Lattice Wess-Zumino model with Ginsparg-Wilson fermions: One-loop results and GPU simulations

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with

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Student research assistants

Chen Chen (3rd yr 2010-2011)

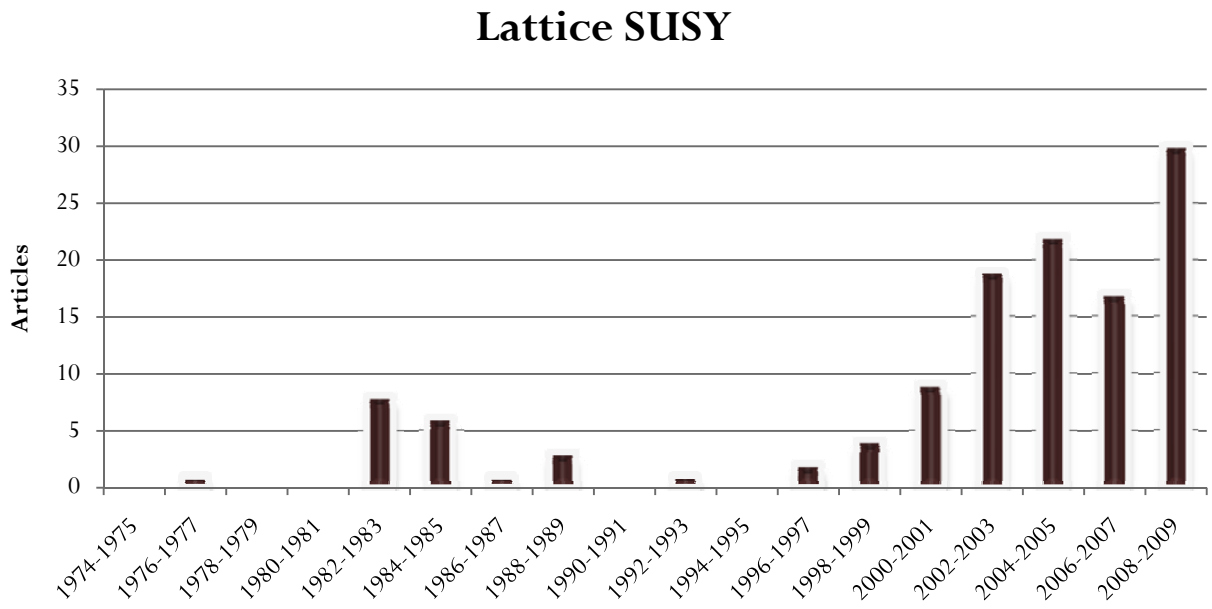
- **Lattice Wess-Zumino model: one-loop counterterms**
- **Lattice Wess-Zumino model: Pfaffian sign problem?**

Eric Dzienkowski [M.S. May 2010; Summer 2010 RPI; UCSB Fall 2010]

- Lattice Wess-Zumino model: GPU code development
- Twisted N=4 Lattice SUSY counterterms (w/ Catterall, Syracuse)

Lattice SUSY: Emerging Field

Computational resources now allow dynamical fermions



Lattice formulations that reduce/eliminate fine-tuning

Lattice Wess-Zumino model

- With Ginsparg-Wilson fermions [Fujikawa, Ishibashi 01; Fujikawa 02; Bonini, Feo 04; Kikukawa, Suzuki 04]
- Preserves $\mathsf{U(1)_R}$ symmetry (limits counterterms)
- 1-loop nonrenormalization (Chen Chen) and

$$Z_\phi = Z_\chi \neq Z_F$$ Consistent with previous studies

- GPU simulation code (Eric Dzienkowski)
- Early results collected in [1005.3276]
- Very small violation of SUSY, g=1/10:

$$\tfrac{1}{V}\sum_x \langle F(x)\rangle = \mathcal{O}(10^{-3})$$

Lattice derivative operators

$$
\begin{aligned}
A &= 1 - aD_W, \quad D_W = \frac{1}{2}\gamma_\mu(\partial^*_\mu + \partial_\mu) + \frac{1}{2}a^2\partial^*_\mu\partial_\mu \\
D_1 &= \frac{1}{2}\gamma_\mu(\partial^*_\mu + \partial_\mu)(A^\dagger A)^{-1/2} \\
D_2 &= \frac{1}{a}\left[1 - \left(1 + \frac{1}{2}a^2\partial^*_\mu\partial_\mu\right)(A^\dagger A)^{-1/2}\right] \\
D &= D_1 + D_2 = \frac{1}{a}\left(1 - A(A^\dagger A)^{-1/2}\right)
\end{aligned}
$$

Bonini, Feo 2004

Lattice action without fine-tuning

$$S = a^4 \sum_x \left\{ -\frac{1}{2}\chi^T C M \chi + \frac{2}{a}\phi^* D_2 \phi + (m^*\phi^* + g^*\phi^{*2})(1 - \frac{a}{2}D_2)(m\phi + g\phi^2) \right\}$$

$$M = \not{D} + mP_+ + m^*P_- + 2g\phi P_+ + 2g^*\phi^* P_-, \quad \not{D} = (1 - \frac{a}{2}D_2)^{-1} D_1$$

$\mathsf{U(1)}_\mathrm{R}$ symmetry at m=0

$$\delta\chi = i\alpha\gamma_5\chi, \quad \delta\phi = -2i\alpha\phi$$

$$\gamma_5 \not{D} = -\not{D}\gamma_5$$

Lattice action with fine-tuning (massive)

$$
\begin{aligned}
S \;=\; & a^4 \sum_x \Big\{ -\frac{1}{2}\chi^T C(\not{D} + m_1 P_+ + m_1^* P_-)\chi + \frac{2}{a}\phi^* D_2 \phi \\
& + m_2^2|\phi|^2 + \lambda_1|\phi|^4 + \left(m_3^2\phi^2 + g_1\phi^3 + g_2\phi\phi^{*2} + \lambda_2\phi^4 + \lambda_3\phi\phi^{*3} + \text{h.c.}\right) \\
& - \chi^T C(y_1\phi P_+ + y_1^*\phi^* P_-)\chi - \chi^T C(y_2\phi P_- + y_2^*\phi^* P_+)\chi \Big\}
\end{aligned}
$$

- Impose CP, gives only real parameters
- Fix m_1, y_1
- Still have 8 parameters to fine-tune

Lattice action with fine-tuning (massless)

In the limit $m_1 \to 0$ we can impose the $U(1)_R$ symmetry. This restricts the action to

$$\begin{aligned} S &= a^4 \sum_x \Big\{ -\frac{1}{2}\chi^T C \not{D} \chi + \frac{2}{a}\phi^* D_2 \phi + m_2^2 |\phi|^2 + \lambda_1 |\phi|^4 \\ &\quad -\chi^T C (y_1 \phi P_+ + y_1^* \phi^* P_-)\chi \Big\} \end{aligned}$$

Fix y_1, fine-tune two parameters

One-loop calculation

- No mass counterterms, no coupling counterterms, only wavefunction renormalization

$$Z_\phi - 1 = \lim_{p\to 0} \Sigma_\phi(p)/p^2$$

$$Z_\chi - 1 = \lim_{\not{p}\to 0} \Sigma_\chi(p)/\not{p}$$

$$Z_F - 1 = \lim_{p\to 0} \Sigma_F(p)$$

Example: scalar self energy

ma	$(Z_\phi - 1)(\infty)/\|g\|^2$	$(Z_\phi - 1)(8)/\|g\|^2$
0.5	-0.00589	-0.006355
0.25	-0.00884	-0.009532
0.125	-0.01205	-0.013358
0.0625	-0.01573	-0.017541
0.03125	-0.01986	-0.021837
0.015625	-0.02573	-0.026205
0.0078125	-0.02956	-0.030583
0.00390625	-0.03370	—
0.001953125	-0.03671	—

Table 1: Wavefunction counterterm from self-energy for the scalar, for $L = \infty$ and $mL = 8$.

We have fit the $L = \infty$ numerical data with $ma \leq 0.125$ to

$$f(ma) = c_0 \ln(ma) + c_1 ma + c_2 (ma)^2$$

Giving the data points equal weight, the fit for the scalar self energy is

$$c_0 = 0.00604(7), \quad c_1 = 0.024(20), \quad c_2 = -0.15(17)$$

while for the fermion the fit is

$$c_0 = 0.00597(5), \quad c_1 = 0.061(15), \quad c_2 = -0.39(12)$$

Thus we see that at $L \to \infty$ the log divergences of the scalar $Z_\phi - 1$ and the fermion $Z_\chi - 1$ match. For the auxiliary field we obtain instead

$$c_0 = -0.0261(5), \quad c_1 = 0.87(11), \quad c_2 = -3.9(1.0)$$

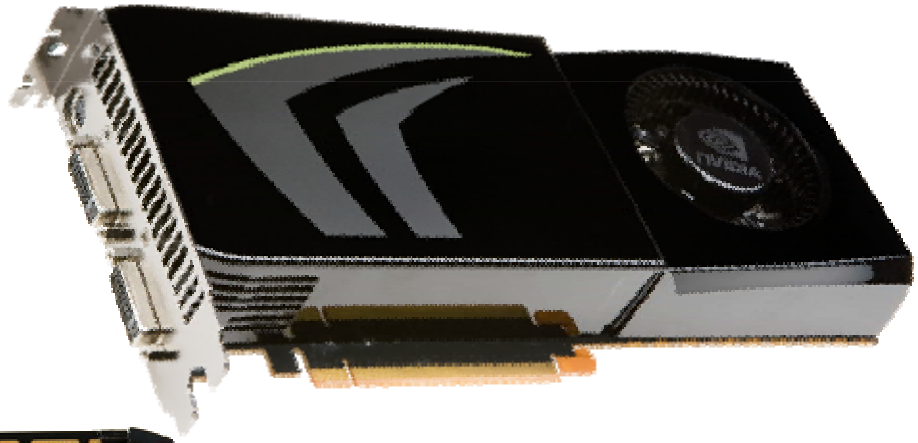
so that its $Z_F - 1$ does not match the scalar and fermion. This is consistent with the results found in [Kikukawa, Suzuki 04].

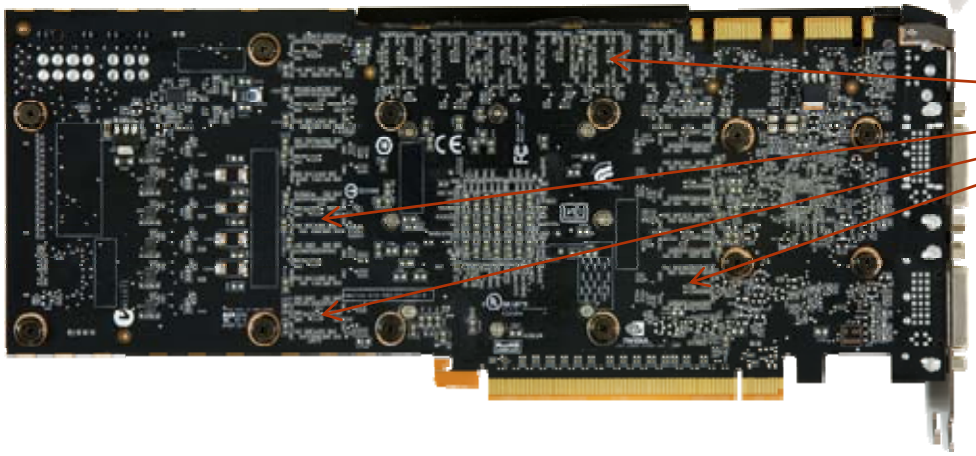
Monte Carlo simulation

- In addition to the perturbative analysis, we have developed simulation code to run on graphics processing units (GPUs) that are CUDA enabled
- What we have is RHMC code that works with scalars and spinors that reside on the card
- We have benchmarks for the code and have run various checks
- Serious simulations and analysis still needs to be done

Nvidia GTX 285

- 240 cores, $450, 200W





4 of the 30 MPUs

Each MPU has 8 single prec cores and 1 double prec core

http://www.nvidia.com/object/product_geforce_gtx_285_us.html

"The most powerful single GPU on the planet for gaming and beyond."

GPU Benchmarks

Lattice	CPU single	GPU (CUDA) single	GPU (Ours) single	CPU double	GPU (CUDA) double	GPU (Ours) double
$8^3 \times 32$	1.1	6.9	24	0.94	4.4	11
$16^3 \times 32$	0.88	14	71	0.69	10	35
$32^3 \times 64$	0.11	20	—	0.085	10	—

Table 1: Comparison of timing, Gflop/s, for fast fourier transform of the spinor field. Both single and double precision results are given.

Preconditioning

At weak couplings, we expect that preconditioning by the inverse of the free theory fermion matrix M_0 will improve convergence of the conjugate gradient solver. For this purpose, we re-express the problem

$$M^\dagger M x = b$$

as follows:

$$(M_0^{-1\dagger} M^\dagger)(M M_0^{-1})(M_0 x) = (M_0^{-1\dagger} b)$$

Large speed-up

Lattice	Precision	NPC secs.	NPC iters.	Gflop/s	PC secs.	PC iters.	Gflop/s	speed-up
$8^3 \times 32$	single	1.3	830	14	0.038	8	17	34
$8^3 \times 32$	double	4.9	1600	7.2	0.13	15	8.1	38
$16^3 \times 32$	single	7.1	870	25	0.20	8	31	36
$16^3 \times 32$	double	31	1800	12	0.74	15	13	42
$32^3 \times 64$	single	420	1600	14	6.8	8	15	62
$32^3 \times 64$	double	2200	3900	6.5	25	15	7.0	86

Table 1: Timing benchmarks at $m = 1$, $g = 1/5$. PC indicates preconditioning whereas NPC is the inversion without preconditioning. The time in seconds and the number of iterations (iters.) is for convergence. The criterion for convergence is that the relative residual is less than 1×10^{-6} for single precision and less than 1×10^{-12} for double precision. The speed-up is the ratio of NPC time to PC time.

Conclusions

- Reasonable fine-tuning in massless limit
- Working, fairly fast GPU code
- Large speed-up from preconditioning
- Next: parameter space scans

$$m_2^2, \quad \lambda_1$$

to obtain $\langle \partial_\mu S_\mu(x) \mathcal{O}(0) \rangle \approx 0$

and degeneracy in fermion/boson effective mass