Improving hadron creation operators on the lattice

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Lattice 2010

Sardinia

1. Introduction - quark smearing
2. Distillation
3. Making hadron measurements with distillation
4. Some first results
5. Stochastic estimation

Hadron Spectrum Collaboration

Carnegie Mellon
Justin Foley, David Lenkner, Colin Morningstar,
Chik-Him Wong

Jefferson Lab
Jo Dudek, Robert Edwards, David Richards,
Christopher Thomas

U Maryland
Steve Wallace

U Pacific
Keisuke Jimmy Juge

TIFR, Mumbai
Nilmani Mathur

Trinity College Dublin
MP, Sinéad Ryan

U Washington
Huey-Wen Lin

DESY Zeuthen
John Bulava

The method

1. "A Novel quark-field creation operator construction for hadronic physics in lattice QCD."
   MP *et.al.* Phys.Rev.D80:054506,2009.

Results using the method

1. "Highly excited and exotic meson spectrum from dynamical lattice QCD"
   J. Dudek *et.al.* Phys.Rev.Lett.103:262001,2009.
2. "Nucleon, Δ and Ω excited states in $N_f = 2 + 1$ lattice QCD."
   J. Bulava *et.al* arXiv:1004.5072
3. "Toward the excited meson spectrum of dynamical QCD"
   J. Dudek *et.al.* arXiv:1004.4930

. . . and a number of conference proceedings

Other presentations at this meeting

1. Justin Foley (CMU)
2. Keisuke Jimmy Juge (U Pacific)
3. Sinéad Ryan (TCD)
4. Christopher Thomas (JLab)
5. Stephen Wallace (U Maryland)
6. Chik-Kim Wong (CMU)

Motivation

- Computing quark propagation in configuration generation and observable measurement is expensive.
- Objective: extract as much information from correlation functions as possible.

Two problems:

1. Most correlators: signal-to-noise falls exponentially
2. Making measurements can be costly:
    - Variational bases
    - Exotic states using more sophisticated creation operators
    - Isoscalar mesons
    - **Multi-hadron states**

- Good operators are **smeared**; helps with problem 1, can it help with problem 2?

Smearing

- **Smeared field:** $\tilde{\psi}$ from ψ, the "raw" quark field in the path-integral:

$$\tilde{\psi}(t) = \Box[U(t)] \;\; \psi(t)$$

- Extract the essential degrees-of-freedom.
- Smearing should preserve symmetries of quarks.
- Now form creation operator (e.g. a meson):

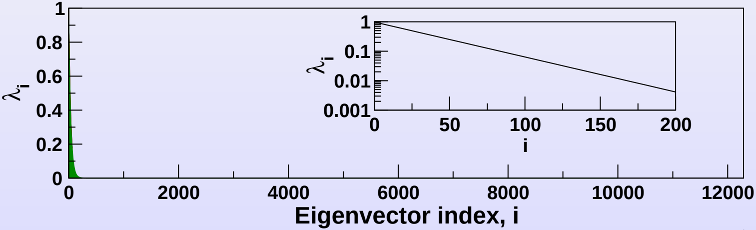
$$O_M(t) = \bar{\tilde{\psi}}(t)\Gamma\tilde{\psi}(t)$$

- Γ: operator in $\{\underline{s}, \sigma, c\} \equiv \{\text{position,spin,colour}\}$
- Smearing: overlap $\langle n|O_M|0\rangle$ is large for low-lying eigenstate $|n\rangle$

- Many recipes in use. One popular gauge covariant choice is **gaussian** smearing:

$$\lim_{n\to\infty}\left(1+\frac{\sigma\nabla^2}{n}\right)^n=\exp(\sigma\nabla^2)$$

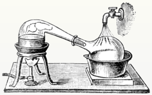
- This acts in the space of coloured scalar fields on a time-slice: $N_s \times N_c$



- Data from $a_s \approx 0.12\text{fm}\ 16^3$ lattice: $16^3 \times 3 = 12288$.

Distillation

"*distill*: to extract the quintessence of" [OED]



- Distillation: **define** smearing to be explicitly a very low-rank operator. Rank is $N_{\mathcal{D}}(\ll N_s \times N_c)$.

Distillation operator

$$\Box(t) = V(t)V^{\dagger}(t)$$

with $V^{a}_{\underline{x},c}(t)$ a $N_{\mathcal{D}} \times (N_s \times N_c)$ matrix

- Example (used to date): $\Box_{\nabla}$ the **projection operator into $\mathcal{D}_{\nabla}$, the space spanned by the lowest eigenmodes of the 3-D laplacian**
- Projection operator, so idempotent: $\Box_{\nabla}^{2} = \Box_{\nabla}$
- $\lim_{N_{\mathcal{D}} \to (N_s \times N_c)} \Box_{\nabla} = I$
- Eigenvectors of ∇^2 not the only choice. . .

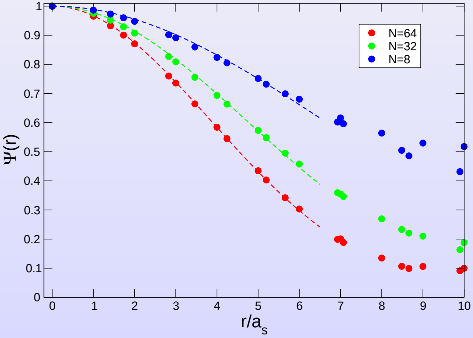
Distillation: preserve symmetries

- Using eigenmodes of the gauge-covariant laplacian **preserves lattice symmetries**

$$U_i(\underline{x}) \xrightarrow{g} U_i^g(\underline{x}) = g(\underline{x})U_i(\underline{x})g^\dagger(\underline{x}+\hat{\underline{i}})$$

$$\Box_\nabla(\underline{x},\underline{y}) \xrightarrow{g} \Box_\nabla^g(\underline{x},\underline{y}) = g(\underline{x})\Box_\nabla(\underline{x},\underline{y})g^\dagger(\underline{y})$$

- Translation, parity, charge-conjugation symmetric
- O_h symmetric
- Close to $SO(3)$ symmetric
- “local” operator



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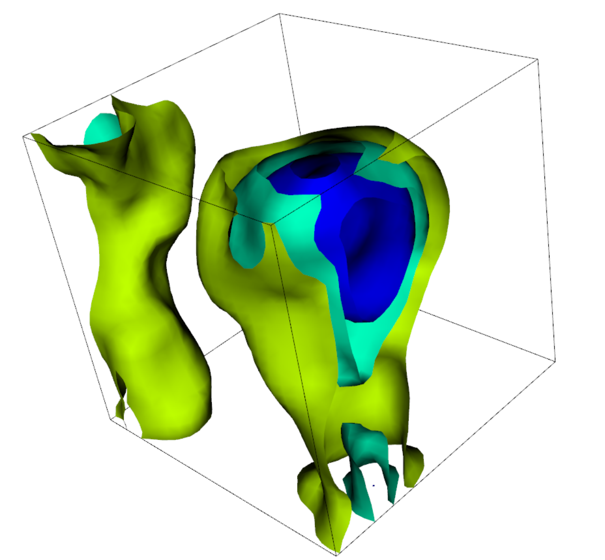
Eigenmodes of the laplacian



- Lowest mode on a $32^3 \equiv (3.8\ \mathrm{fm})^3$ lattice.

- Consider an isovector meson two-point function:

$$C_M(t_1 - t_0) = \langle\langle \bar{u}(t_1)\Box_{t_1}\Gamma_{t_1}\Box_{t_1} d(t_1) \quad \bar{d}(t_0)\Box_{t_0}\Gamma_{t_0}\Box_{t_0} u(t_0)\rangle\rangle$$

- Integrating over quark fields yields

$$C_M(t_1 - t_0) =$$
$$\langle \mathrm{Tr}_{\{\underline{s},\sigma,c\}} \left(\Box_{t_1}\Gamma_{t_1}\Box_{t_1} M^{-1}(t_1, t_0)\Box_{t_0}\Gamma_{t_0}\Box_{t_0} M^{-1}(t_0, t_1) \right) \rangle$$

- Substituting the low-rank distillation operator $\Box$ reduces this to a **much smaller** trace:

$$C_M(t_1 - t_0) = \langle \mathrm{Tr}_{\{\sigma,\mathcal{D}\}} \left[\Phi(t_1)\tau(t_1, t_0)\Phi(t_0)\tau(t_0, t_1) \right] \rangle$$

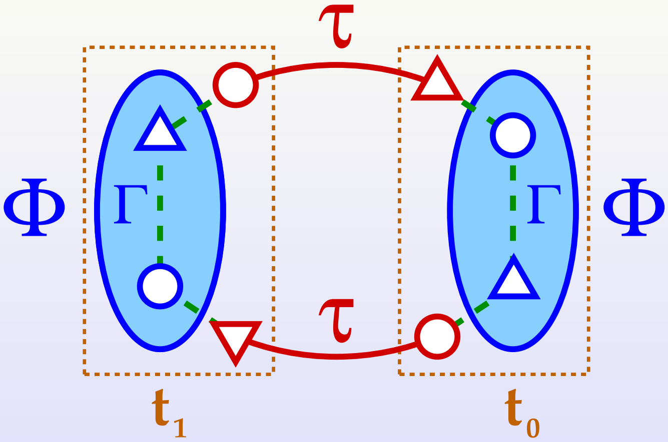
- $\Phi^{\alpha,a}_{\beta,b}$ and $\tau^{\alpha,a}_{\beta,b}$ are $(N_\sigma \times N_\mathcal{D}) \times (N_\sigma \times N_\mathcal{D})$ matrices.

$$\Phi(t) = V^\dagger(t)\Gamma_t V(t)$$

$$\tau(t, t') = V^\dagger(t) M^{-1}(t, t') V(t')$$

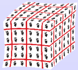
The "perambulator"

Meson two-point function

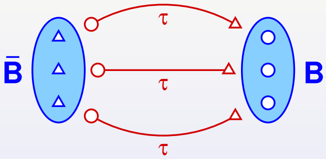


Distilled meson two-point correlation function

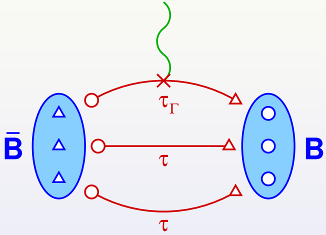
$$C_M(t_1 - t_0) = \mathrm{Tr}_{\{\sigma,D\}} \left[\Phi(t_1)\, \tau(t_1, t_0)\, \Phi(t_0)\, \tau(t_0, t_1)\right]$$



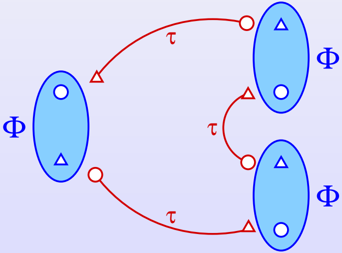
More diagrams



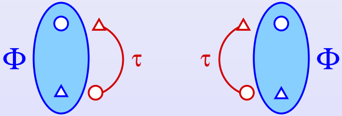
$\bar{B}_{abc}\tau_{aa'}\tau_{bb'}\tau_{cc'}B_{a'b'c'}$



$\bar{B}_{abc}\tau_{aa'}\tau^{\Gamma}_{bb'}\tau_{cc'}B_{a'b'c'}$



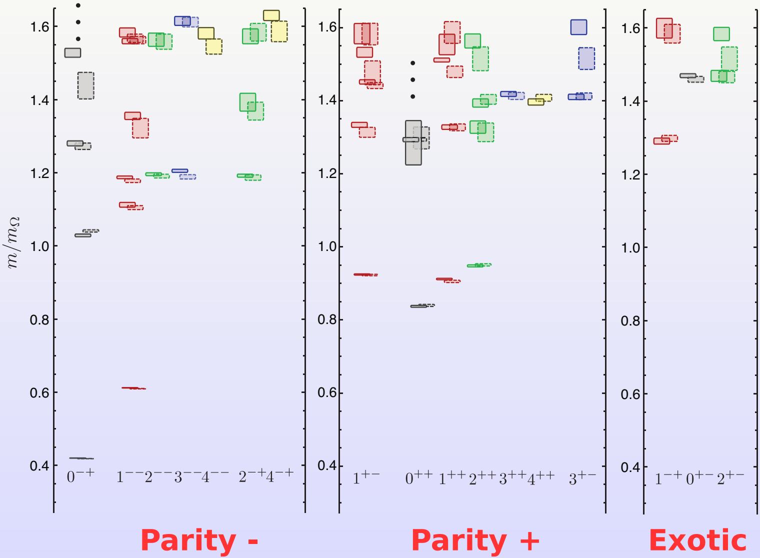
$\mathrm{Tr}\left[\Phi\tau\Phi\tau\Phi\tau\right]$



$\mathrm{Tr}\left[\Phi\tau\right]\ \mathrm{Tr}\left[\Phi\tau\right]$

Isovector meson spectrum

HadSpec Collaboration [Dudek *et.al.* arXiv:1004.4930]



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Talk: Christopher Thomas

Nucleon spectrum

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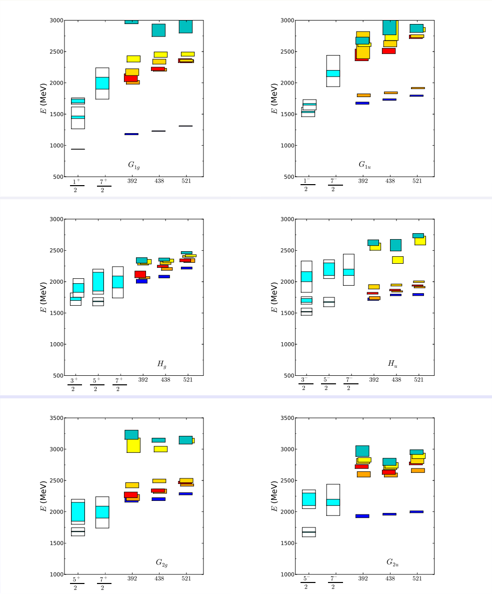


Talk: Steve Wallace

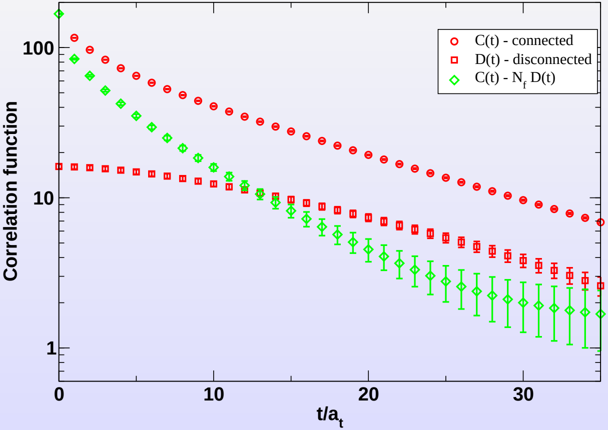
HadSpec Collaboration

Bulava *et.al.*

`arXiv:1004.5072`

Isoscalar mesons

- 16^3 anisotropic lattice, $N_{\mathcal{D}} = 64$, 447 configs,
- $O_M = \bar{\tilde{\psi}}\gamma_5\tilde{\psi}$. PRELIMINARY.
- Scale: $30a_t \approx 1\text{fm}$



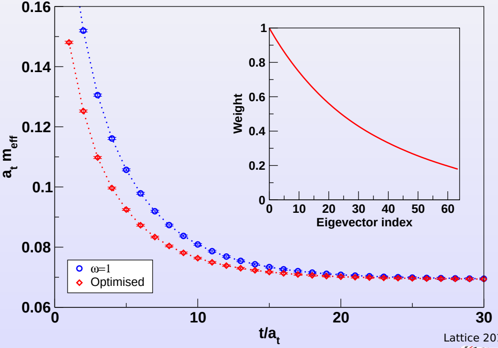
- Statistical precision in $I = 0$ mass fit $\approx 1 - 2\%$

Optimising distillation

- Free to put operator ω into the distillation space

$$\square = VV^\dagger = V_\nabla \omega \omega^\dagger V_\nabla^\dagger$$

- Determine $\omega_{ab} = \kappa_a \delta_{ab}$ via variational calculation



- Simple test: C_π
- 25% improvement in $\langle \pi | \bar{\tilde{u}} \gamma_5 \tilde{d} | 0 \rangle$
- Exponential fall-off in κ_a with a

- So $\square_\nabla$ works well, but is not optimal.

Bad news - the price tag

- So far - good results on modest lattice sizes $N_s = 16^3 \equiv (1.9\text{fm})^3$.
- Used $N_{\mathcal{D}} = 64$ for mesons, $N_{\mathcal{D}} = 32$ for baryons

The problem:

- To maintain constant resolution, need $N_{\mathcal{D}} \propto N_s$
- **Budget:**

Fermion solutions	construct τ	$\mathcal{O}(N_s^{2})$
Operator constructions	construct Φ	$\mathcal{O}(N_s^{2})$
Meson contractions	$\text{Tr}[\Phi\tau\Phi\tau]$	$\mathcal{O}(N_s^{3})$
Baryon contractions	$\bar{B}\tau\tau\tau B$	$\mathcal{O}(N_s^{4})$

- 32^3 lattice: $64 \times (\frac{32}{16})^3 = 512$ — too expensive.
- Some benefits in reduction in variance with N_s
- **Can stochastic estimation technology help?**

Stochastic estimation in the distillation space

- Construct a **stochastic identity matrix in $\mathcal{D}$**: introduce a vector η with $N_{\mathcal{D}}$ elements and with

$$E[\eta_i] = 0 \text{ and } E[\eta_i \eta_j^*] = \delta_{ij}$$

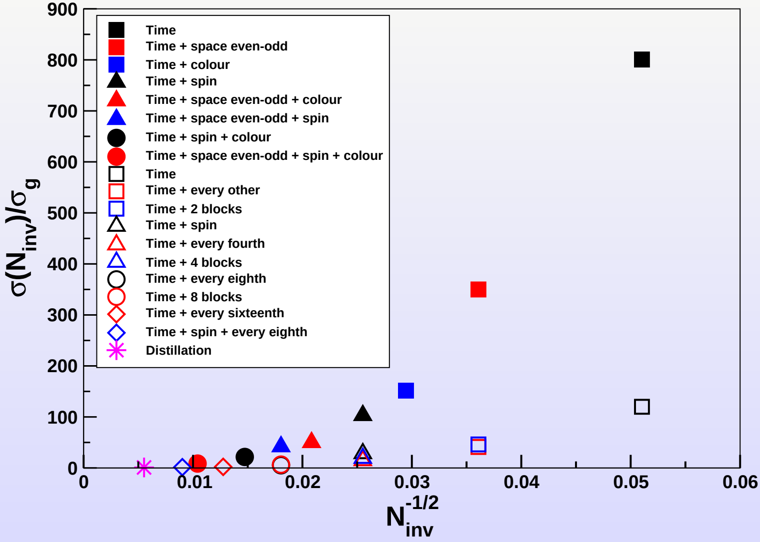
- Now the distillation operator is written

$$\Box = E[V\eta\eta^\dagger V^\dagger] = E[WW^\dagger]$$

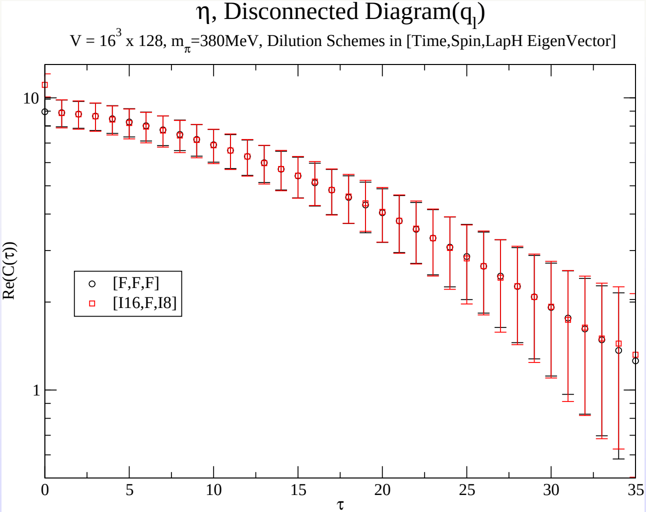
- Introduces noise into computations
- **Dilution:** “thin out” the stochastic noise with N_η orthogonal projectors to make a variance-reduced estimator of $I_{\mathcal{D}} = E[WW^\dagger] = \sum_{k=1}^{N_\eta} E[V\mathcal{P}_k\eta\eta^\dagger\mathcal{P}_k V^\dagger]$, with $W_k = V\mathcal{P}_k\eta$, a $N_\eta \times (N_s \times N_c)$ matrix

First test: baryon correlation function

Talk: Justin Foley

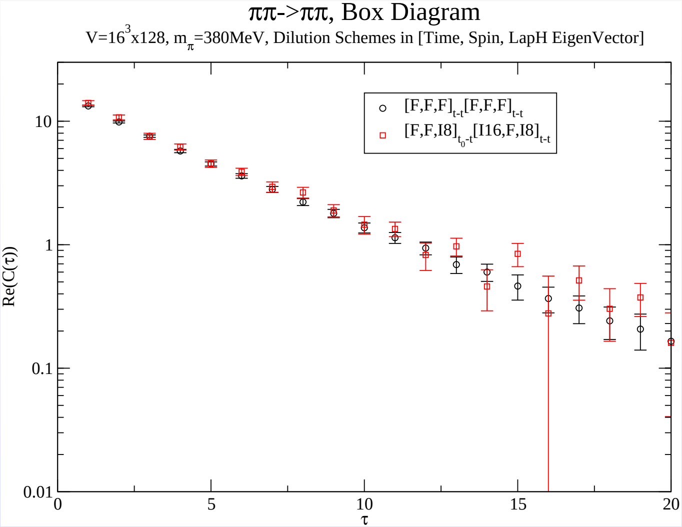


Talk: Chik-Him Wong



- Same signal, factor 32 cost reduction.

Talk: Chik-Him Wong, Jimmy Juge



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Summary

Distillation is a smearing algorithm that enables construction of many previously challenging measurements

- Exploits the low-rank nature of good smearing operators
- First results using the method have shown its usefulness
- The rapid cost growth as the volume is increased is a **problem**. One **solution** is to use stochastic estimation in the distillation space
- Many possibilities in this framework to improve the method