Effects of a potential fourth fermion generation on the Higgs boson mass bounds

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Introduction

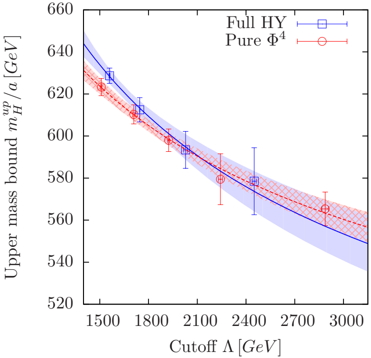
- A heavy fourth fermion generation may be a way to...
  - increase the CP-violating phase in SM by several orders of magnitude. [Hou et al.]
  - strengthen electroweak phase transition supporting scenario of electroweak baryogenesis. [Carena et al.]
- Interest in fourth generation repeatedly vanished and reappeared:
  - 4th generation not excluded by electroweak precision data if mass splitting allowed in 4th doublets. [Holdom et al.]
- A heavy 4th fermion generation would have very strong (non-perturbative?) effect on Higgs boson mass.

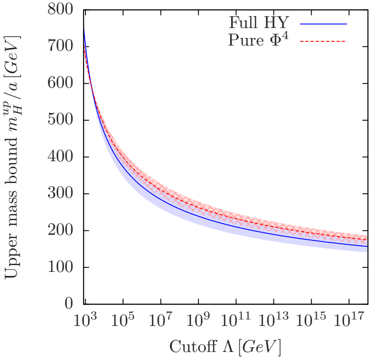
Aim of this investigation

Study the influence of the 4th fermion generation on the Higgs boson mass non-perturbatively in a lattice Higgs-Yukawa model.

Upper Higgs boson mass bound in SM3

- Higgs-Sector of SM is a trivial field theory.
  - Cutoff Λ must remain finite. (Otherwise no interaction.)
  - Consider SM as effective theory valid up to energy scale Λ.
- Upper mass bound in SM3:
  - How are bounds shifted in the presence of a 4th generation (t',b')?





Targeted coupling structure in SM

- Higgs-Fermion coupling in SM:
  - φ complex scalar doublet and $\tilde{\varphi} = i\tau_2\varphi$.
  - y_t, y_b, $\ldots$: Yukawa coupling constants.

$$L_Y = y_b \cdot (\bar{t}, \bar{b})_L \varphi b_R \; + y_t \cdot (\bar{t}, \bar{b})_L \tilde{\varphi} t_R \; + \; h.c. + \ldots$$

- Higgs-Higgs self-interaction in SM:
  - λ: Quartic coupling constant

$$L_\varphi = \lambda(\varphi^\dagger\varphi)^2$$

- Higgs-dynamics dominated by ...
  - coupling to heaviest fermions (4th generation).
  - quartic self-coupling (if $\lambda \gg 1$).
- In this study: Pure Higgs-fermion sector of SM:
  - All gauge fields neglected.

Circumventing the No-Go-theorem via overlap fermions

- Lattice model, obeying global $SU(2)_L \times U(1)_Y$ symmetry:

$$S = \sum_{x,\mu} \frac{1}{2} \nabla^f_\mu \varphi^\dagger_x \nabla^f_\mu \varphi_x + \sum_x \frac{1}{2} m_0^2 \varphi^\dagger_x \varphi_x + \sum_x \lambda \left(\varphi^\dagger_x \varphi_x\right)^2$$
$$+ \sum_{x,y} \bar{t}_x \mathcal{D}^{(ov)}_{x,y} t_y + \sum_{x,y} \bar{b}_x \mathcal{D}^{(ov)}_{x,y} b_y + \sum_x y_b \cdot (\bar{t}_{L,x}, \bar{b}_{L,x}) \varphi_x b_{R,x} + y_t \cdot (\bar{t}_{L,x}, \bar{b}_{L,x}) \tilde{\varphi}_x t_{R,x} + h.c.$$

A lattice version of chiral symmetry

- Need chiral invariance on lattice (to define $t_L, t_R, \ldots$)
  $\Rightarrow$ Ginsparg-Wilson fermions satisfying GW-relation (here: overlap op. $\mathcal{D}^{(ov)}$)

$$\gamma_5 \mathcal{D}^{(ov)} + \mathcal{D}^{(ov)} \hat{\gamma}_5 = 0 \quad \text{with} \quad \hat{\gamma}_5 = \gamma_5 \left(1 - \frac{1}{\rho} \mathcal{D}^{(ov)}\right)$$

- Use modified projectors $\hat{P}_\pm = \frac{1}{2}(1 \pm \hat{\gamma}_5)$ to define $t_L, \ldots$:

Continuum	Lattice
$\begin{pmatrix} t \\ b \end{pmatrix}_{L,R} = P_\mp \begin{pmatrix} t \\ b \end{pmatrix}$	$\begin{pmatrix} t \\ b \end{pmatrix}_{L,R} = \hat{P}_\mp \begin{pmatrix} t \\ b \end{pmatrix}$
$(\bar{t}, \bar{b})_{L,R} = (\bar{t}, \bar{b}) P_\pm$	$(\bar{t}, \bar{b})_{L,R} = (\bar{t}, \bar{b}) P_\pm$

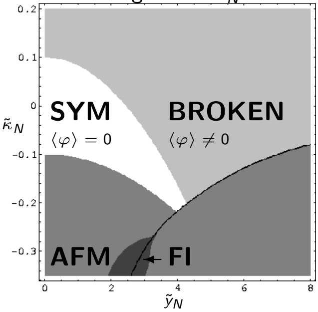
- Use here overlap operator $\mathcal{D}^{(ov)}$:

$$\mathcal{D}^{(ov)} = \rho \left\{1 + \frac{A}{\sqrt{A^\dagger A}}\right\}, \quad A = \mathcal{D}^{(W)} - \rho, \quad \mathcal{D}^{(W)} : \text{Wilson Dirac operator}$$

Strategy for mass bound determination

- Idea: For given cutoff $\Lambda = a^{-1}$ find min. and max. Higgs masses in HY-model consistent with phenomenology.
- Considered phenomenology:
  - SSB: $\langle\varphi\rangle/(a\sqrt{Z_G}) \equiv v_r = 246$ GeV
    $\rightarrow$ Fixes cutoff $\Lambda = a^{-1}$.
  - t' quark mass: $m_t/a = 700$ GeV
    $\rightarrow$ Fixes Yukawa coupling constant y_t.
  - b' quark mass: $m_b/a = 700$ GeV
    $\rightarrow$ Fixes Yukawa coupling constant y_b.
- 4 param. - 3 cond. = 1 freedom
  $\rightarrow \lambda$ undetermined.
- From tree-level: $m_H^2 \propto \lambda v^2$
  $\Rightarrow$ Smallest m_H at small λ. (Weak coupling.)
  Largest m_H at $\lambda \rightarrow \infty$. (Strong coupling.)

Phase diagram at $\tilde{\lambda}_N = 0.1$



Considered observables

- On lattice: Always $\langle\varphi\rangle \equiv 0$. To study SSB:
  $\longrightarrow$ Rotate each φ-configuration: $\varphi_x^{rot} = U\varphi_x$, $U \in SU(2)$ such that

$$\sum_x \varphi_x^{rot} = \begin{pmatrix} 0 \\ \left|\sum_x \varphi_x\right| \end{pmatrix}. \text{ Then define } \langle\varphi^{rot}\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}.$$

- Define Higgs-/Goldstone-modes: $\varphi_x^{rot} = \begin{pmatrix} g_x^2 + ig_x^1 \\ v + h_x - ig_x^3 \end{pmatrix}$
- Propagators: $\tilde{G}_H(p) = \langle \tilde{h}_p \tilde{h}_{-p} \rangle$, $\tilde{G}_G(p) = \frac{1}{3}\sum_{\alpha=1}^{3} \langle \tilde{g}_p^\alpha \tilde{g}_{-p}^\alpha \rangle$
- Goldstone mass and Z_G (analogous for Higgs boson):

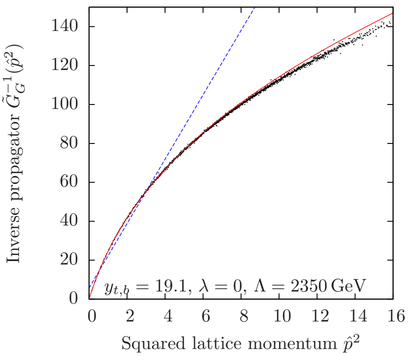
$$Z_G^{-1} = \frac{\mathrm{d}}{\mathrm{d}p_c^2}\left[\tilde{G}_G^c(p_c^2)\right]^{-1}\Big|_{p_c^2=-m_G^2} \quad \text{and} \quad \left[\tilde{G}_G^c(p_c^2)\right]^{-1}\Big|_{p_c^2=-m_{Gp}^2} = 0$$

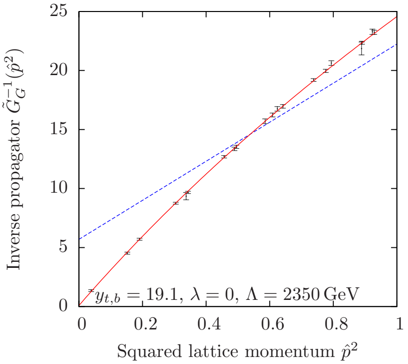
- Top and bottom quark mass: m_t, m_b
  $\longrightarrow$ From exponential decay of time correlation functions

$$C_t(\Delta t) = \sum_{\vec{x},\vec{y}} \left\langle 2\,\mathrm{Re\,Tr}\left(t_{L,\Delta t,\vec{x}} \cdot \bar{t}_{R,0,\vec{y}}\right)\right\rangle \quad \text{and} \quad C_b(\Delta t) = \sum_{\vec{x},\vec{y}} \left\langle 2\,\mathrm{Re\,Tr}\left(b_{L,\Delta t,\vec{x}} \cdot \bar{b}_{R,0,\vec{y}}\right)\right\rangle$$

Determination of Z_G

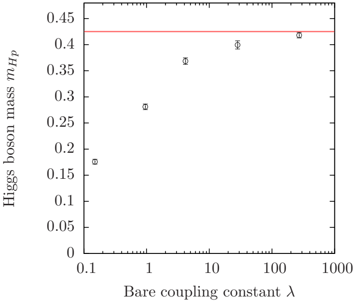
- Continuous fit function for discrete lattice Goldstone propagator needed, to derive Z_G from its derivative.
- Use 1-loop result from renormalized PT (red) with ren. quantities being free parameters as fit ansatz. (Blue: linear fit for comparison.)





Bare model parameters for upper bound on m_H

- Degenerate Yukawa constants:
  (Otherwise $\det(\mathcal{M}) \in \mathbb{C}$)
  Tuned to yield: $m_{t,b} = 700$ GeV
- m_H rises monoton. with $\lambda \to \infty$
  $\Rightarrow$ Choose $\lambda = \infty$
- Accessible energy scales:
  Require: $\hat{m} > 0.5$ and $\hat{m} \cdot L_{s,t} > 4$
  (for all $\hat{m} = m_H, m_t, m_b$)
  $\Rightarrow \Lambda = 1500 - 4000\ GeV$ accessible



$16^3 \times 32$-lattice, $\Lambda \approx 1500\ GeV$, $y_{t,b} = 0.71138$
Red band: $\lambda = \infty$ result

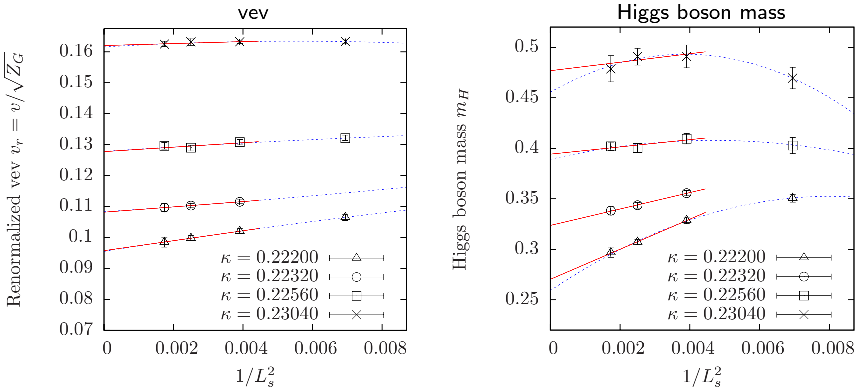
κ	L_s	L_t	N_f	λ	y_t	y_b/y_t	$1/v$	Λ
0.22200	12,16,20,24	32	1	∞	3.18	1	≈ 10.4	$\approx$ 1520 GeV
0.22320	12,16,20,24	32	1	∞	3.17	1	≈ 9.3	$\approx$ 1930 GeV
0.22560	12,16,20,24	32	1	∞	3.16	1	≈ 7.8	$\approx$ 2280 GeV
0.23040	12,16,20,24	32	1	∞	3.12	1	≈ 6.2	$\approx$ 2570 GeV

Infinite volume extrapolation

- Goldstone modes induce algebraic FSE of order $O(L_s^{-2})$, $O(L_s^{-4})$, ...
- Perform infinite volume extrapolation with fit ansatz

$$\text{Linear:}\quad f_m^{(l)}(L_s^{-2}) = A_m^{(l)} + B_m^{(l)} \cdot L_s^{-2} \quad \text{for } L \geq 16 \text{ (red)}$$

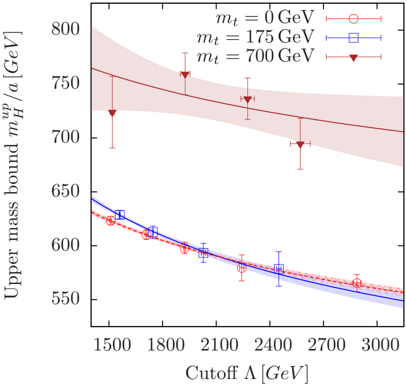
$$\text{Parabolic:}\quad f_m^{(p)}(L_s^{-2}) = A_m^{(p)} + B_m^{(p)} \cdot L_s^{-2} + C_m^{(p)} \cdot L_s^{-4} \quad \text{for all } L \text{ (blue)}$$



Upper Higgs boson mass bound at $m_t = 700$ GeV

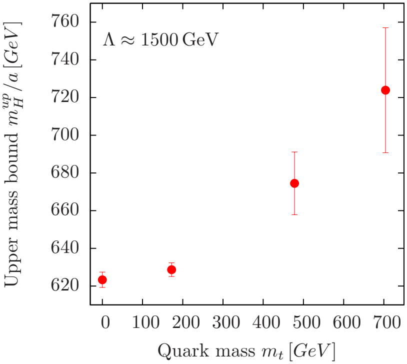
- Colored curves:
  Fits (A_m, B_m free fit parameter) with expected cutoff-dependence

$$\frac{m_H}{a} = A_m \cdot \left[\log(\Lambda^2/\mu^2) + B_m\right]^{-1/2}$$



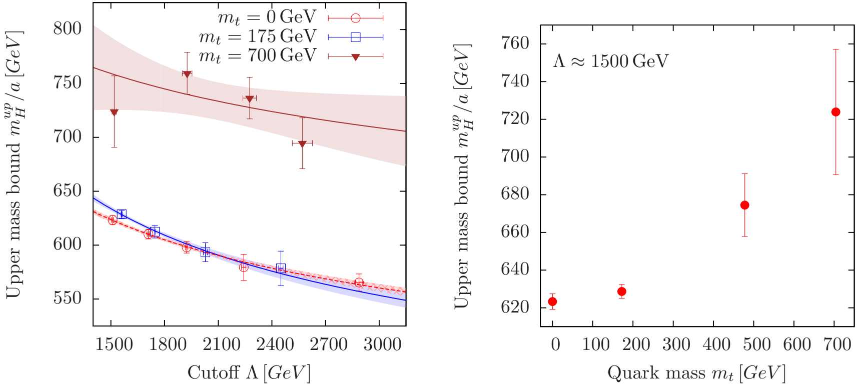
Quark mass dependence of m_H^{up}

- Dependence of m_H^{up} on quark mass m_t at $\Lambda \approx 1500\,\text{GeV}$:



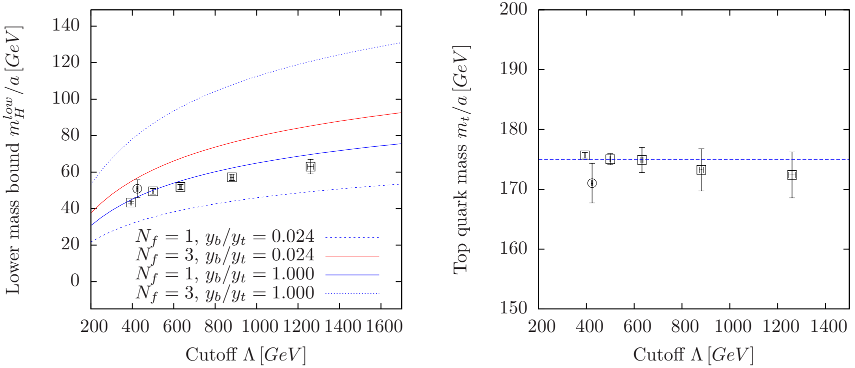
Summary and Outlook

- Effect of heavy 4th fermion generation on m_H^{up} investigated:
  $m_H^{up}(m_t = 175\,\text{GeV}, \Lambda = 1.5\,\text{TeV}) = (630 \pm 5)\,\text{GeV}$,
  $m_H^{up}(m_t = 700\,\text{GeV}, \Lambda = 1.5\,\text{TeV}) = (730 \pm 35)\,\text{GeV}$
- Next: Find largest possible quark mass through $y_{t,b} \to \infty$.



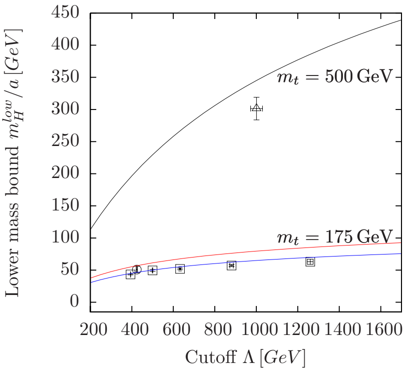
Lower Higgs boson mass bound

- m_H: Colored lines: CEP-results for $V = \infty$, different physical setups
  Red curve closest to situation in SM
- Circular symbols: Series of lattice runs in non-degenerate case
  i.e. $m_b = 4.2\ GeV \Rightarrow y_b/y_t = 0.024$.
  Caution: Unknown systematic uncertainties due to $\det(M) \in \mathbb{C}$, if $y_t \neq y_b$



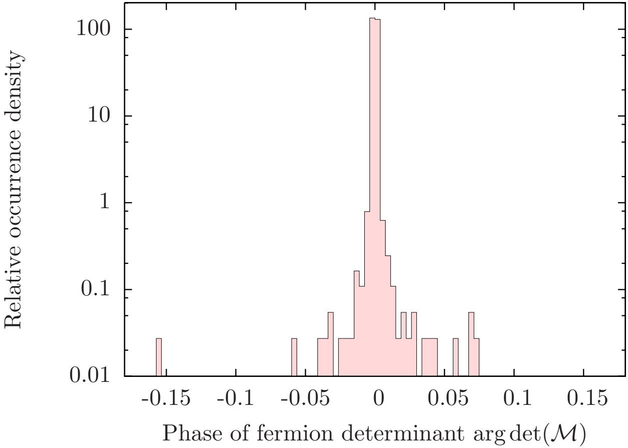
Fourth generation: Some preliminary results

- Assume mass-degenerate fourth quark doublet at 500 GeV
  Triangular symbols: m_H at $m_t = m_b = 500\,\text{GeV}$, $N_f = 1$ on $16^3 \times 32$ lattice
- Colored lines show CEP-results for $V = \infty$:
  - Black: $m_t = m_b = 500\,\text{GeV}$, $N_f = 1$
  - Blue: $m_t = m_b = 175\,\text{GeV}$, $N_f = 1$
  - Red: $m_t = 175\,\text{GeV}$, $m_b = 4.2\,\text{GeV}$, $N_f = 3$



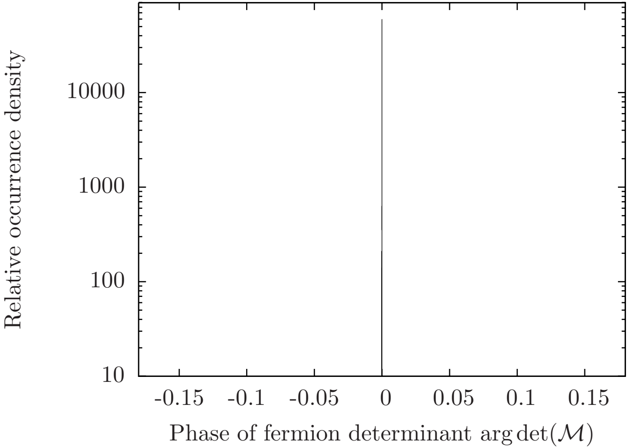
Complex phase of fermion determinant

$y_t/y_b \approx 40$, 4^4-lattice, φ Gauss sampled.



Complex phase of fermion determinant

$y_t/y_b \approx 40$, 4^4-lattice, φ from MC-sim. in broken phase.



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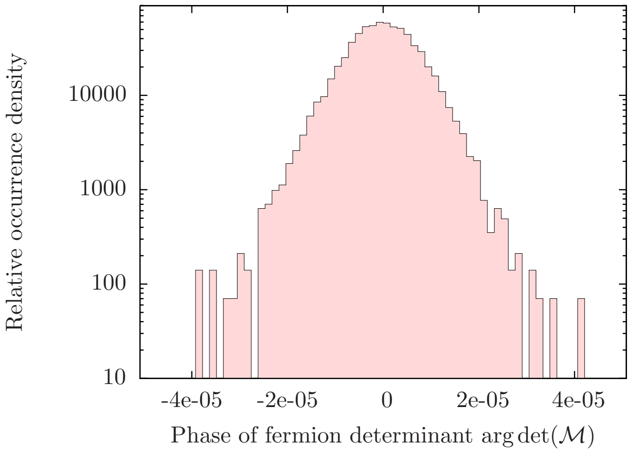

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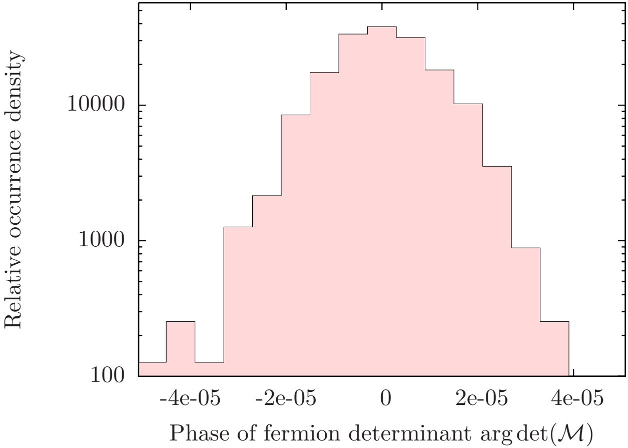

Phase diagram in large N_f-limit

- Analytical calculation based on Constraint Effective Potential (CEP)
- Use lattice parameters $\kappa, \hat{\lambda}, \hat{y}_{t,b}$ related to $m_0, \lambda, y_{t,b}$ through

$$\lambda = \frac{\hat{\lambda}}{4\kappa^2}, \quad m_0^2 = \frac{1 - 2N_f\hat{\lambda} - 8\kappa}{\kappa}, \quad y_{t,b} = \frac{\hat{y}_{t,b}}{\sqrt{2\kappa}}$$

- Consider limit $N_f \to \infty$, while scaling $\kappa, \hat{\lambda}, \hat{y}_{t,b}$ according to:

$$\hat{y}_{t,b} = \frac{\tilde{y}_N}{\sqrt{N_f}}, \quad \hat{\lambda} = \frac{\tilde{\lambda}_N}{N_f}, \quad \varphi = N_f^{1/2}\tilde{\varphi}, \quad \kappa = \tilde{\kappa}_N, \qquad \tilde{y}_N, \tilde{\lambda}_N, \tilde{\kappa}_N, \tilde{\varphi} = \text{const}$$

- Order parameters:
  $\langle\varphi\rangle$ and $\langle\varphi\rangle_s \equiv \langle \frac{1}{V}\sum_x (-1)^{\sum_\mu x_\mu}\varphi_x\rangle, \quad V = L_s^3 \times L_t$
- Four phases:
  - SYM: $\langle\varphi\rangle = 0, \ \langle\varphi\rangle_s = 0$
  - FM: $\langle\varphi\rangle \neq 0, \ \langle\varphi\rangle_s = 0$
  - AFM: $\langle\varphi\rangle = 0, \ \langle\varphi\rangle_s \neq 0$
  - FI: $\langle\varphi\rangle \neq 0, \ \langle\varphi\rangle_s \neq 0$