right|_\Lambda = \frac{\gamma^{match}(g_\star)}{1-\tau(g_\star)}$:

$$C_{PS} \overset{M\to\infty}{\sim} \left(2b_0 g_\star^2\right)^{-\gamma_0/(2b_0)} \sim \left[\log(M/\Lambda)\right]^{\gamma_0/(2b_0)}$$

C_{PS} perturbatively under control? [3-loop AD by Chetyrkin & Grozin, 2003]



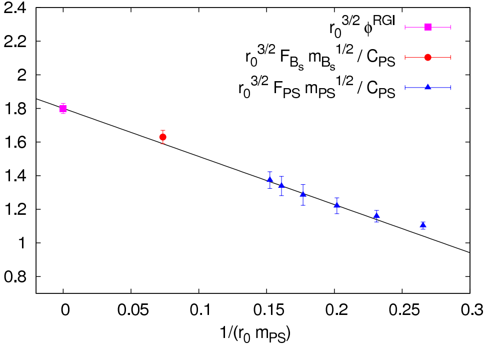
$N_f = 0$

- Full (logarithmic) mass dependence $\in C_{PS}$
- Fig. seems to indicate that the remaining $O(\bar{g}^6(m_b))$ errors are relatively small
  $\rightarrow$ however: a premature conclusion . . .
- For B-Physics: $\Lambda_{\overline{MS}}/M_b \approx 0.04$

An application ($N_f = 0$)

Interpolation between the static limit and the charm region

Della Morte, Dürr, Guazzini, H., Jüttner & Sommer, JHEP0802(2008)078
Blossier, Della Morte, Garron, von Hippel, Mendes, Simma & Sommer, in preparation



Looks good: under a reasonable smoothness assumption, *interpolate* the mass dependence (linearly) in the inverse PS mass to the physical point:

- F_{B_s} follows the heavy quark scaling law, no $1/(r_0 m_{PS})^2$ – effects are visible
  $\rightarrow$ $1/m$ – expansion appears to work very well even for charm quarks
  $\leftarrow$ surprising; needs further confirmation, as the perturbative C_{PS} is used
- Question: What is the accuracy of perturbation theory involved in this ?

Accuracy of perturbation theory in the matching

Bekavac, Grozin, Marquard, Piclum, Seidel & Steinhauser, NPB833(2010)46

From a recent 3-loop computation of $\gamma_{\Gamma}^{\mathrm{match}}$, *ratios* of conversion functions (such as $C_{\mathrm{PS/V}} = C_{\mathrm{PS}}/C_{\mathrm{V}}$) are now known to 4-loop precision

$\Rightarrow$ Outcome: PT is badly behaved for beauty and even worse for charm

"We find that the perturbative series for f_{B^*}/f_B *and* $f_{B^*}^{T}/f_{B^*}$ *converge very slowly at best."* [quote from Bekavac at al., 2010]

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Freedom to "optimize" the scale: [R. Sommer, private communication]

$$\mu = s^{-1} m_\star = \overline{m}(m_\star)\ ,\ \hat{g} = \bar{g}\left(s^{-1} m_\star\right) \qquad C_\Gamma\left(M/\Lambda\right) = \exp\left\{\int^{\hat{g}} dx\, \frac{\hat{\gamma}_\Gamma^{\rm match}(x)}{\beta(x)}\right\}$$

- Matching below $m_\star$, i.e., expect $s > 1$ is better, s.th. decrease of terms in perturbative series is improved once $s \gtrsim 4$
- However: $\alpha(m_b/4)$ is not small then, series unreliable again
- Effective scale is well below $\mu = m_b$; asymptotic convergence of PT only improved far beyond m_b, where it is of limited use for B-physics

$\Rightarrow$ Accuracy is hard to assess, error estimates in the literature too optimistic ?

Mass dependence in finite-volume QCD ($N_f = 2$)

Della Morte, Fritzsch, H. & Sommer, PoS LATTICE2008(2008)226
Fritzsch & H., in progress

Non-perturbative computation of the *heavy quark mass dependence* of heavy-light meson observables in the continuum limit of finite-volume QCD

$\longrightarrow$ Explicit pure theory tests that HQET is an *effective* theory of QCD

$\longrightarrow$ Constraining the large-mass behaviour of QCD by the static limit

- QCD with Schrödinger Functional boundary conditions (T, L, θ)
- $N_f = 2$ NP'ly $O(a)$ improved Wilson action, massless sea quarks
- Evaluation of QCD heavy-light valence quark correlation functions with relativistic heavy quarks from charm to beyond bottom
  (in SF simulations: set light PCAC masses to zero, $m_{\rm light}^{\rm valence} = m^{\rm sea} = 0$)
- Renormalization [ALPHA Collaboration, 2005-2008]
  - Fix $\bar{g}^2(L_1) = 4.484$ s.th. $L_1 \approx 0.5\,\mathrm{fm}$, $L_1/a = 20, 24, 32, 40$, $L_2 = 2L_1$
  - Fix RGI (heavy) quark masses via its NP relation to bare parameters:

$$z \equiv L_1 M = Z_m \frac{M}{\overline{m}(\mu_0)} (1 + b_m a m_q) \times L_1 m_q \qquad Z_m = \frac{Z(g_0)\, Z_A(g_0)}{Z_P(g_0, a\mu_0)}$$

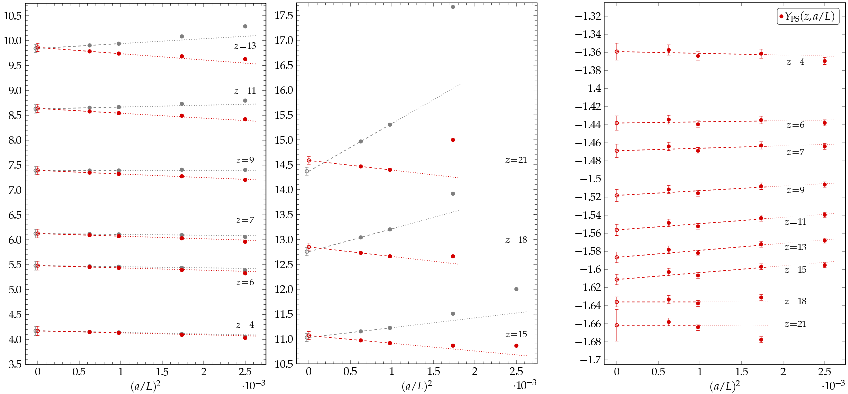
[Fritzsch, H. & Tantalo, arXiv:1004.3978]

Mass dependence in finite-volume QCD ($N_f = 2$)

Della Morte, Fritzsch, H. & Sommer, PoS LATTICE2008(2008)226
Fritzsch & H., in progress

The B-system in finite-volume QCD ($L = L_1$)

- $L_1 = 0.5\,\text{fm}$, z–values covering the b-quark down to the charm quark region
- Removal of all $O\left(\left(\frac{a}{L}\right)^n\right)$ effects at tree-level: $O \rightarrow O_{\text{impr}}(a/L) = \frac{O(a/L)}{1+\delta(a/L)}$
- Examples of continuum extrapolations (B-meson mass & decay constant):

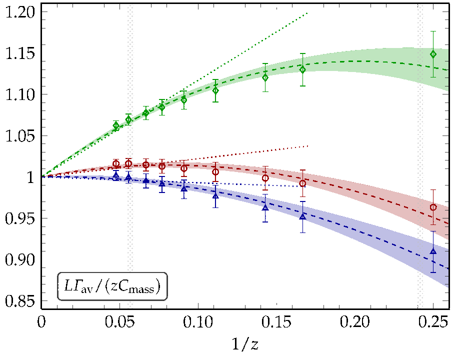


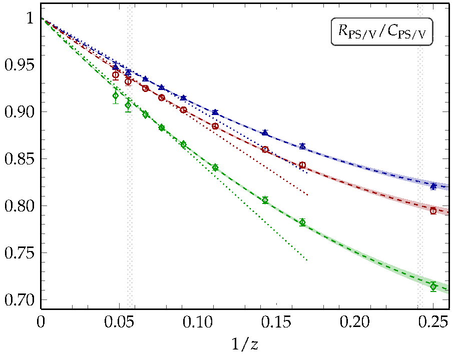
Mass dependence in finite-volume QCD ($N_f = 2$)

Della Morte, Fritzsch, H. & Sommer, PoS LATTICE2008(2008)226
Fritzsch & H., in progress

The B-system in finite-volume QCD ($L = L_1$)

- Tests of HQET: validating and demonstrating the applicability of HQET
- Verification of the approach to the spin-symmetric limit:
  (B-meson mass & ratio of PS to V decay constants)





$\Rightarrow$ Large-mass asymptotics ($1/z \to 0$) confirms HQET predictions

Mass dependence in finite-volume QCD ($N_f = 2$)

Della Morte, Fritzsch, H. & Sommer, PoS LATTICE2008(2008)226

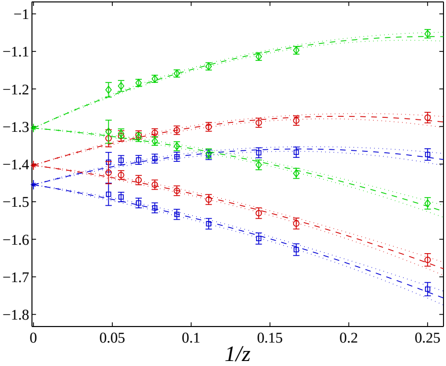
Fritzsch & H., in progress

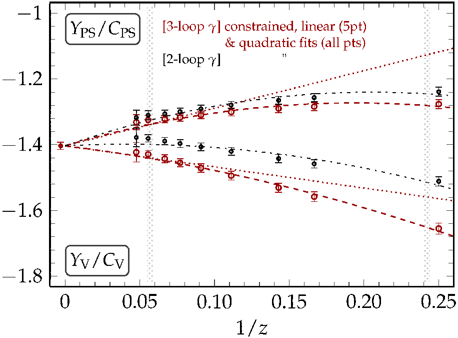
The B-system in finite-volume QCD ($L = L_1$)

- But: some numerical evidence for the previous doubts in the reliability of PT in the b-quark region is found with Y_{PS}, Y_V and its effective theory predictions

$$Y_{PS}(L,z)/C_{PS}(M/\Lambda) \;=\; X_{RGI}(L) \;+\; O(1/z)$$

$$Y_{PS}(L,z;\theta) \;\propto\; Z_A \frac{f_A(L/2,\theta)}{\sqrt{f_1(\theta)}} \qquad X_{RGI}(L;\theta) \;\propto\; Z^{stat}_{A,RGI} \underbrace{\frac{f^{stat}_A(L/2,\theta)}{\sqrt{f^{stat}_1(\theta)}}}_{=X^{stat}(\theta)}$$





Mass dependence in finite-volume QCD ($N_f = 2$)

Della Morte, Fritzsch, H. & Sommer, PoS LATTICE2008(2008)226

Fritzsch & H., in progress

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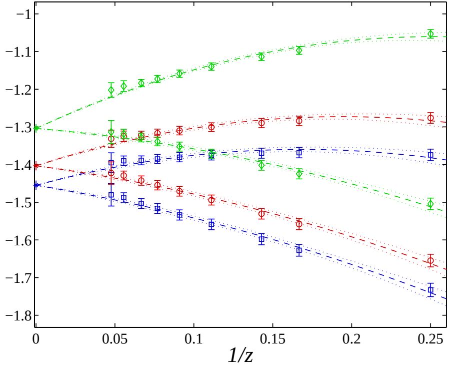
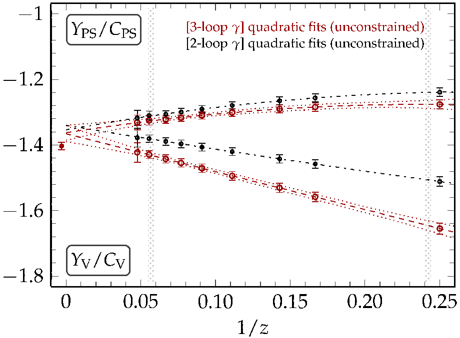
$$Y_{PS}(L,z)/C_{PS}(M/\Lambda) \quad = \quad X_{RGI}(L) \; + \; O(1/z)$$

$$Y_{PS}(L,z;\theta) \; \propto \; Z_A \frac{f_A(L/2,\theta)}{\sqrt{f_1(\theta)}} \qquad X_{RGI}(L;\theta) \; \propto \; Z^{stat}_{A,RGI} \underbrace{\frac{f^{stat}_A(L/2,\theta)}{\sqrt{f^{stat}_1(\theta)}}}_{=X^{stat}(\theta)}$$





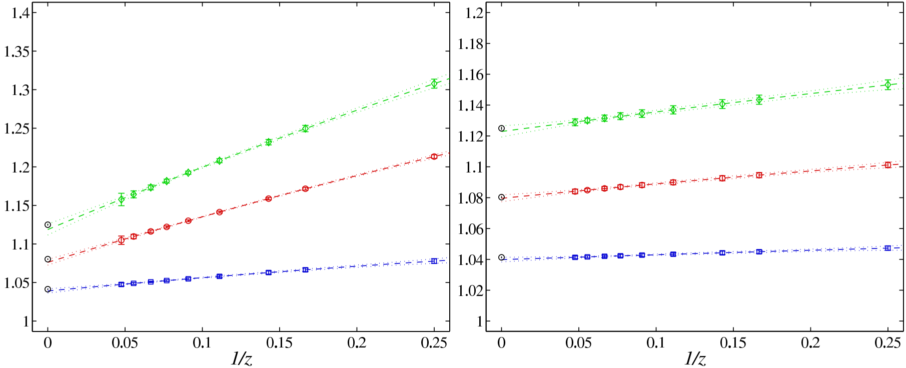
Mass dependence in finite-volume QCD ($N_f = 2$)

Della Morte, Fritzsch, H. & Sommer, PoS LATTICE2008(2008)226
Fritzsch & H., in progress

The B-system in finite-volume QCD ($L = L_1$)

- Consider *ratios* instead, where C_{PS} cancels completely:

$$\frac{Y_{PS}(z;\theta_1)}{Y_{PS}(z;\theta_2)} = \frac{X^{stat}(\theta_1)}{X^{stat}(\theta_2)} + O(1/z)$$



$\Rightarrow$ These turn smoothly & unconstrained into effective theory predictions

Determination of HQET parameters at $\mathrm{O}(1/m)$

Blossier, Della Morte, Garron & Sommer, arXiv:1001.4783

Vector of the $N_{\mathrm{HQET}} = 5$ parameters in $S_{\mathrm{HQET}}, A_0^{\mathrm{HQET}}$ up to $\mathrm{O}(1/m_b)$:

$$\omega = \begin{pmatrix} \omega^{\mathrm{stat}} \\ \omega^{(1/m)} \end{pmatrix}$$

$$\omega^{\mathrm{stat}} = \left(m_{\mathrm{bare}} \, , \, \ln(Z_A^{\mathrm{HQET}}) \right)^t$$

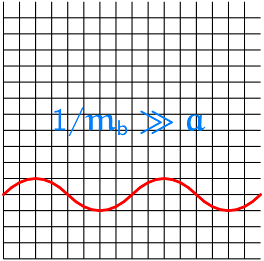
$$\omega^{(1/m)} = \left(c_A^{\mathrm{HQET}} \, , \, \omega_{\mathrm{kin}} \, , \, \omega_{\mathrm{spin}} \right)^t$$

ω_i	classical value	static value
m_{bare}	m_b	$m_{\mathrm{bare}}^{\mathrm{stat}}$
$\ln(Z_A^{\mathrm{HQET}})$	0	$\ln(Z_{A,RGI}^{\mathrm{stat}} C_{PS})$
c_A^{HQET}	$-1/(2m_b)$	$a c_A^{\mathrm{stat}}$
ω_{kin}	$1/(2m_b)$	0
ω_{spin}	$1/(2m_b)$	0

$\Rightarrow$ Trick: *non-perturbative* matching of HQET to QCD in a *finite* volume

[H. & Sommer, JHEP0402(2004)022]

QCD

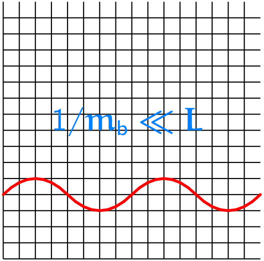


Matching conditions

$$\Phi_i^{\mathrm{QCD}} = \Phi_i^{\mathrm{HQET}}$$

for observables Φ_i

(renormal. quantities, computable for $a \to 0$)



HQET

NP matching in $L = L_1$

Suitable observables in the Schrödinger functional, $L = T = L_1 \approx 0.5\,\mathrm{fm}$

$$\Phi_i(L_1, M, a) \qquad i = 1, \ldots, N_{\mathrm{HQET}}$$

Matching conditions for $i = 1, \ldots, N_{\mathrm{HQET}}$ (note: $a \leftrightarrow g_0$)

$$\lim_{a\to 0} \Phi_i^{\mathrm{QCD}}(L_1, M, a) = \Phi_i^{\mathrm{QCD}}(L_1, M, 0) = \Phi_i^{\mathrm{HQET}}(L_1, M, a)$$

Conveniently, one chooses observables linear in ω_i, e.g.

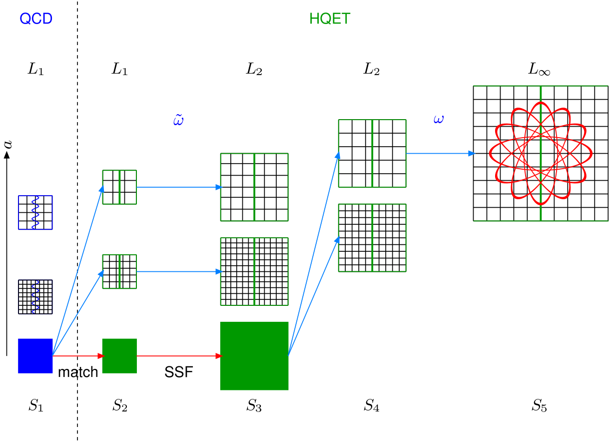
$$\begin{aligned}
\Phi(L, M, a) &= \eta(L, a) + \phi(L, a)\, \omega(M, a) \\
\Phi_1 &= L \,\langle\, B(L) \,|\, \mathbb{H} \,|\, B(L) \,\rangle \overset{L\to\infty}{\sim} L m_B \\
\Phi_2 &= \ln\left(L^{3/2} \,\langle\, \Omega(L) \,|\, A_0 \,|\, B(L) \,\rangle \right) \overset{L\to\infty}{\sim} \ln\left(L^{3/2} F_B \sqrt{m_B/2} \right) \\
&\ldots
\end{aligned}$$

$$\eta = \begin{pmatrix} \Gamma^{\mathrm{stat}} = \langle\, B(L) \,|\, \mathbb{H} \,|\, B(L) \,\rangle_{\mathrm{stat}} \\ \zeta_A = \ln\left(L^{3/2} \,\langle\, \Omega(L) \,|\, A_0 \,|\, B(L) \,\rangle_{\mathrm{stat}} \right) \\ \ldots \end{pmatrix} \qquad \phi = \begin{pmatrix} L & 0 & \cdots \\ 0 & 1 & \cdots \\ \ldots & & \end{pmatrix}$$

Step scaling to $L = L_2$

Matching volume $L_1 \approx 0.5\,\mathrm{fm}$ has very small a, but larger a are needed

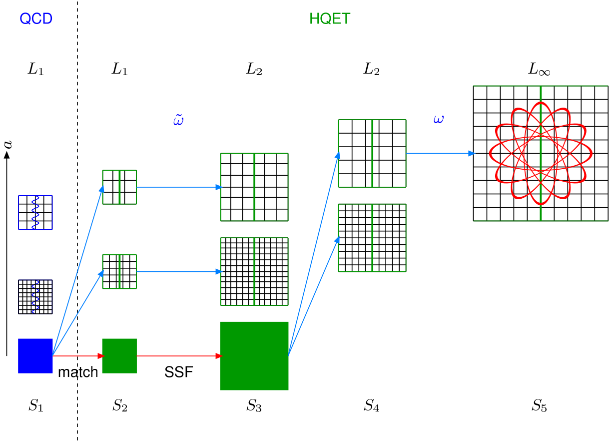
$\Rightarrow$ Gap to large volume & practicable lattice spacings, where physical quantities (m_B, F_B) are extracted, bridged by finite-size scaling steps



Fully NP, CL can be taken everywhere, $L \to 2L$ via Step Scaling Functions

$$\Phi_i^{\mathrm{HQET}}(2L) = \sigma_i\left(\left\{\Phi_j^{\mathrm{HQET}}(L), j = 1, \ldots, N_{\mathrm{HQET}}\right\}\right) \qquad 2L = 2L_1 \approx 1.0\,\mathrm{fm}$$

Step scaling to $\mathrm{L} = \mathrm{L}_2$



Finite-size scaling to $\mathrm{L}_2 = 2\mathrm{L}_1$:

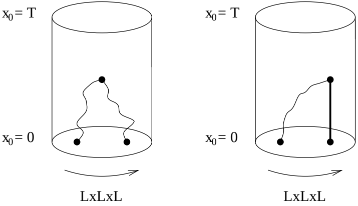
- Amounts to solve a matrix equation to obtain the HQET parameters at larger lattice spacings . . .
- . . . corresponding to β–values for simulations in large volume, "L_∞", where a B-meson in HQET fits comfortably

Computational setup

- Convenient finite-volume framework: QCD Schrödinger Functional
  [Lüscher et al., 1992; Sint, 1994]

  $\exists$ HQET expansions of (renormalized) SF CFs up to first order in $1/m$, including $m_{\rm bare}, Z_A^{\rm HQET}$ and insertions $c_A^{\rm HQET}\delta A_0^{\rm stat}$, $\omega_{\rm kin}\mathcal{O}_{\rm kin}$, $\omega_{\rm spin}\mathcal{O}_{\rm spin}$

  $f_A(x_0) =$ $\qquad$ $f_A^{\rm stat}(x_0) =$



- High numerical accuracy of NP HQET thanks to technical advances:
  - HYP-smeared static actions, giving improved statistical precision
    [Hasenfratz & Knechtli, 2001; ALPHA Collaboration 2004/05]

    $\rightarrow$ this change of action does not introduce large cutoff effects
  - In large V, evaluate them solving the Generalized EigenValue Problem:
    [Michael & Teasdale, 1983; Lüscher & Wolff, 1990; ALPHA Collaboration, Blossier et al., 2009]
    Analysis of matrix correlators s.th. a larger gap dominates the excited state corrections and these disappear more quickly with growing x_0

$$E_n^{\rm eff}(t,t_0) \;=\; E_n \,+\, \beta_n(t_0)\,e^{-(E_{N+1}-E_n)\,t}$$

Use of the HQET parameters

These HQET parameters can finally be exploited for phenomenological applications in the $B_{(s)}$−meson system, e.g.

- to calculate the b-quark mass and the $B_{(s)}$−meson decay constant:

$$m_B = m_{bare} + E_{stat} + \omega_{kin} E_{kin} + \omega_{spin} E_{spin}$$

$$\frac{\Phi}{\sqrt{2}} \equiv F_B \sqrt{m_B/2} = Z_A^{HQET} \left(1 + b_A^{stat} a m_q\right) p_{stat} \times \left(1 + c_A^{HQET} p_{\delta A} + \omega_{kin} p_{kin} + \omega_{spin} p_{spin}\right)$$

- Mass splittings, such as (radial) excitation energies of $B_{(s)}$−states and the $B_{(s)} - B^*_{(s)}$ mass difference to $O(1/m_b)$:

$$\Delta E_{n,1}^{HQET} = \left(E^n_{stat} - E^1_{stat}\right) + \omega_{kin}\left(E^n_{kin} - E^1_{kin}\right) + \omega_{spin}\left(E^n_{spin} - E^1_{spin}\right)$$

$$\Delta E_{P-V} = \frac{4}{3}\,\omega_{spin} E^1_{spin}$$

E^i_y, p_y : plateau averages of (bare) effective HQET energies and matrix elements in large volume

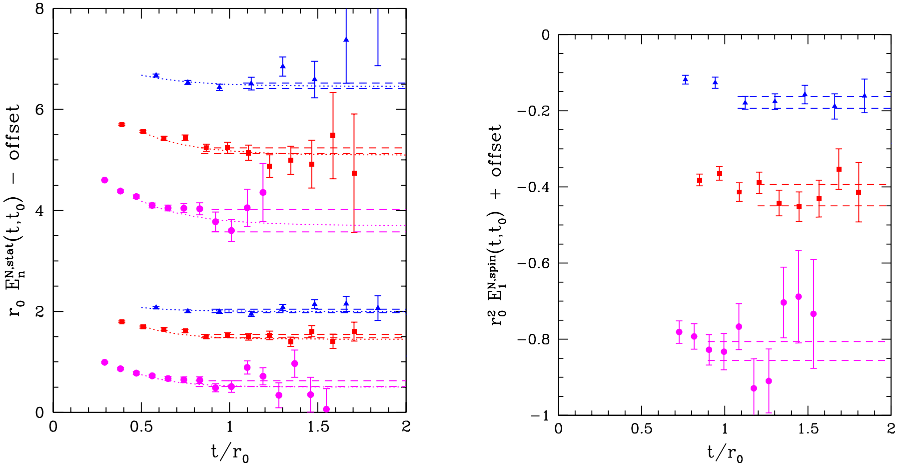
- Note: The power-divergent δm drops out in energy *differences*

Some examples of $N_f = 0$ results

Blossier, Della Morte, Garron, von Hippel, Mendes, Simma & Sommer, arXiv:1004.2661

Excited state energy levels, $a \approx (0.1, 0.08, 0.05)$ fm, $L \approx 1.5$ fm, $T = 2L$

- CF matrices $C^{stat}_{ij}(t) = \sum_{x,\mathbf{y}} \langle O_i(x_0 + t, \mathbf{y})\, O^*_j(x) \rangle_{stat}$ & $\mathcal{O}_{spin/kin}$ insertions
- GEVP: all-to-all propagators, $t-$dilution, Gaussian smeared variational basis



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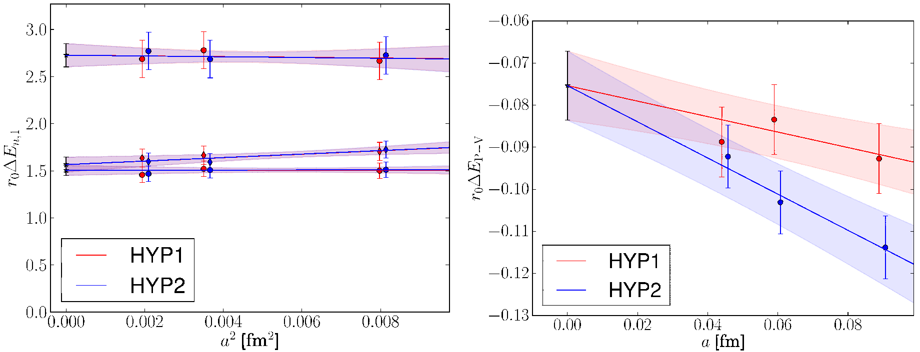


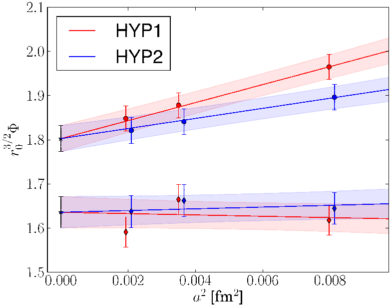
- Linear a–term suppressed by $1/m_b$, physical $O(1/m_b)$ corrections are small
- Divergences cancel after proper NP renormalization

  $\Rightarrow$ Strong *numerical* evidence for the renormalizability of HQET

Some examples of $N_f = 0$ results

Blossier, Della Morte, Garron, von Hippel, Mendes, Simma & Sommer, in preparation

Computation of F_{B_s} in HQET matches at m_{B_s} with interpolating between the charm sector (around F_{D_s}) and $F_{B_s}^{stat}$



- HYP & GEVP lead to (2 – 3)% precision for F_{B_s} in the continuum limit, i.e., $r_0 = 0.5$ fm: $F_{B_s}^{stat} = 229(3)$ MeV , $F_{B_s}^{stat+1/m} = 212(5)$ MeV
(using $r_0 = 0.45$ fm leads to $\simeq$ 15% increase, but $O(1/m_b^2)$ corrections are small)
- Given the unclear precision of PT, interpolation methods have to be taken with care; the inherent perturbative error remains to be estimated
- Data points beyond charm difficult for $N_f > 0$, obtain slope directly in HQET

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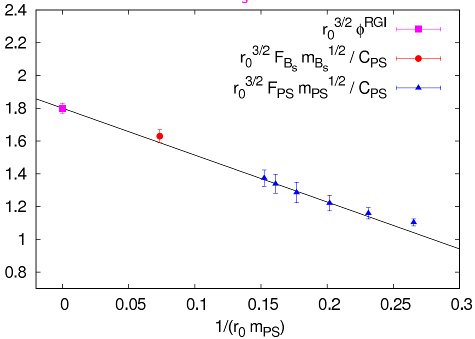

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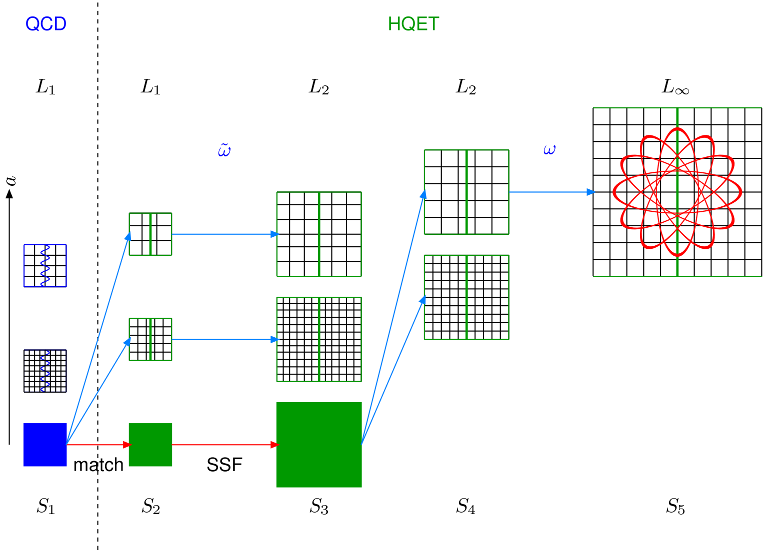
First physical results in the two-flavour theory

Which ingredients are needed ?

Recall the strategy . . .



First physical results in the two-flavour theory

Which ingredients are needed ?

S_1 NP matching of HQET to QCD in finite volume with a relativistic b, to perform the power-divergent subtractions

- Crucial element of this step:
  Calculation of the *heavy quark mass dependence* of heavy-light meson observables in the continuum limit of finite-volume QCD (L_1)
- . . . already discussed above

$S_{2,3,4}$ HQET computations in small & intermediate volumes

- Evaluation of the HQET step scaling functions to connect the small matching ($L_1 \approx 0.5\,\text{fm}$) to the intermediate volume ($L_2 = 2L_1 \approx 1\,\text{fm}$)
- Interpolation of the resulting HQET parameters to the large-volume "L_∞" lattice spacings ($\beta = 5.2, 5.3, 5.5$)

S_5 HQET computations in large volume

- Extract HQET energies & matrix elements, using $N_f = 2$ dynamical configurations in large volume ("L_∞", periodic b.c.'s) produced by CLS
- Action: NP'ly $O(a)$ improved $N_f = 2$ Wilson ; algorithm: DD-HMC
- Problem of slow sampling of topology less relevant here, since HQET can afford to work with much coarser lattices

HQET energies & matrix elements (preliminary)

ALPHA Collaboration, talk by B. Blossier

CLS based

Preliminary $N_f = 2$ HQET results in large volume

- Gauge configuration ensembles with $N_f = 2$ $O(a)$ improved Wilson fermions

β	a [fm]	$L^3 \times T$	m_π [MeV]	#	traj. sep.
5.2	0.08	$32^3 \times 64$	700	110	16
		$32^3 \times 64$	370	160	16
5.3	0.07	$32^3 \times 64$	550	152	32
		$32^3 \times 64$	400	600	32
		$48^3 \times 96$	300	192	16
		$48^3 \times 96$	250	350	16
5.5	0.05	$32^3 \times 64$	430	250	20
		$48^3 \times 96$	430	30	16

- Use of HYP-smearing & variant of the stochastic all-to-all propagator method for the light quarks (8 noise sources, full time-dilution) [Foley et al., 2005]
- GEVP: cleanly quantify systematic errors from excited state contaminations (variational basis of interpolating fields through Gaussian smearing levels)
- Energies, splittings, ground & excited state matrix elements of the B, . . .

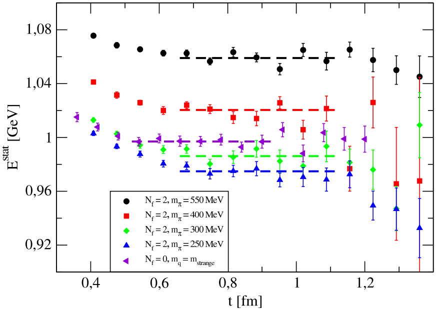
HQET energies & matrix elements (preliminary)

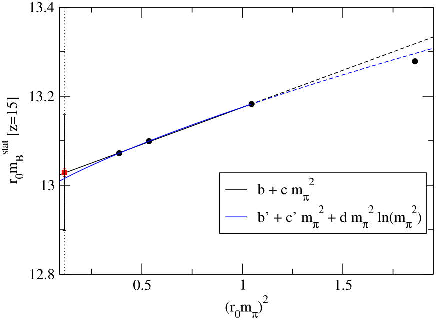
ALPHA Collaboration, talk by B. Blossier

CLS based

Static energies ($\beta = 5.3$, $a \approx 0.07$ fm) & extrapolation to the chiral limit, where the r_0/a uncertainty is still large

[Scale prelim.; talk by B. Leder]



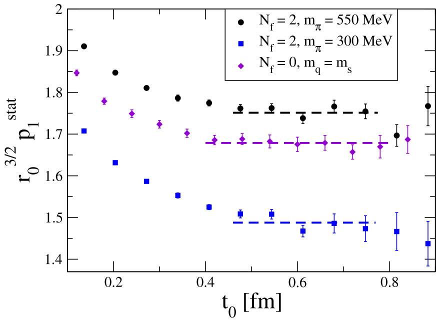


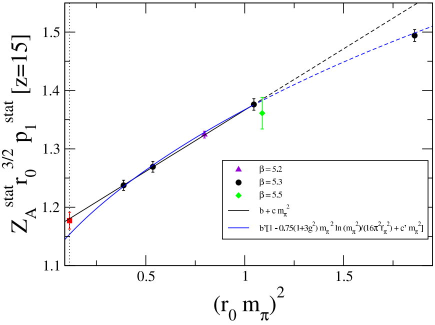
HQET energies & matrix elements (preliminary)

ALPHA Collaboration, talk by B. Blossier

CLS based

F_B: renormalized (not $O(a)$ improved) matrix element of $A_0^{\rm stat}$, data well described by HMχPT



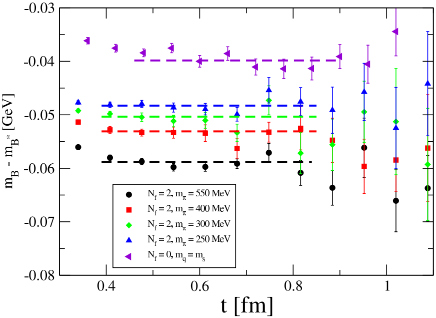


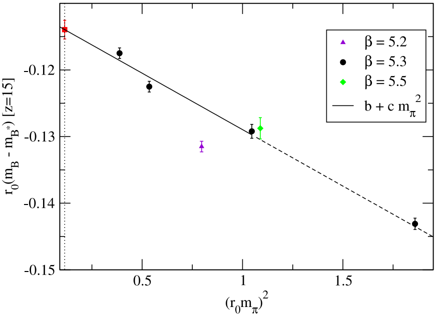
HQET energies & matrix elements (preliminary)

ALPHA Collaboration, talk by B. Blossier

CLS based

Spin-splitting: situation for $O(1/m)$ terms of energies is encouraging





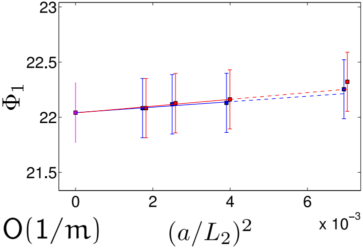
HQET parameters (preliminary)

ALPHA Collaboration, talk by N. Garron

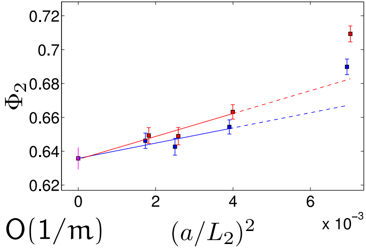
After evolution to L_2 where $5.3 \lesssim \beta \lesssim 5.8$

$\Phi_1 = L \langle B(L) | \mathbb{H} | B(L) \rangle$ $\Phi_2 = \ln \left(L^{3/2} \langle \Omega(L) | A_0 | B(L) \rangle \right)$

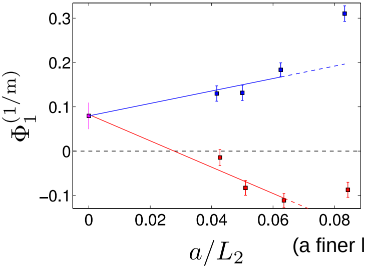
$O(m)$



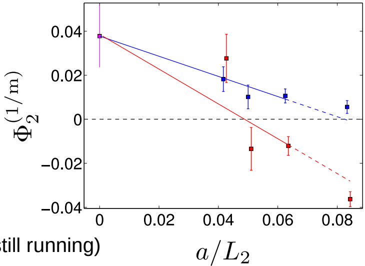
$O(1)$



$O(1/m)$



$O(1/m)$



(a finer lattice resolution is still running)

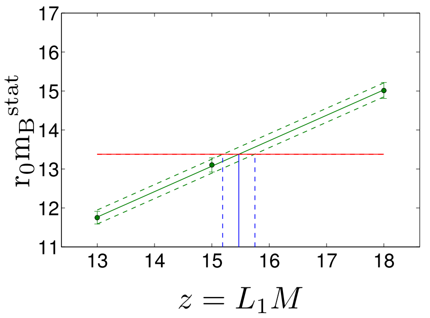
b-quark mass interpolation (preliminary)

ALPHA Collaboration, talk by N. Garron

Now insert $\omega_1 \in \omega(M, a)$ for $N_f = 2$:

$$
\begin{aligned}
m_B &= \omega_1 + E_{stat} = m_{bare} + E_{stat} = \omega_1 + E_{stat} \\
&= \lim_{a\to 0} \left[E_{stat} - \Gamma^{stat}(L_2, a) \right] \qquad a = (0.1 - 0.05)\,\mathrm{fm} \\
&\quad + \lim_{a\to 0} \left[\Gamma^{stat}(L_2, a) - \Gamma^{stat}(L_1, a) \right] \qquad a = (0.05 - 0.025)\,\mathrm{fm} \\
&\quad + \frac{1}{L_1} \lim_{a\to 0} \Phi_1(L_1, M_b, a) \qquad a = (0.025 - 0.012)\,\mathrm{fm}
\end{aligned}
$$

Analysis with $r_0 m_B^{(exp)}$, $r_0 = (0.475 \pm 0.025)$ fm [Scale prelim.; talk by B. Leder]



- $\overline{m}_b^{\overline{MS}}(\overline{m}_b)^{stat} = 4.255(25)_{r_0}(50)_{stat+renorm}(?)_a$ GeV
- NP renormalization; no CL yet in the large volume part (only $\beta = 5.3$)
- Error dominated by $\approx 1\%$ on Z_M in $L_1 M = Z_M Z (1 + b_m a m_q) \times L_1 m_q$
- Dependence on the matching kinematics is very small

b-quark mass interpolation (preliminary)

ALPHA Collaboration, talk by N. Garron

Now insert $\omega_1 \in \omega(M, a)$ for $N_f = 2$:

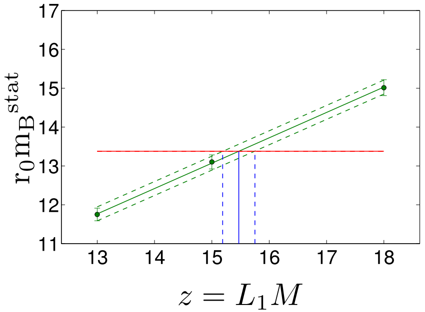
$$m_B = \omega_1 + E_{stat} = m_{bare} + E_{stat} = \omega_1 + E_{stat}$$

$$= \lim_{a\to 0} \left[E_{stat} - \Gamma^{stat}(L_2, a) \right] \qquad a = (0.1 - 0.05)\,\text{fm}$$

$$+ \lim_{a\to 0} \left[\Gamma^{stat}(L_2, a) - \Gamma^{stat}(L_1, a) \right] \qquad a = (0.05 - 0.025)\,\text{fm}$$

$$+ \frac{1}{L_1} \lim_{a\to 0} \Phi_1(L_1, M_b, a) \qquad a = (0.025 - 0.012)\,\text{fm}$$

Analysis with $r_0 m_B^{(exp)}$, $r_0 = (0.475 \pm 0.025)$ fm [Scale prelim.; talk by B. Leder]



- $\overline{m}_b^{\overline{MS}}(\overline{m}_b)^{stat+1/m} = 4.276(25)_{r_0}(50)_{stat+renorm}(?)_a$ GeV
- NP renormalization; no CL yet in the large volume part (only $\beta = 5.3$)
- Error dominated by $\approx 1\%$ on Z_M in $L_1 M = Z_M\, Z\,(1 + b_m a m_q) \times L_1 m_q$
- Dependence on the matching kinematics is very small

b-quark mass interpolation (preliminary)

ALPHA Collaboration, talk by N. Garron

Now insert $\omega_1 \in \omega(M, a)$ for $N_f = 2$:

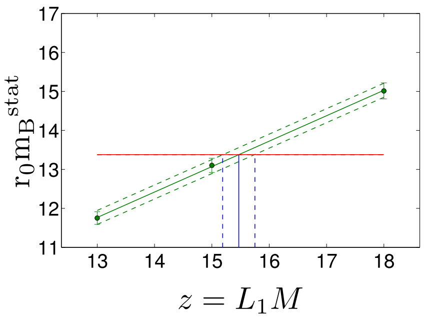
$$m_B = \omega_1 + E_{stat} = m_{bare} + E_{stat} = \omega_1 + E_{stat}$$

$$= \lim_{a\to 0} \left[E_{stat} - \Gamma^{stat}(L_2, a) \right] \qquad a = (0.1 - 0.05)\,\mathrm{fm}$$

$$+ \lim_{a\to 0} \left[\Gamma^{stat}(L_2, a) - \Gamma^{stat}(L_1, a) \right] \qquad a = (0.05 - 0.025)\,\mathrm{fm}$$

$$+ \frac{1}{L_1} \lim_{a\to 0} \Phi_1(L_1, M_b, a) \qquad a = (0.025 - 0.012)\,\mathrm{fm}$$

Analysis with $r_0 m_B^{(exp)}$, $r_0 = (0.475 \pm 0.025)\,\mathrm{fm}$ [Scale prelim.; talk by B. Leder]



- $\overline{m}_b^{\overline{MS}}(\overline{m}_b)^{stat+1/m} = 4.320(40)_{r_0}(48)\,\mathrm{GeV}$ ($N_f = 0$!)
- NP renormalization; no CL yet in the large volume part (only $\beta = 5.3$)
- Error dominated by $\approx 1\%$ on Z_M in $L_1 M = Z_M\, Z\,(1 + b_m a m_q) \times L_1 m_q$
- Dependence on the matching kinematics is very small

Unquenching effect is presently not significant

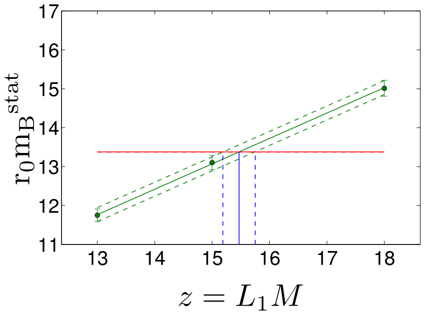
b-quark mass interpolation (preliminary)

ALPHA Collaboration, talk by N. Garron

Now insert $\omega_1 \in \omega(M, a)$ for $N_f = 2$:

$$\begin{aligned} m_B \;=\;& \omega_1 + E_{stat} = m_{bare} + E_{stat} = \omega_1 + E_{stat} \\ =\;& \lim_{a\to 0}\left[E_{stat} - \Gamma^{stat}(L_2, a)\right] && a = (0.1 - 0.05)\,\text{fm} \\ &+ \lim_{a\to 0}\left[\Gamma^{stat}(L_2, a) - \Gamma^{stat}(L_1, a)\right] && a = (0.05 - 0.025)\,\text{fm} \\ &+ \frac{1}{L_1}\lim_{a\to 0}\Phi_1(L_1, M_b, a) && a = (0.025 - 0.012)\,\text{fm} \end{aligned}$$

Analysis with $r_0 m_B^{(exp)}$, $r_0 = (0.475 \pm 0.025)$ fm [Scale prelim.; talk by B. Leder]



- $\overline{m}_b^{\overline{MS}}(\overline{m}_b)^{stat+1/m} = 4.276(25)_{r_0}(50)_{stat+renorm}(?)_a$ GeV
- NP renormalization; no CL yet in the large volume part (only $\beta = 5.3$)
- Error dominated by $\approx 1\%$ on Z_M in $L_1 M = Z_M\, Z\,(1 + b_m a m_q) \times L_1 m_q$
- Dependence on the matching kinematics is very small

Unquenching effect is presently not significant

Conclusions

- Lattice heavy flavour physics has become a precision field
- Lattice QCD inputs have to be pushed to few-% level (incl. reliable assessment of all systematics), to contribute to uncovering signals for BSM physics in CKM analyses and resolve / support current tensions
- Dynamical quark simulations ($N_f = 2, 2+1, 2+1+1$) are routine: $m_\pi \sim 500$ MeV (2001) $\rightarrow$ $m_\pi \lesssim 250$ MeV (2010), but the behaviour of algorithms at small lattices spacings needs to be understood
- Lattice artefacts are being investigated, but there are not yet always systematic continuum limit extrapolations
- Non-perturbative renormalization & matching in HQET is doable with considerable accuracy
- Cross-checks between different calculations employing different techniques are demanded to ensure credibility in our lattice results and its impact for phenomenology

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