STRING EFFECTS IN THE YANG-MILLS THEORY

M I C H E L E   P E P E

Istituto Nazionale di Fisica Nucleare

I N F N

Sez. M i l a n o – B i c o c c a

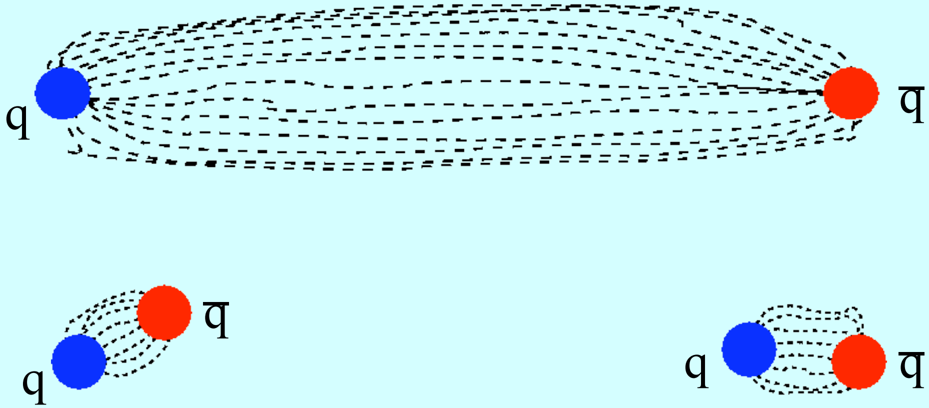
Milan (I t a l y)

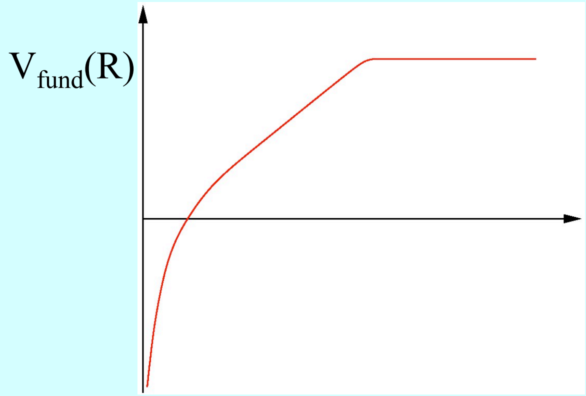
PLAN OF THE TALK

- Introduction
- Numerical results
  - the width of the flux tube
  - the Lüscher term for k-strings
  - the decay of unstable strings
- Conclusions

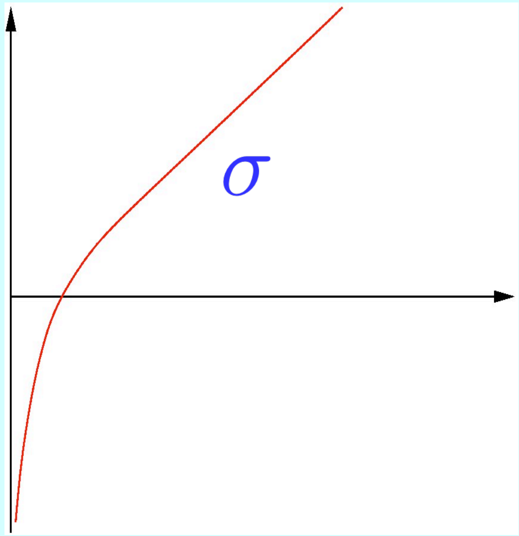
Introduction

At low temperature quarks are confined inside hadrons





If $\mathbf{m_q} \to \infty$ the string between two fundamental charges becomes stable



As strong as a cm-thick steel cable but 13 orders of magnitude thinner

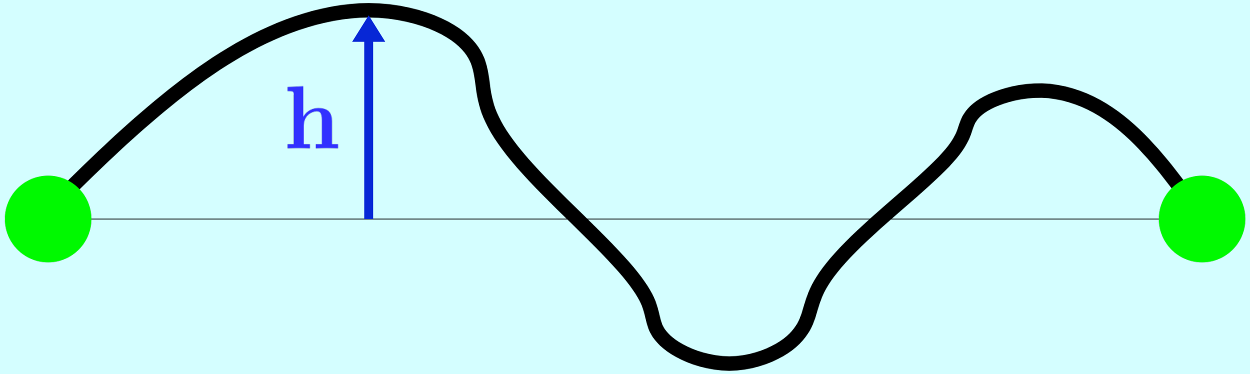
$$\sigma \simeq (\mathbf{0.4Gev})^{\mathbf{2}} \simeq \mathbf{10^5 N}$$

$$\sigma = \mathbf{M\,g}$$

$$\mathbf{M} \sim \mathbf{100}\ \text{people}$$

The Yang-Mills cableway

Low-energy effective string description



The vibrating string: no relevance of the internal structure.

No dependence on the gauge group

The free string

$$\mathbf{S_2} = \frac{\sigma}{\mathbf{2}} \int (\partial_\mu \mathbf{h_i}\, \partial_\mu \mathbf{h_i})\, \mathbf{dx}\, \mathbf{dt}$$

$$\mathbf{i} = \mathbf{1}, \ldots, \mathbf{d} - \mathbf{2}$$

$$\mathbf{V(R)} = \mathbf{a} + \sigma\, \mathbf{R} - \frac{(\mathbf{d}-\mathbf{2})\pi}{\mathbf{24\ R}}$$

M. Lüscher, K. Symanzik, and P. Weisz (1980)

M. Lüscher (1981)

$$\mathbf{w^2(R)} = \frac{(\mathbf{d}-\mathbf{2})}{\mathbf{2}\pi\sigma} \log(\mathbf{R}) + const$$

M. Lüscher, G. Münster, and P. Weisz (1981)

- The broadening of the color flux tube at T=0 and finite T

SU(2) Yang-Mills

- Sources in larger representations of the gauge group

stable strings

the Lüscher term of k-strings

SU(4) Yang-Mills

simplest case with a stable string different from the fundamental string

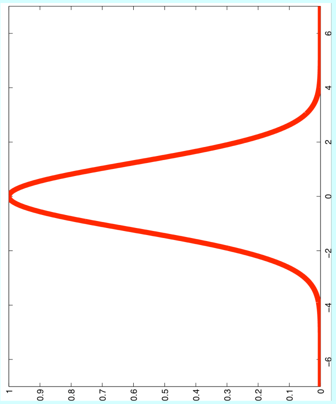
unstable strings

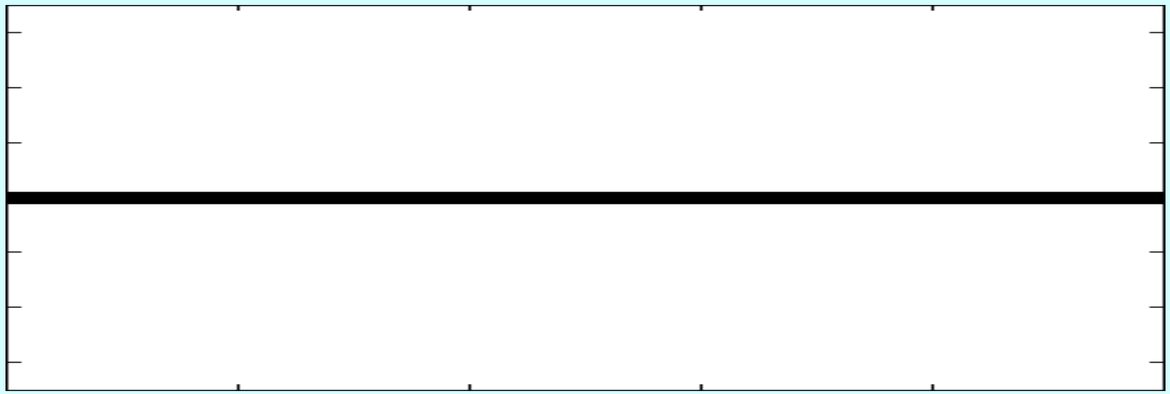
observation of string decay

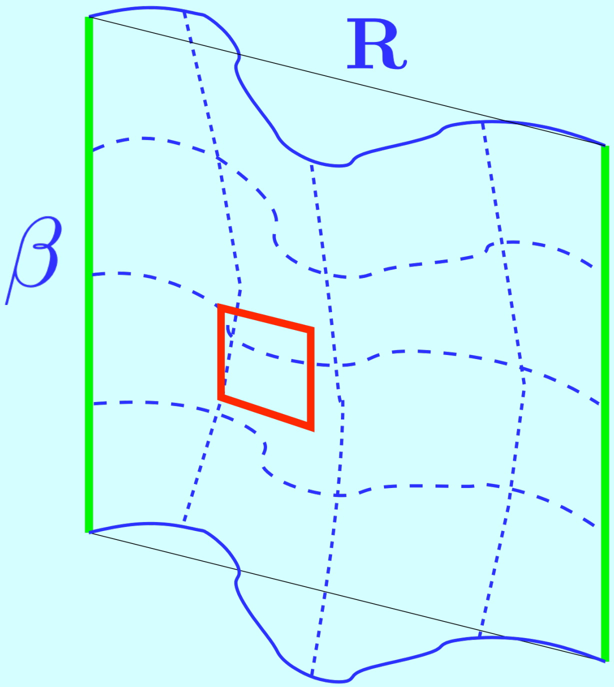
SU(2) Yang-Mills

only the fundamental string is stable

The width of the flux tube







$$\mathbf{w^2(R)} = \frac{\int \mathbf{h^2}\,\mathcal{P}(\mathbf{R},\mathbf{h})\,\mathbf{dh}}{\int \mathcal{P}(\mathbf{R},\mathbf{h})\,\mathbf{dh}}$$

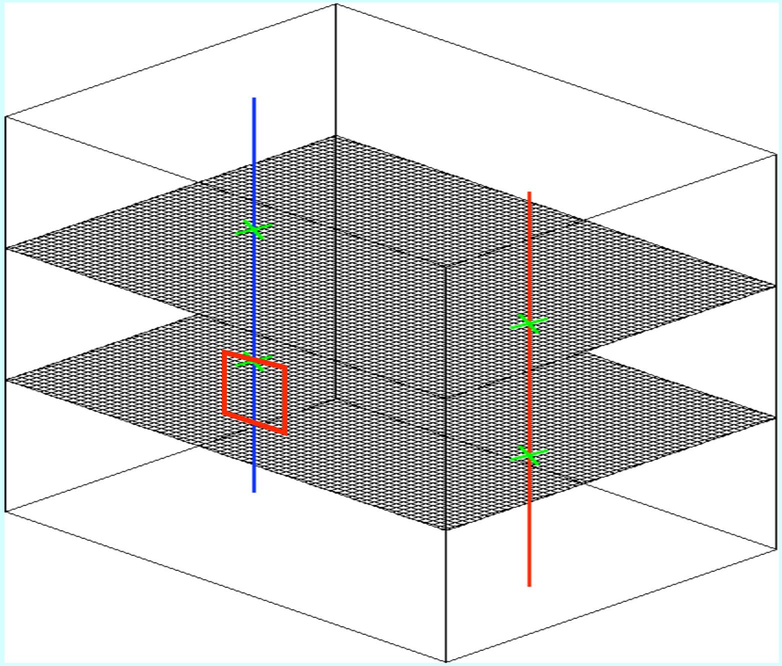
$$\mathcal{P}(\mathbf{R},\mathbf{h}) = \frac{\langle \mathbf{\Phi(0)\,\Phi(R)\,p(\frac{R}{2},h)} \rangle}{\langle \mathbf{\Phi(0)\,\Phi(R)} \rangle} - \langle \mathbf{p} \rangle$$

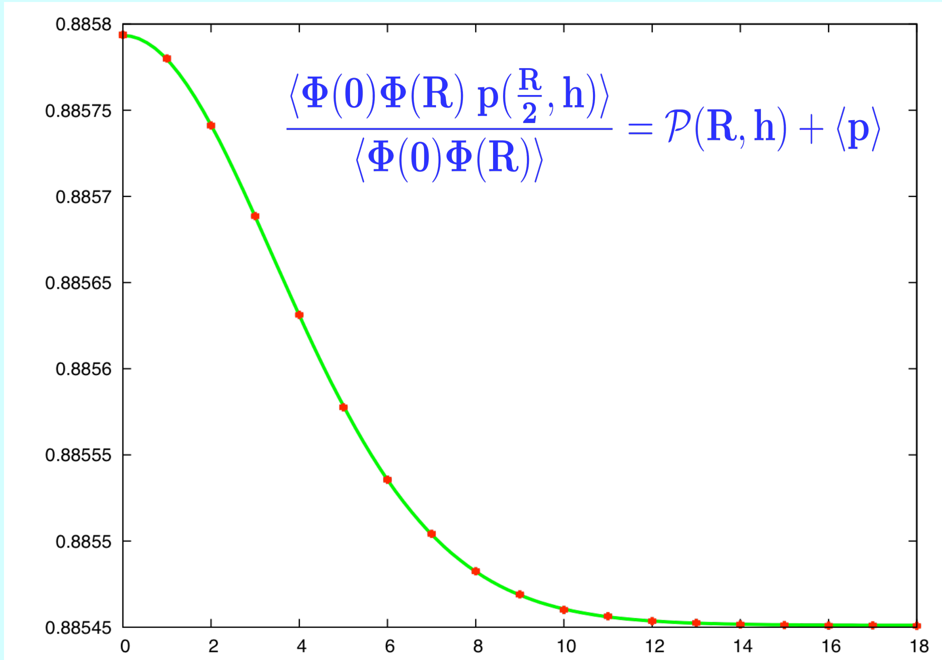
$$\mathcal{P}(\mathbf{R},\mathbf{h}) \approx \mathrm{e}^{-\frac{\mathbf{h^2}}{\mathbf{2w^2(R)}}}$$

The numerical study

Very challenging measurement: it is the ratio of two exponentially small signals

We use the multilevel Lüscher-Weisz technique





h

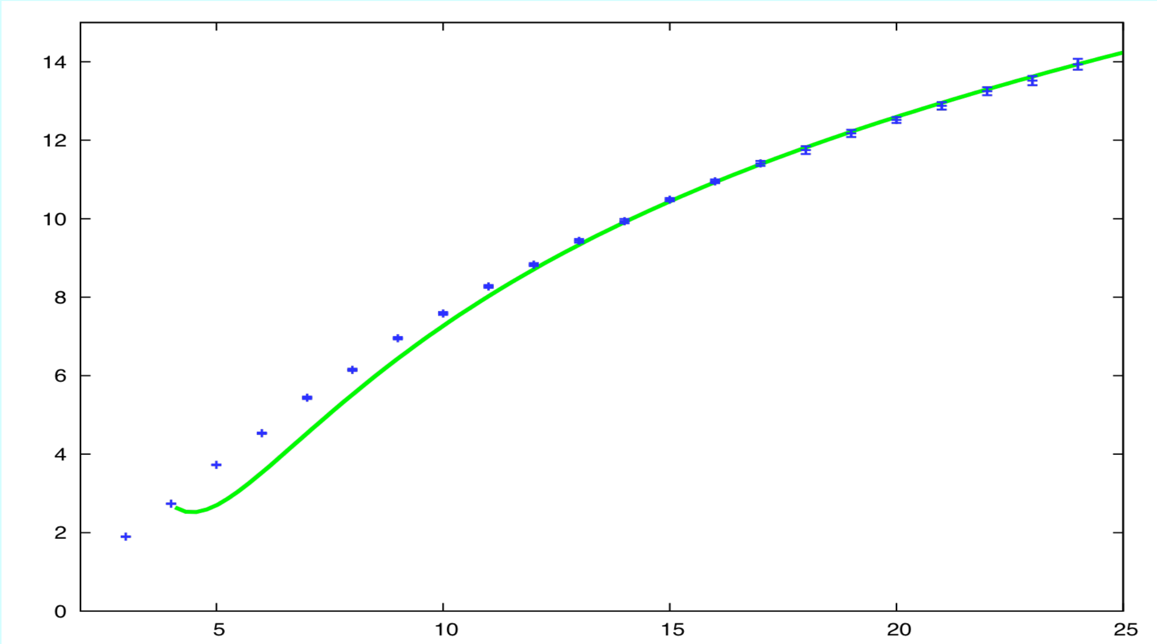
$$\mathcal{P}(\mathbf{R},\mathbf{h}) = \mathbf{a}\,\mathbf{X}\,\frac{\mathbf{1}+\mathbf{b}\,\mathbf{X}}{\mathbf{1}+\mathbf{c}\,\mathbf{X}}; \qquad \mathbf{X} = e^{-\frac{\mathbf{h}^2}{\mathbf{s}}} \qquad \mathbf{w}^2(\mathbf{R}) = \frac{\int \mathbf{h}^2\,\mathcal{P}(\mathbf{R},\mathbf{h})\,\mathbf{dh}}{\int \mathcal{P}(\mathbf{R},\mathbf{h})\,\mathbf{dh}}$$

The results: T=0

F. Gliozzi, M. Pepe, and U.-J. Wiese
PRL 104, 232001 (2010)
arXiv:1006.2252

$$\mathbf{w_{lo}^2(R/2)} = \frac{\mathbf{1}}{\mathbf{2}\pi\sigma}\log\left(\frac{\mathbf{R}}{\mathbf{R_0}}\right) + \frac{\mathbf{1}}{\pi\sigma}\log\left(\frac{\eta(\mathbf{2u})}{\eta^{\mathbf{2}}(\mathbf{u})}\right)$$

SU(2) YM: $54^2 \times 48$; $\beta=9$



$$\mathbf{u} = \frac{\beta}{\mathbf{2R}} \qquad \mathbf{q} = \mathrm{e}^{-\mathbf{2}\pi\mathbf{u}}$$

$$\eta(\mathbf{u}) = \mathbf{q}^{\mathbf{1/24}} \prod_{\mathbf{n=1}}^{\infty} (\mathbf{1} - \mathbf{q^n})$$

$$\mathbf{E_2(u)} = \mathbf{1} - \mathbf{24} \prod_{\mathbf{n=1}}^{\infty} \frac{\mathbf{n\,q^n}}{(\mathbf{1} - \mathbf{q^n})}$$

Higher orders in the effective string theory: $\mathbf{S[h]} = \mathbf{S_2[h]} + \frac{\mathbf{1}}{\mathbf{8}} C \sigma \int [(\partial_\mu \mathbf{h})^{\mathbf{2}}]^{\mathbf{2}}\, \mathbf{dx\,dt}$

M. Lüscher and P. Weisz (2004)
O. Aharony and E. Karzbrun (2009)

$$\mathbf{w^2(R/2)} = \left(\mathbf{1} + \frac{\mathbf{4}\pi\,\mathbf{f(u)}}{\sigma\mathbf{R^2}}\right)\mathbf{w_{lo}^2(R/2)} - \frac{\mathbf{f(u)} + \mathbf{g(u)}}{(\sigma\mathbf{R})^{\mathbf{2}}}$$

$$\mathbf{f(u)} = \frac{\mathbf{E_2(u)} - \mathbf{4\,E_2(2u)}}{\mathbf{48}}; \qquad \mathbf{g(u)} = \pi\mathbf{u}\left(\mathbf{q}\frac{\mathbf{d}}{\mathbf{dq}} - \frac{\mathbf{E_2(u)}}{\mathbf{12}}\right)\left(\mathbf{f(u)} + \frac{\mathbf{E_2(u)}}{\mathbf{16}}\right) + \frac{\mathbf{E_2(u)}}{\mathbf{96}}$$

The results: finite T

Inversion transformation: $\eta(\mathbf{u}) = \frac{1}{\sqrt{\mathbf{u}}}\,\eta(\frac{1}{\mathbf{u}});\qquad \mathbf{E_2}(\mathbf{u}) = -\frac{1}{\mathbf{u^2}}\,\mathbf{E_2}(\frac{1}{\mathbf{u}}) + \frac{\mathbf{6}}{\pi\mathbf{u}}$

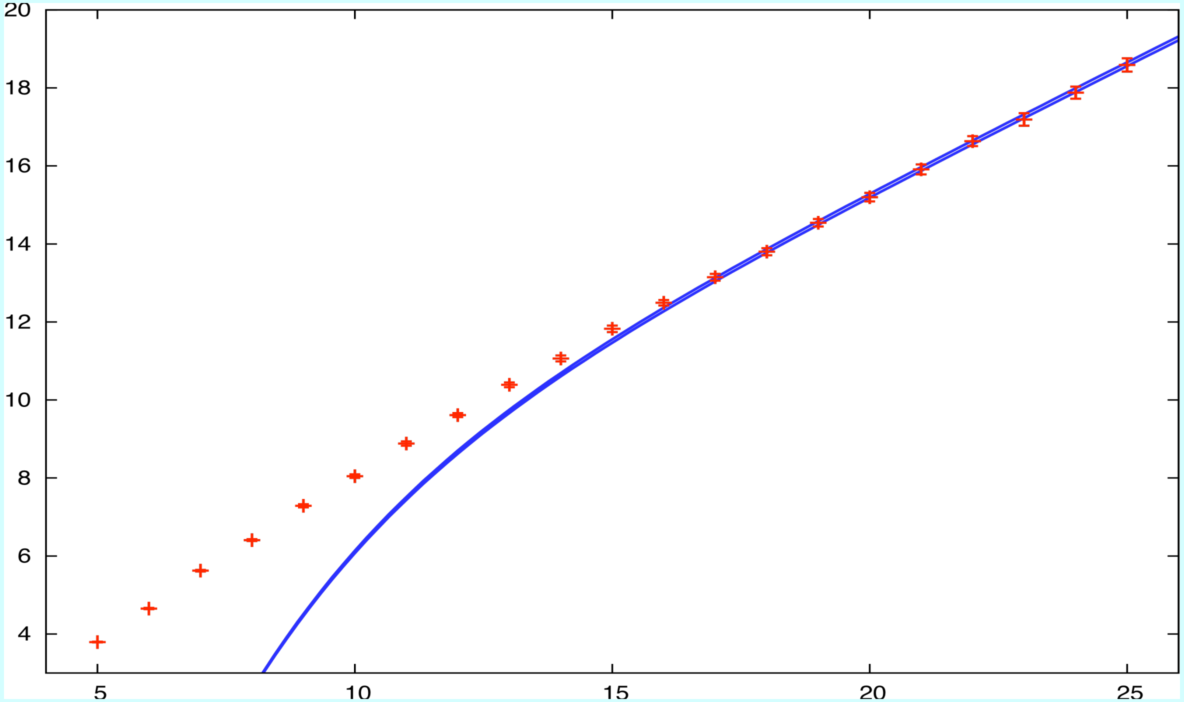
$$\frac{\beta}{\mathbf{2R}} = \mathbf{u} \longrightarrow \frac{\mathbf{1}}{\mathbf{u}} = \frac{\mathbf{2R}}{\beta}$$

A. Allais and M. Caselle (2009)

$\beta \gg \mathbf{R}$ — T = 0

$\mathbf{R} \gg \beta$ — T finite

$$\mathbf{w^2(R/2)} = \frac{\mathbf{1}}{\mathbf{2}\pi\sigma}\log\left(\frac{\beta}{\mathbf{4R_0}}\right) + \frac{\mathbf{R}}{\mathbf{4}\beta\sigma} + \dots$$

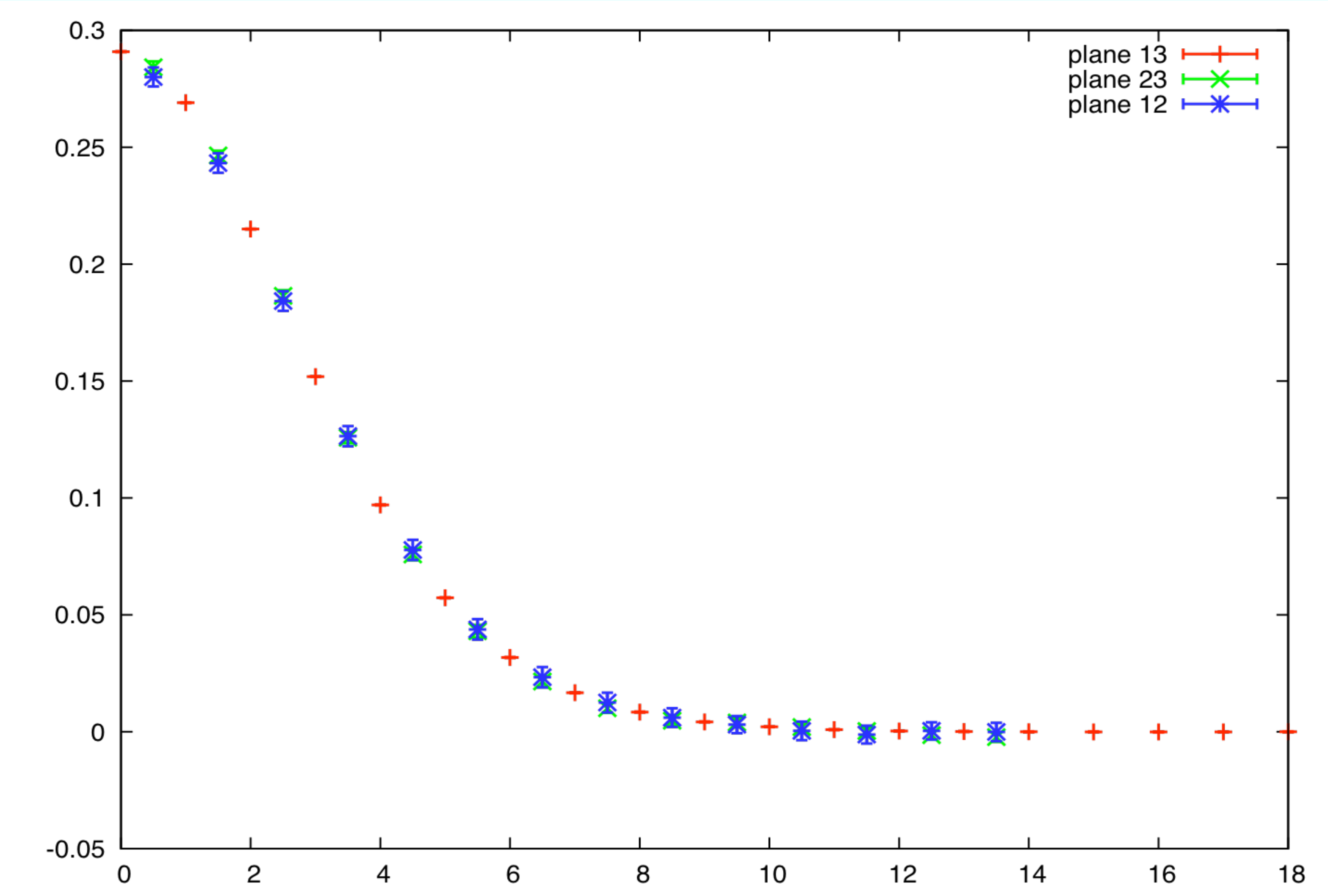


No adjustable parameter!

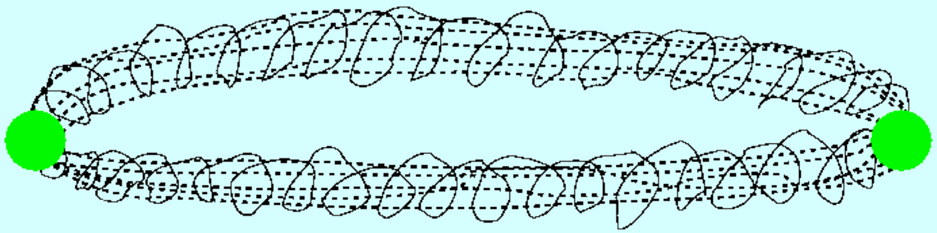
F. Gliozzi, M. Pepe, U.-J. Wiese
To be published

The plaquette orientation

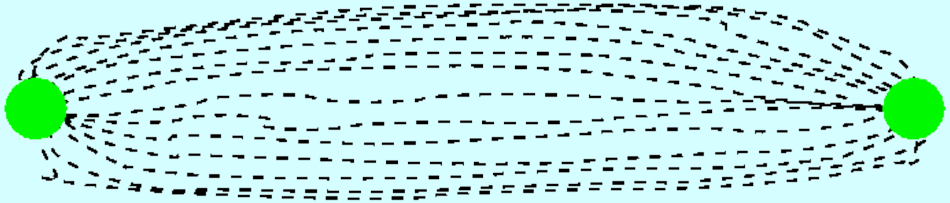
Normalized probability distribution $$\frac{\mathcal{P}_{\mu\nu}(\mathbf{R},\mathbf{h})}{\int \mathcal{P}_{\mu\nu}(\mathbf{R},\mathbf{h})\,\mathbf{dh}}$$



Sources in larger representations



Bound states of fundamental strings



Single string without an internal structure

SU(N) representations branch in N sectors (N-ality sectors)

$$\mathcal{R}_k \otimes \{\mathsf{adj}\} \otimes \{\mathsf{adj}\} \ldots = \mathcal{R}'_k \oplus \ldots \Longrightarrow V_{\mathcal{R}_k} \sim V_{\mathcal{R}'_k}$$

For every N-ality sector there is a stable string: k-string

All other strings in that sector are unstable and decay into the stable one as $\mathbf{R} \rightarrow \infty$

The Lüscher term for k-strings

SU(4) Yang-Mills reps: {4} the {6}

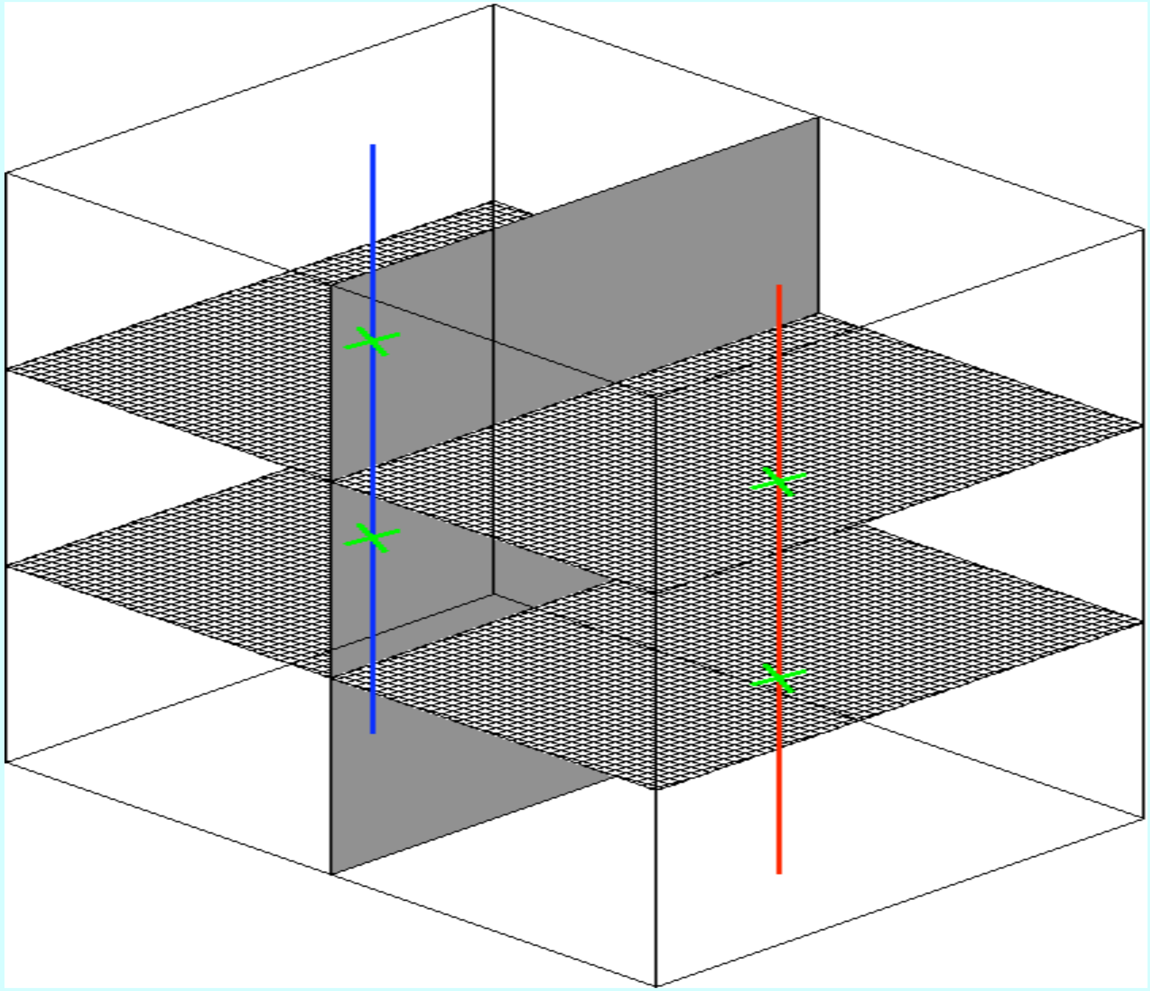
The decay of unstable strings

SU(2) Yang-Mills rep: {2}

The numerical technique

Wilson action: $\mathcal{S}_{YM}[U] = -\beta \sum_P \mathrm{Tr}\,(U_P)$

$$\mathcal{O} = \frac{\int \mathcal{D}U\ \phi_{\mathcal{R}}(R)\,\phi_{\mathcal{R}}(0)\ \mathrm{e}^{-\mathcal{S}_{YM}}}{\int \mathcal{D}U\ \ \mathrm{e}^{-\mathcal{S}_{YM}}}$$



$$\mathcal{O} \sim \mathrm{e}^{-V_0 T} \sim \left(\mathrm{e}^{-V_0 \tau}\right)^{T/\tau}$$

$\mathcal{O} \rightarrow$ tensor product of the two segments

$$\mathrm{e}^{-V_0 \tau} \sim \left(\mathrm{e}^{-V_0 \tau/2}\right)^2$$

The Lüscher term of k-string

Teper et al.

Gliozzi et al.

Shifman et al.

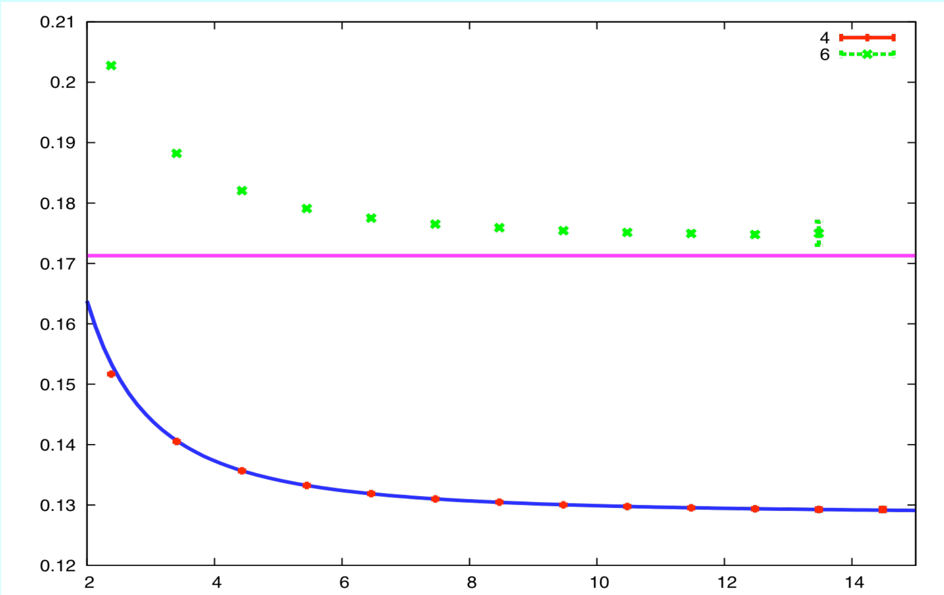
Conjectures

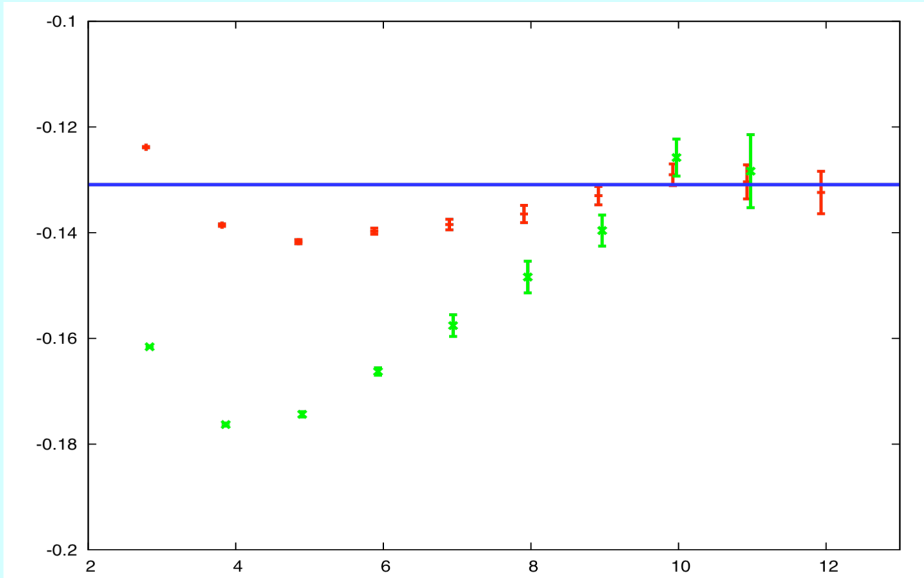
$$-\frac{(d-2)\pi}{24}$$

$$-k\frac{(d-2)\pi}{24}$$ k transparent fundamental strings

?

SU(4) Yang-Mills: 32^3 at $\beta = 21$.





Force: $F = V'(R) = \sigma + \frac{c}{R^2} + \ldots$

Lüscher term: $-\frac{1}{2}R^3V''(R) = c + \ldots$

The decay of unstable strings

Many groups:
C. Michael et al.,
Bali et al., Sommer,
Stephenson,
Philipsen et al.,
de Forcrand et al,
Gliozzi et al.,
Vicari et al., …

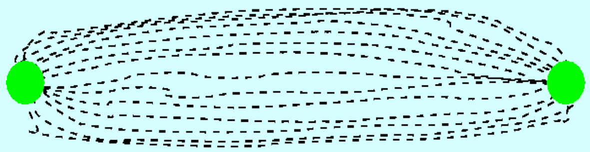
- Observing the decay of unstable strings is an important step in studying the phenomenology of confinement

- SU(2): 2 sectors → {0} sources of integer spin
  → {1} sources of half-integer spin

- SU(2) Yang-Mills theory in (2+1)-d: $32^2 \times 64$ at $\beta = 6.0$ (Wilson action)

- Very challenging numerically: only adjoint sources

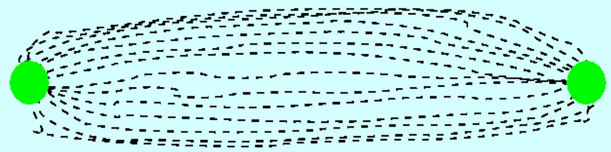
Q = 1



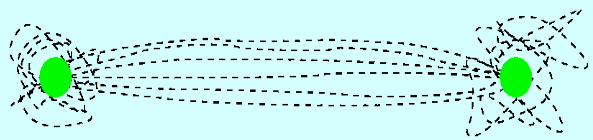
The color charge is completely screened and the string breaks

Many groups and de Forcrand and Kratochvila (2003)

Q = 3/2

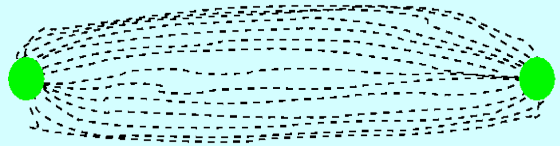


The color charge is partially screened and the string decays into the Q=1/2 string state

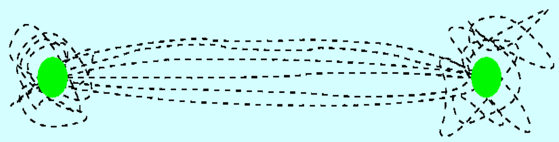


The Q=1/2 string is stable

Q = 2



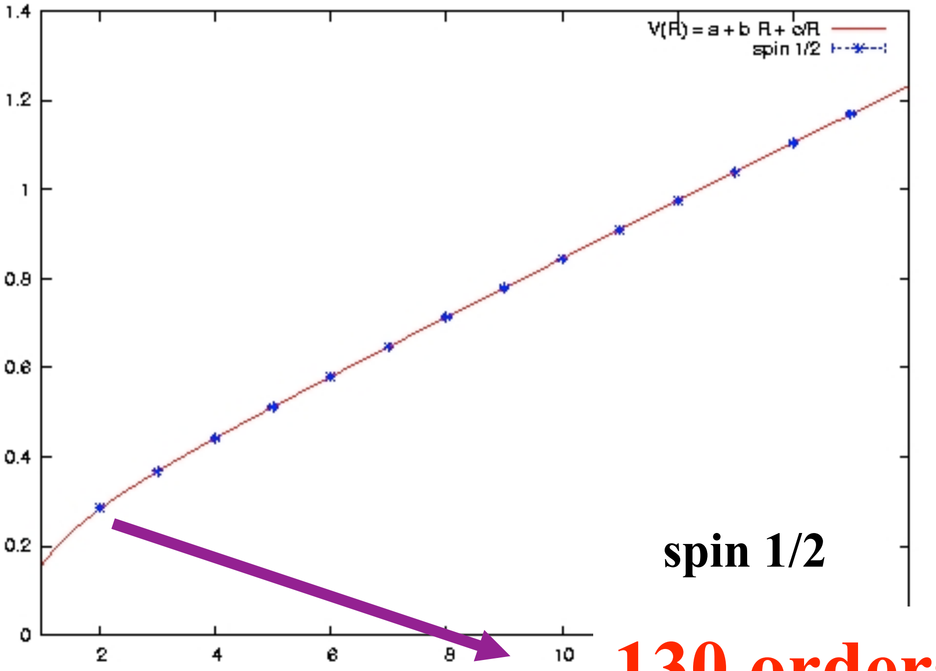
The color charge is partially screened and the string decays into the Q = 1 string state

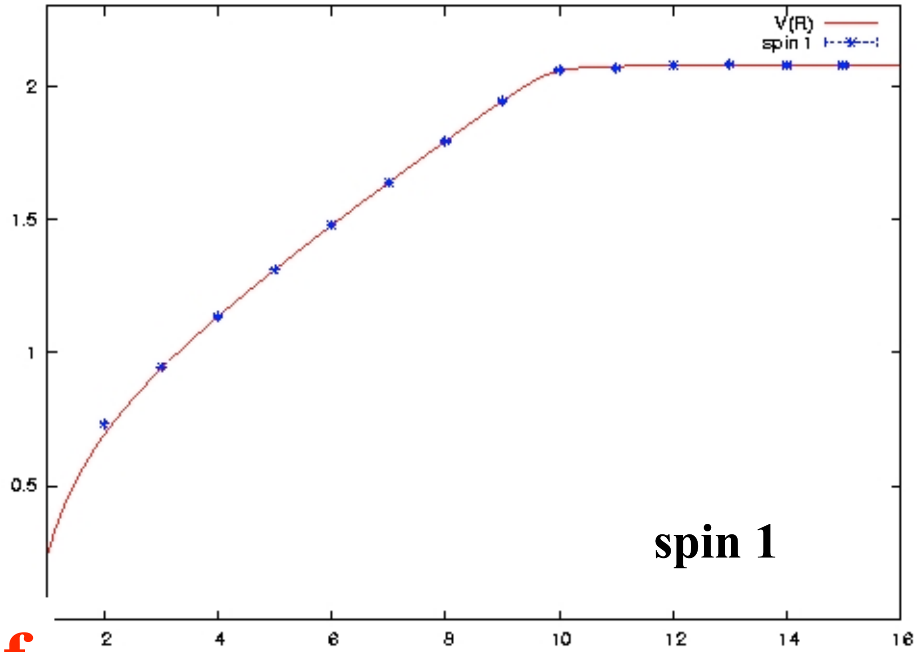


The partially screened color charge Q = 1 is now completely screened and the string breaks

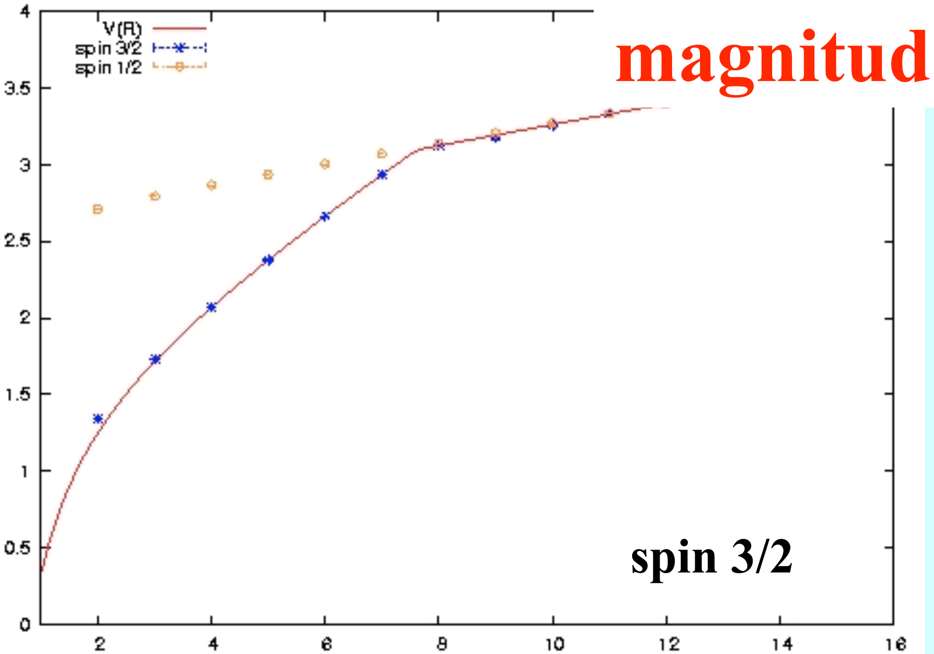
N-ality = {-1} # Potential N-ality = {1}

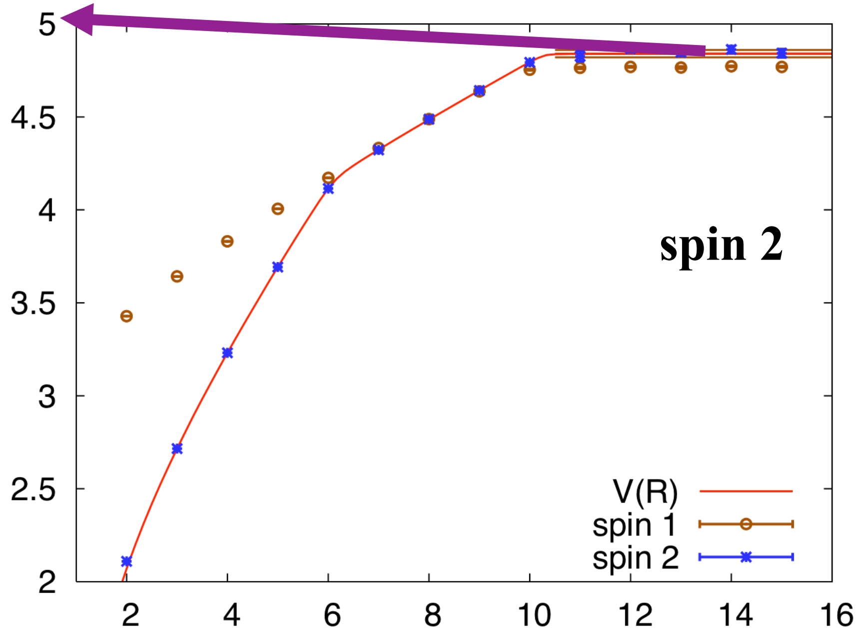
M. Pepe and U.-J. Wiese
PRL 102, 101601 (2009)





130 orders of magnitude!

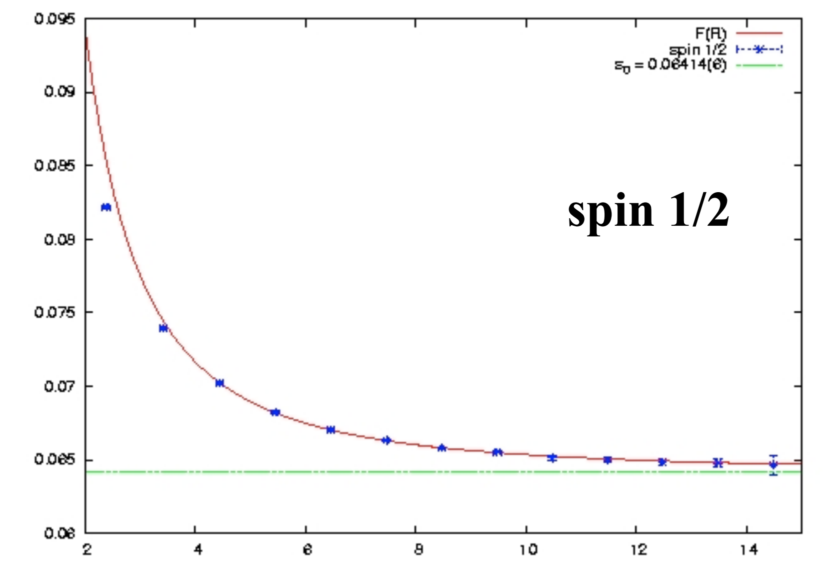


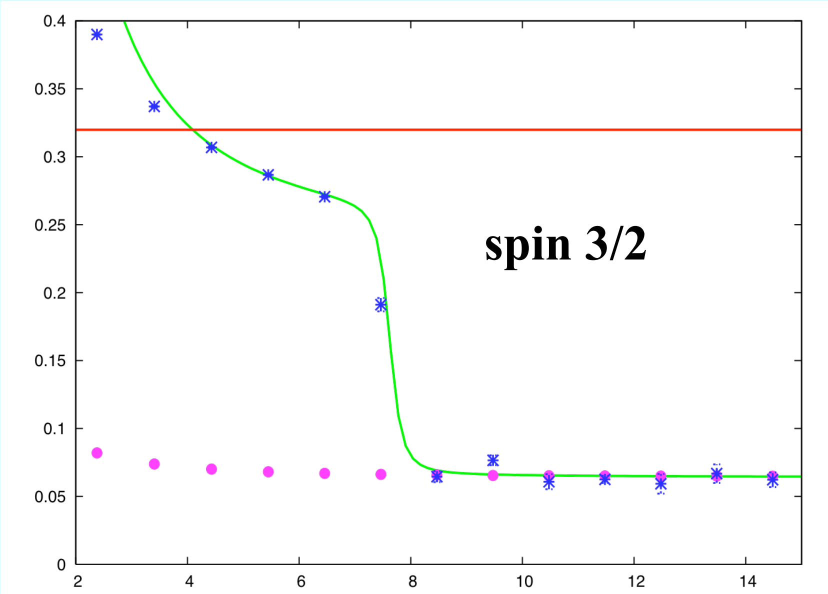


Force

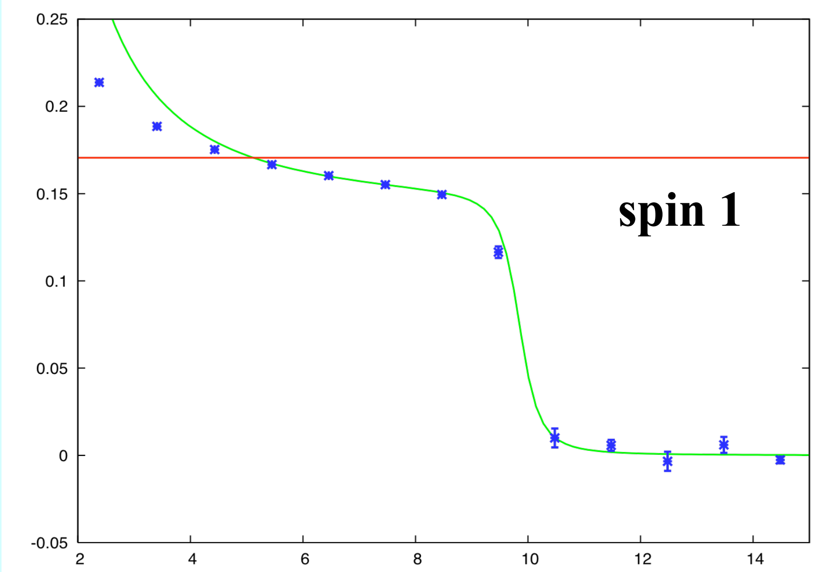
$$V(R) \sim \sigma R - \frac{c}{R} + k \quad \Longrightarrow \quad F(R) \sim \sigma + \frac{c}{R^2}$$

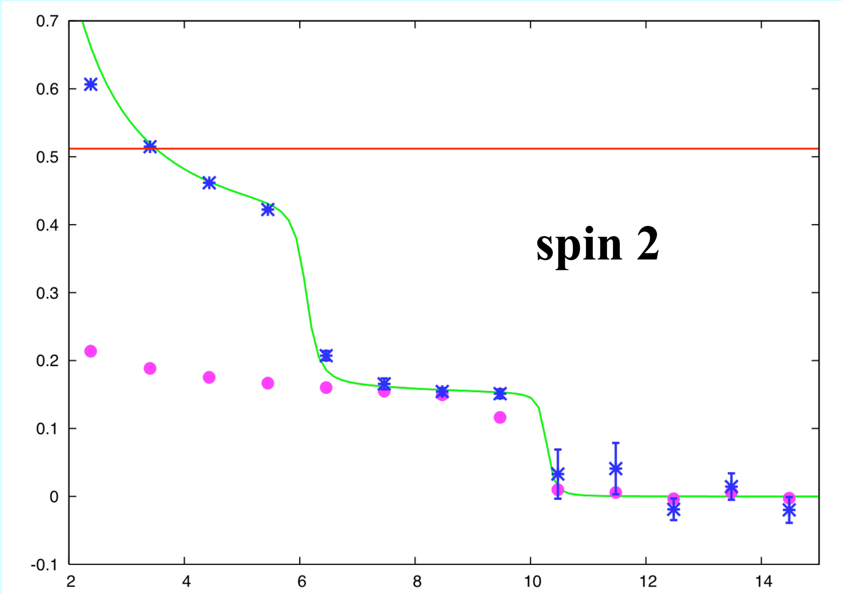
N-ality = {-1}





N-ality = {1}





Conclusions

- In Yang-Mills theory there is a systematic low-energy effective string description (analogous to chiral p.t. in QCD) which describes the dynamics of the Goldstone modes of the spontaneously broken translation symmetry of the world-sheet.

- The effective theory has been tested at the next-to-leading order with very high precision. First clear observation of the broadening of the color flux tube both at T=0 (logarithmic) and at finite T (linear).

- Strings between static charges in higher reps are stable or decay and may break.

- We have studied the Lüscher term of the 1-string and the 2-string in SU(4) YM.

- We have observed the decay of unstable strings in SU(2) Yang-Mills theory

  We have measured the 2-point function for the reps {2}, {3}, {4}, {5}:

  decay of $\{4\} \longrightarrow \{2\}$ double decay $\{5\} \longrightarrow \{3\} \longrightarrow \{1\}$

- Casimir scaling for the string tensions is ruled out ((2+1)-d)

- Numerically very challenging: used the multilevel Lüscher-Weisz algorithm