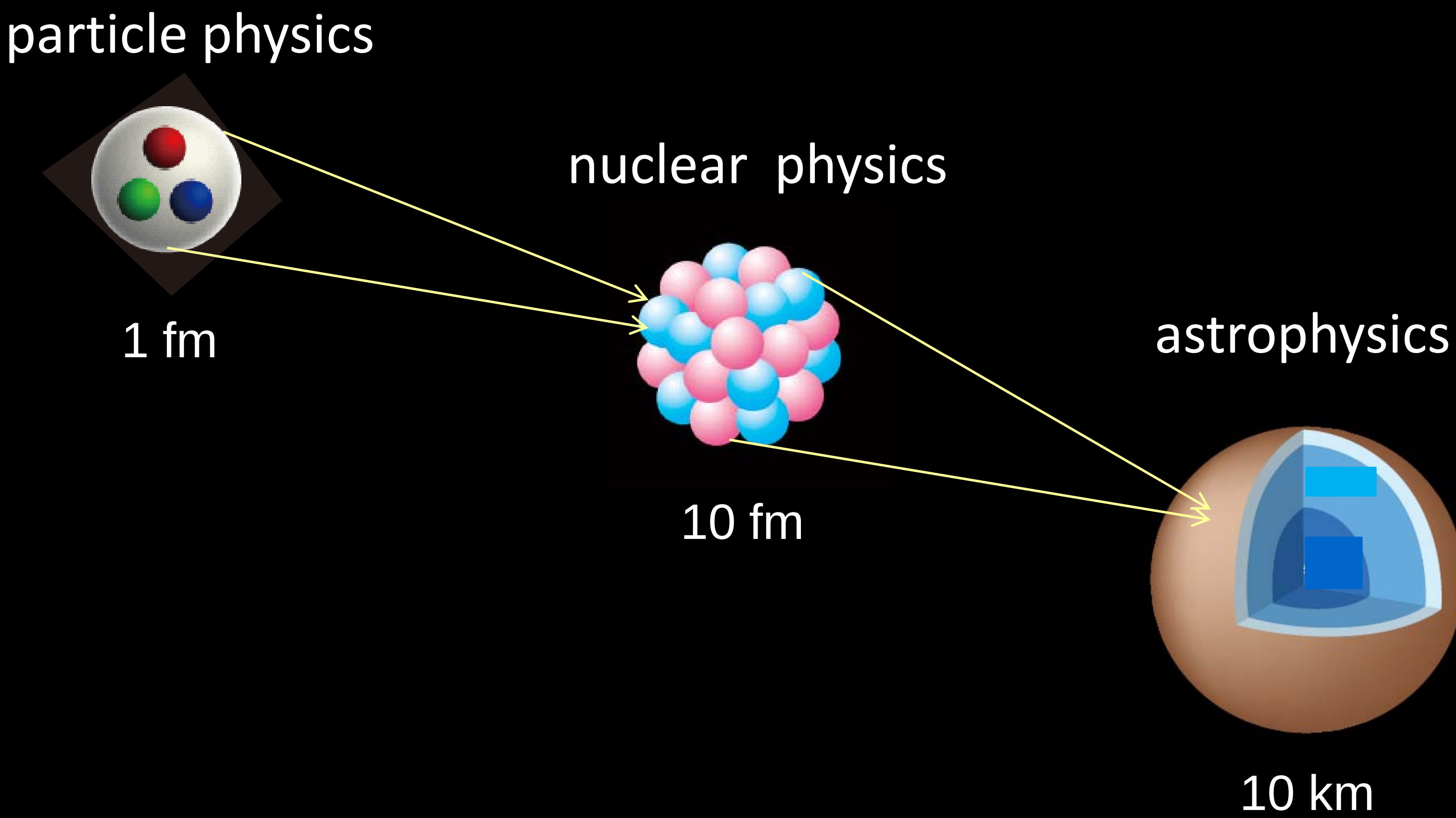
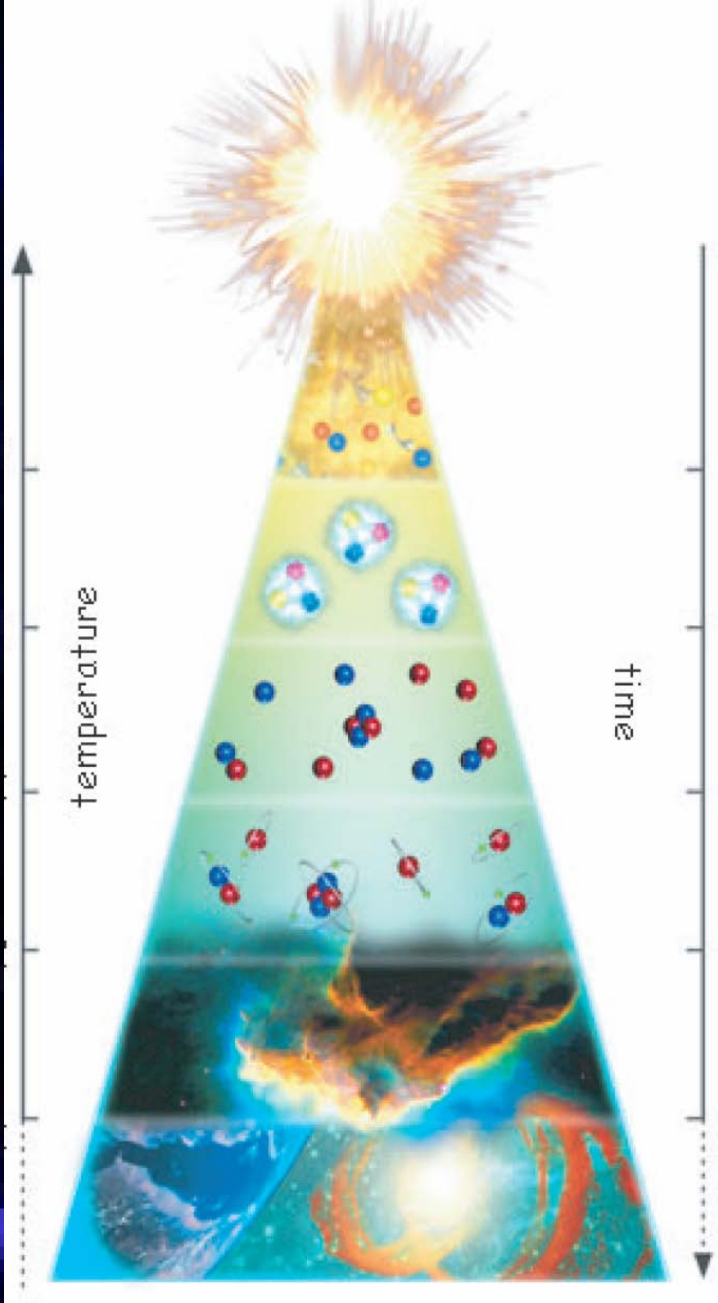
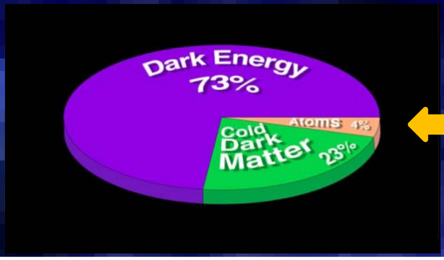
Nuclear Physics from Lattice QCD



Tetsuo Hatsuda (Univ. Tokyo)
Lattice 2010, June 19, 2010



Major Challenges in Nuclear Physics



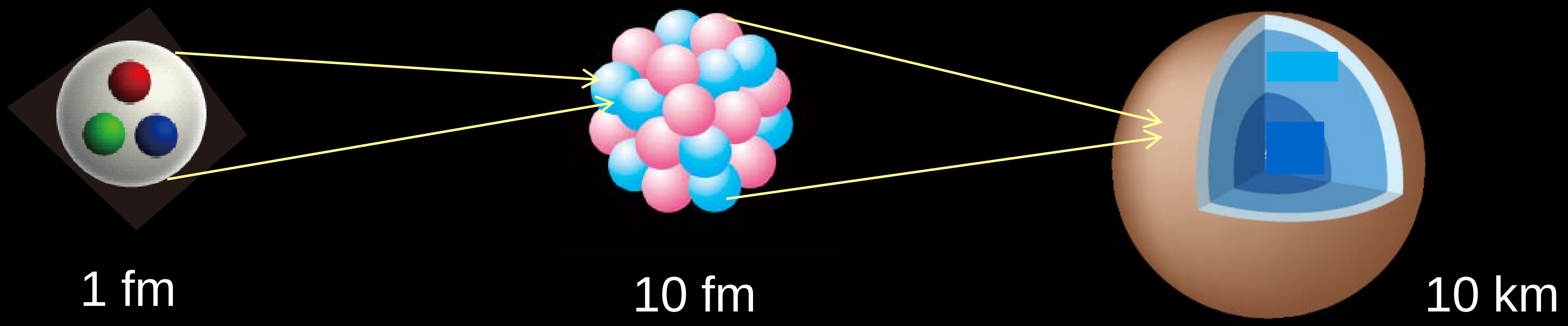
origin & evolution of baryonic matter

- **hot matter** ⇔ RHIC, LHC
  quark-gluon plasma in early universe
  Kanaya & Gupta [Plenary, Thu.]

- **origin of elements** ⇔ Radioactive Beams
  nucleosynthesis
  in big-bang, stars, supernovae, …

- **dense matter** ⇔ J-PARC, FAIR
  neutron stars, exotic nuclei, …

LATTICE QCD inputs are crucial



Outline

[1] nuclear force – nuclei and neutron stars

[2] nuclear force from lattice QCD

[3] hyperon force – hyperonic matter and neutron star core

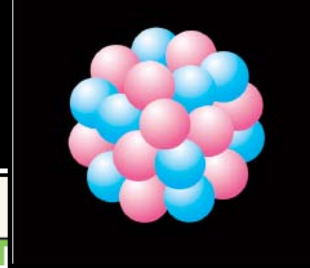
[4] Hyperon force from lattice QCD

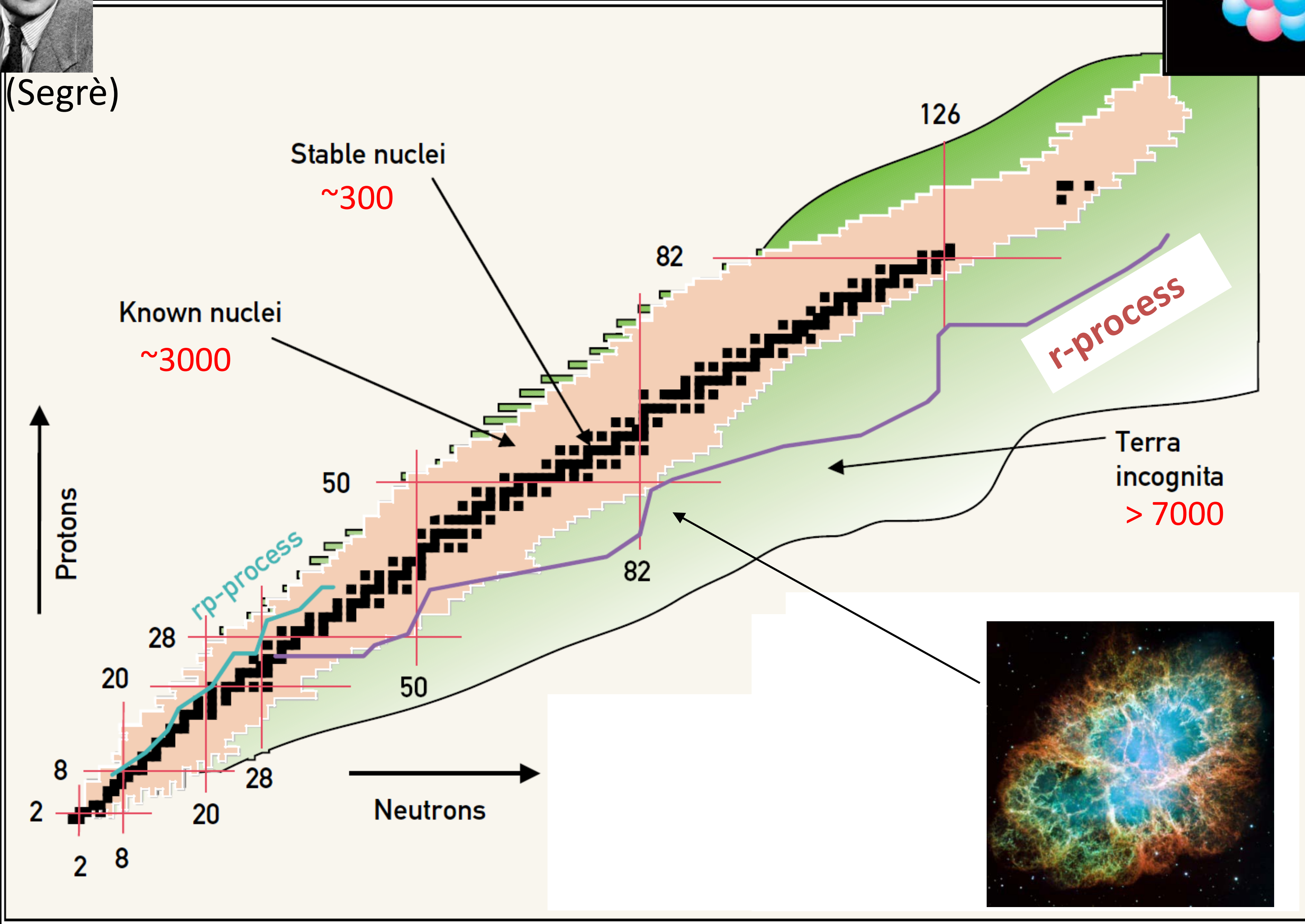
[5] origin of repulsive core and the Pauli principle

[6] Summary and Future

2D (N-Z) Nuclear Chart

(Segrè)

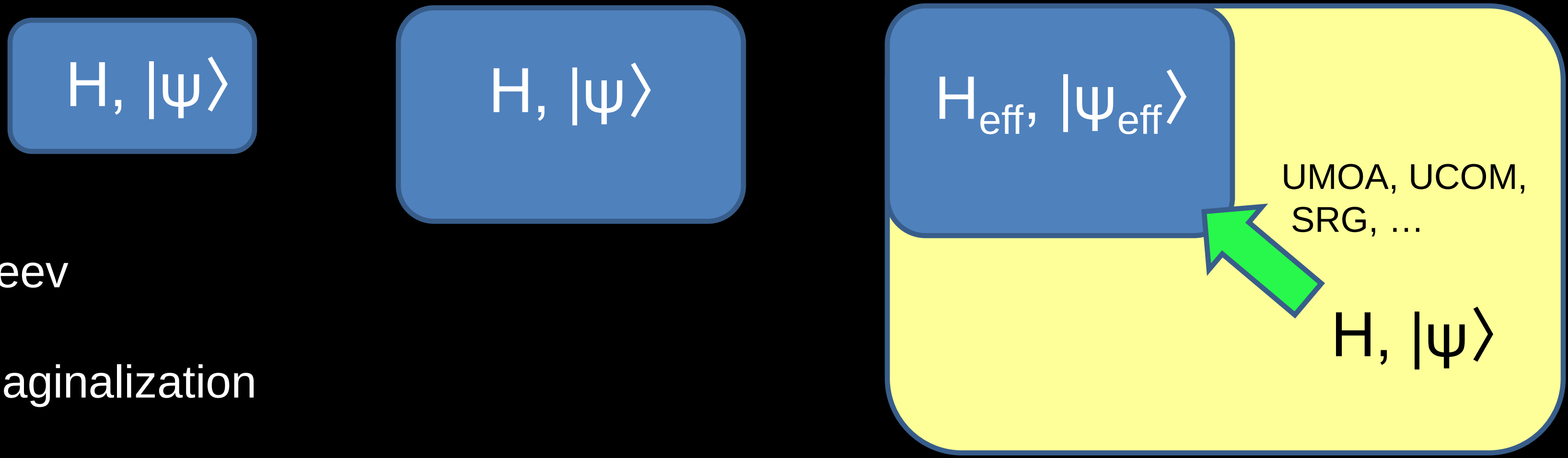




ab initio nuclear A-body calculations (2001-)

$$\mathcal{H}\ \Psi(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_A;s_1,s_2,\cdots,s_A;t_1,t_2,\cdots,t_A)$$
$$= E\ \Psi(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_A;s_1,s_2,\cdots,s_A;t_1,t_2,\cdots,t_A)$$

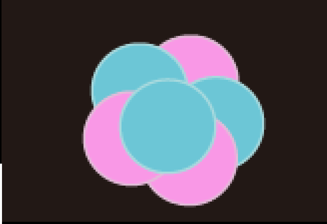
A ≦ 5 A ≦ 12 A ≦ 56



- Faddeev
- diaginalization
- Green's function Monte Carlo
- diagonalization after reduction (coupled cluster, NCSM, NC-MCSM, …)

Benchmark Calculations of ^{4}He by 7 methods
→ agreement within 0.5%
Phys. Rev. C64, 044001 (2001) [arXiv:nucl-th/0104057].

Example: Green's Function Monte Carlo for light nuclei



$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

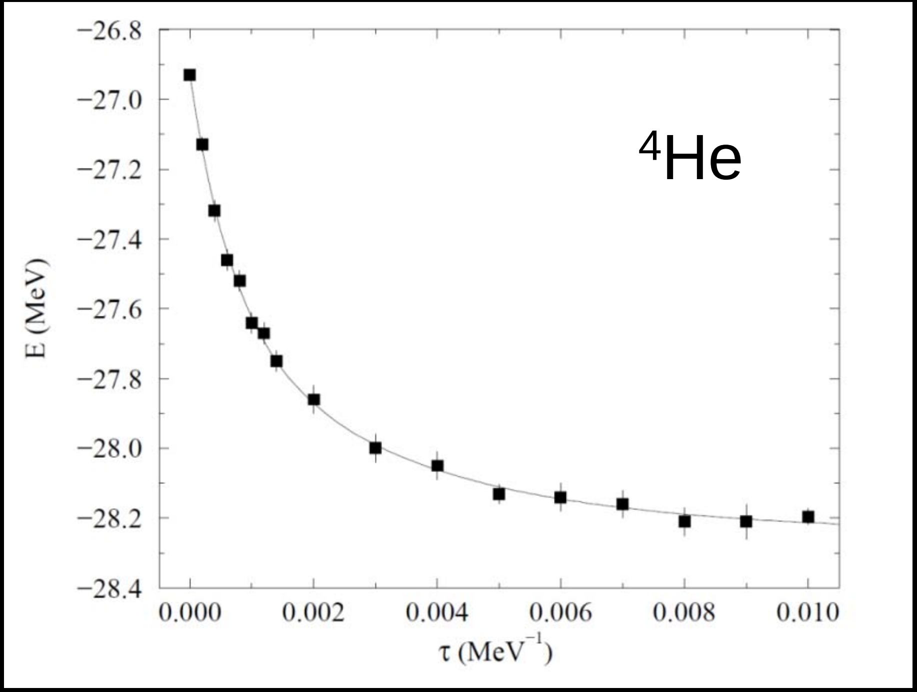
$$\Psi(\tau) = \left[e^{-(H-E_0)\triangle\tau} \right]^n \Psi_T$$

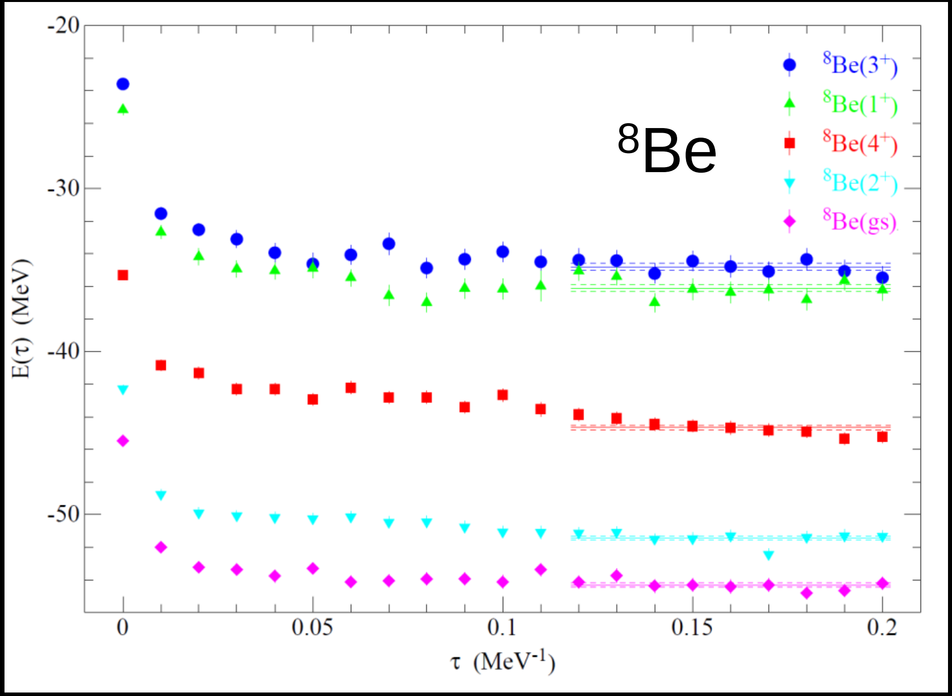
$$\lim_{\tau \to \infty} \Psi(\tau) \propto \Psi_0$$

NN scattering data (v_{ij})

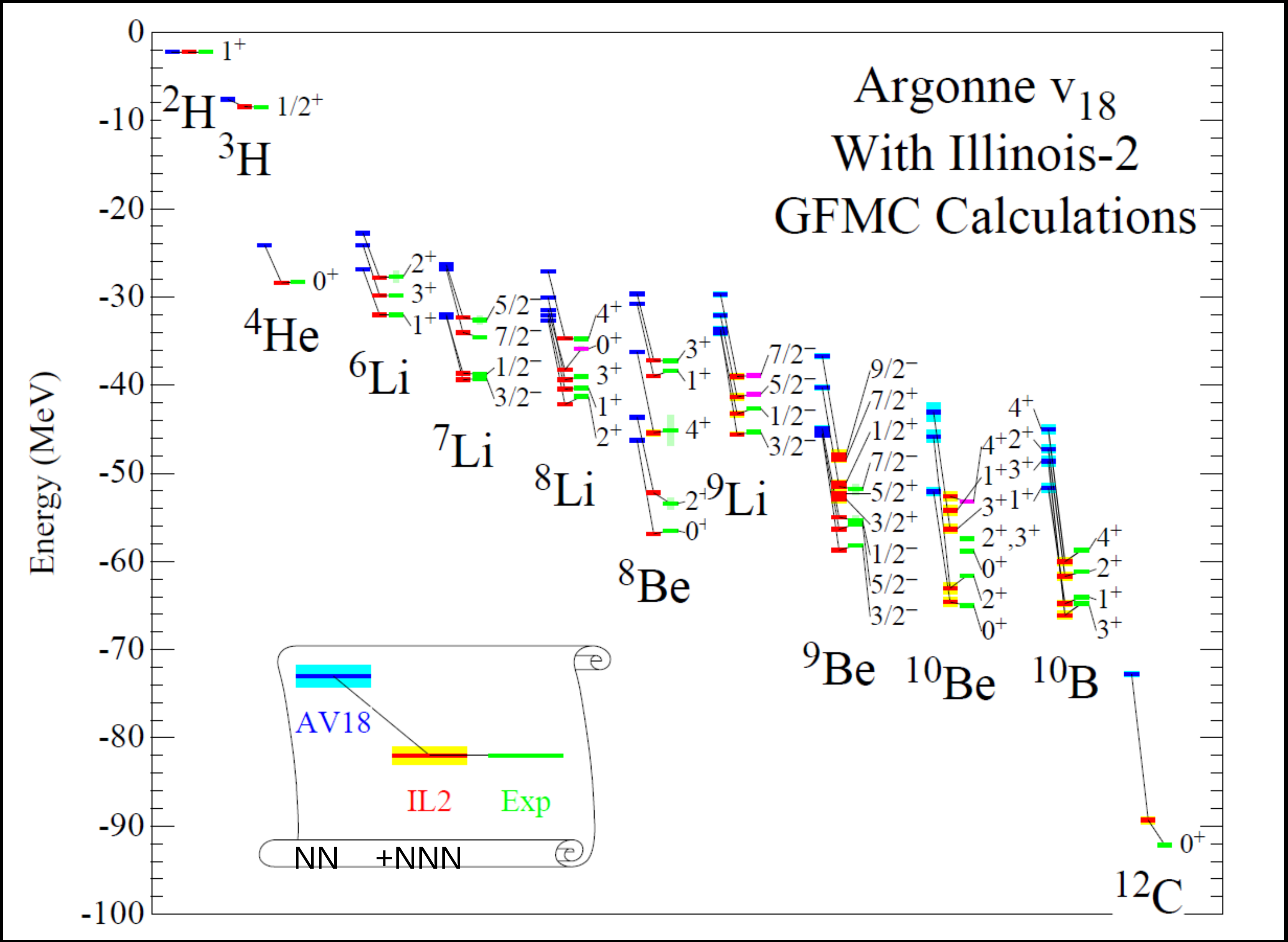
Phenomenological inputs (V_{ijk})

n = (a few) x 100





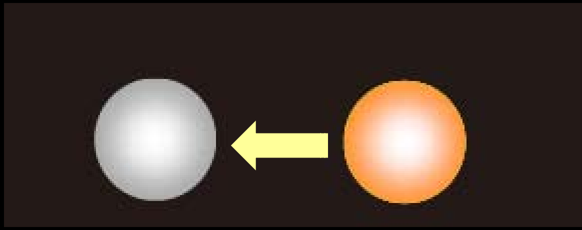
S.C.Pieper, ``Quantum Monte Carlo Calculations of Light Nuclei,''
Riv. Nuovo Cim. 031, 709 (2008) [arXiv:0711.1500 [nucl-th]].

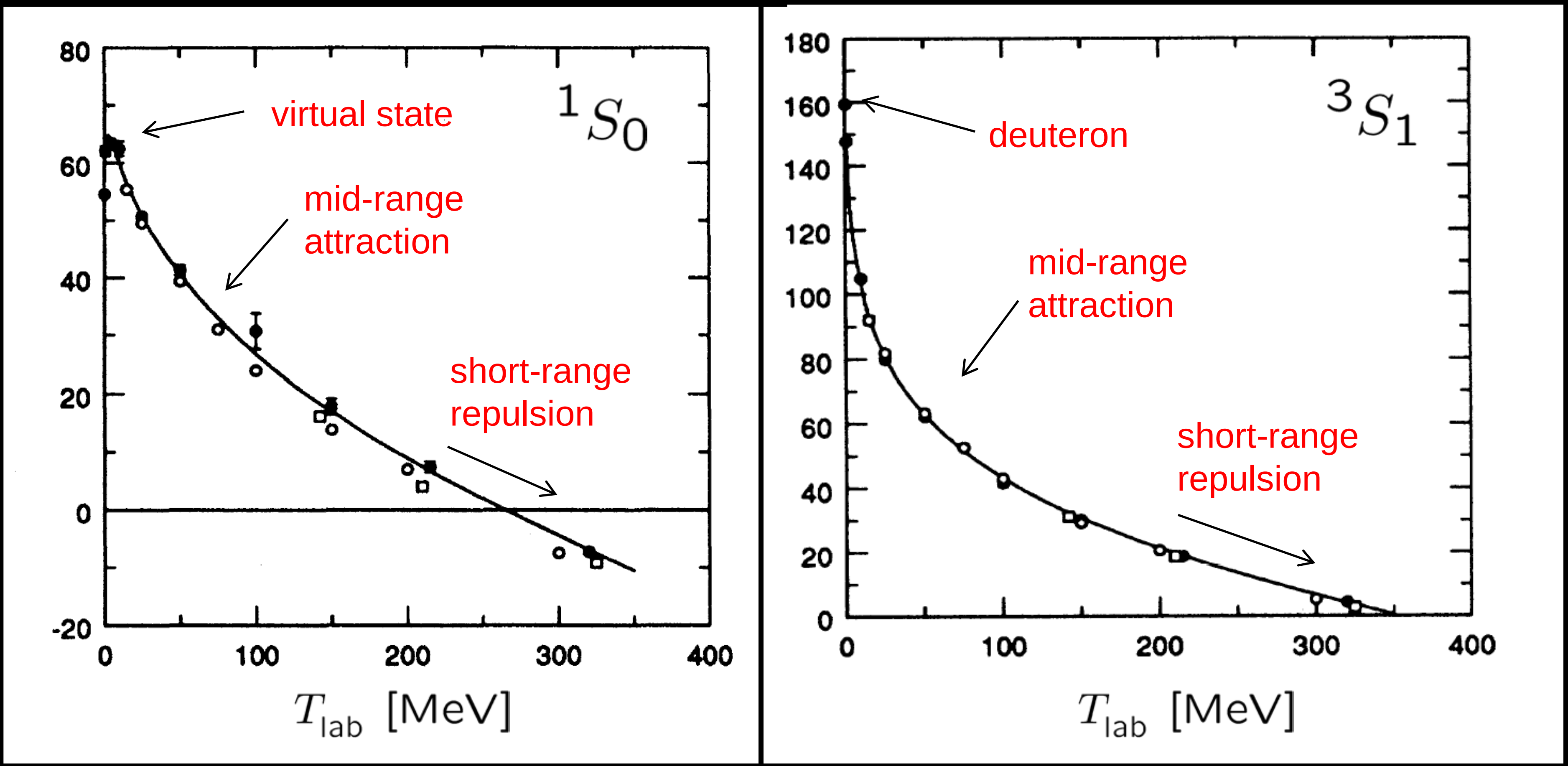


S.C.Pieper, Riv. Nuovo Cim. 031, 709 (2008) [arXiv:0711.1500 [nucl-th]].

NN interactions
critical inputs in nuclear physics

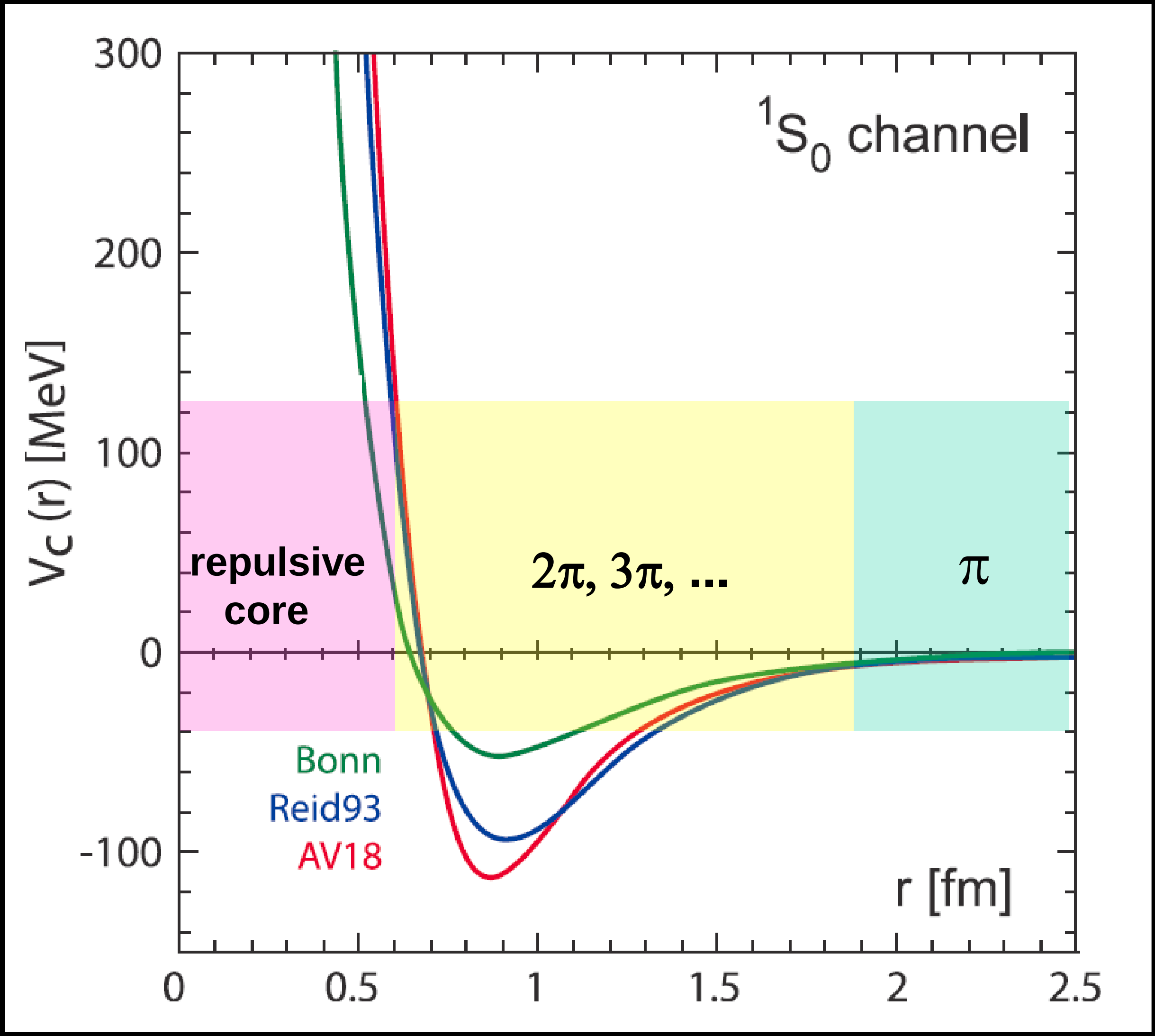
$^{2S+1}L_J$





Nijmegen partial-wave analysis,
Stoks et al., Phys.Rev. C48 (1993) 792

Key features of the Nuclear force



- One-pion exchange
  Yukawa (1935)

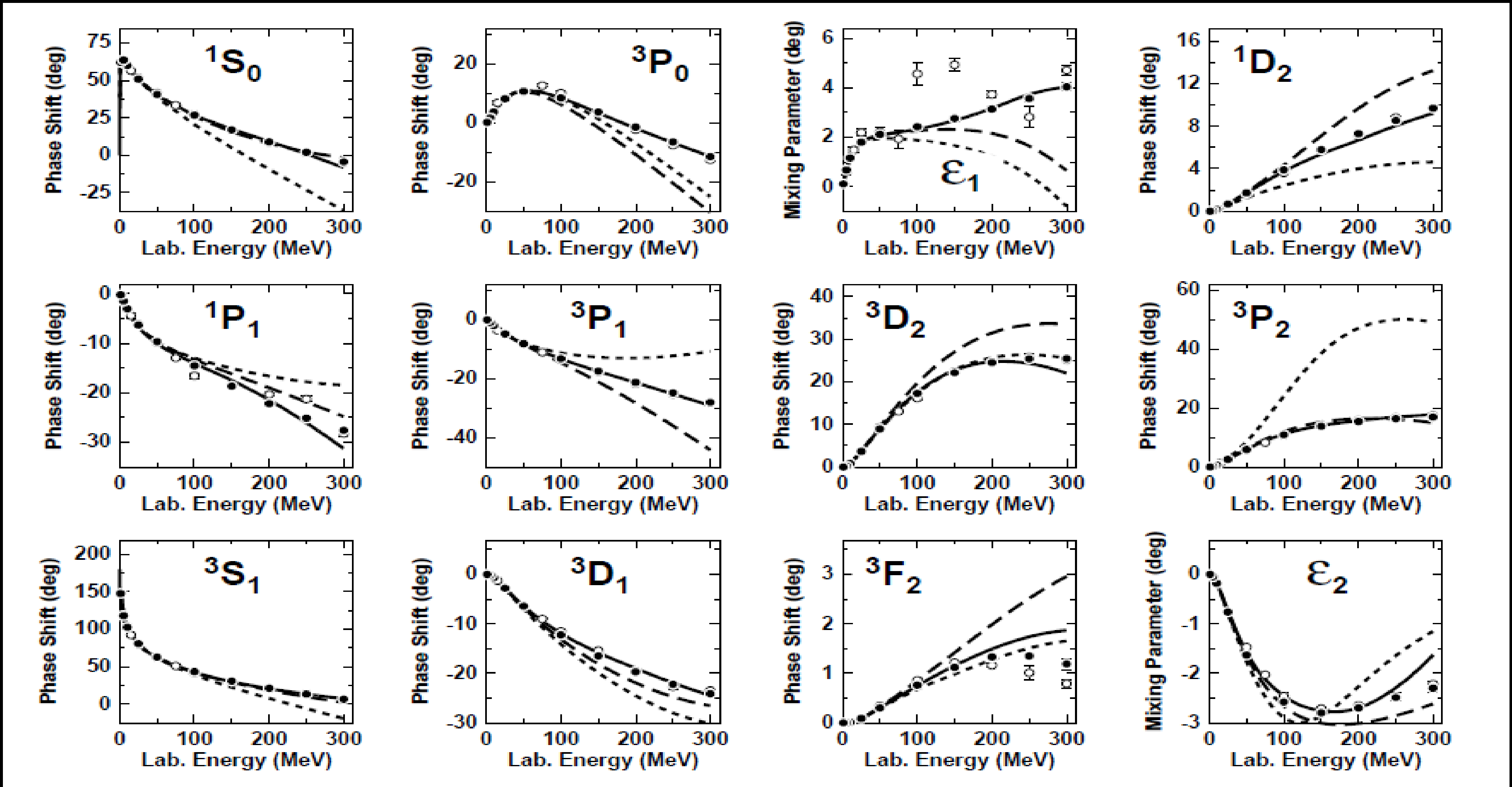
- Multi-pions
  Taketani et al. (1951)

- Repulsive core
  Jastrow (1951)

Modern high precision NN forces (90's-)

phenomenological NN interactions
-- how many parameters ? --

R. Machleidt, arXiv:0704.0807 [nucl-th]



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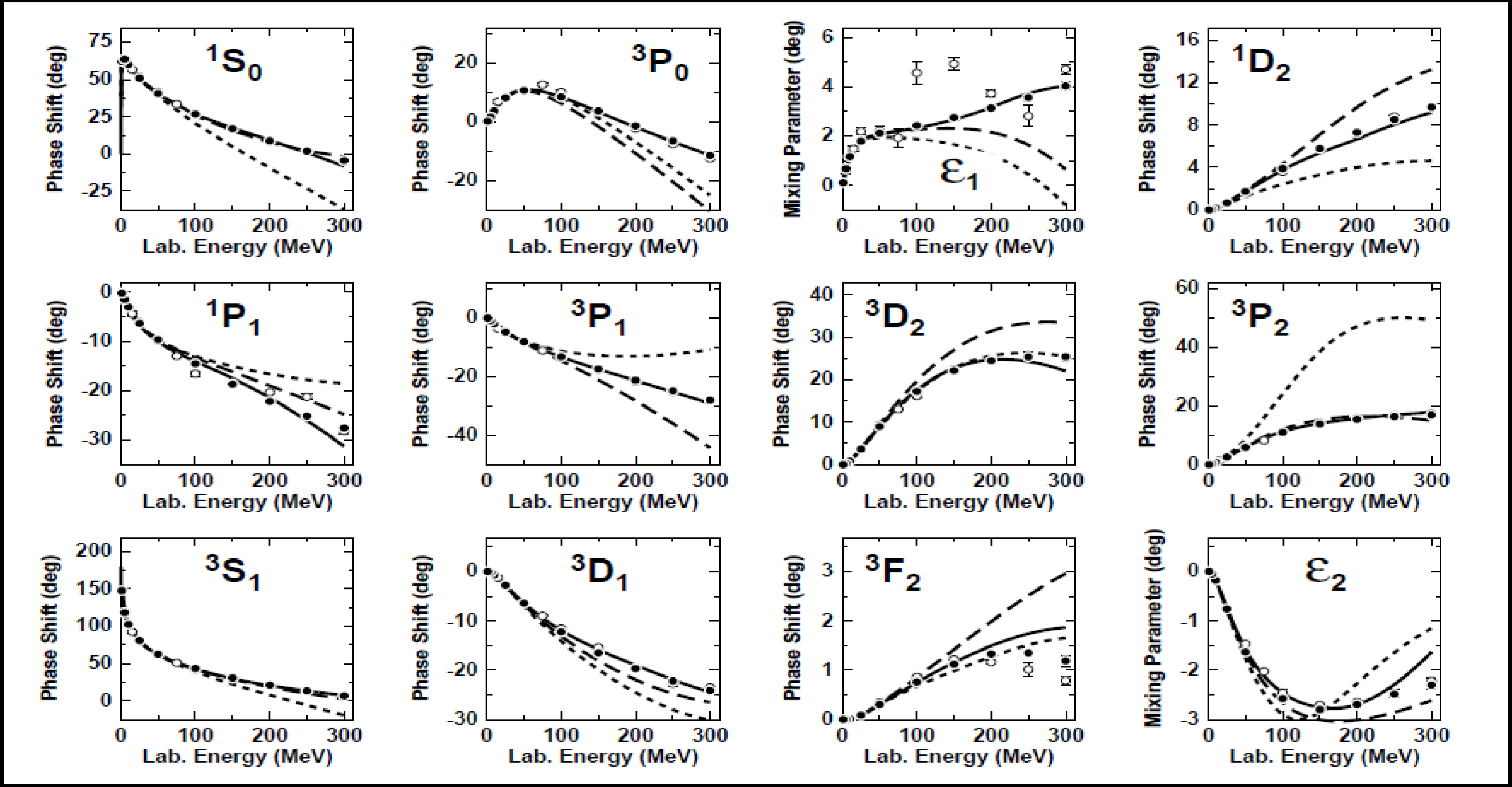


~ 4500 np and pp scattering data (T_{lab} < 300 MeV)

NNN, YN, YY: data very limited

phenomenological NN interactions
-- how many parameters ? --

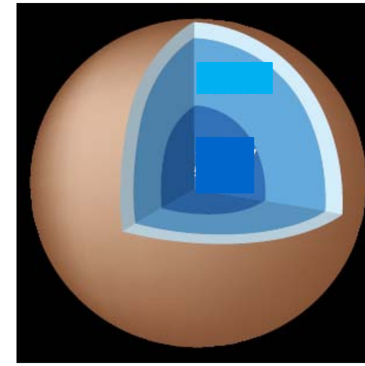
R. Machleidt, arXiv:0704.0807 [nucl-th]

high precision NN interactions		# of parameters	χ^2/dof
CD Bonn	(p space)	38	~ 1
AV18	(r space)	40	~1
EFT in N^3LO	(nπ+contact)	24	~ (1-2)

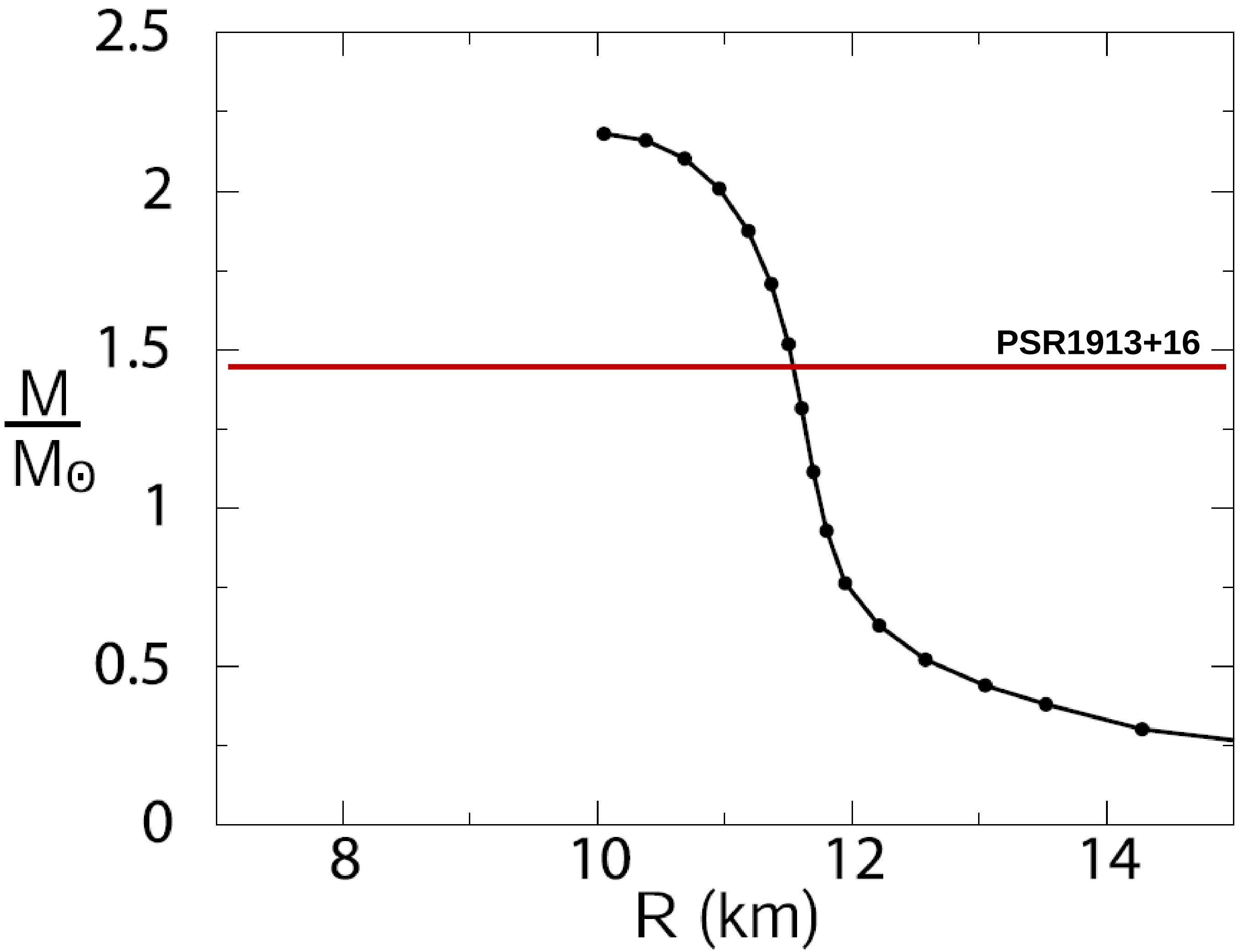
~ 4500 np and pp scattering data (T_{lab} < 300 MeV)

NNN, YN, YY: data very limited

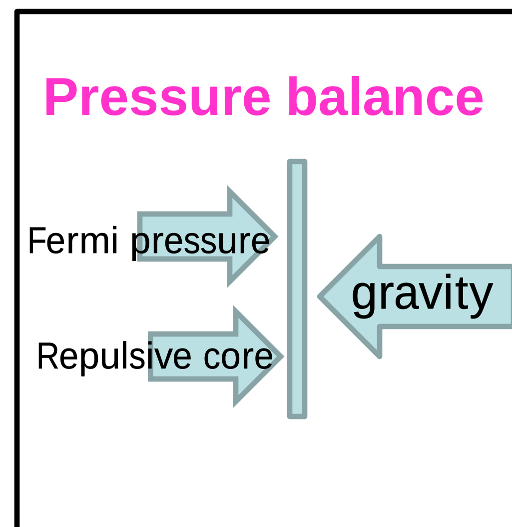
Nuclear Force and Neutron Star



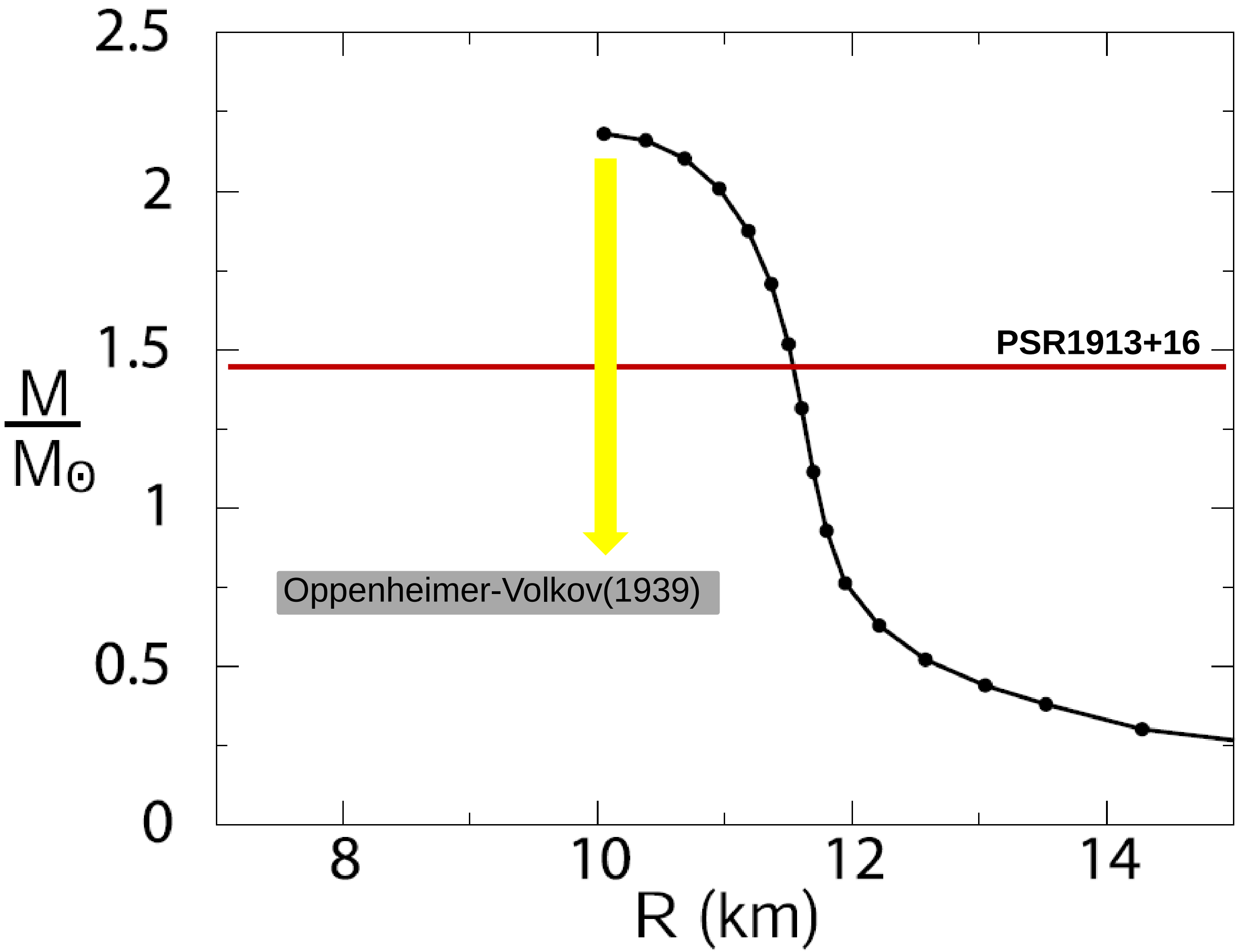
$(\rho_{max} \sim 6\rho_0)$



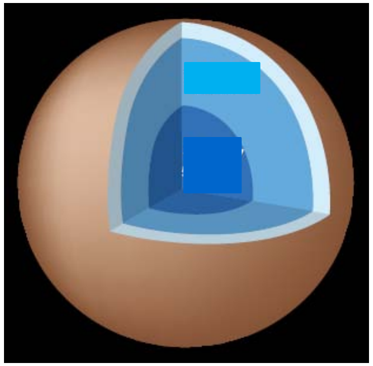
Neutron star binary



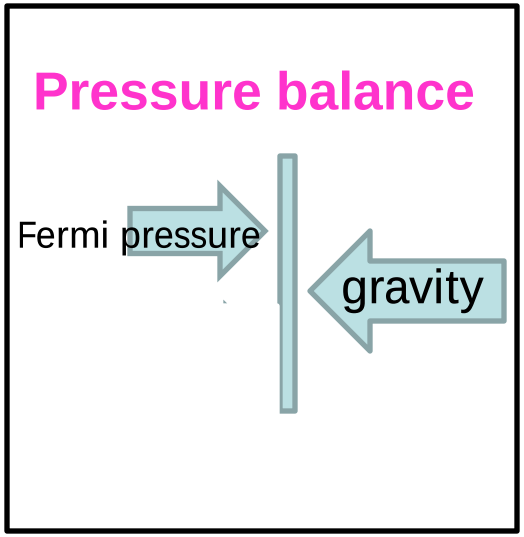
Nuclear Force and Neutron Star



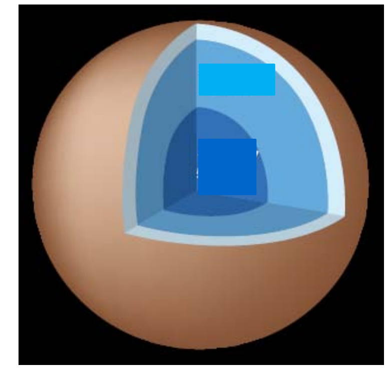
Neutron star binary



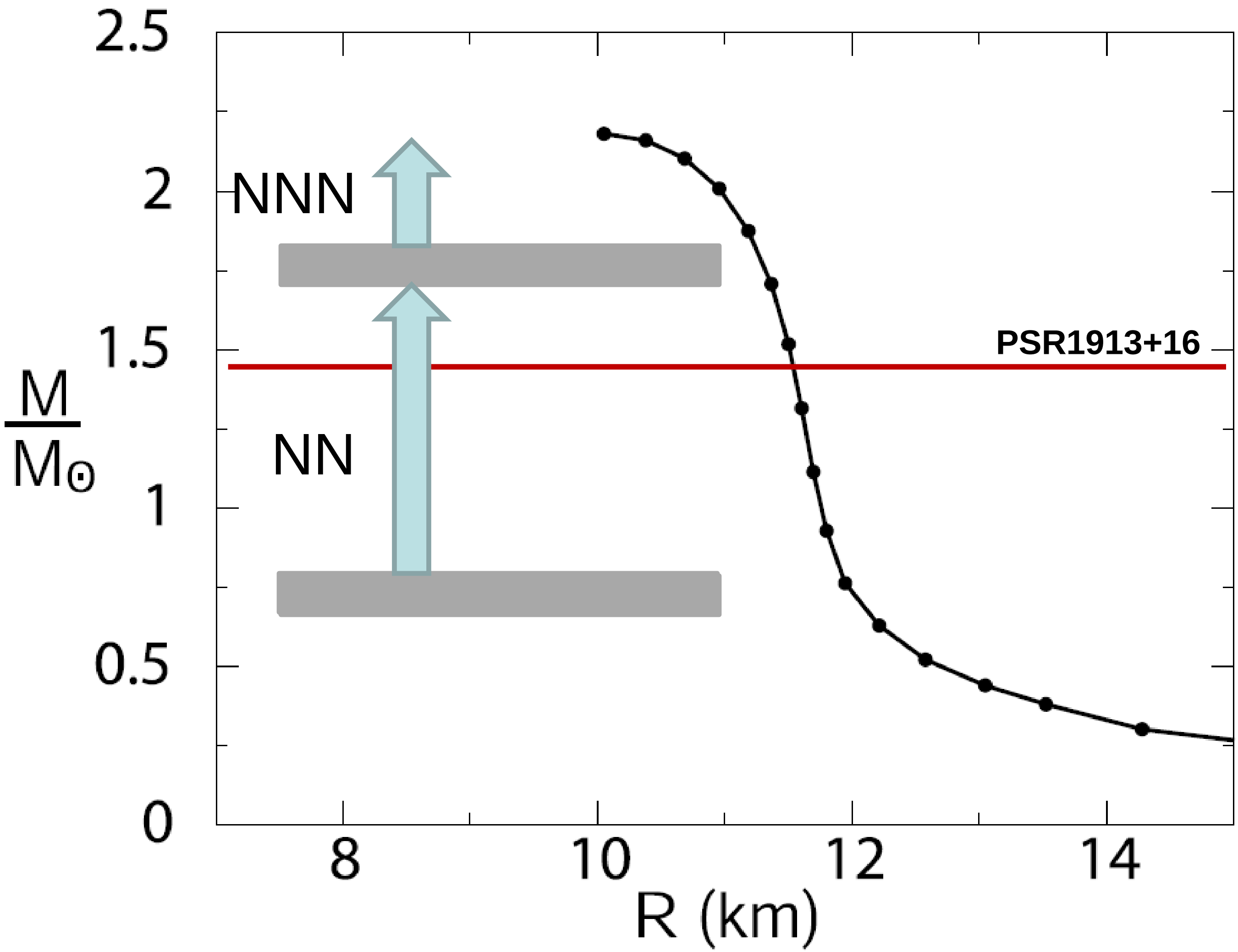
($\rho_{max} \sim 6\rho_0$)



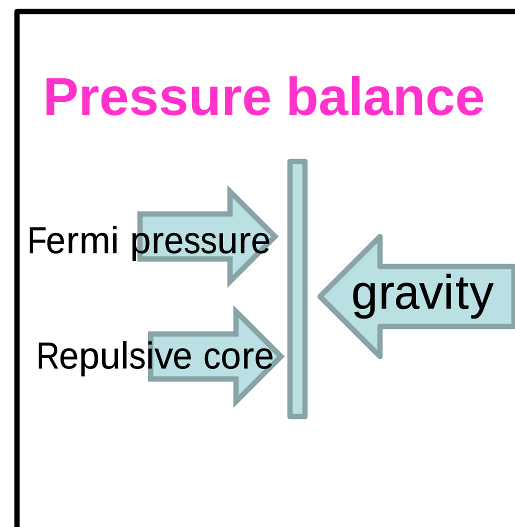
Nuclear Force and Neutron Star

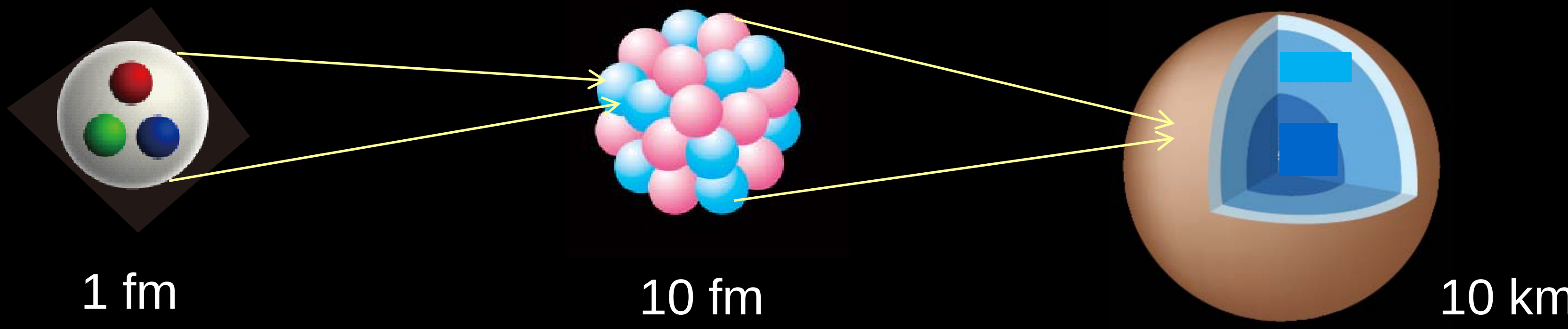


$(\rho_{max} \sim 6\rho_0)$



Neutron star binary





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[6] Summary and Future

Nuclear Physics for Lattice QCD

1. NN Phase shift (Lüscher's formula)
2. BS wave function →Lattice NN potential
3. Light nuclei
4. Strong coupling

1. Kuramashi et al. [arXiv:hep-lat/9501024].

1.2. Ishizuka et al. (CP-PACS Coll.) [arXiv:hep-lat/0503025].

1. Beane et al. (NPLQCD Coll), [arXiv:hep-lat/0602010].

2. Ishii, Aoki and Hatsuda, [arXiv:nucl-th/0611096].

3. Yamazaki et al . (PACS-CS Coll.) arXiv:0912.1383 [hep-lat].
   Yamazaki [Plenary, Thur.]

4. Miura, Nakano, Ohnishi and Kawamoto, PR D80 (2009) 074034
   de Forcrand and Fromm, [arXiv:0907.1915 [hep-lat]].

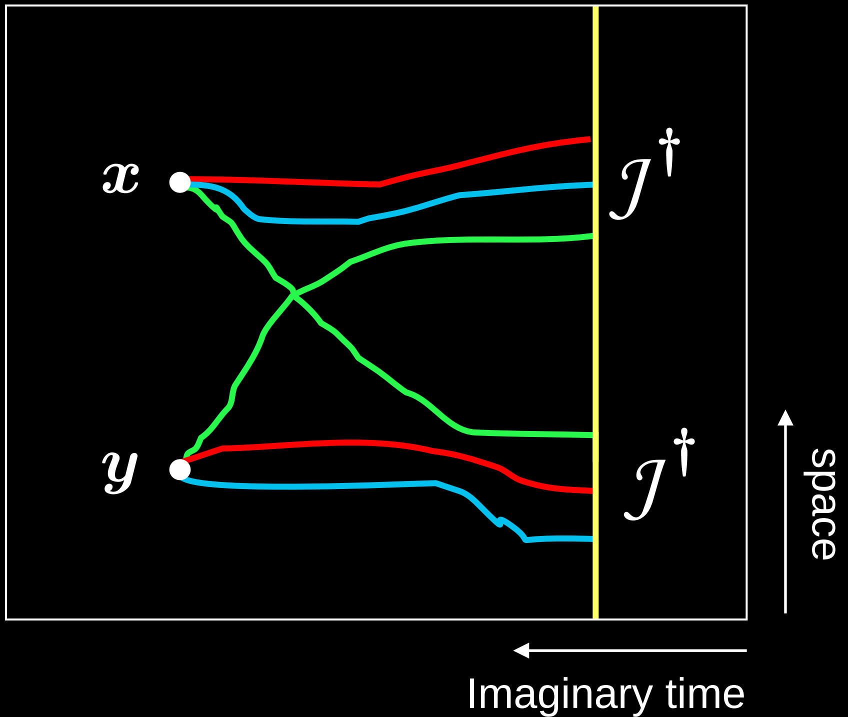
Nuclear Physics for Lattice QCD

1. NN Phase shift (Lüscher's formula)
2. BS wave function →Lattice NN potential
3. Light nuclei
4. Strong coupling

N. Ishii, T. Hatsuda (Tokyo)
T. Doi, K. Sasaki, S. Aoki (Tsukuba)
K. Murano (KEK), T. Inoue (Nihon)
Y. Ikeda (RIKEN), H. Nemura (Tohoku)

Hadrons to Atomic nuclei
HAL
from Lattice QCD

Equal-time BS amplitude $\phi(\mathbf{r})$ in lattice QCD



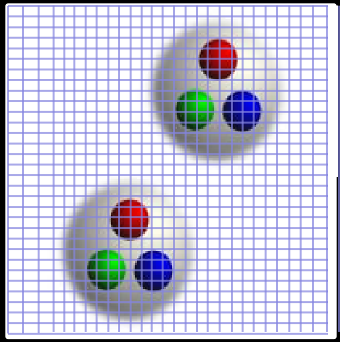
+ all possible combinations

$$
\begin{aligned}
C_4(\mathbf{r};t) &= \langle N_1(\mathbf{x},t)N_2(\mathbf{y},t)\mathcal{J}_1^\dagger(0)\mathcal{J}_2^\dagger(0)\rangle \\
&= \sum_n \langle 0|N_1(\mathbf{x})N_2(\mathbf{y})|n\rangle A_n\, e^{-E_n t} \longrightarrow \phi(\mathbf{r})A_0 e^{-E_0 t}
\end{aligned}
$$

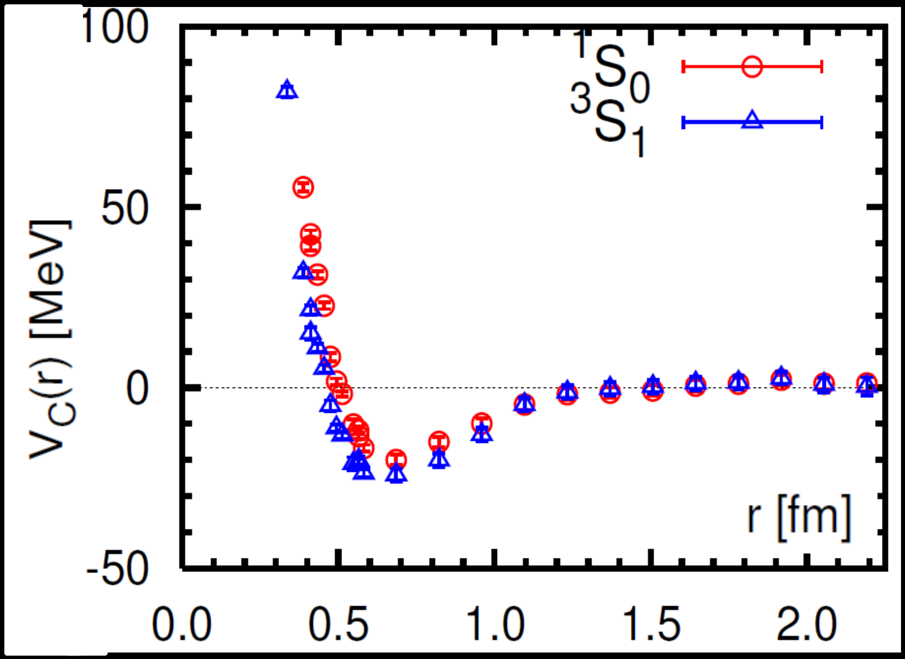
$\phi(r > R)$ → phase shift : Lüscher, Nucl. Phys. B354 (1991) 531

$\phi(r < R)$ → potential : Ishii, Aoki & Hatsuda, PRL 99 (2007) 022001

Lattice NN potential

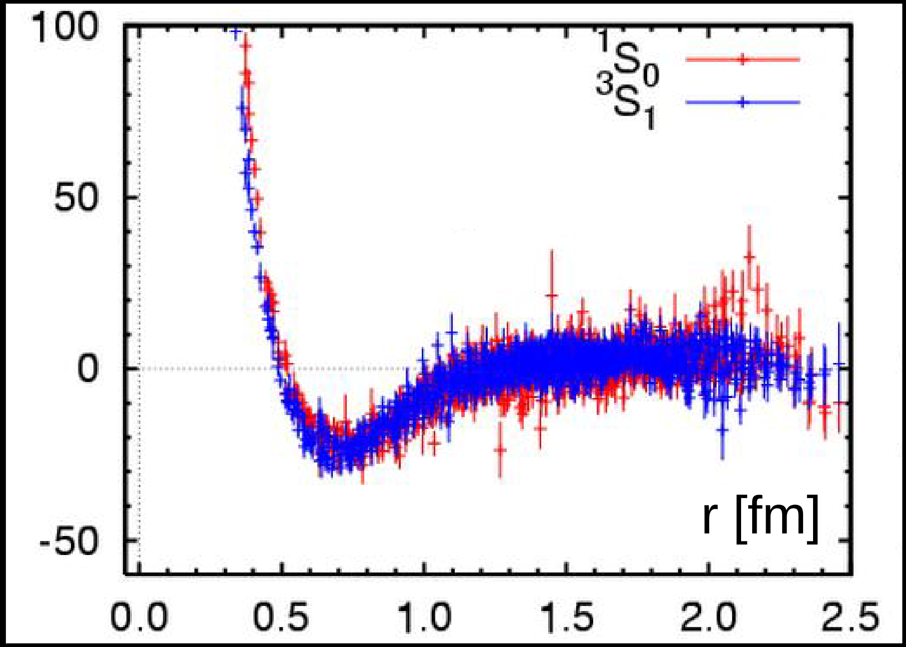


Quenched QCD
(m_π=530MeV, L=4.4 fm)



Ishii, Aoki & Hatsuda,
PRL 99 (2007) 022001

(2+1)-flavor QCD : Iawasaki+clover
(m_π=570MeV, L=2.9 fm)



Ishii, Aoki & Hatsuda,
arXive 0903.5497 [hep-lat]

HAL QCD procedure : 5 steps to go

Aoki, Hatsuda & Ishii, PTP 123 (2010) 89-128 [0909.5585 [hep-lat]],

(i) Choose a composite operator: e.g. $N(x) = \epsilon_{abc} q^a(x) q^b(x) q^c(x)$

(ii) Measure the BS amplitude: $\phi_n(\vec{r}) = \langle 0 | N(\vec{x}+\vec{r}) N(\vec{x}) | (6q)_n \rangle$

(iii) Calculate off-shell T-matrix: $L_n(\vec{r}) = (k_n^2 + \nabla^2)\phi_n(\vec{r})$

(iv) Derive non-local potential: $U(\vec{r},\vec{r}') = \sum_{n,n'}^{n_c} L_n(\vec{r}) \mathcal{N}_{nn'}^{-1} \phi_{n'}^*(\vec{r}')$

$$(k_n^2 + \nabla^2)\phi_n(\vec{r}) = \int U(\vec{r},\vec{r}')\phi_n(\vec{r}')d^3r'$$

(v) Make derivative expansion: $U(\vec{r},\vec{r}') = m_N V(\vec{r},\nabla)\delta(\vec{r}-\vec{r}')$

$$V(\vec{r},\nabla) = V_{\rm C}(r) + S_{12}V_{\rm T}(r) + \vec{\rm L}\cdot\vec{\rm S}\, V_{\rm LS}(r) + \{V_{\rm D}(r), \nabla^2\} + \cdots$$

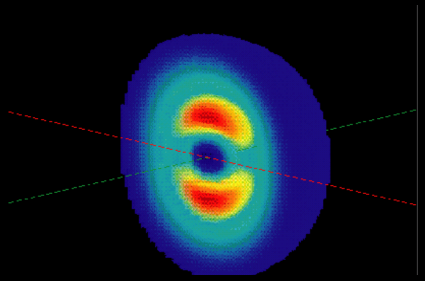
LO LO NLO NNLO

LO LO NLO NNLO

$$V(\vec{r},\nabla) = V_{\rm C}(r) + S_{12}V_{\rm T}(r) + \vec{\rm L}\cdot\vec{\rm S}\; V_{\rm LS}(r) + \{V_{\rm D}(r),\nabla^2\} + \cdots$$

central tensor spin-orbit

central	tensor	spin-orbit
Nuclear Binding Nuclear saturation S-wave superfluidity	Deuteron binding	P-wave superfluidity



1. *U(r,r')* reproduces phase shift, and is E-independent
2. { *N(x), U(r,r')* } is a pair to reproduce observables
3. Validity of *(p/Λ)*^{2} -expansion needs to be checked

Murano [Parallel 38, Thur.]

4. Coupled channel potential

Sasaki [Parallel 49, Fri.]
Ishii [Parallel 50, Fri.]

5. NNN force from $\phi(r, \rho)$

Doi [Parallel 49, Fri.]

LO potentials : $V_C(r)$ & $V_T(r)$

mixing between $^3S_1$ and $^3D_1$ through the tensor force

$$|\phi\rangle = |\phi_S\rangle + |\phi_D\rangle$$

$$|\phi_S\rangle = \mathcal{P}|\phi\rangle = \frac{1}{24}\sum_{\mathcal{R}\in\mathcal{O}} \mathcal{R}|\phi\rangle$$

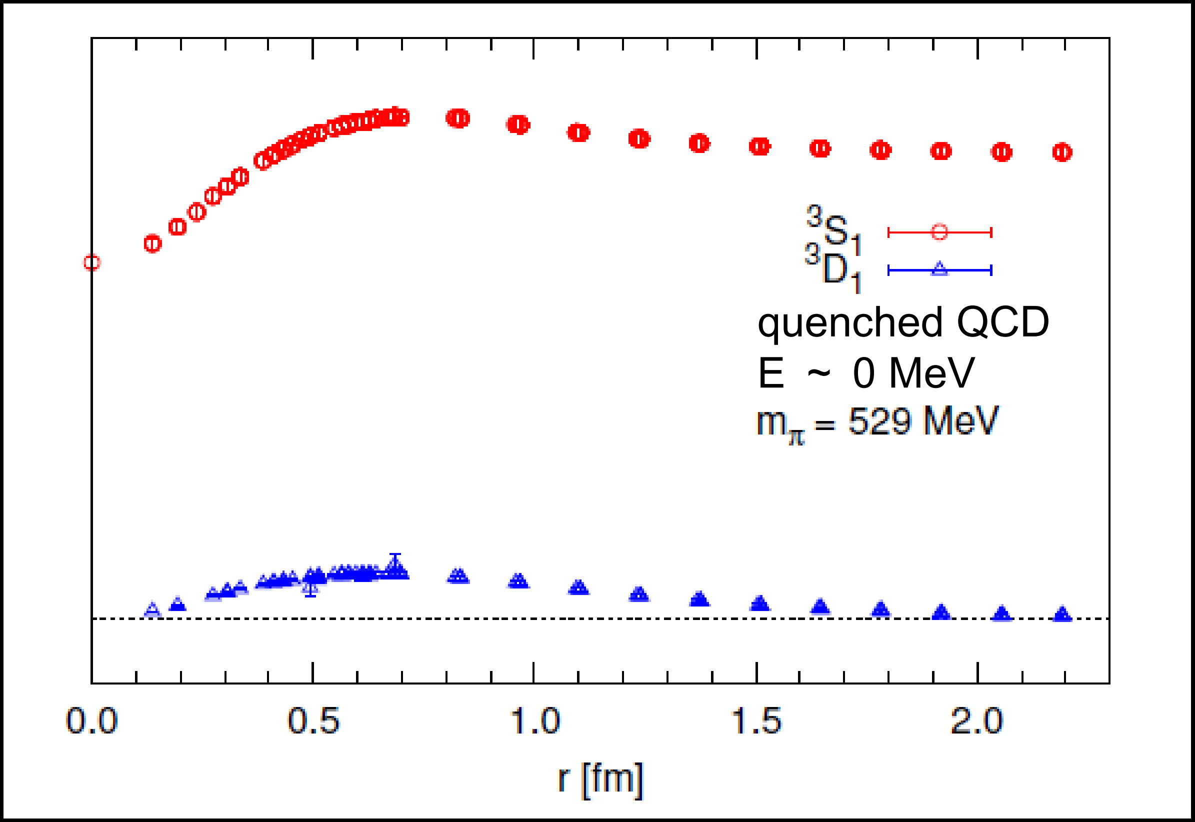
$$|\phi_D\rangle = \mathcal{Q}|\phi\rangle = (1-\mathcal{P})|\phi\rangle$$

$$\mathcal{P}(H_0 + V_C + S_{12}V_T)|\phi\rangle = E\mathcal{P}|\phi\rangle$$

$$\mathcal{Q}(H_0 + V_C + S_{12}V_T)|\phi\rangle = E\mathcal{Q}|\phi\rangle$$

LO potentials : $V_C(r)$ & $V_T(r)$

Aoki, Hatsuda & Ishii,
0909.5585 [hep-lat]
PTP 123 (2010) 89-128



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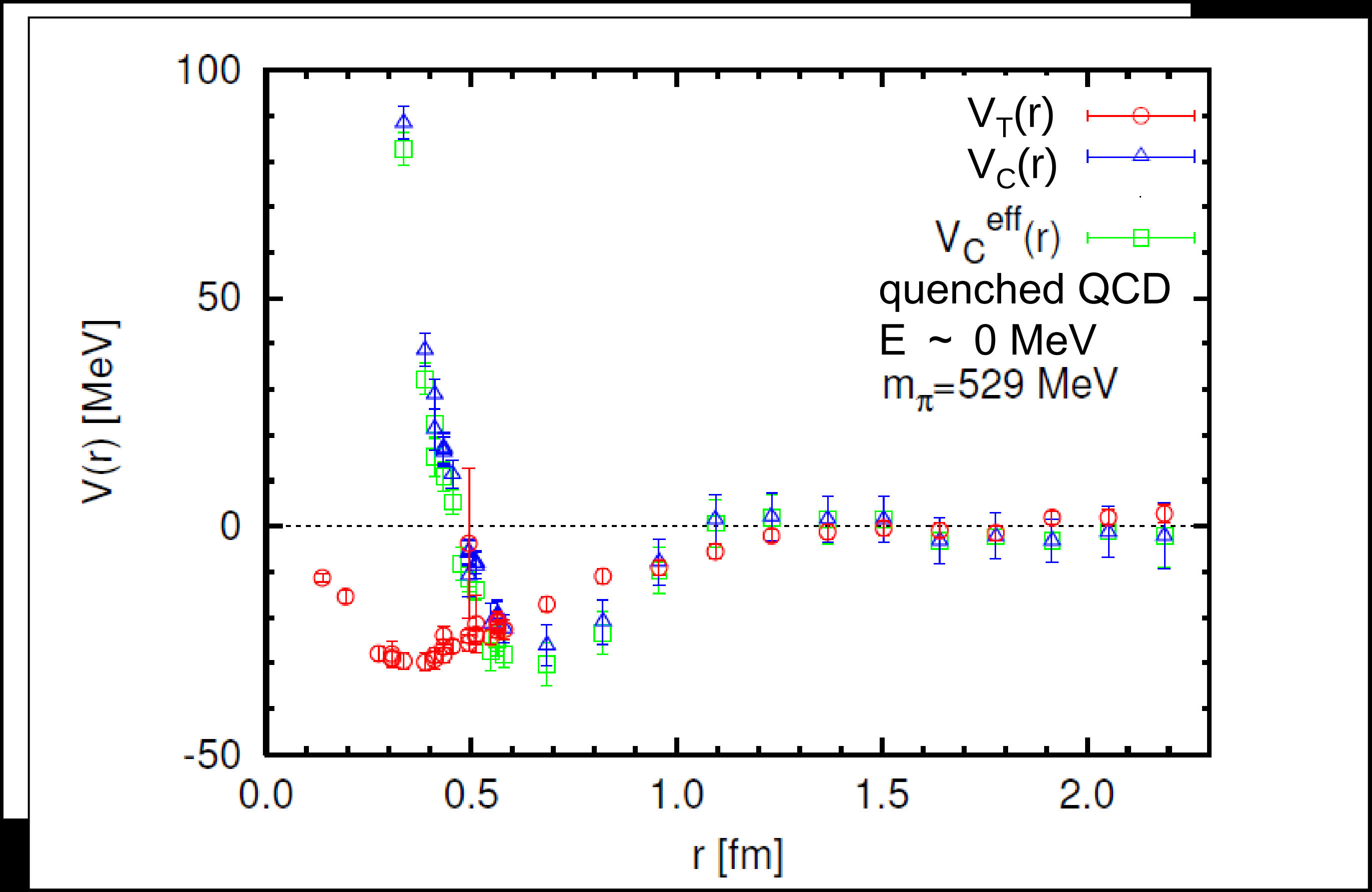
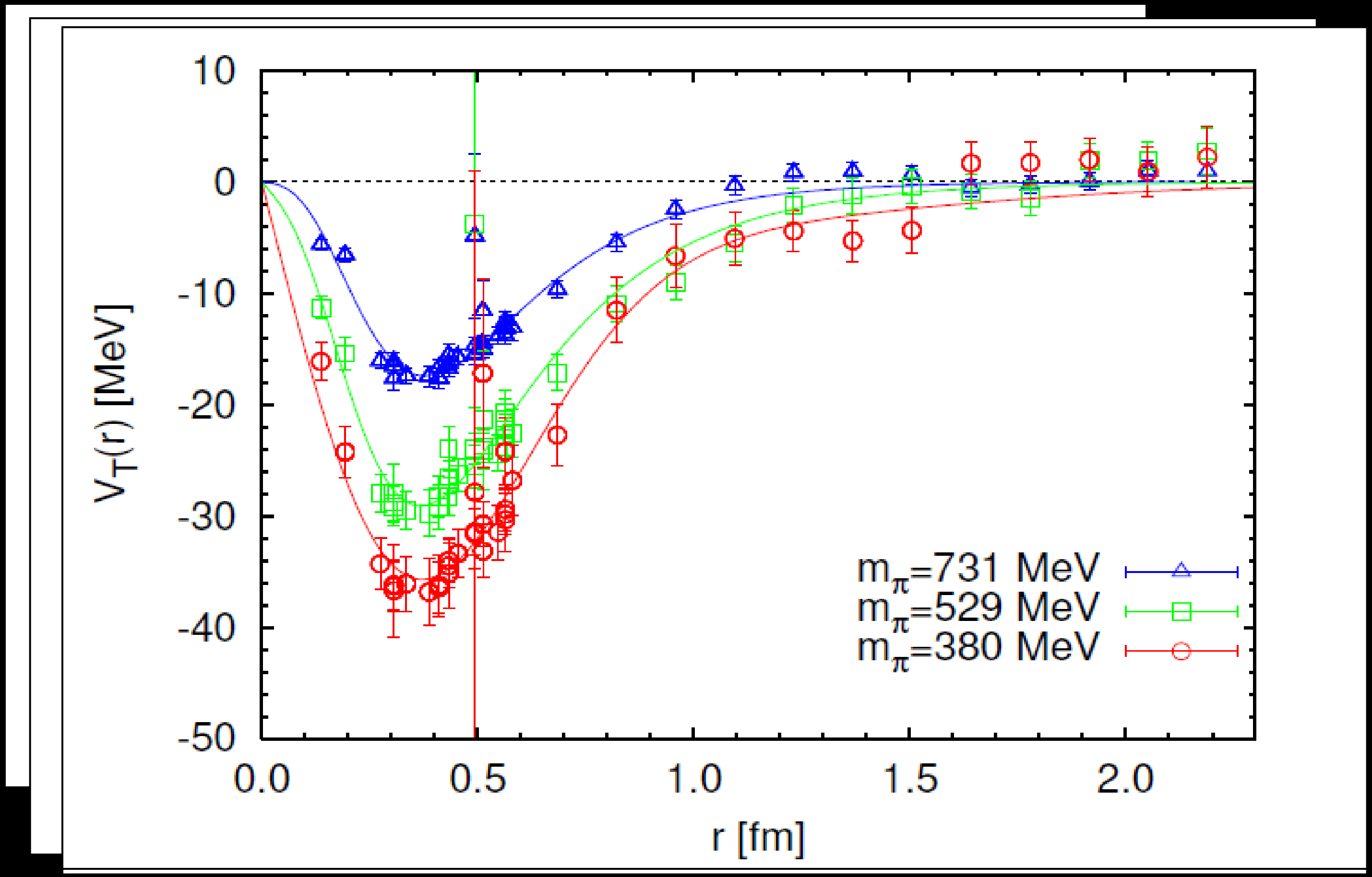


$V_c(r \to 0) \sim (\log r)^{\beta}/r^2$, $V_T(r \to 0) \to 0$ from OPE
Aoki, Balog & Weisz, JHEP 1005, 008 (2010)

LO potentials : $V_C(r)$ & $V_T(r)$

Aoki, Hatsuda & Ishii,
0909.5585 [hep-lat]
PTP 123 (2010) 89-128



fit function

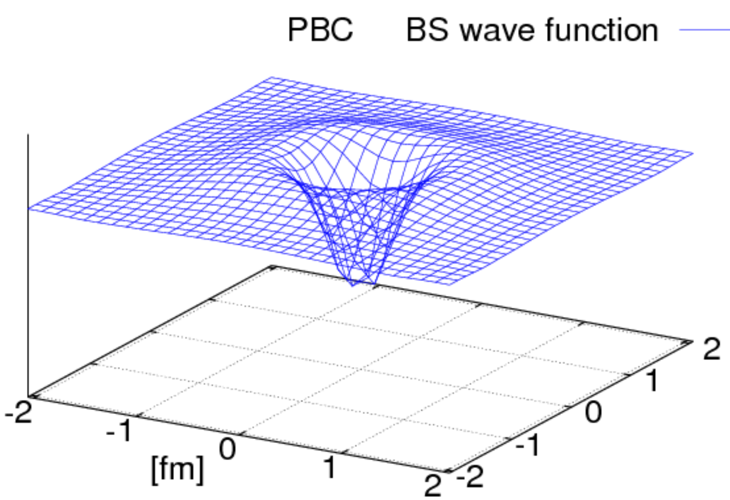
$$V_T(r) = b_1(1-e^{-b_2 r^2})^2\left(1+\frac{3}{m_\rho r}+\frac{3}{(m_\rho r)^2}\right)\frac{e^{-m_\rho r}}{r} + b_3(1-e^{-b_4 r^2})^2\left(1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2}\right)\frac{e^{-m_\pi r}}{r},$$

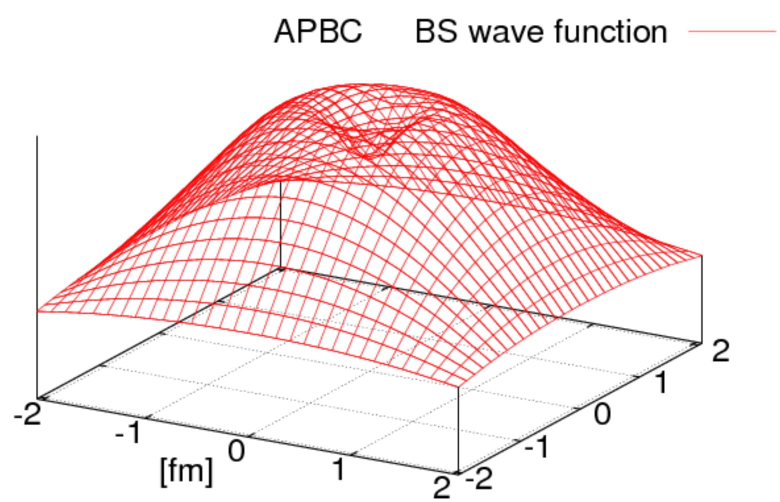
- Rapid quark-mass dependence of $V_T(r)$
- Evidence of the one-pion-exchange

NNLO potential of O(∇^2): how large ?

$$V(\vec{r},\nabla) = V_{\rm C}(r) + S_{12}V_{\rm T}(r) + \vec{\rm L}\cdot\vec{\rm S}\ V_{\rm LS}(r) + \{V_{\rm D}(r),\nabla^2\} + \cdots$$

- PBC (T_{Lab}∼0 MeV)
- APBC (T_{Lab}∼100 MeV)





Murano [Parallel 38, Thur.]

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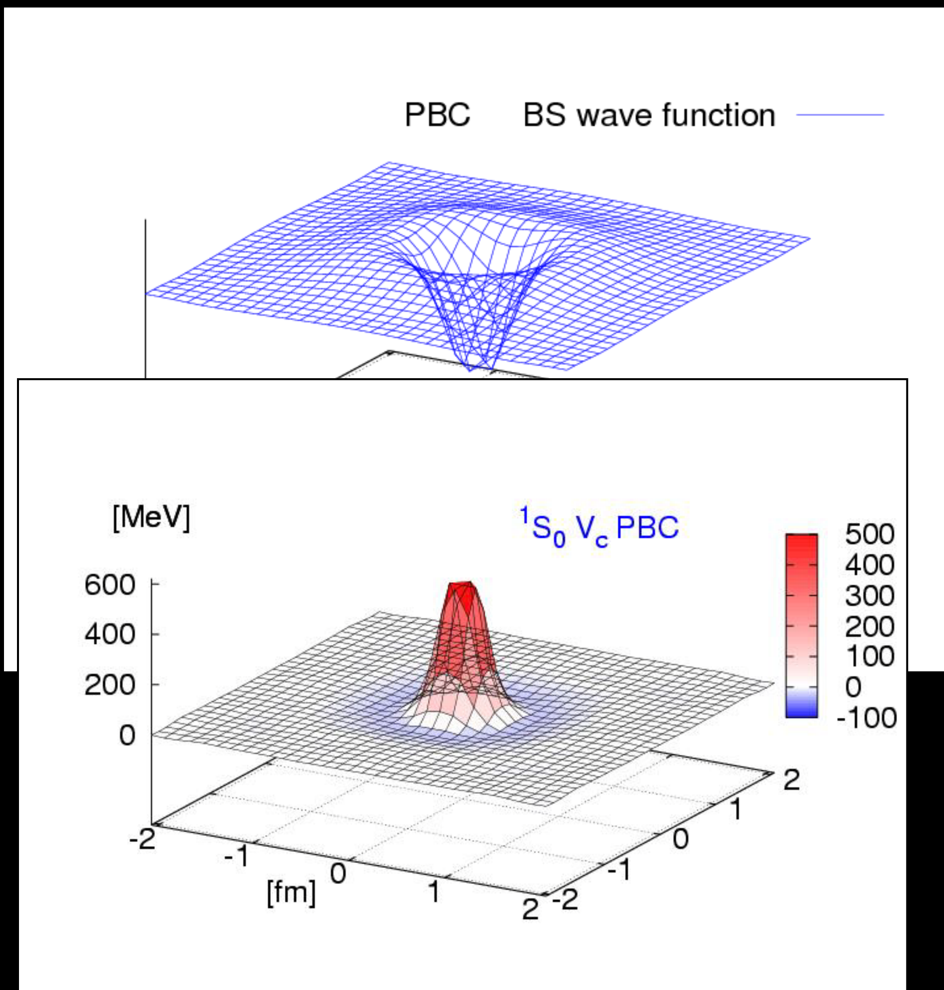

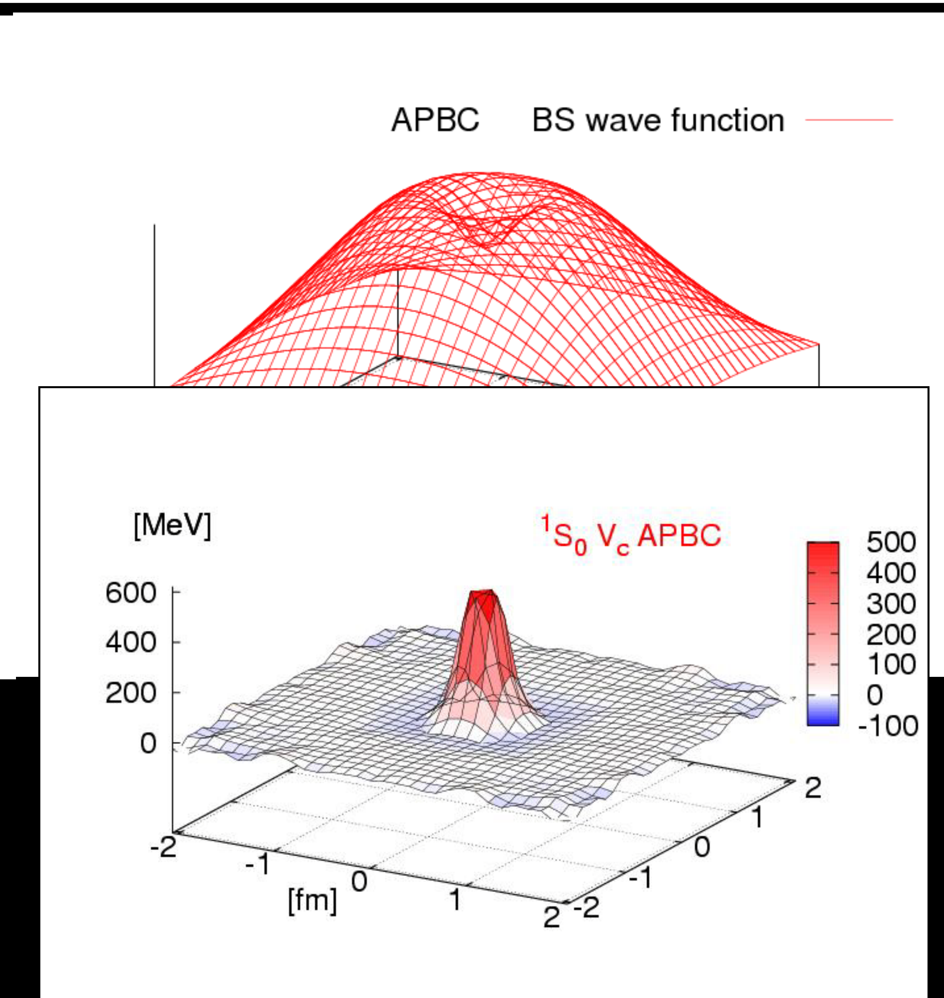

Murano [Parallel 38, Thur.]

NNLO potential of O(∇^2): how large ?

$$V(\vec{r},\nabla) = V_C(r) + S_{12}V_T(r) + \vec{L}\cdot\vec{S}\,V_{LS}(r) + \{V_D(r),\nabla^2\} + \cdots$$

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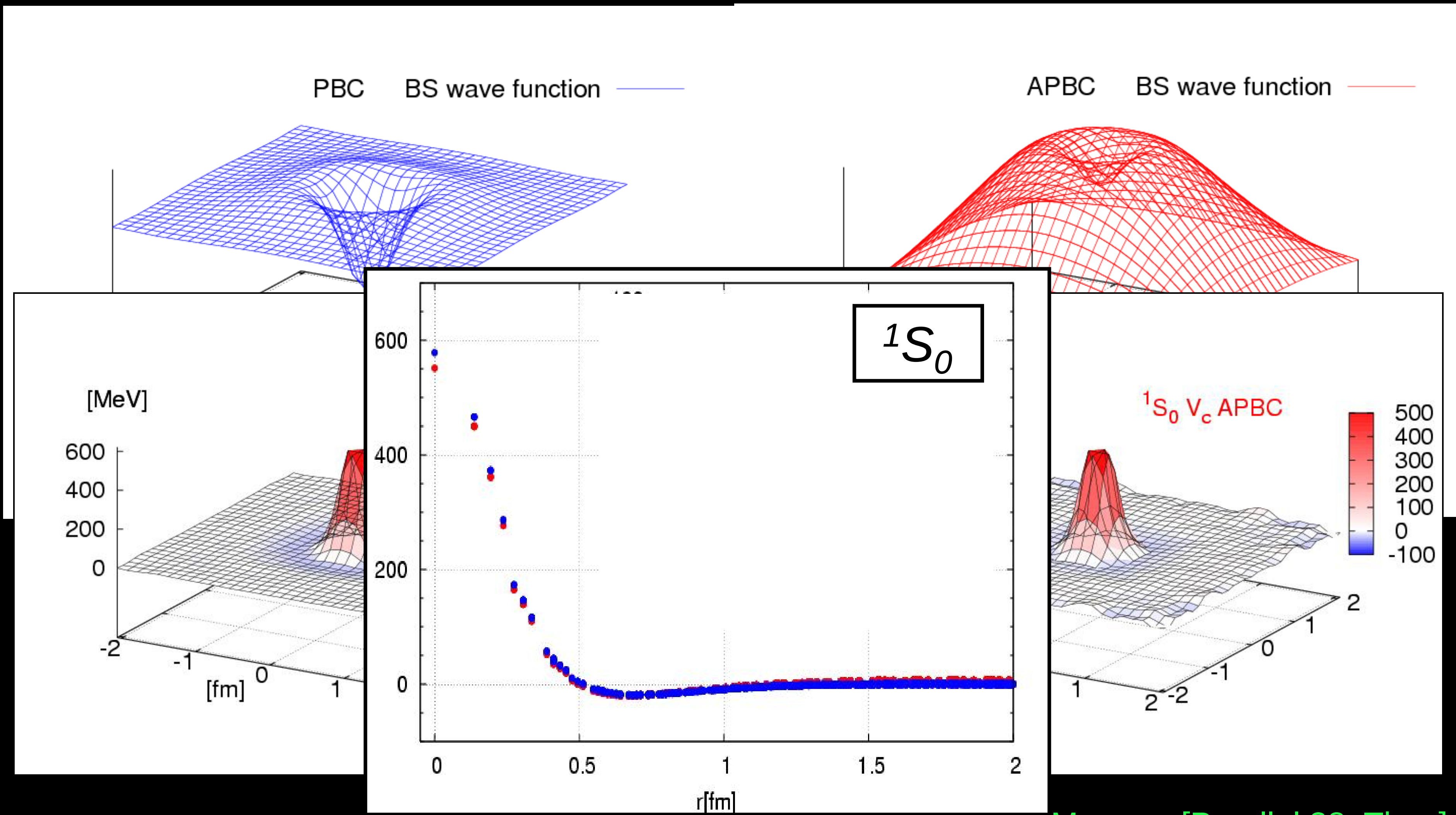
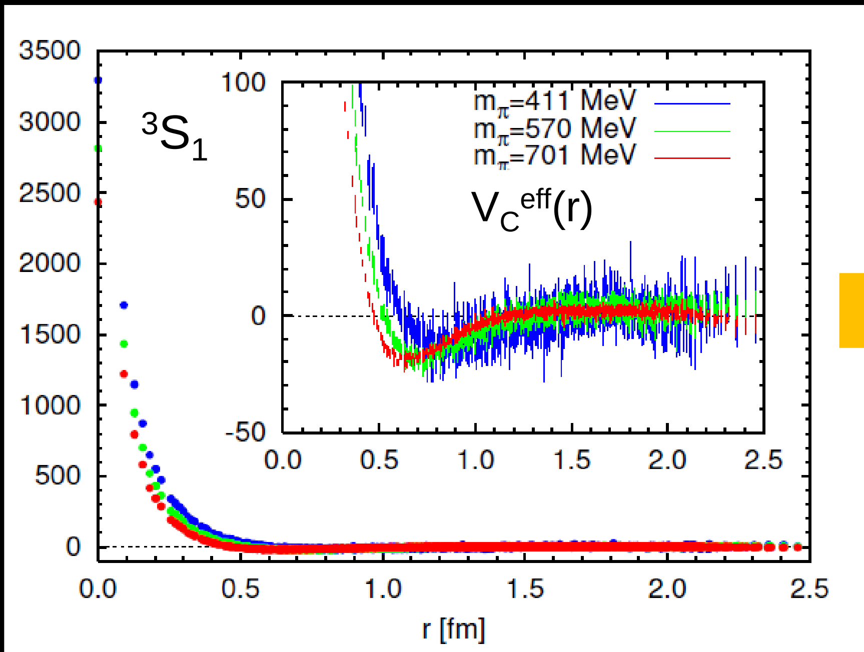
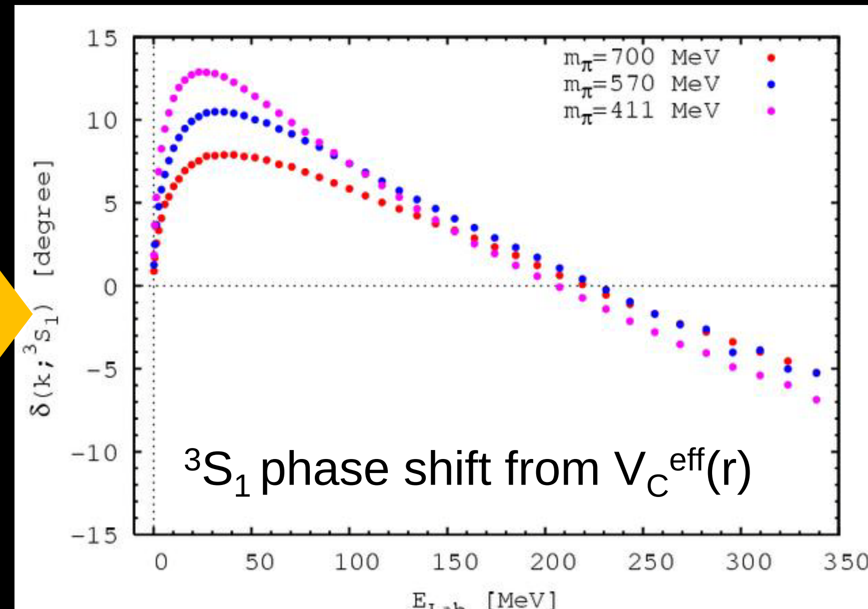
- APBC (T_{Lab}∼100 MeV)

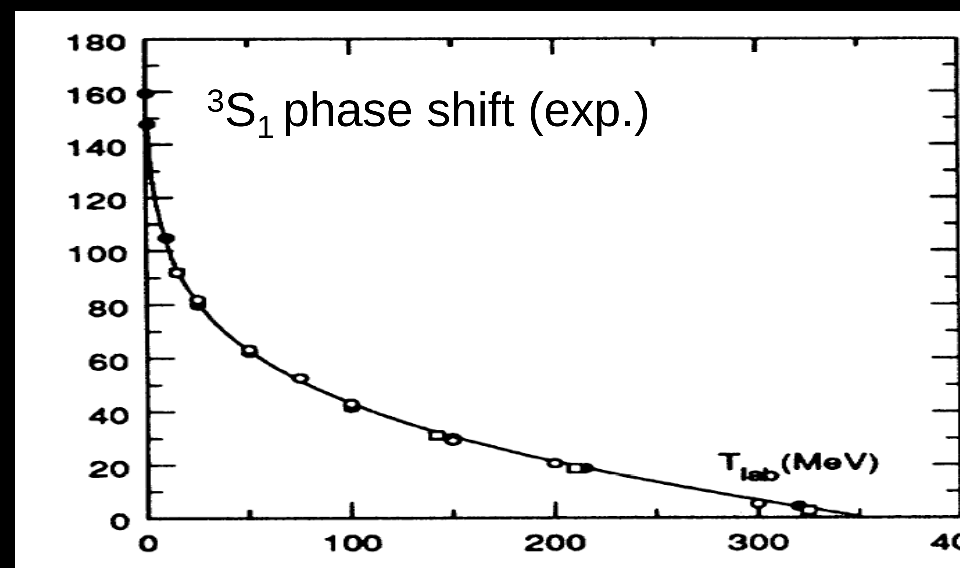


Murano [Parallel 38, Thur.]

Phase shifts from V(r) in (2+1)-flavor QCD





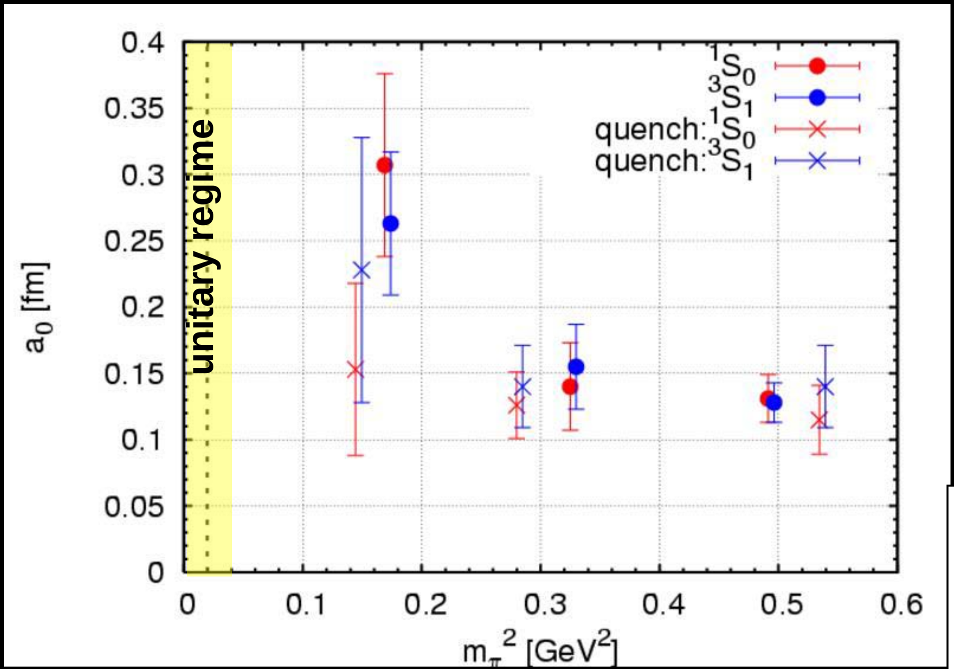


deuteron not bound
for $m_\pi \geqq 410$ MeV

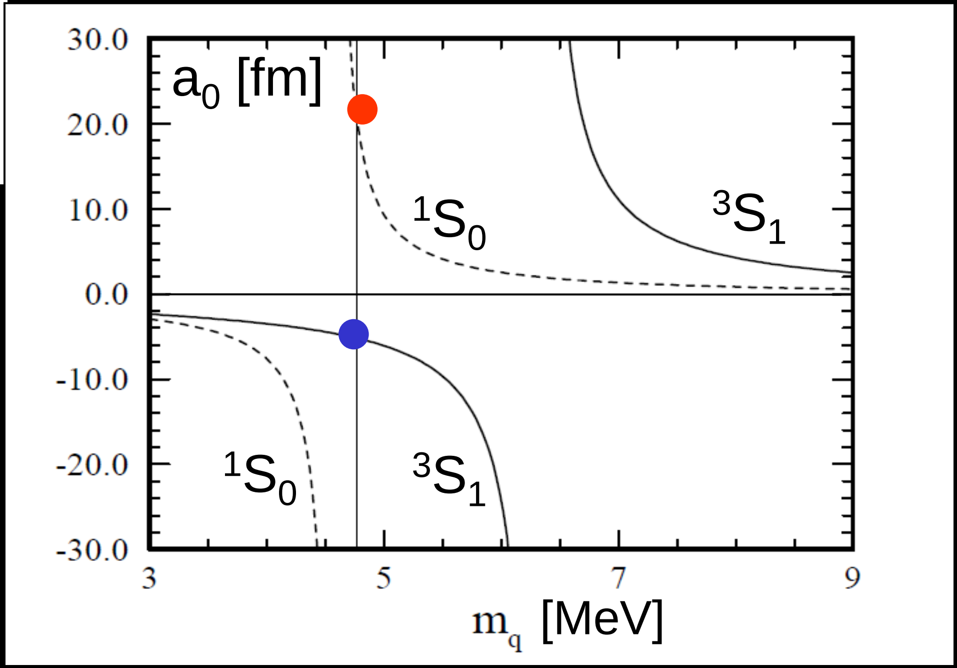
Ishii et al. (HAL QCD Coll.),
arXiv:1004.0405 [hep-lat]

NN scattering lengths in full QCD

BS wave func. $\rightarrow q^2 \rightarrow$ Luscher's formula



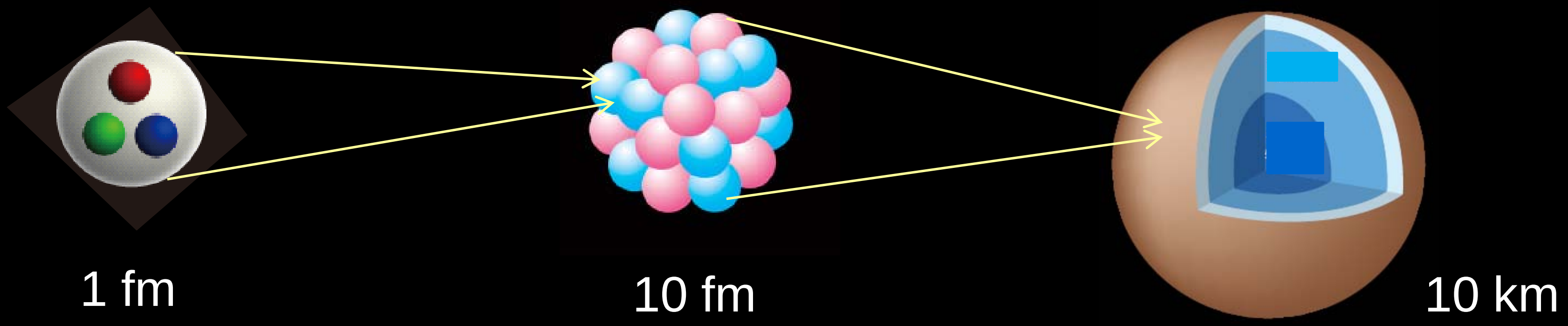
Kuramashi Plot [hep-lat/9510025]



NN interaction

- net attraction at low energy
- still far from "unitary regime"
- V(r) : mild func. of m_q

  a_0 : highly sensitive to m_q



Outline

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[2] nuclear force from lattice QCD

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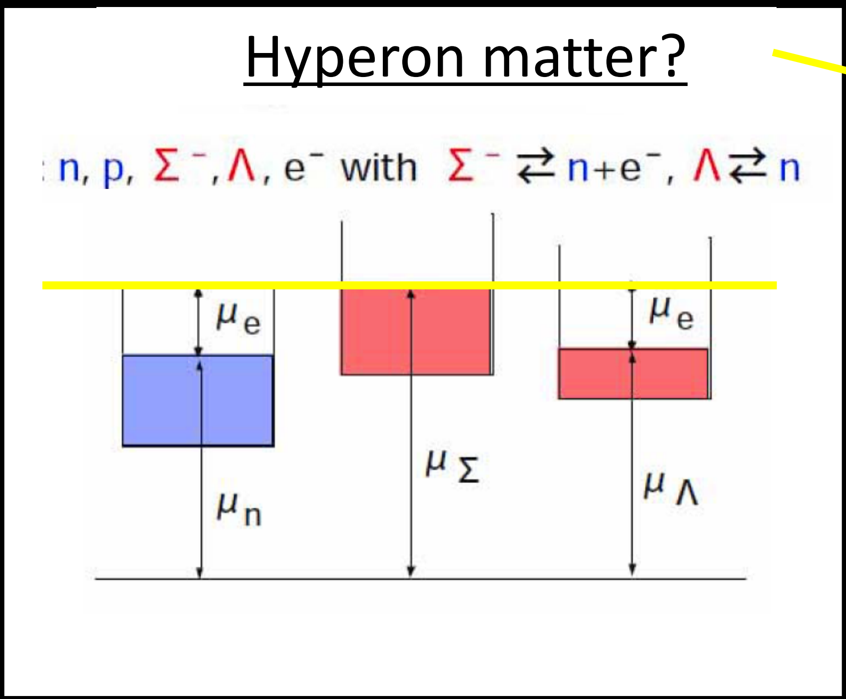
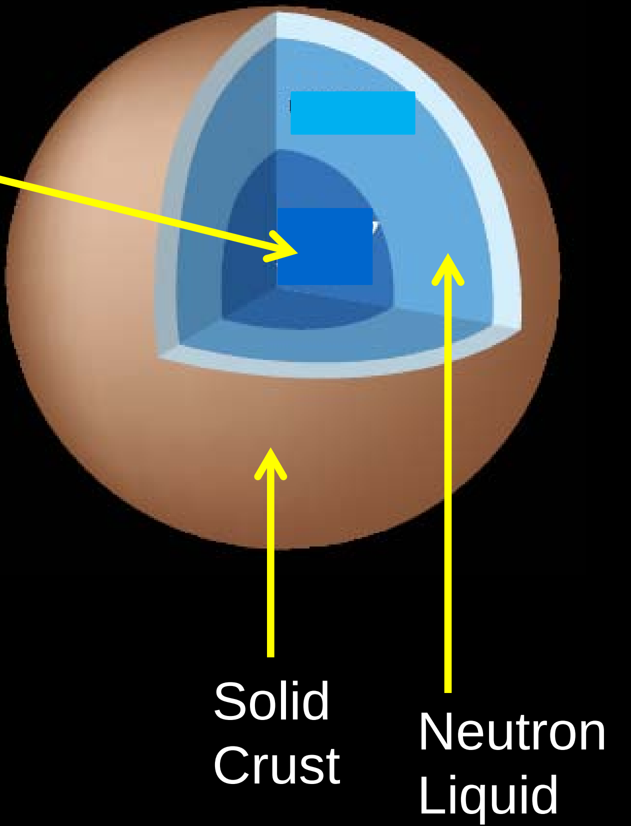
[6] Summary and Future

YN and YY interactions

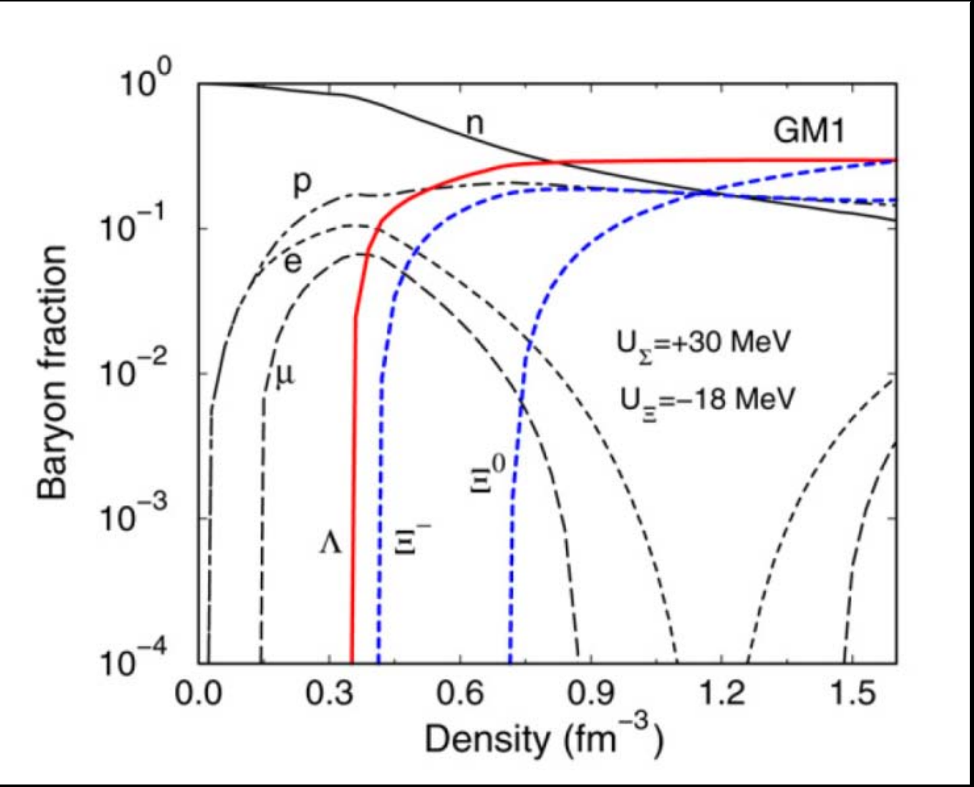
Radius ～ 10 km
Mass ～ solar mass
Central density ～ 10^{12} kg/cm^3

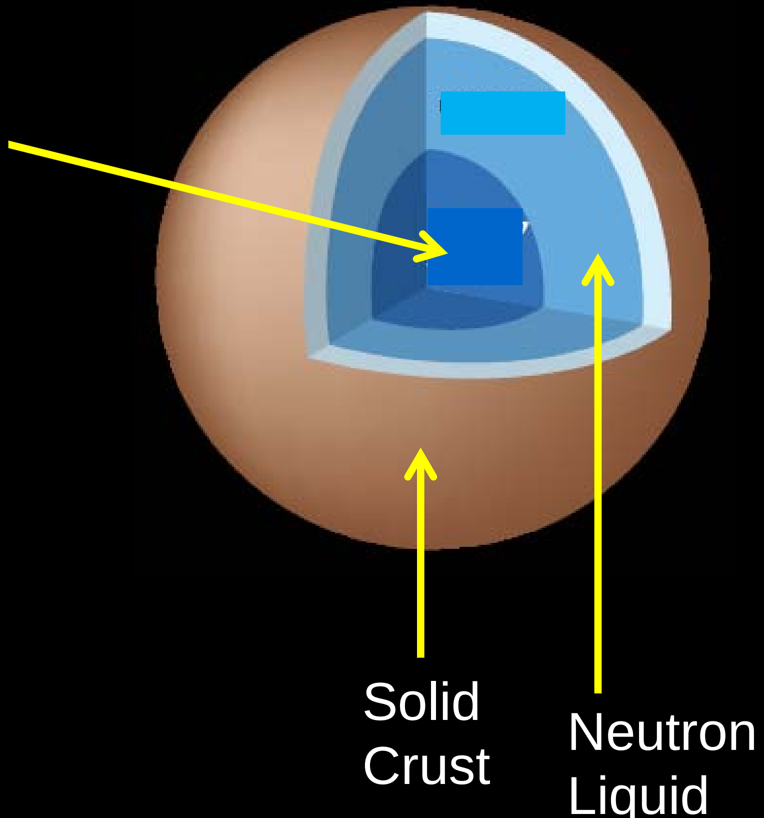




YN and YY interactions

Radius ∼ 10 km
Mass ∼ solar mass
Central density ∼ 10^{12} kg/cm^3





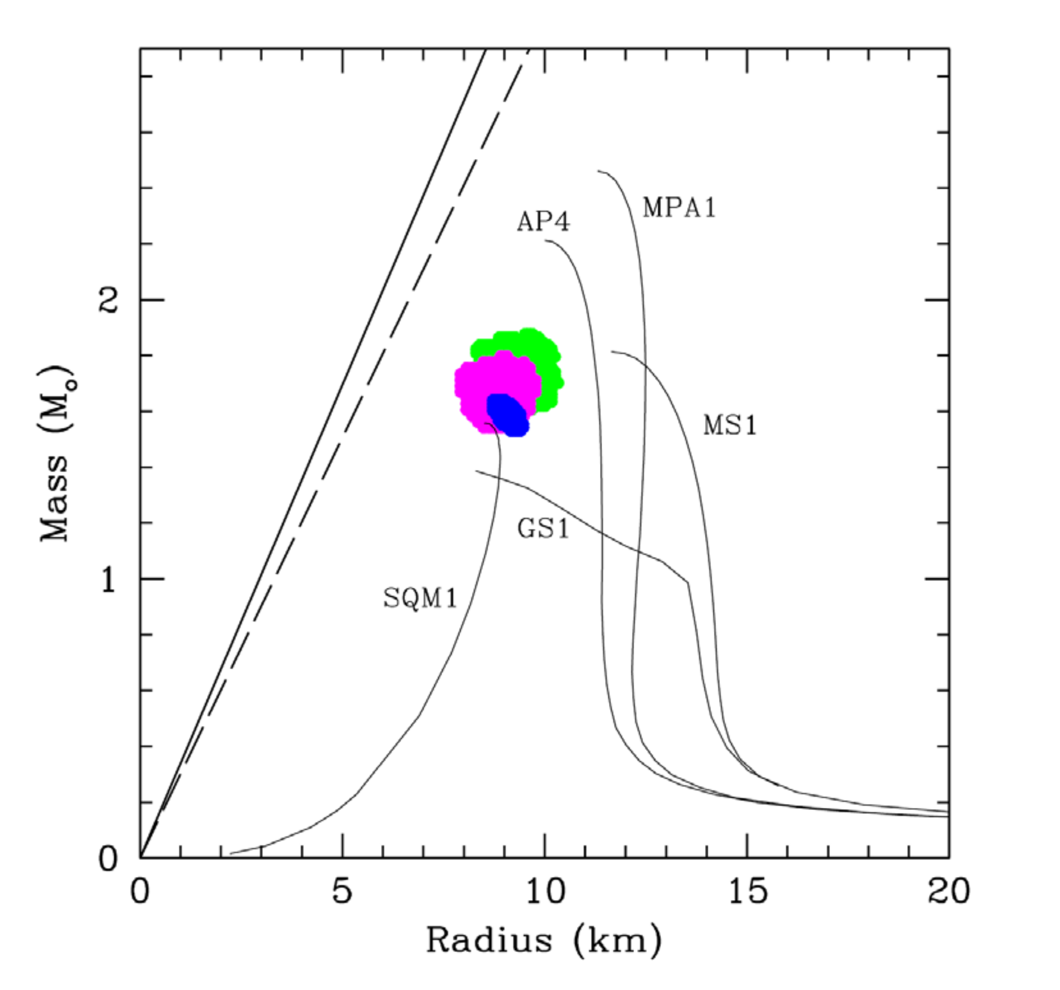
Schaffner-Bielich, ``Strangeness in Compact Stars,''

Nucl. Phys.A 835, 279 (2010) [arXiv:1002.1658 [nucl-th]].

M-R relation of Neutron Stars and dense EOS

Thermonuclear Burst in X-ray Binaries

4U 1608-248 EXO 1745-248 4U 1820-30



(i) Apparent surface area

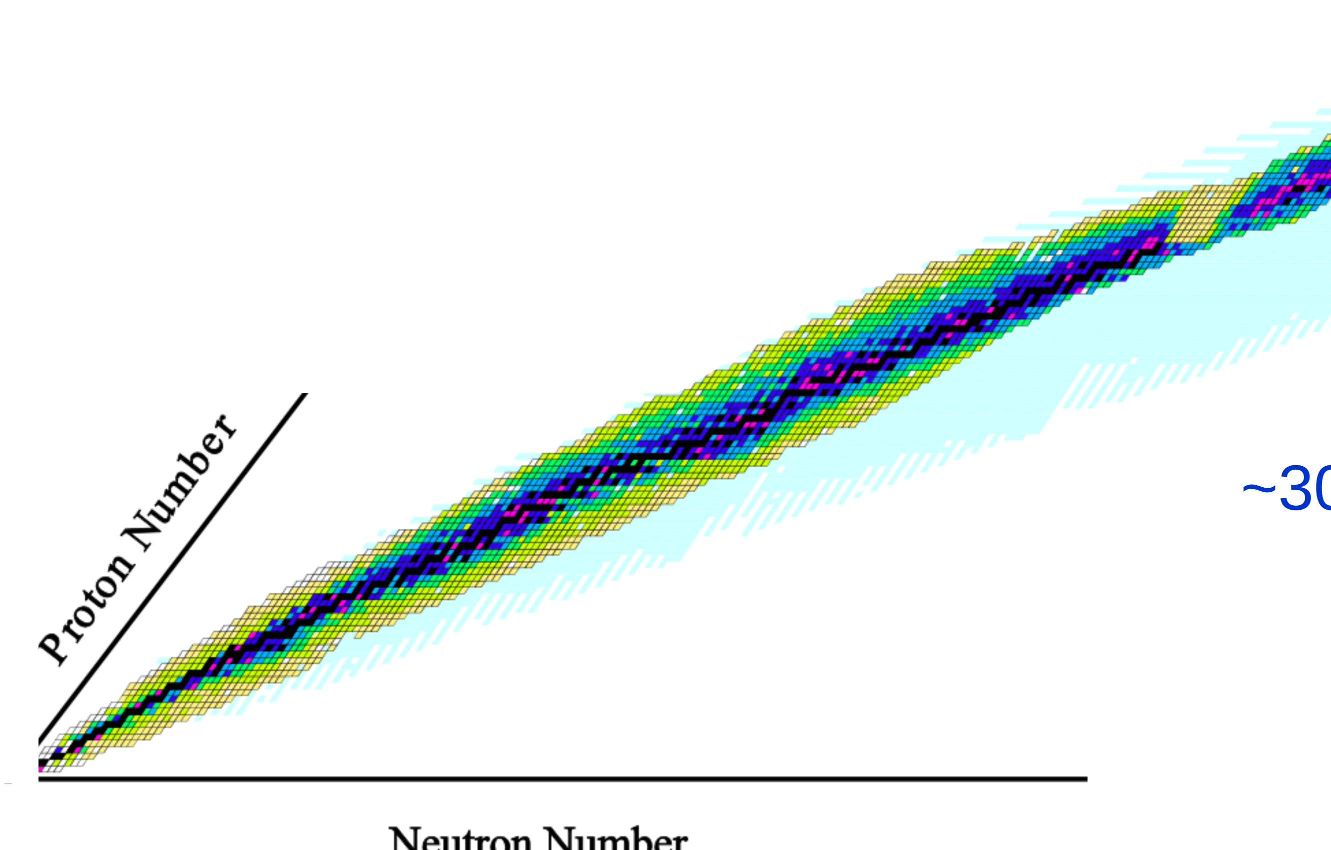
$$A = \frac{R^2}{D^2 f_c^{\,4}} \left(1 - \frac{2GM}{R}\right)^{-1}$$

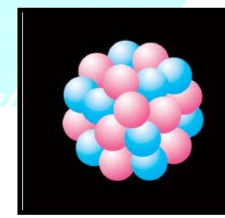
(ii) Eddington limit

$$F_{edd} = \frac{4\pi GM}{\kappa_{cs} D^2} \left(1 - \frac{2GM}{R}\right)^{1/2}$$

Ozel, Baym & Guver, arXiv: 1002.3153 [astro-ph.HE]

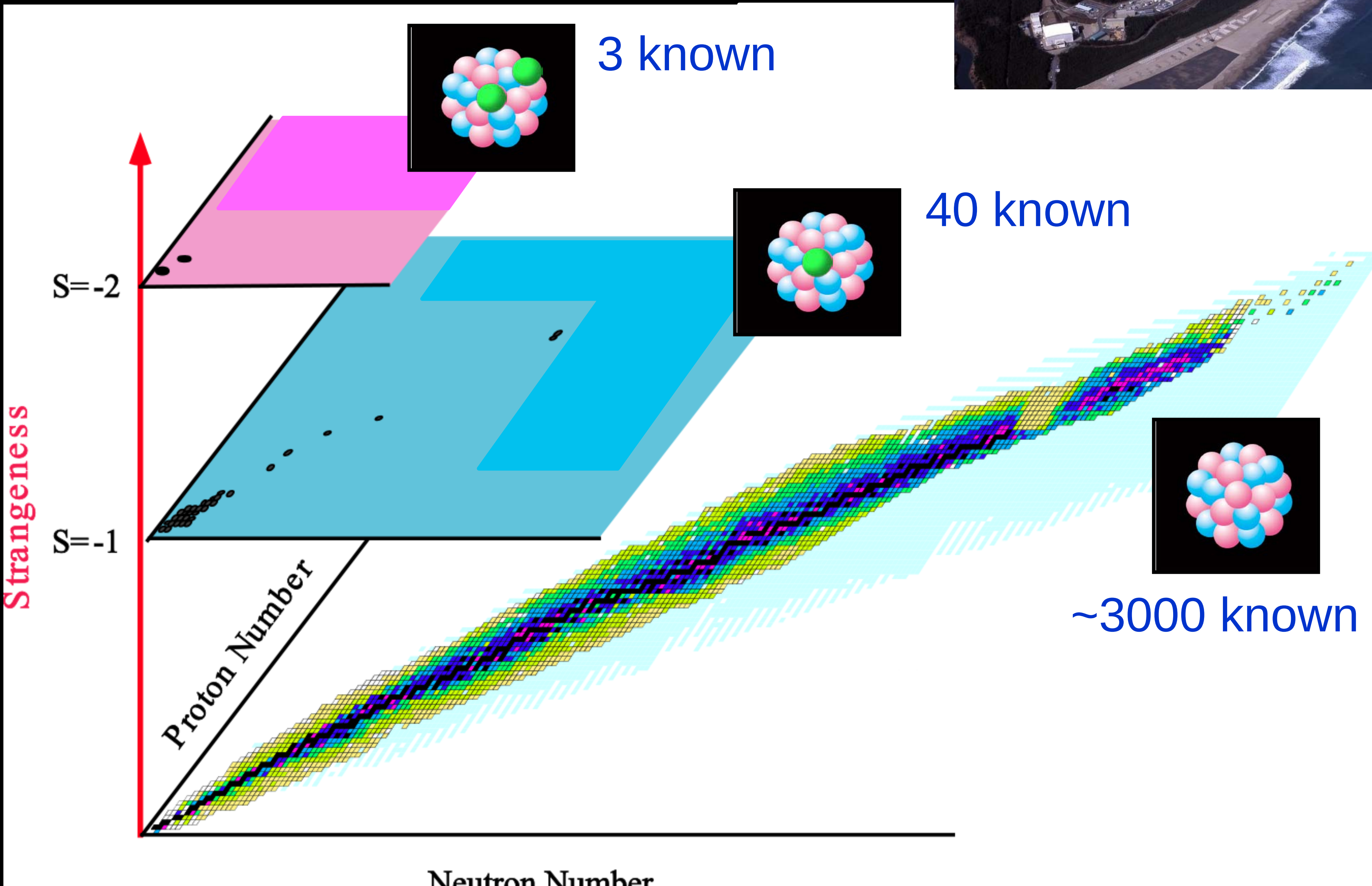
2D (N-Z) Nuclear Chart



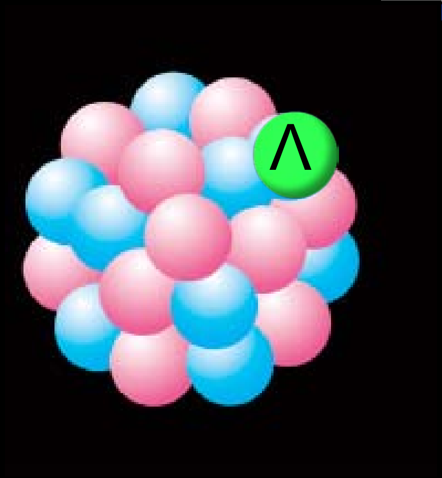


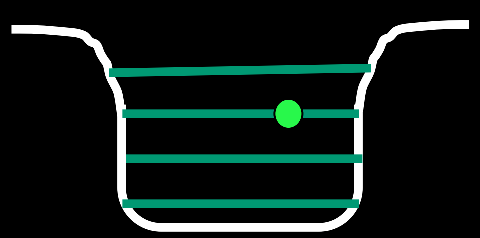
~3000 known

2D (N-Z) Nuclear Chart

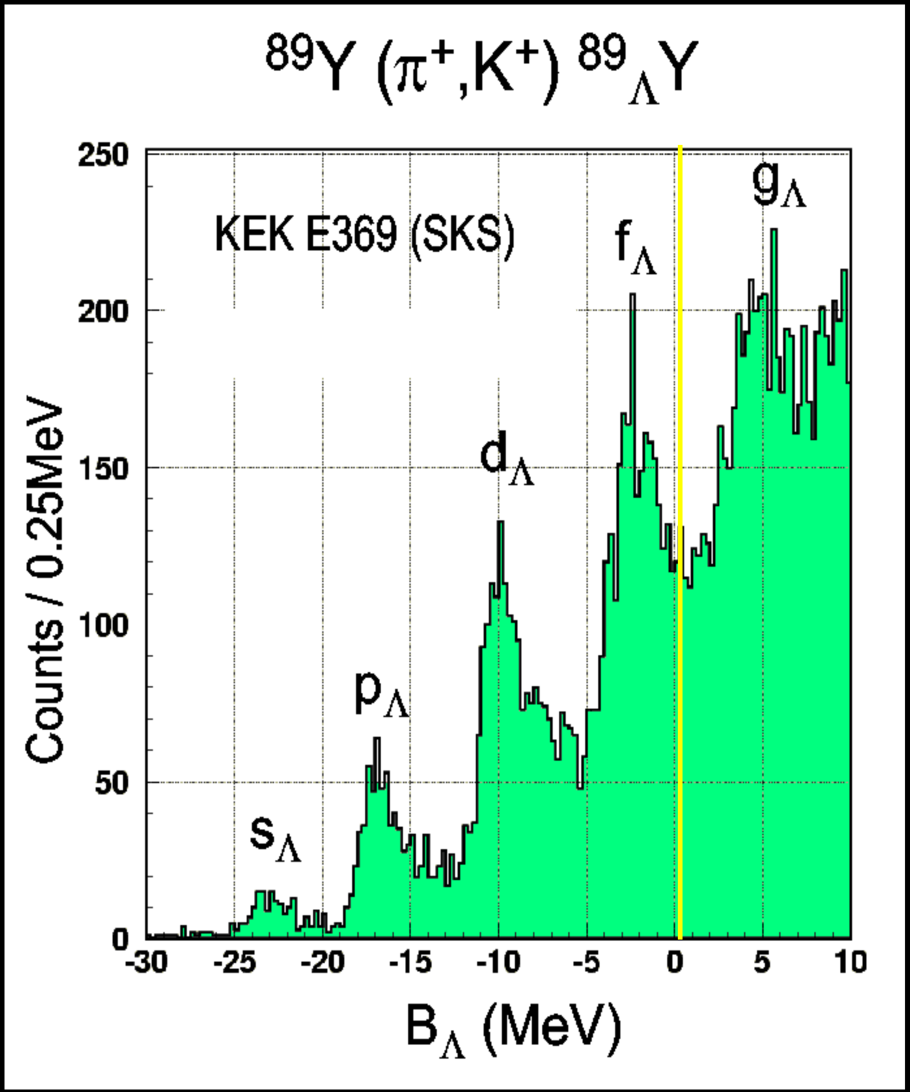


Λ hypernuclei



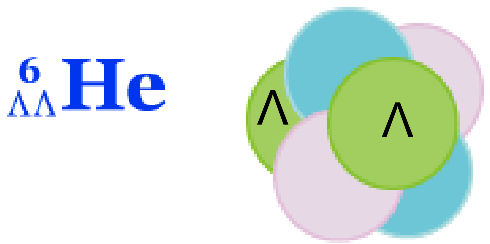


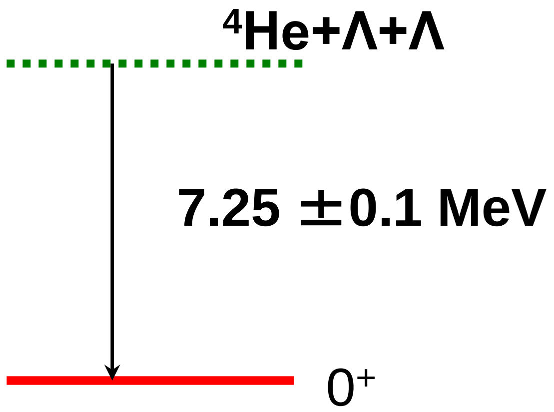
U_Λ = - 30 MeV
(U_N = - 50 MeV)



Hotchi et al., PRC 64 (2001) 044302

double-Λ hypernuclei

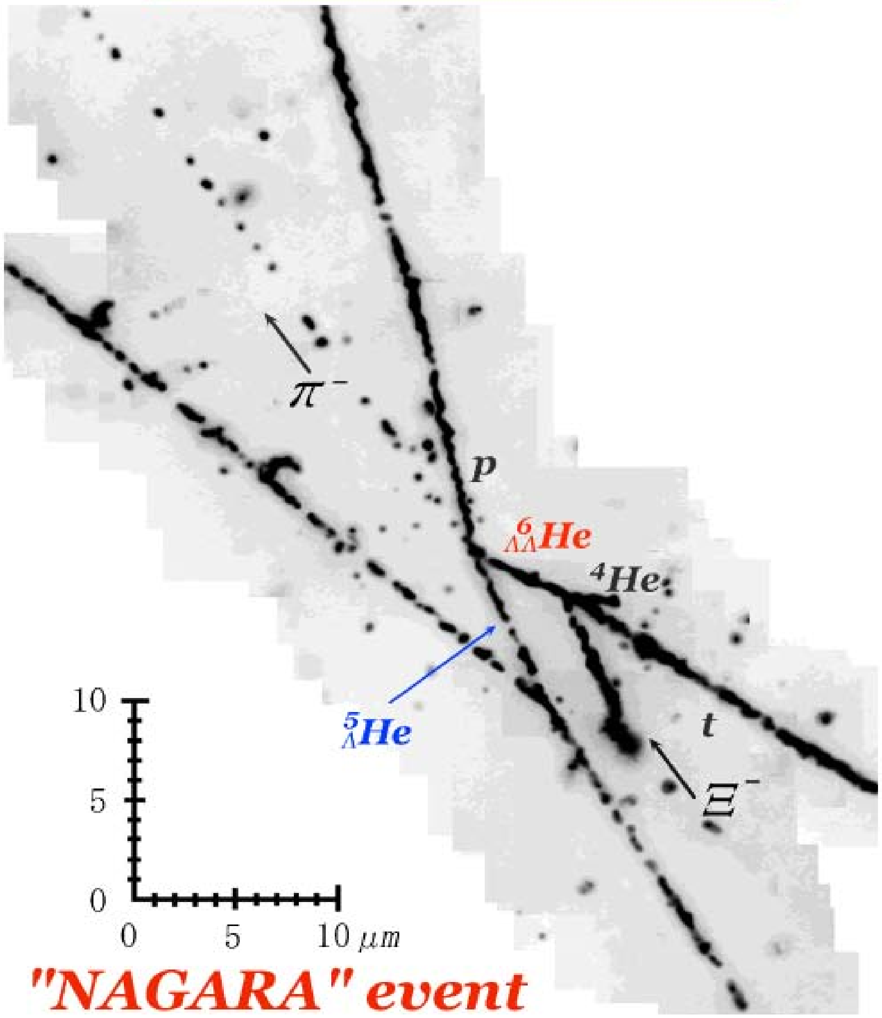




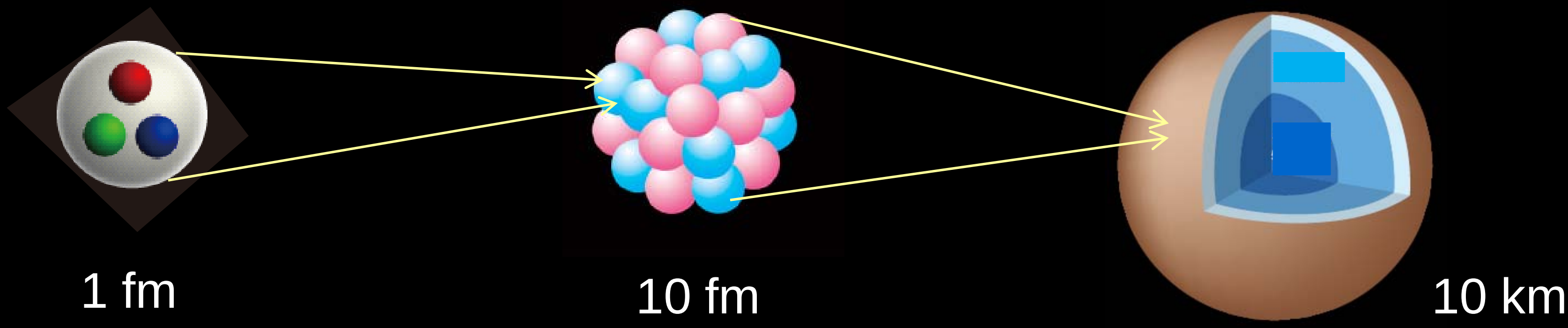
- ΛN attraction
- ΛΛ weak attraction
- No deeply bound H-dibaryon

H. Takahashi *et al*., PRL 87, 212502-1 (2001)

$$\Xi^- + {}^{12}C \longrightarrow {}^{6}_{\Lambda\Lambda}He + {}^{4}He + t$$
$$ {}^{6}_{\Lambda\Lambda}He \longrightarrow {}^{5}_{\Lambda}He + p + \pi^-$$



"NAGARA" event



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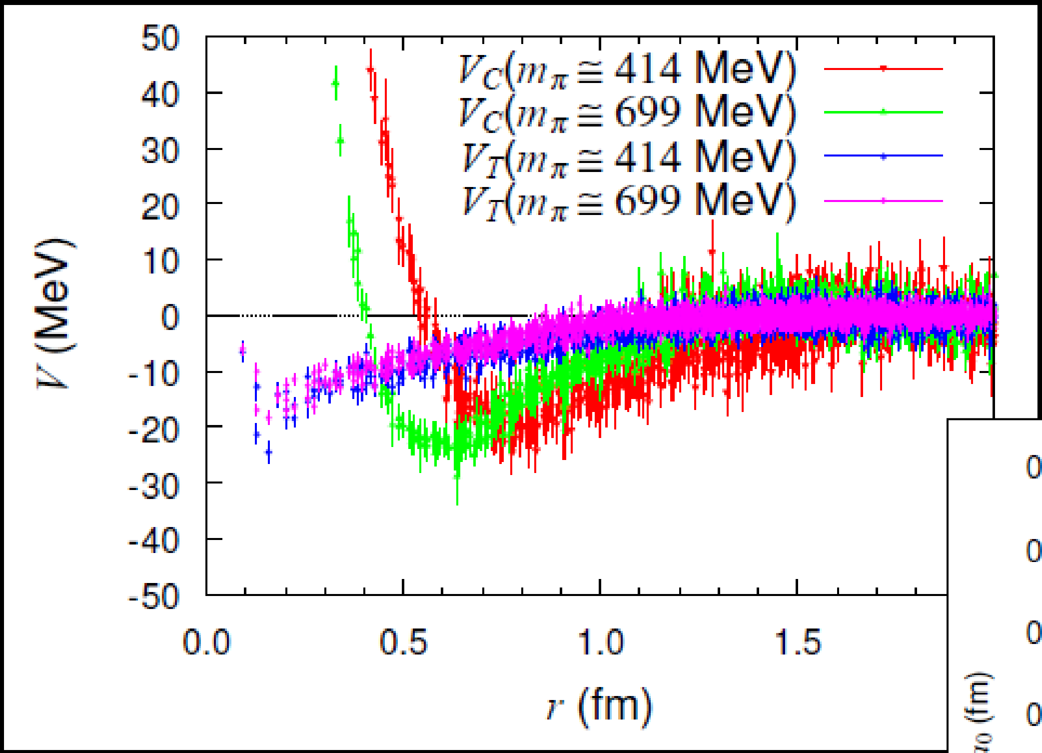
ΛN interaction in (2+1)-flavor QCD

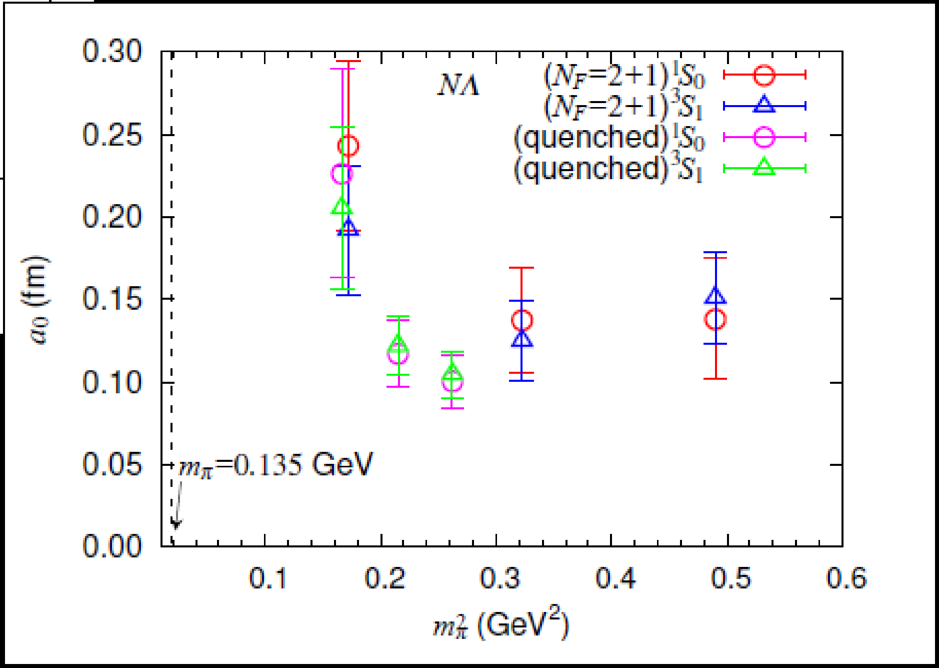
LO potentials from BS wave function

(2+1)-flavor, Iwasaki + clover (PACS-CS)
L=2.9fm, a=0.09fm, 32^3x64

Nemura et al. (HAL QCD Coll.)
arXiv: 1005.5352 [hep-lat]



Scattering length
from Lüscher's formula
with k from BS wave function

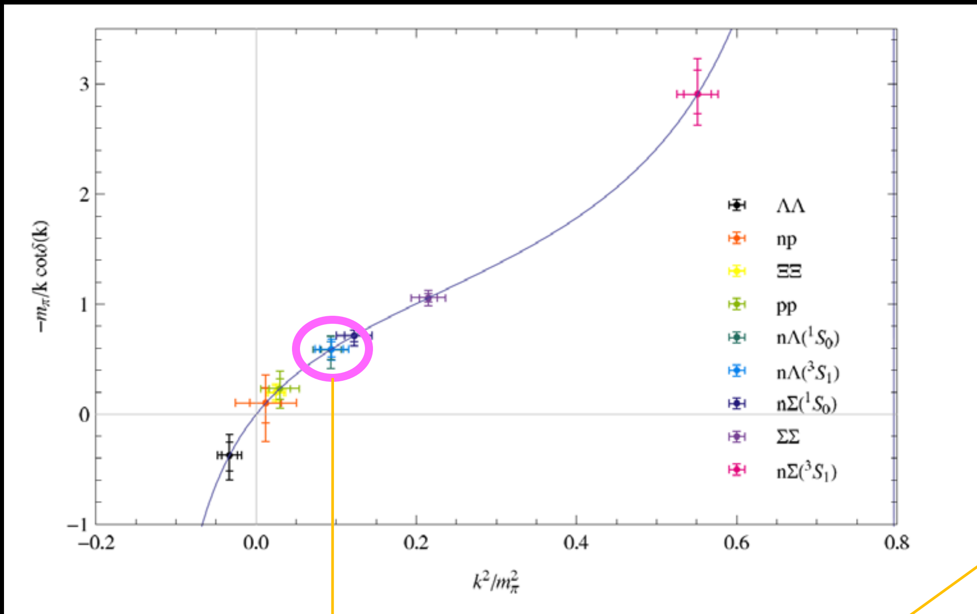


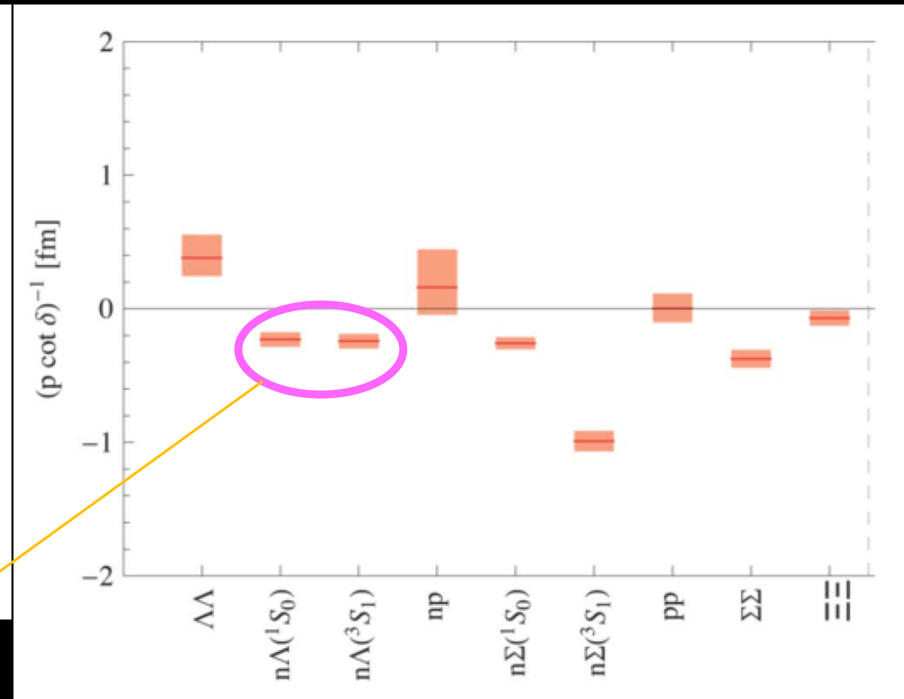
<u>ΛN interaction</u>

- repulsive core + attractive well
- net attraction at low energy

Beane et al., (NPLQCD), arXiv:1004.2935 [hep-lat].
Parreno (NPL QCD), Nuc.Phys. A835 (2010) 184

(2+1)-flavor anisotropic clover
20^3 x120, a_s=0.12 fm
m_π~360MeV



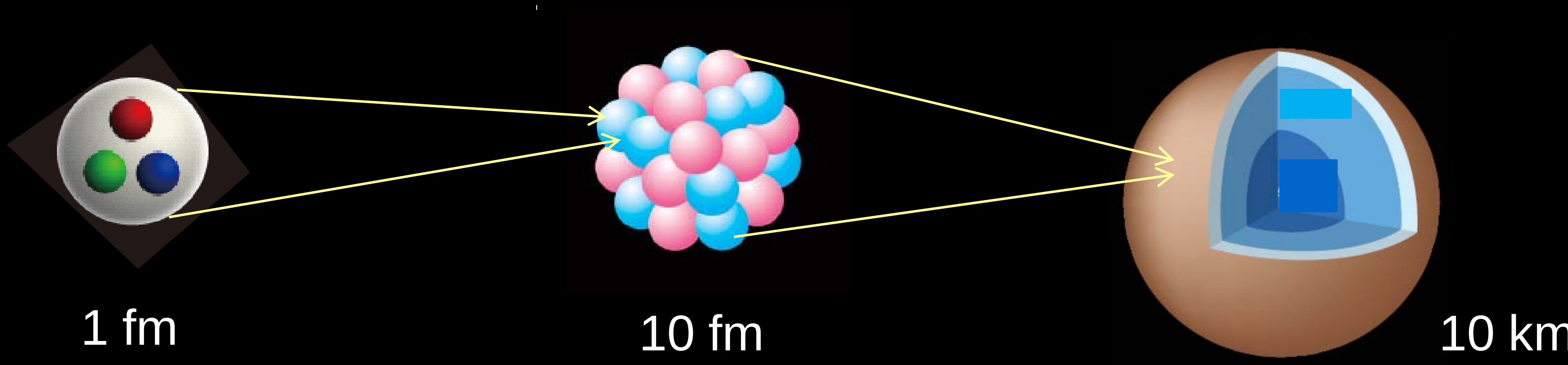


ΛN interacti

repulsive (NPLQCD)

Sign problem

attractive (HAL QCD)



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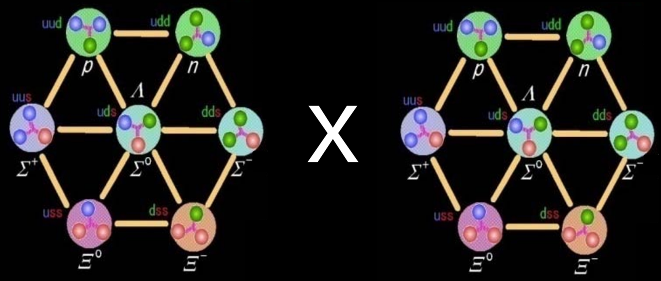
BB interactions
in a SU(3) symmetric world



1. First step to predict YN, YY interactions not accessible in exp.
2. Origin of the repulsive core (universal or not)

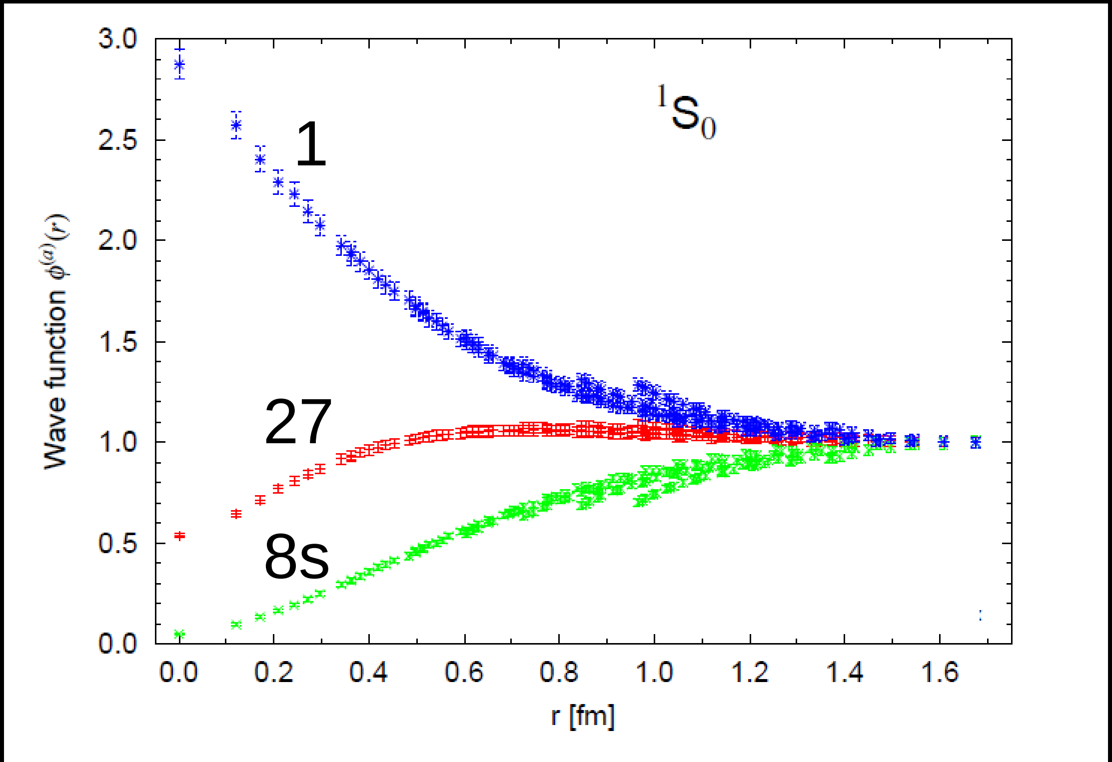
$$8 \times 8 = \underbrace{27 + 8s + 1}_{\text{Symmetric}} + \underbrace{10^* + 10 + 8a}_{\text{Anti-symmetric}}$$

Six independent potentials in flavor-basis

$$V^{(27)}(r),\ V^{(8s)}(r),\ V^{(1)}(r) \rightarrow {}^1S_0$$

$$V^{(10^*)}(r),\ V^{(10)}(r),\ V^{(8a)}(r) \rightarrow {}^3S_1$$

Equal-time BS amplitudes in the SU(3) limit



Iwasaki + clover (CP-PACS/JLQCD)
L=1.9 fm, a=0.12 fm, 16^3x32
m_π=835 MeV, m_B=1752 MeV

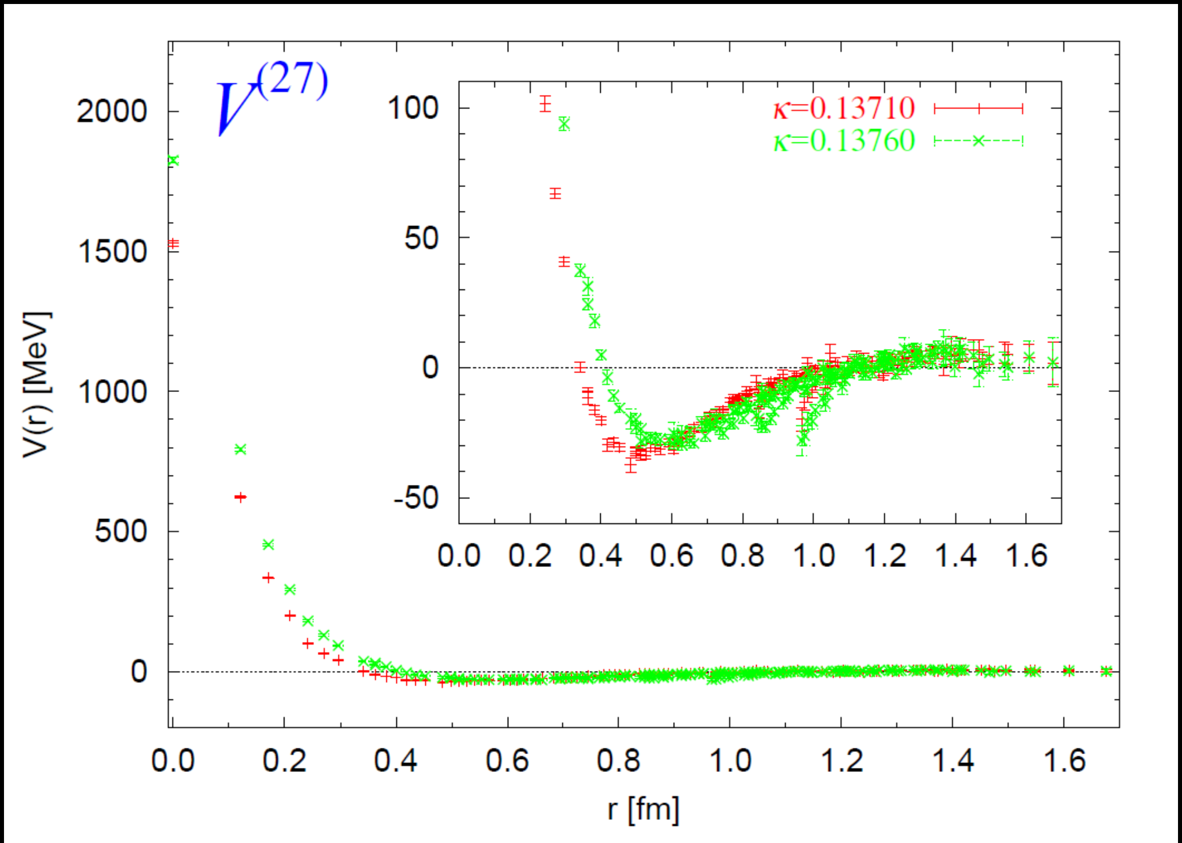
Inoue [Parallel 49, Fri.]

Pauli principle at work !

1 : allowed
27 : partially blocked
8s : almost blocked

c.f. Oka, Shimizu, Yazaki ,
Nucl. Phys. A464 (1987) 700

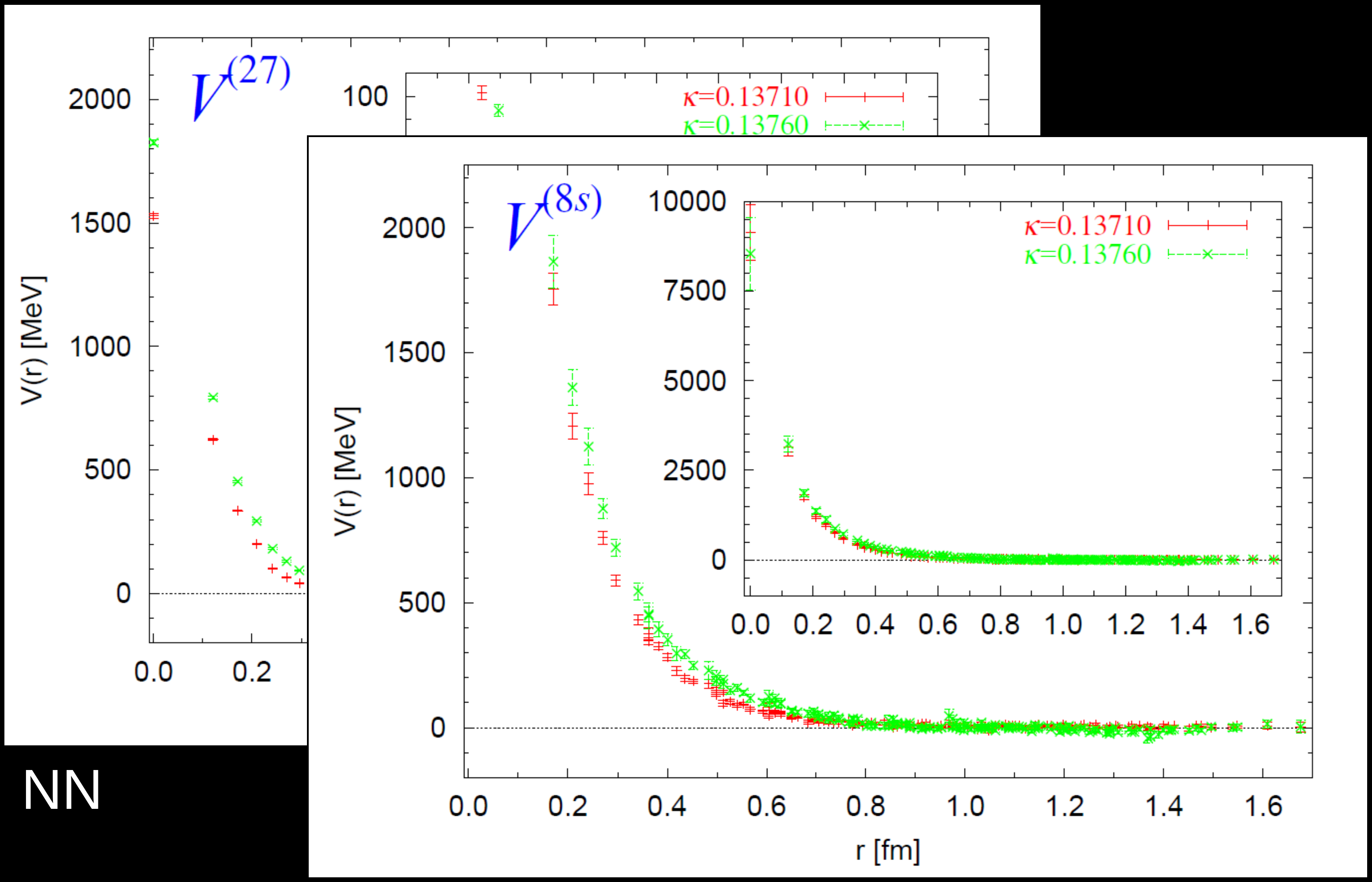
BB potentials in flavor-basis ($^1S_0$ channel)



NN

Inoue [Parallel 49, Fri.]

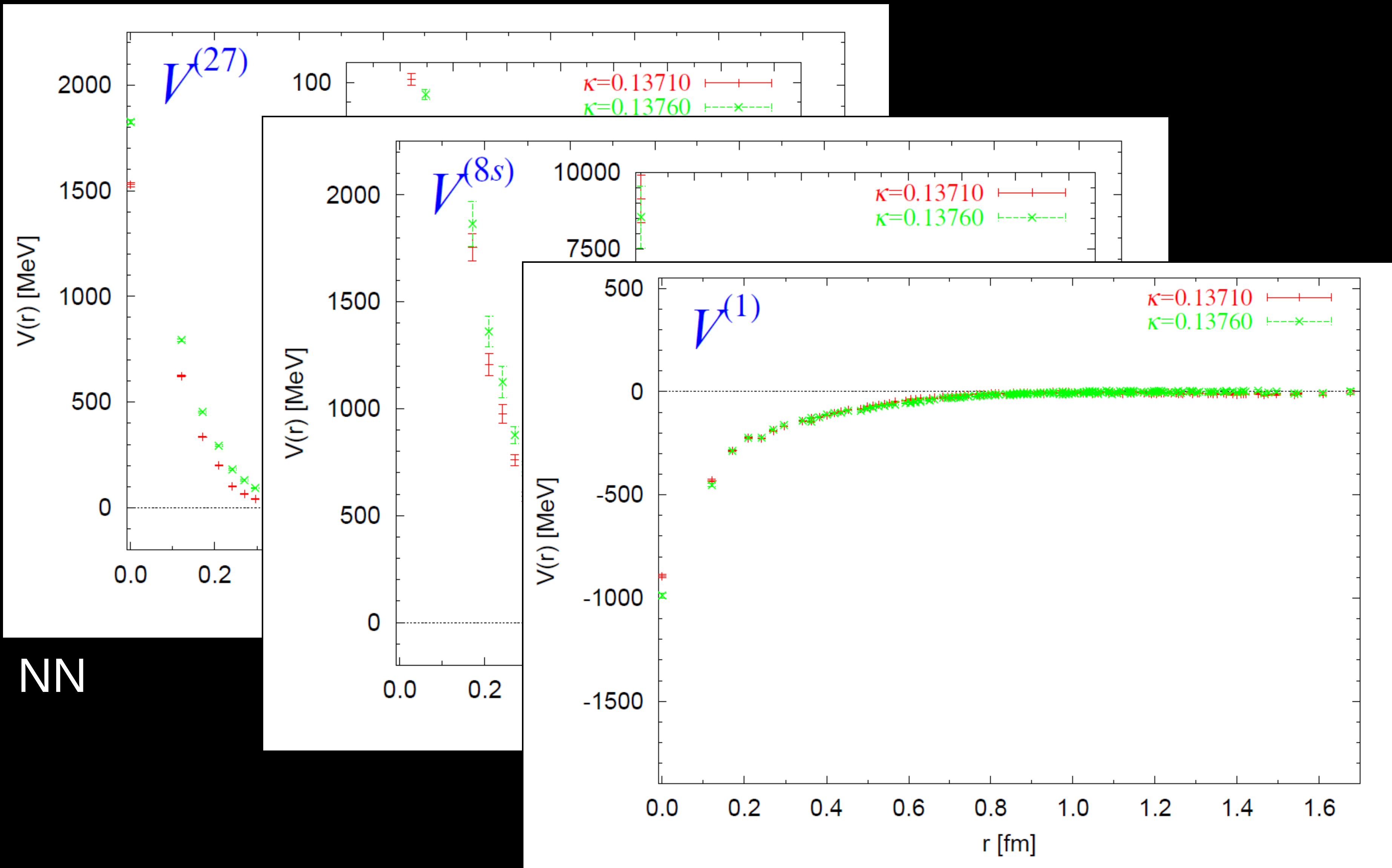
BB potentials in flavor-basis ($^1S_0$ channel)



NN

Inoue [Parallel 49, Fri.]

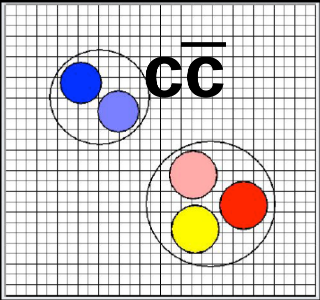
BB potentials in flavor-basis ($^1S_0$ channel)



NN

Inoue [Parallel 49, Fri.]

S-wave η_c-N interaction



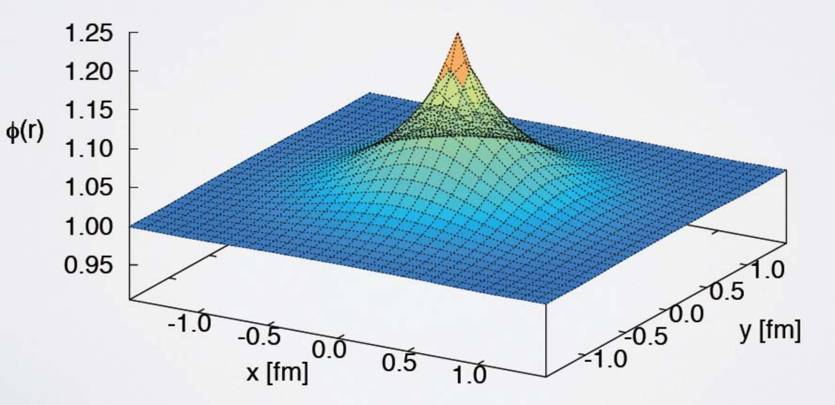
no Pauli-blocking + QCD van der Walls attraction
→ charmonium-nucleus bound state ?

Brodsky et al., PRL 64 (1990) 1011

Quenched QCD: 32^3x48, L = 3 fm
(2+1)-flavor QCD on-going

Sasaki &Kawanai [Parallel 49, Fri.]

Potential
from BS wave function



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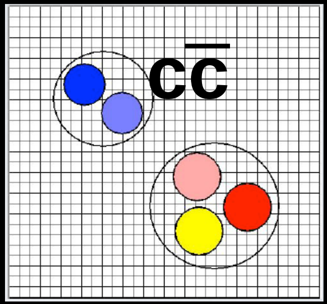

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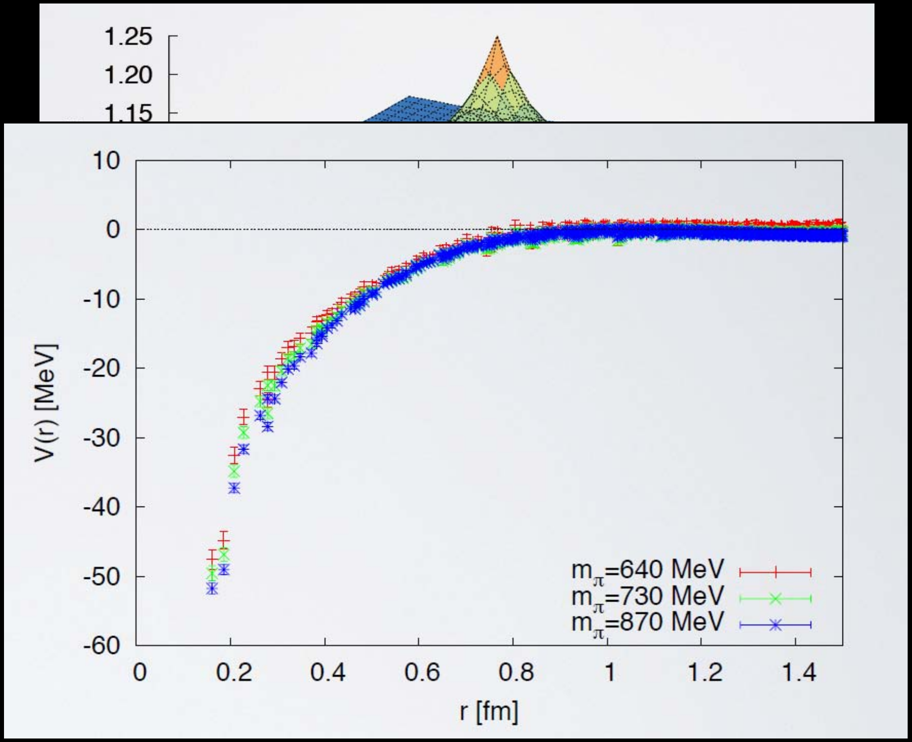
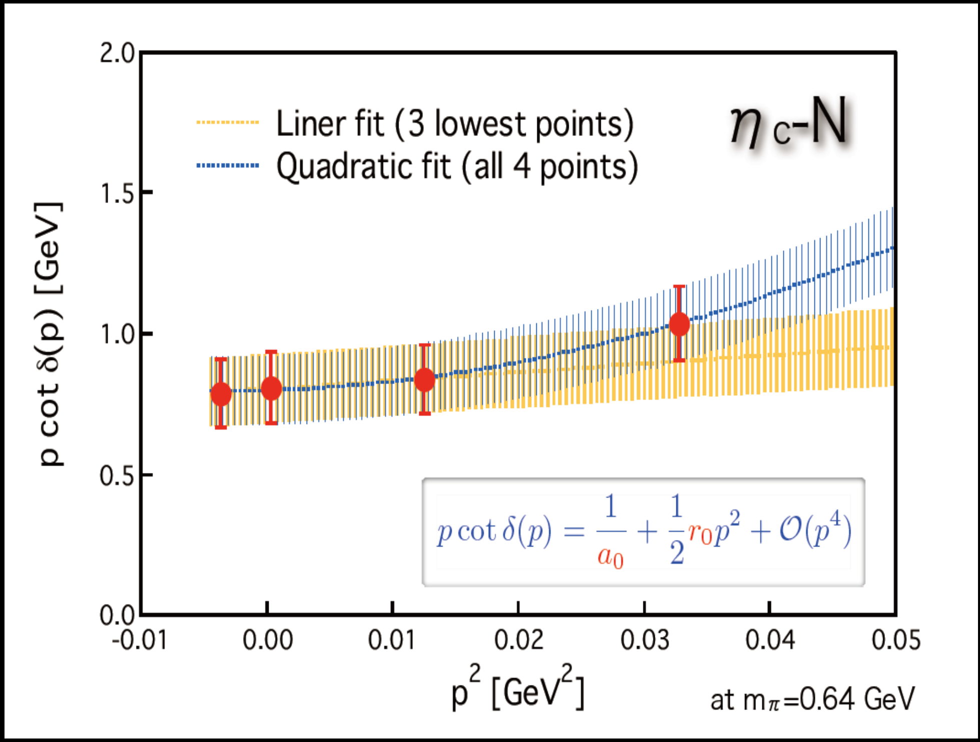
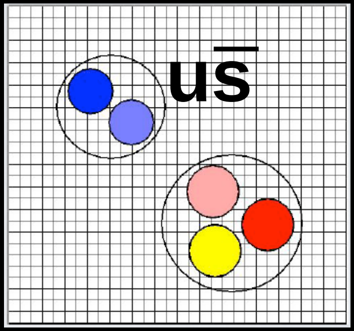
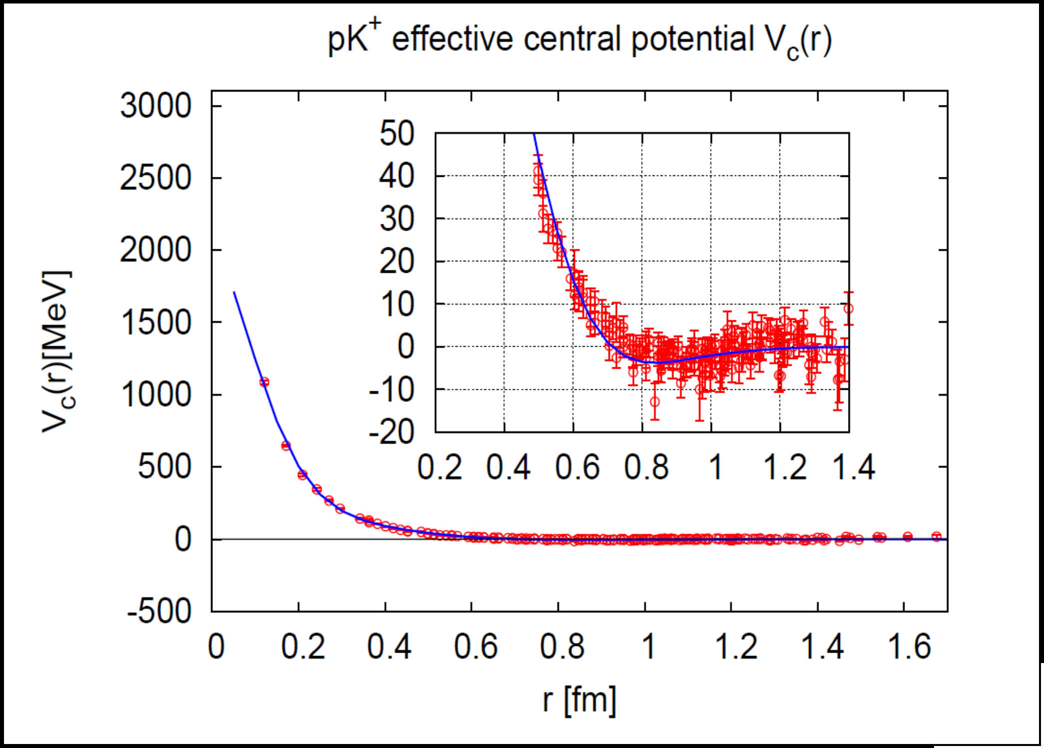


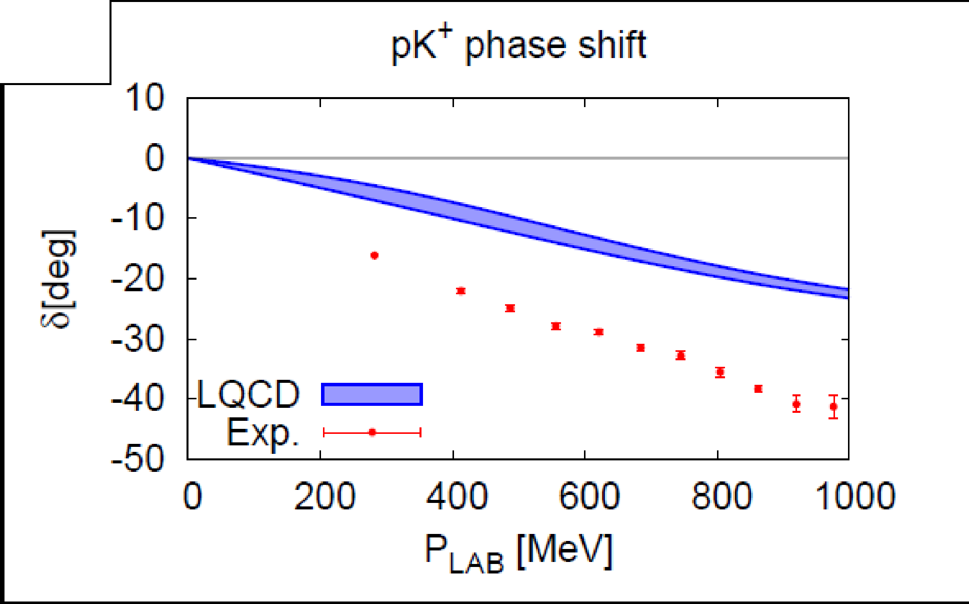
Phase shift from Lüscher's formula with wisted boundary



S-wave K^+-p interaction





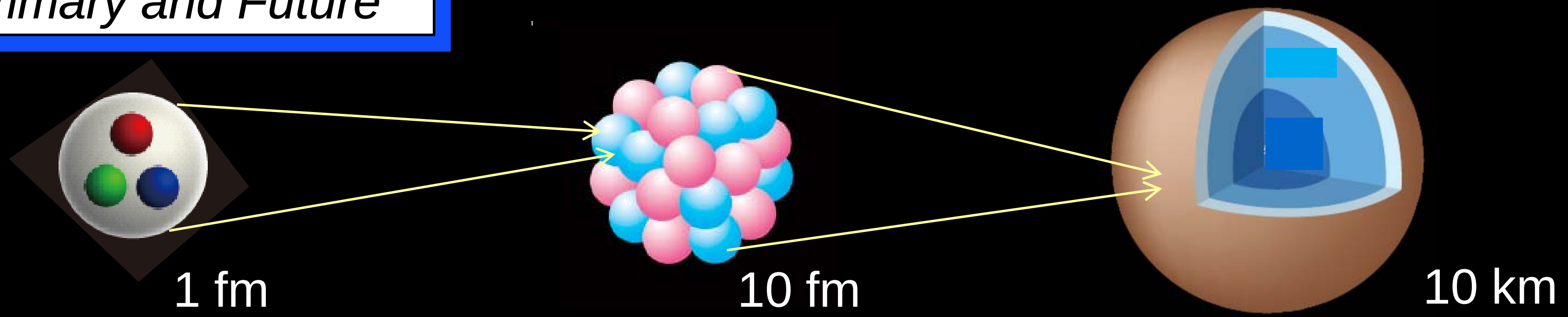


Iwasaki + Clover (CP-PACS/JLQCD)
a= 0.12 fm, L=1.9 fm, 16^3x32

m_π = 871 MeV, m_K = 912 MeV
m_N = 1796 MeV

Ikeda et al. (HALQCD Coll.)

Summary and Future



1. Nuclear physics needs QCD inputs
   Lattice NN, NNN, YN, YY, YYY interactions
   → ab initio nuclear calculations, neutron/hyperon matter

2. Different approaches available
   - phase shifts
   - lattice potentials from BS amplitude
   - lattice nuclei

3. Imaginary nuclei with large quark mass ?
   lattice nuclei vs. lattice pot.+ab initio cal.

4. Full QCD in large volume at physical point
   e.g. L=6 fm, m_π=135 MeV (PACS-CS)