Egalitarian Improvement to Democracy

Quark Renormalization Constants from Paris

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Basics

The Goal is to extract the operators relevant to quark momenta inside nucleus

$$\begin{aligned} \langle x^n\rangle_q &= \int_0^1 \mathrm{d}x\, x^n\big(q(x)+(-1)^{n+1}\bar{q}(x)\big) \\ \langle x^n\rangle_{\Delta q} &= \int_0^1 \mathrm{d}x\, x^n\big(\Delta q(x)+(-1)^{n}\Delta\bar{q}(x)\big) \\ \langle x^n\rangle_{\delta q} &= \int_0^1 \mathrm{d}x\, x^n\big(\delta q(x)+(-1)^{n+1}\delta\bar{q}(x)\big) \end{aligned} \quad (1)$$

$$q = q_\uparrow + q_\downarrow, \Delta q = q_\uparrow - q_\downarrow, \delta q = q_\top + q_\perp$$

x is momentum fraction carried by the quark

Operators

Helicity even:

$$O_{\mu\nu} = \frac{1}{2}\left\{\bar{q}\left[D^0_\mu\gamma^0_\nu + D^0_\nu\gamma^0_\mu - \frac{1}{2}\delta_{\mu\nu}\not{D}\right] q\right\} \quad (2)$$

$$\begin{aligned}\Gamma_{\mu\nu}(p) &= \frac{1}{2}\Sigma_1(p^2)\left[p_\mu\gamma_\nu + p_\nu\gamma_\mu - \frac{1}{2}\delta_{\mu\nu}\not{p}\right] \\ &+ \Sigma_2(p^2)\not{p}\left[p_\mu p_\nu - \frac{1}{4}\delta_{\mu\nu}p^2\right] \end{aligned} \quad (3)$$

Helicity odd:

$$O_{5\mu\nu} = \frac{1}{2}\left\{\bar{q}\gamma_5\left[D_\mu\gamma_\nu + D_\nu\gamma_\mu - \frac{1}{2}\delta_{\mu\nu}\not{D}\right] q\right\} \quad (4)$$

Propagators and Green Functions:

$$
\begin{aligned}
S_{u(d)}(p) &= \sum_x e^{ip\cdot x}\langle \psi^{u(d)}(x)\bar{\psi}^{u(d)}(0)\rangle, \\
G^c_\Gamma(p) &= \sum_{x,y} e^{ip\cdot(x-y)}\langle \psi^u(x)\bar{\psi}^u(0)\Gamma\psi^d(0)\bar{\psi}^d(y)\rangle.
\end{aligned}
$$

RI-MOM renormalisation conditions are the following:

$$
\frac{1}{Z_q}\frac{i}{12}\mathrm{Tr}\left(\frac{\sum_\mu \gamma_\mu \sin(ap_\mu)\,S^{-1}(p)}{\sum_\mu \sin^2(ap_\mu)}\right)_{p^2=\mu^2} = 1,
$$

$$
\frac{Z_\Gamma}{Z_q}\frac{1}{12}\mathrm{Tr}(\Lambda^\Gamma(p)P_\Gamma)_{p^2=\mu^2} = 1, \qquad (5)
$$

$$
\Lambda^\Gamma(p) = S_u^{-1}(p)G^c_\Gamma(p)S_d^{-1}(p), \quad \Gamma P_\Gamma = 1.
$$

Cubic and Spheric

- Breaking of rotational invariance is very important
- Because our computers are too good
- And statistical errors on two-point functions are very small
- So systematics become very visible
- In particular for the renormalization constants
- Need some ways to make things smooth

Diagonal Thinking

- Democracy is a popular choice (Leinweber'98)
- Pick momenta p such that
- $\frac{p^{[4]}}{(p^2)^2} < 0.3$ where
- $p^{[n]} = \sum_\mu p^n$, and $a^2 p^2 < 3$.
- Works quite well, but as usual in democracy
- Voter turnout is small
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- We loose information from many momenta
- Also, democracy is mathematically impossible (Arrow'50)

Rubick's Hypercube

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Rubick's Hypercube

- First law of theoretical physics:
- If you don't know what to do - use group theory
- O(4) is broken down to H(4), discrete rotational group
- Operates on discrete momenta $p \equiv \frac{2\pi}{La} \times (n_1, n_2, n_3, n_4)$
- finite group with 4 invariants
- $p^{[n]} \equiv \sum_\mu p_\mu^n, \quad n = 2, 4, 6, 8$

Orbitology

- Any polynomial function of p can be expresses as
- $F_L(p) \equiv F_L(p^{[2]}, p^{[4]}, p^{[6]}, p^{[8]}) = \frac{1}{\|O(p)\|} \sum_{p \in O(p)} F_L(p)$
- where $\|O(p)\|$ is the cardinal number of orbit $\|O(p)\|$
- which split into different $H(4)$ orbits
- we can catalogue them by $p^{[n]}$ invariants
- $\widehat{p}^2 \approx \; p^2 - \frac{a^2}{12} p^{[4]} + \frac{a^4}{360} p^{[6]} - \frac{a^6}{20160} p^{[8]} + \cdots$
- "Democracy" is "trivial" H4 method, minimizing $p^{[4]}$

Globalization

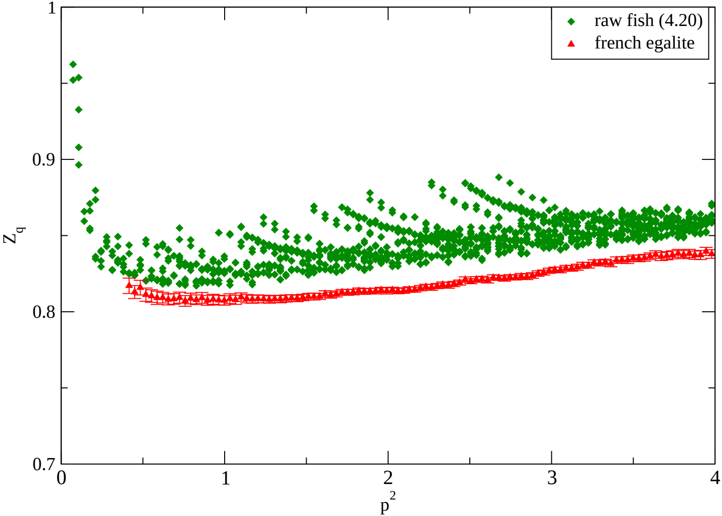
- Second law of Theoretical Physics:
- If group theory does not help - assume and expand
- Assume that lattice form-factor is more-or-less smooth function of $p^{[n]}$
- Expand around continuum

$$bF_L(p^2, p^{[4]}, p^{[6]}, p^{[8]}) \approx F_L(p^2) + p^{[4]}\frac{\partial F_L}{\partial p^{[4]}}(p^2)+ p^{[6]}\frac{\partial F_L}{\partial p^{[6]}}(p^2) + (p^{[4]})^2\frac{\partial^2 F_L}{\partial^2 p^{[4]}}(p^2) + \cdots$$

H4 Global Prescription

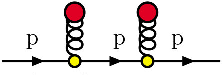
- extrapolate the lattice data to get $F_L(p^2, 0, 0, 0)$
- by using linear regression at fixed p^2
- impossible to do for all values of p^2
- so need to pick window of momenta
- scale it according to χ^2
- check validity by analyzing smoothness of the result
- watch the disappearance of the rib-cage

Fish and Chips



$< A^2 >$ correction to the quark propagator

Diagram:



where red bubble is VEV. At zero quark mass:

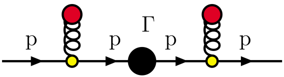
$$\frac{-i\not{p}}{p^2}\left(\sum_{\mu=1,a=1}^{\mu=4,a=8} ig\frac{\lambda_a}{2}\not{A}^a\frac{-i\not{p}}{p^2}ig\frac{\lambda_a}{2}\not{A}^a\right)\frac{-i\not{p}}{p^2} = -\frac{g^2}{12}\frac{< A^2 >}{p^2}\times\frac{-i\not{p}}{p^2} \tag{6}$$

so that non-perturbative contribution at tree level is:

$$< (A_\mu^a)^2 >=< A^2/32 >, \qquad < (A\cdot\hat{p})^2 >=< A^2/4 >$$

$< A^2 >$ correction to the vertex function

Now consider:



where black bubble is an inserted operator $(1,\ \gamma_\mu,\ \gamma_5$ etc.)
One can show that for cases $\Gamma = 1, \gamma_5$ the non-perturbative contribution is, after averaging over all directions:

$$-g^2 \frac{4}{3 \times 8} \frac{f_{ave}(p)}{(p^2)^2} < f_{ave}(A) > \qquad (7)$$

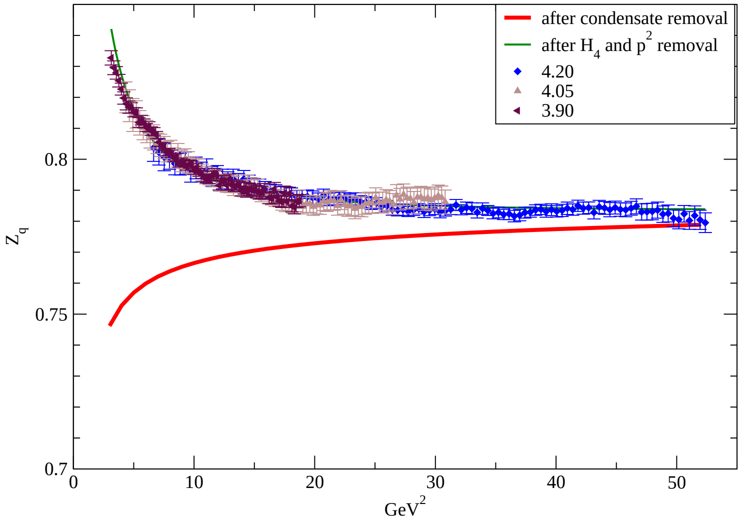
where $f_{ave}(p) = \pm p^2$ and $< f_{ave}(A) >= \pm < A_a^2 >$ for scalar/pseudoscalar channels.

Numerics

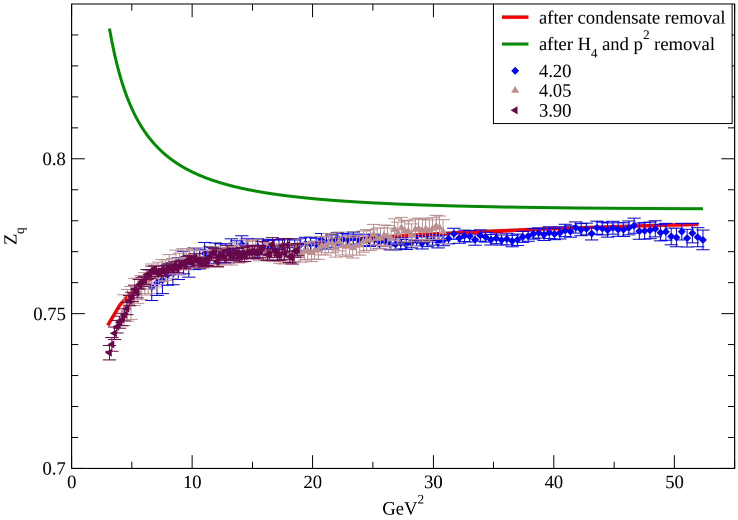
V	a, fm	β	$a\mu$
$24^3 \times 48$	0.055	4.2	0.002
$24^3 \times 48$	0.0675	4.05	0.006
$24^3 \times 48$	0.083	3.9	0.004

- two-flavour ETMC lattices
- 100 thermalized configurations
- distance 10 between configurations
- all data is rescaled by overall matching coefficient
- which is usually roughly 1.01(1)
- compatible with contribution from leading log (Chetyrkin/Maier)

Z_q


after condensate removal
after H4 and p2 removal
4.20
4.05
3.90
Zq
GeV2
0.7
0.75
0.8
0
10
20
30
40
50

Z_q


after condensate removal
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4.20
4.05
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40
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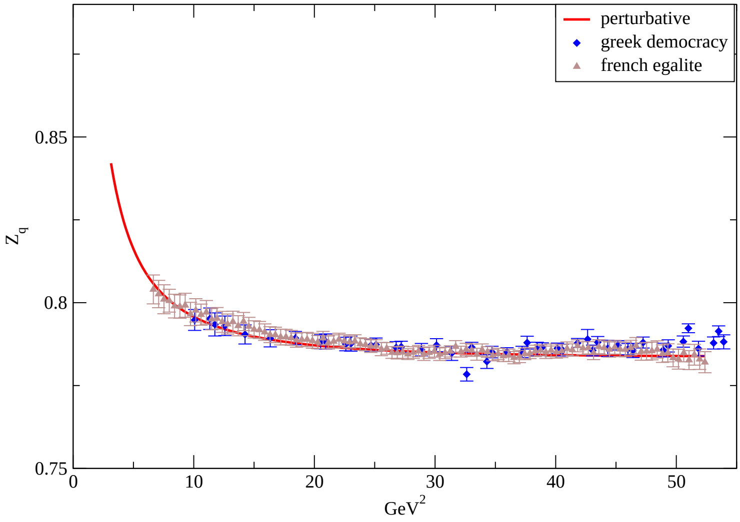
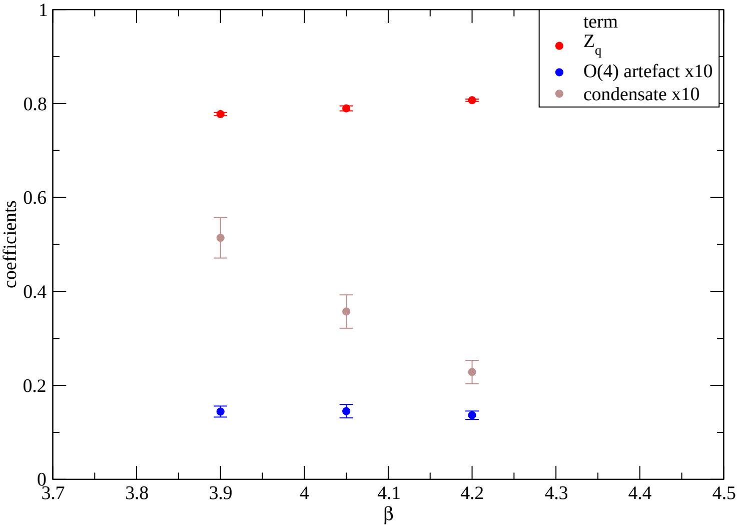
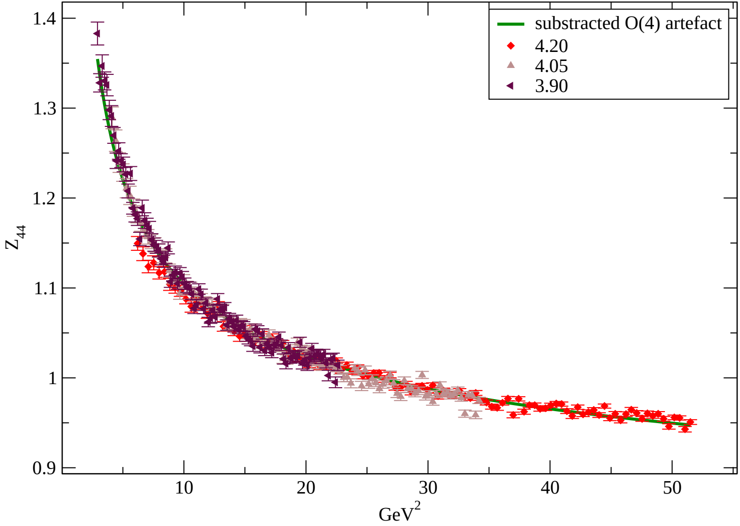
Greek Democracy vs. French Egalite



Austerity and Stability



Z44


substracted O(4) artefact
4.20
4.05
3.90
1.4
1.3
1.2
1.1
1
0.9
Z44
10
20
30
40
50
GeV2

Conclusions and Outlook

- Thorough analysis of quark renormalization constants
- Solid method for elimination of hypercubic artefacts
- $O(4)$ symmetric artefact is a-independent, as expected
- Good agreement with perturbation theory
- Condensate exists for Z_q, questionable for Z_{44}
- Need to complete analysis for Z14/Z514
- And extend it to the 2+1+1 using all momenta
- Likely to be performed on our upcoming FermiSea cluster (4x4) at CEA