Improvements in calculating the strange quark content of the nucleon
Lattice 2010

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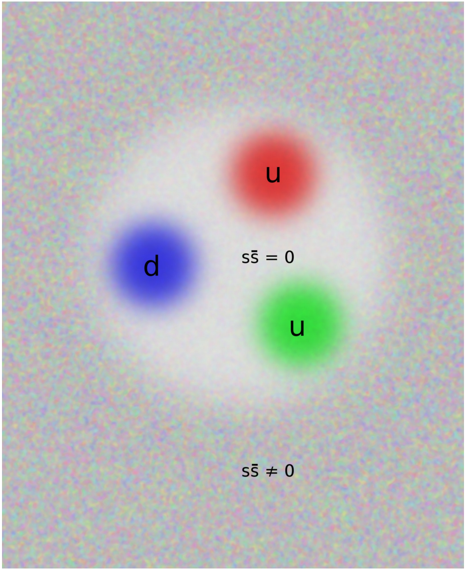
June 15, 2010

Introduction

- "Nucleon strangeness": the matrix element $\langle N|\bar{s}s|N\rangle - \langle 0|\bar{s}s|0\rangle$
- Important for understanding interactions between some dark matter candidates and nuclear matter (e.g. in detectors)
- Also important for understanding nucleon and QCD-sea structure
- Calculated in 2009 using MILC data (see arXiv:0905.2432, Phys. Rev. Lett. 103.122002)
- Errors in the 10% range were improvement on previous results, but can they be improved further?

Meaning of "nuclear strangeness"



- Does not imply that there are a great many virtual $\bar{s}s$ pairs in the nucleon
- Nucleon strangeness is actually the *suppression* of the vacuum strange quark condensate
- Nucleon like an air bubble in glass

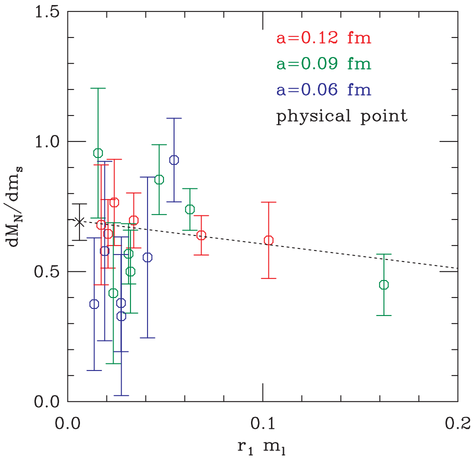
Previous calculation: method

- Differentiate the partition function to get $\langle N|\bar{s}s|N\rangle - \langle 0|\bar{s}s|0\rangle = \frac{\partial M_N}{\partial m_s}$ (Feynman-Hellman theorem)
  - Does not mean that physical M_N depends greatly on m_s
  - Changing m_s changes *all* dimensionful lattice quantities; we interpret this as an overall rescaling of a
- M_N just a complicated function of propagator $P(t)$ over chosen range of t (since M_N is gotten by a fit to $P(t)$)
- Chain rule: $\frac{\partial M_N}{\partial m_s} = \sum_t \frac{\partial M_N}{\partial P(t)} \frac{\partial P(t)}{\partial m_s}$
  - Get the first of these from changing $P(t)$ and seeing how M_N changes
  - Get the second from $\frac{\partial P(t)}{\partial m_s} = \langle P(t)\bar{s}s\rangle - \langle P(t)\rangle \langle \bar{s}s\rangle$
- Key idea: **look for correlations between strange condensate and propagator**

Previous calculation: data

- Use the large preexisting library of MILC Asqtad lattices with dynamical u, d and s quarks
- Range of dynamical quark masses: $0.1m_s < m_{light} < 1.2m_s$
- Range of lattice spacings: $a = 0.12$, 0.09, 0.06 fm used here
- High statistics (cheap staggered quarks)
- Nucleon propagators already computed on most of them
- Several stochastic estimations of $\bar{s}s$ on each lattice

Results for $\frac{\partial M_N}{\partial m_s}$



- $\frac{\partial M_N}{\partial m_s}$ ($\overline{MS}(2GeV)$) after extrapolation to physical m_s, with chiral fit shown
- Best statistics from coarsest ($a = 0.12fm$) ensembles
- Weak dependence on m_l

Result and error budget

$$\frac{\partial M_N}{\partial m_s} = 0.69 \pm 0.07_{stat} \pm 0.09_{systematic}$$

Error source	Estimate
statistical	0.070
Excited states	0.069 (10%)
Finite volume	0.021 (3%)
Higher order χPT	0.049 (7%)
Error in Z_m	0.028 (4%)
Combined systematic	0.09

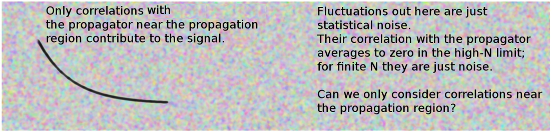
- Excited states: estimated (conservatively) from alternate choices of d_{min}
- Finite volume: 1% difference in precision nucleon mass measurements; we take 3% as a worst-case value
- Higher order dependence on m_l: 7% effect in nucleon mass fitting from adding extra terms past constant-plus-linear
- Error in Z_m: from Mason *et al.*, hep-ph/0511160

How can we do better?

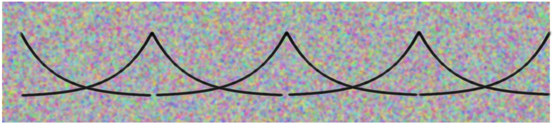
- Large source of statistical error: fluctuations of $\bar{s}s$ that are not correlated with the nucleon propagator/mass
- Each propagator covers only a small region of the lattice
  - Typical $a = 0.12fm$ ensemble: $n_t = 64$
  - M_N determined by fit to propagators with length from 5-15
  - Use entire lattice by averaging many propagators at different source times
- No physical reason for $\bar{s}s$ to be correlated with $P(t)$ far from propagation region
- In the limit of high statistics, correlations far from this region will average to zero...
- ... but for finite N they contribute statistical noise

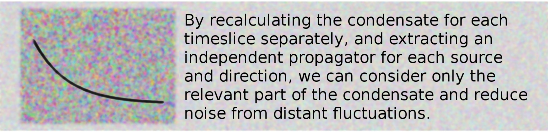
Modifying the calculation

- Only consider the condensate at times between source and sink operators of the propagator, plus some “padding” of a few time units at each end



In the previous calculation the condensate samples are global averages, and the nucleon propagators are averages of many sources spread over the whole lattice.



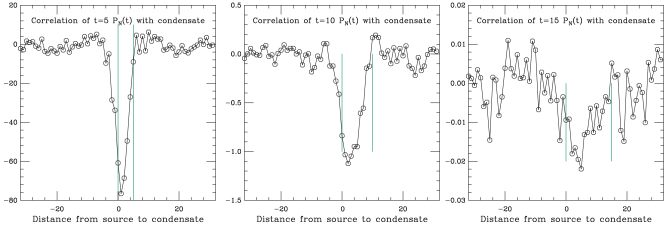


Modifying the calculation: issues

- Introduction of bias: how much $\bar{s}s$ can we safely discard?
- Effectiveness: by how much does this improve statistics?
- Additional computer time required
  - Only have average of all spectrum sources; need each source and direction individually
  - Only have average of $\bar{s}s$ over whole lattice; need separate measurement on each timeslice
  - So far, have only completed the needed measurements on four large $a = 0.12$ ensembles
- Better control of systematics possible
  - Better estimate of error from excited states
  - Continuum and chiral extrapolations

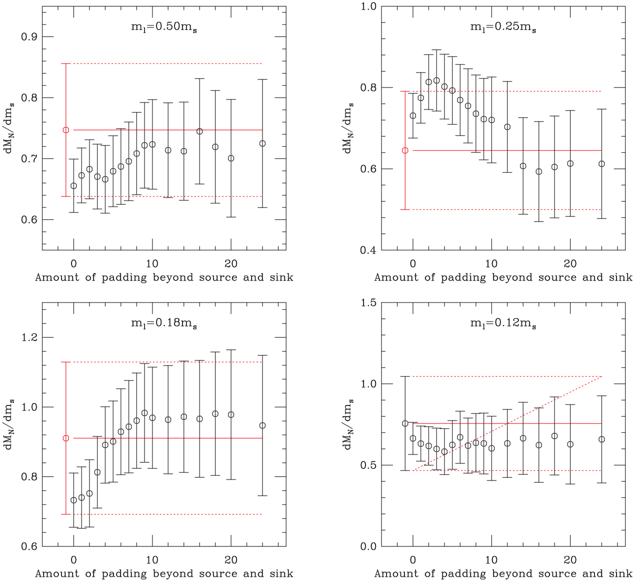
Validity test: propagator-$\bar{q}q$ correlations

- Is $\bar{s}s$ far from the propagation region really uncorrelated with the propagator?
- How far is "far"?

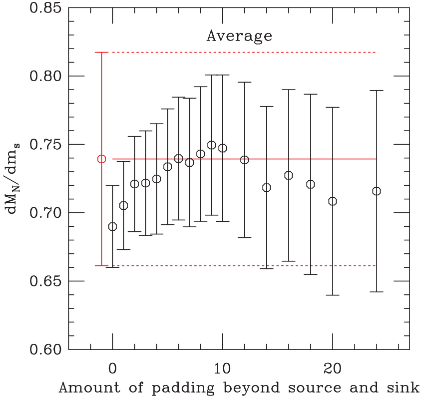


- Success: value of $\bar{s}s$ on timeslices more than 3-4 units from propagator should not matter

Effect of varying the padding width



Amount of required padding

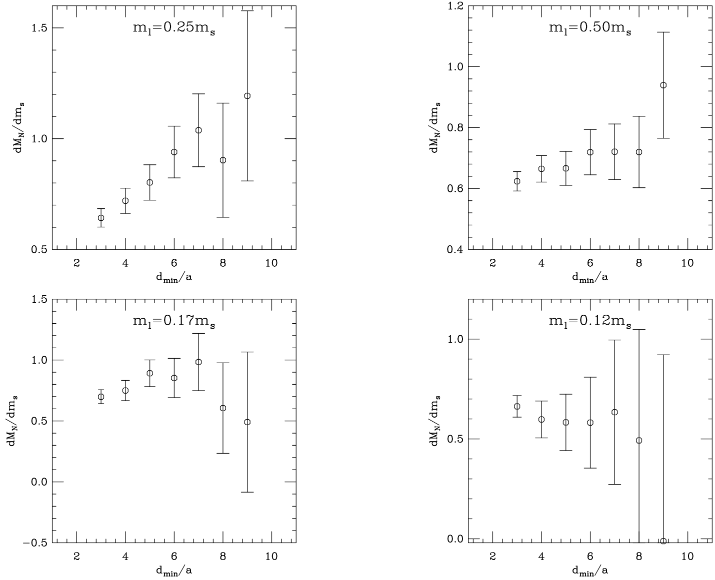


- Expect the systematic error from using too low a padding size to be negative
- As expected, results dip sharply below pad width 3 or so
- Conclude that pad width 4 is acceptable
- Estimate potential systematic error from this as 3%
- Significant decrease in statistical error, by almost 50%!

New estimate of excited states error

- Using too large a minimum distance in nucleon fits gives larger statistical errors
- Using too small a minimum distance gives propagators polluted by excited states
- Choose minimum distance consistent with results at higher t_{min}
- Previous work used $t_{min} = 5a$ on $a = 0.12fm$ ensembles
- Previous systematic error estimate of 10% possibly too conservative
  - Will better statistics let us reduce it?

Results vs. minimum distance



Estimating systematic error from excited states

- Three of these four ensembles, as well as partial data from a fifth, show no strong dependence on d_{min} down to $5a$ or even $4a$
- The smaller statistical errors would allow for a lower upper bound to be placed on the systematic error from excited states
- The $m_l = 0.25m_s$ ensemble shows strong dependence, but this seems more consistent with a statistical fluctuation than a systematic effect
  - The error bars above are highly correlated – a trend is not necessarily indicative of a systematic effect
  - A systematic effect should show up in a similar manner in other ensembles
  - This ensemble showed stronger variation with padding size than others
- Need to use improved procedure on more ensembles, including finer lattice spacings, to support a reduced systematic error estimate

Conclusions

- Substantial reduction in statistical errors possible (by almost half)
- Some systematic errors can also be substantially reduced
  - Better statistics may allow reduced estimate of excited states error once more ensembles are run
  - Using new technique on finer ensembles will allow better control of continuum extrapolation (which had large uncertainty previously)
    - ⋆ This is relatively more expensive in computer time
  - Reduced error bars on ensembles with light quark masses will allow better chiral extrapolation